Multi-material geodynamical modelling using ASPECT

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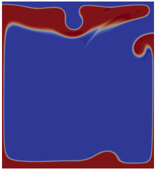


December 11, 2013

NUMERICAL MODELLING IN GEODYNAMICS

- FEM/FDM/FVM/FLAC
- Adaptive Mesh Refinement (AMR) vs. regular grids
- sequential vs. parallel
- direct solvers vs. iterative solvers

ASPECT:
Advanced Solver for Problems in Earth's ConvecTion

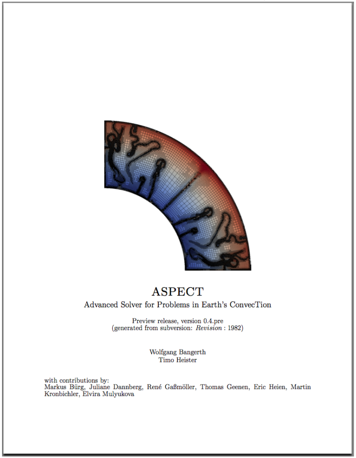
(FEM + AMR + Iterative Solver + Parallel)
solves mass, momentum, energy conservation equations

CIG http://www.geodynamics.org/

High accuracy mantle convection simulation through modern numerical methods, Kronbichler et al., GJI, 2012

Aspect manual

http://www.dealii.org/aspect/



ASPECT
Advanced Solver for Problems in Earth's ConvecTion

Preview release, version 0.4.pre
(generated from subversion: *Revision* : 1982)

Wolfgang Bangerth
Timo Heister

with contributions by:
Markus Bürg, Juliane Dannberg, René Gaßmöller, Thomas Geenen, Eric Heien, Martin Kronbichler, Elvira Mulyukova

- 130 pages
- 10 relevant cookbooks & benchmarks

Aspect manual

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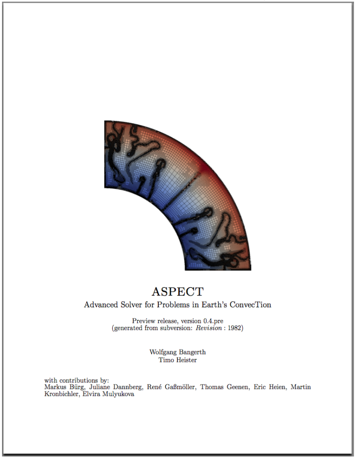

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no benchmark about large deformation of multiple material domains.

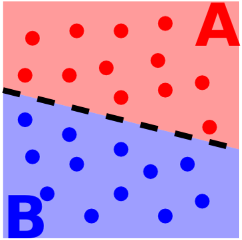
DISCLAIMER

⚠ I am not an ASPECT developer

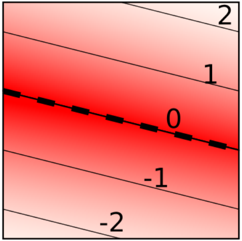
⚠ ASPECT (r1889) as provided to the user

Thanks to:
R. Gassmoeller,
J. Dannberg,
S. Rockel,
E. Mulyukova,
ASPECT team.

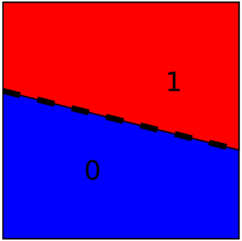
Material advection



marker-in-cell

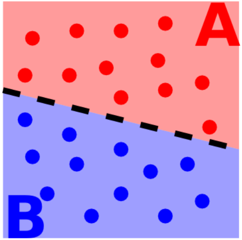


level set function



compositional field

Material advection



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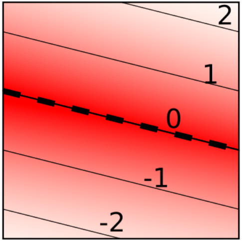

