Constraining the physics governing collective behavior of earthquakes and faults through numerical modeling and observations

Yehuda Ben-Zion (USC)

Fractal earthquakes and faults (power laws)

Planar fault in homogeneous elastic solid (limit cycles)

Evolutionary processes: greater dynamic richness (and complexity), including the above end-member cases and additional regimes!
Phenomenology of earthquakes and faults

Evolution of Fault Zone Structures

Localization to through-going structures and primary slip zones

Creation of granularity and band-limited fractal structures at several hierarchies.

Progressive evolution toward well-developed damage zones and material interfaces

The Punchbowl fault: 44 km of slip (Chester and Chester, 1998)

USGS (1990)
1984–2002 Southern California seismicity (Shearer et al., 2005)

FS statistics for seismicity before the 1980 eruption of Mount St Helens (Main, 1992)
Potency tensor summations for 170,000 SC earthquakes with $0 < M_L < 5$

(Bailey, Becker, Ben-Zion, 2008)

Beachballs show orientations of compression (white) and extension (red).

Size of the CLVD component for each summation type and region

Potency tensor summations for each region and each magnitude range
Detailed observations show clear persistent diversity rather than a single type of behavior.

Improved understanding requires analysis of evolutionary processes and different dynamic regimes.

Key Questions:
• How are geometrical, mechanical, and rheological properties of fault zones and their surrounding media related to different types of earthquake patterns in space-time-energy domains (e.g., localized vs. distributed spatial structures, power-law vs. characteristic frequency-size (FS) statistics, quasi-periodic vs. clustered temporal behavior, Omori-Utsu aftershock sequences vs. swarm response).

• Are there connections between different types of earthquake patterns considered usually in isolation (e.g., are the forms of FS and temporal statistics related, and if yes how)?

• When and how can we extrapolate results of low magnitude seismicity to large earthquake behavior?

• On what time scale is the seismic response to tectonic loading stationary, if at all?

• How are foreshock-mainshock-aftershock properties related to the “brittleness” of a given area and the regional seismic potential?
Frameworks for studying earthquake and fault dynamics

- **Rock strength**
- **Fracture mechanics**
  - $\sigma_1 - \sigma_3$
  - $\sigma_1$
  - $\sigma_3$
  - $\varepsilon$
  - $K, G$

- **Friction studies**
- **Damage rheology**

- **Granular Mechanics**

- **Statistical mechanics**
Non-Linear Continuum Damage Rheology [e.g., Lyakhovsky et al., 97, 01, 05; Ben-Zion and Lyakhovsky, 02, 06; Hamiel et al., 04; Lyakhovsky and Ben-Zion, 08].

- Generalized nonlinear strain energy function extending Hookean elasticity for damaged solids to account for sensitivity of elastic moduli to existing cracks (hysteresis).
- Thermodynamics-based Kinetic equation for a damage state variable ($\alpha$) representing density of microcracks.
- Gradual viscous-like failure beyond a first yielding threshold. Macroscopic brittle instability when the energy function losses convexity.
- Parameters constrained by lab fracture and friction data.
Non-linear Continuum Damage Rheology

(1) Mechanical aspects: sensitivity of elastic moduli to existing cracks (frozen damage) and sense of loading.

\[ \alpha = 0 \quad \text{yielding} \quad \begin{array}{c} \sigma \\ \varepsilon \end{array} \quad 0 < \alpha < \alpha_c \quad \begin{array}{c} \text{peak stress} \\ \text{Tension} \\ \text{Compression} \end{array} \]
This is accounted for by generalizing the strain energy function of a deforming solid

The elastic energy $U$ is written as:

$$U = \frac{1}{\rho} \left( \frac{\lambda}{2} I_1^2 + \mu I_2 - \gamma I_1 \sqrt{I_2} \right)$$

where $\lambda$ and $\mu$ are Lame’ constants; $\gamma$ is an additional elastic modulus

$$I_1 = \varepsilon_{kk}$$
$$I_2 = \varepsilon_{ij} \varepsilon_{ij}$$

$$\sigma_{ij} = \rho \frac{\partial U}{\partial \varepsilon_{ij}} = \left( \lambda - \gamma \frac{\sqrt{I_2}}{I_1} \right) I_1 \delta_{ij} + \left( 2\mu - \gamma \frac{I_1}{\sqrt{I_2}} \right) \varepsilon_{ij}$$

$$\xi = \frac{I_1}{\sqrt{I_2}}$$
Origin of the generalized energy function

Consider a general strain energy function having any second-order term of the type

\[ U \propto I_1^{2x} \cdot I_2^{1-x} \quad \text{with} \quad 0 < x < 1, \; I_1 = \varepsilon_{kk}, \; I_2 = \varepsilon_{ij}\varepsilon_{ij} \]

The limit values \( x = 0 \) and \( x = 1 \) are associated with the standard 2 Hookean terms.

The relation between the mean stress (\( \sigma_{kk} \)) and volumetric deformation (\( I_1 \)) for the assumed general form is

\[ \sigma_{kk} = \rho \frac{\partial U}{\partial I_1} \sim 2xI_1^{2x-1}I_2^{1-x} + \frac{2}{3}(1-x)I_1^{2x+1}I_2^{-x} \]

The first term has a nonphysical singularity for \( 0 < x < 1/2 \).

For \( x > 1/2 \), if the mean stress is zero (\( \sigma_{kk} = 0 \)) the volumetric strain is zero (\( I_1 = 0 \)) for any non-zero shear loading (\( I_2 \neq 0 \)). This is not compatible with material dilation under shear loading.

Thus the only exponent (other than the classical 0 and 1 values) associated with realistic rock deformation is the \( x = 1/2 \) value represented by the third term in our energy function!
Non-linear Continuum Damage Rheology (2) Kinetic aspects associated with damage evolution during deformation

\[ 0 < \alpha < \alpha_c \]

\[ \alpha = \alpha_c \]

Stress vs. Strain diagram showing yield point and ultimate stress.
This is accounted for by making the elastic moduli functions of a damage state variable $\alpha(x, y, z, t)$, representing crack density in a unit volume, and deriving an evolution equation for $\alpha$. 
**Thermodynamics**

Free energy of a solid, $F$, is

$$F = F(T, \varepsilon_{ij}, \alpha)$$

$T$ – temperature, $\varepsilon_{ij}$ – elastic strain tensor, $\alpha$ – scalar damage parameter

**Energy balance**

$