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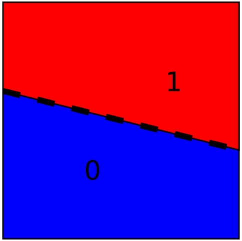

compositional field

compositional field for A :

$$c_A(\boldsymbol{r}) = \begin{cases} 1 & \textit{if} \quad \boldsymbol{r} \in \Omega_A \\ 0 & \textit{if} \quad \boldsymbol{r} \notin \Omega_A \end{cases}$$

Advection of compositional fields

Aspect solves one advection equation per composition c_i:

$$\frac{\partial c_i}{\partial t} + \boldsymbol{v} \cdot \boldsymbol{\nabla} c_i - \boldsymbol{\nabla} \cdot (\nu_h(c) \boldsymbol{\nabla} c) = 0$$

Stabilisation: entropy viscosity method (Guermond et al., 2011)

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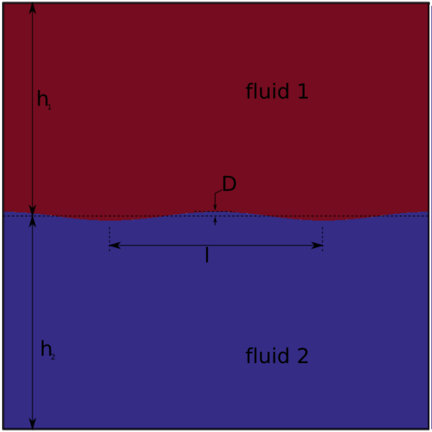
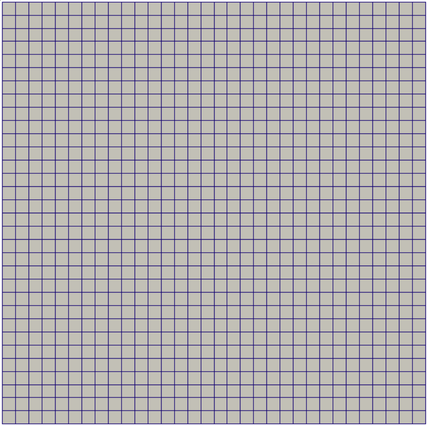
Questions

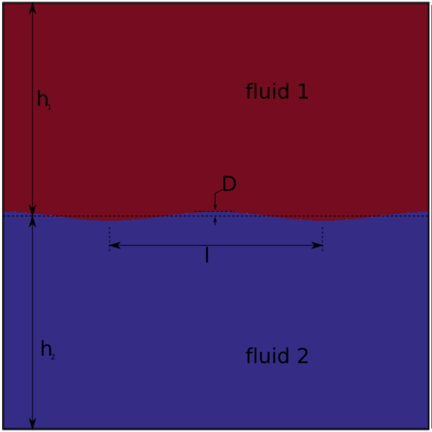
- can we accurately track (many) materials ?
- how does it compare to other methods ?

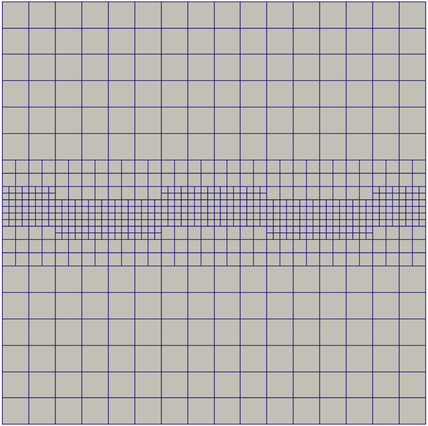
RAYLEIGH-TAYLOR EXPERIMENT





Rayleigh-Taylor experiment





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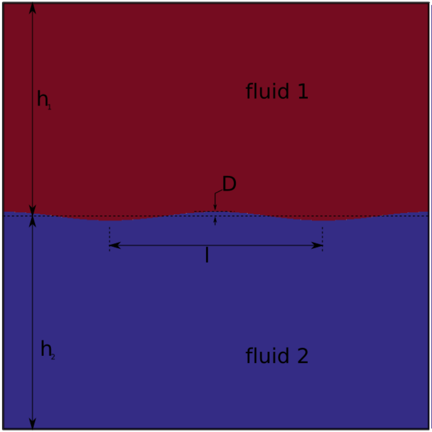

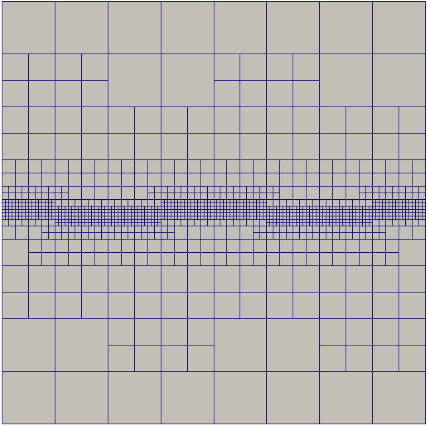

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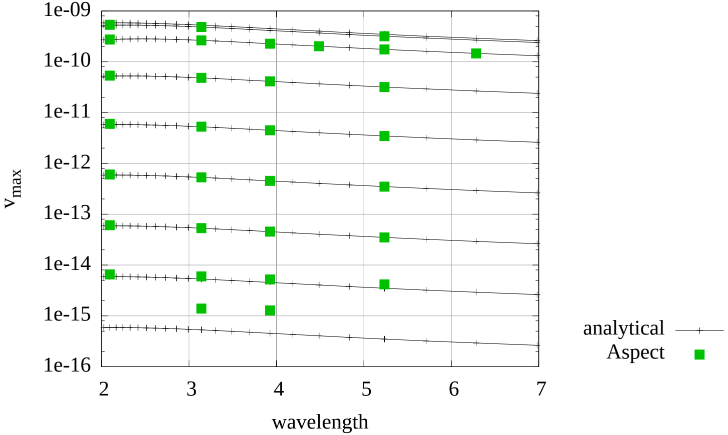
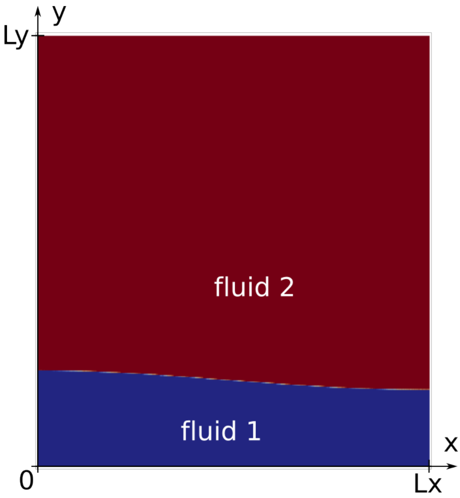


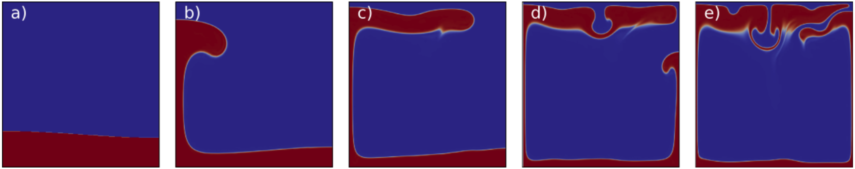
- large viscosity contrast (7 oom ok)
- AMR vs. regular grid yield identical results

Rayleigh-Taylor experiment



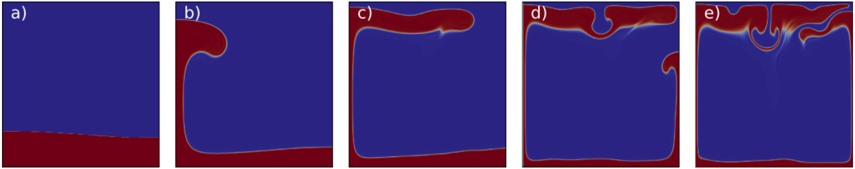
(van Keken et al., JGR, 1997)

Rayleigh-Taylor experiment



t=0 t=500 t =1000 t=1500 t=2000

Rayleigh-Taylor experiment



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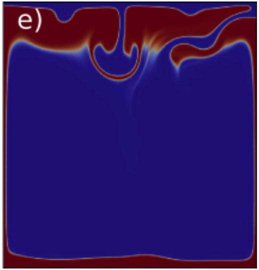
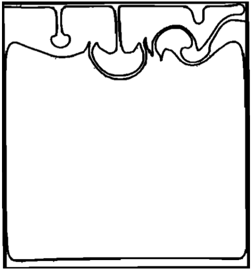


(ASPECT)



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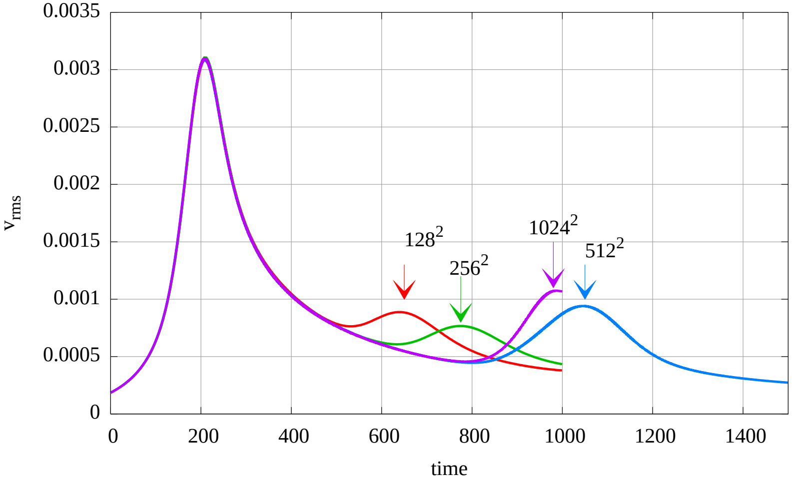
Rayleigh-Taylor experiment



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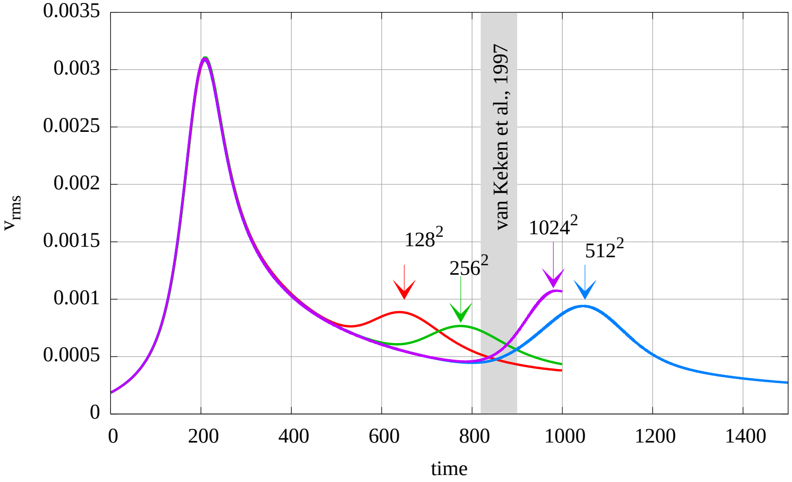

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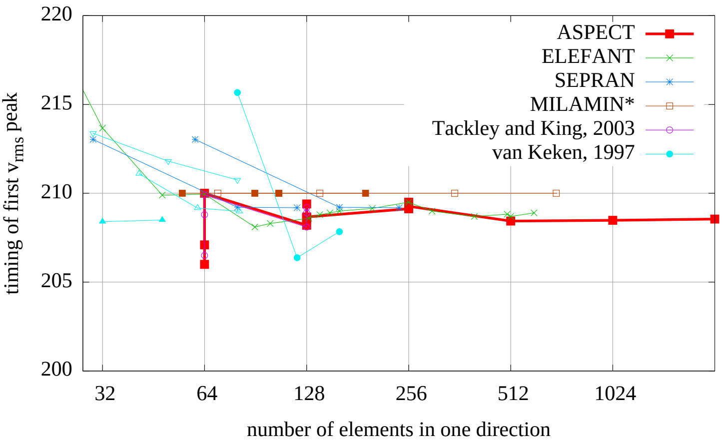

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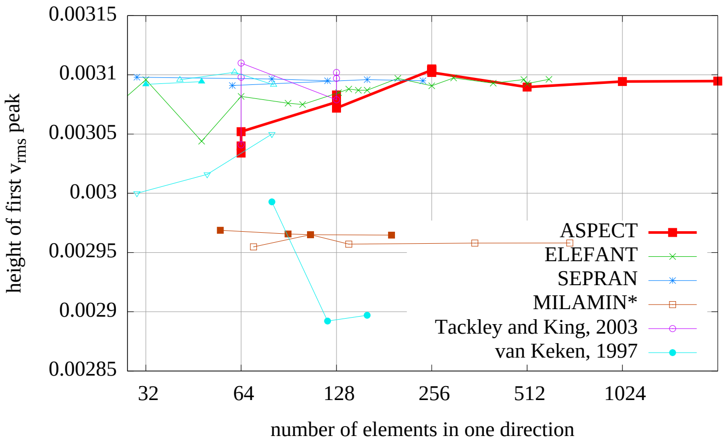

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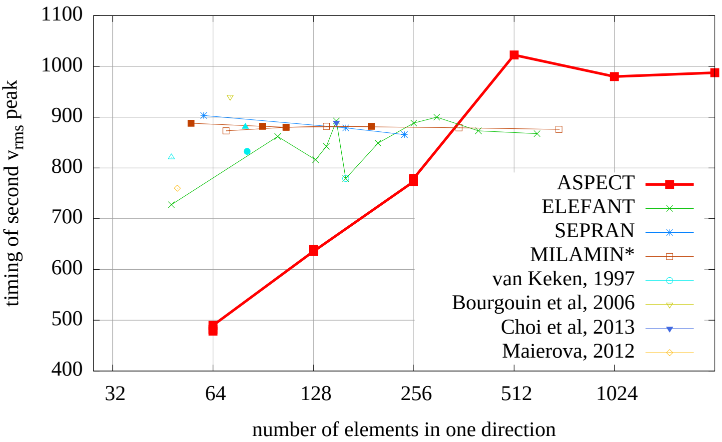

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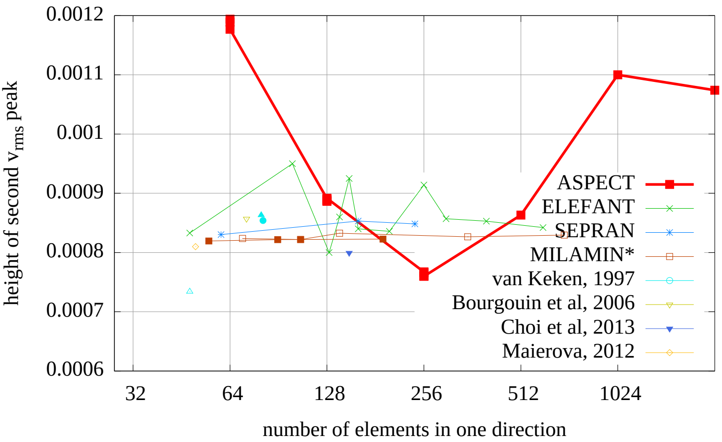

Rayleigh-Taylor experiment

These observations are robust with regards to:

- Courant number $\in [0.1 - 3]$
- Stokes solver tolerance $\times 10^{\pm 2}$
- Composition(s) solver tolerance $\times 10^{\pm 2}$
- Seq vs. parallel
- Diff. supercomputers with diff. compilers
- Diff. (recent) aspect versions
- Stabilisation parameters $\pm 50\%$

Rayleigh-Taylor experiment

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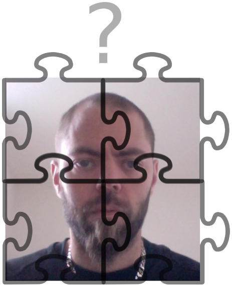
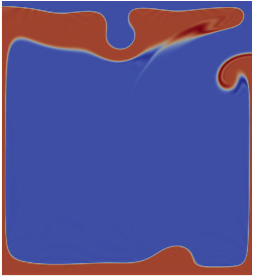


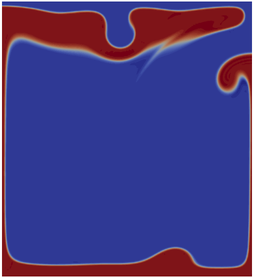
Rayleigh-Taylor experiment

no stabilisation



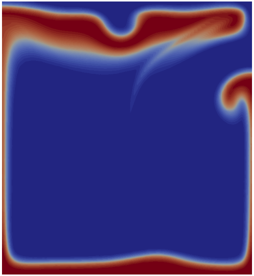
$-0.18 < c < 1.22$

default stabilisation



$-0.07 < c < 1.04$

high stabilisation



$0.00 < c < 1.00$

AVERAGINGS ... AGAIN ?

- At location $\boldsymbol{r}$ each composition i takes value $c_i(\boldsymbol{r})$
- We need to relate viscosity and density to c_i's
  - arithmetic averaging of densities

$$\rho = c_1\rho_1 + c_2\rho_2$$

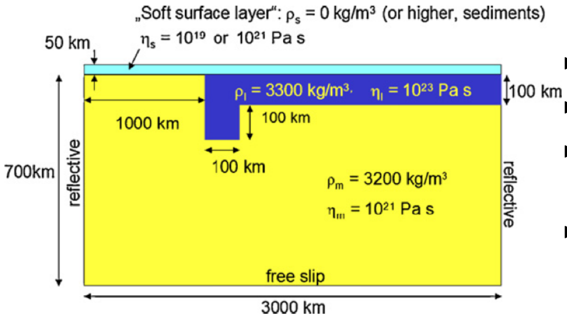
  - geometric averaging of viscosities

$$\log_{10}\mu = c_1\log_{10}\mu_1 + c_2\log_{10}\mu_2$$

- Only two compositions available in material model provided to user
  $\Rightarrow$ User defined extension:

$$\begin{aligned} \rho &= \mathit{function}(\boldsymbol{r}, c_i, T) \\ \mu &= \mathit{function}(\boldsymbol{r}, c_i, T, p, \dot{\epsilon}) \end{aligned}$$

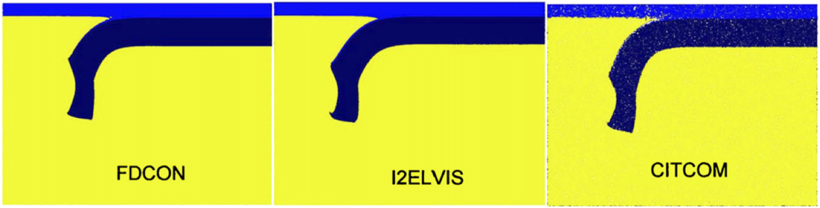
SUBDUCTION BENCHMARK



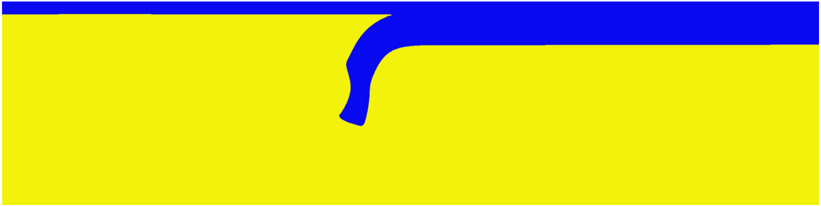
- Schmeling et al, 2008
- 3 compositions
- large viscosity contrasts
- linear viscous materials

SUBDUCTION BENCHMARK

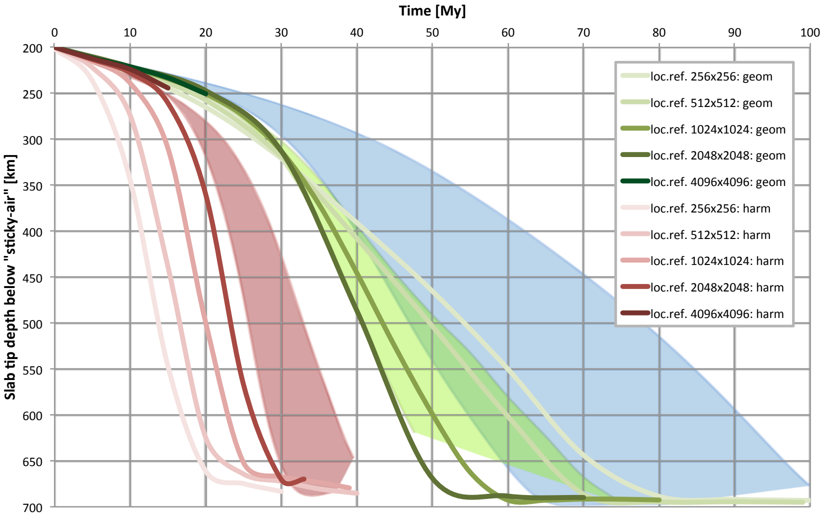
At time = 40Myrs:



ASPECT:

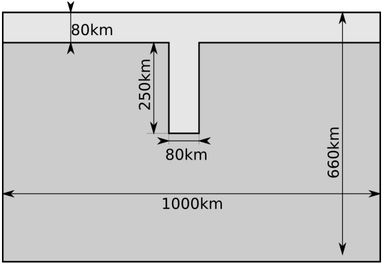


Subduction benchmark

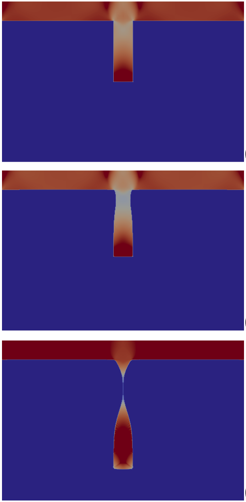


Resolution dependent + averaging dependent

Slab detachment



- Schmalholz, 2011
- nonlinear power-law lithosphere
- linear mantle
- no temperature



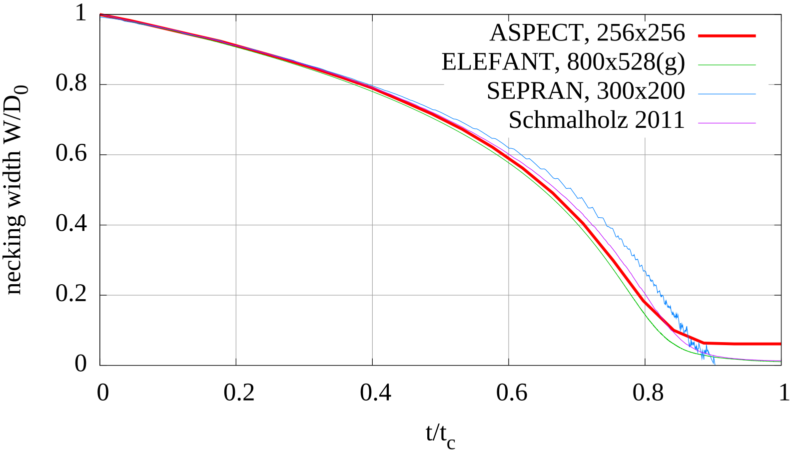
(t=0)

(t≃5Myrs)

(t≃20Myrs)

$21 \leq \log_{10} \mu \leq 25$

Slab detachment



Conclusions

- iterative solver can handle high viscosity contrasts
  $\rightarrow$ use of sticky air possible
- ASPECT passed all benchmarks but
  - puzzling R-T results
  - complex interplay between mass balance, stab. parameters, under/overshoot, comput. time, sharpness of interfaces
  - averaging issue remains
  - tuning of stabilisation parameters is subtle, yet needed for every model setup

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  - tuning of stabilisation parameters is subtle, yet needed for every model setup
- extension to visco-plasticity, application to subduction:
  $\rightarrow$ poster A. Glerum: T31C-2534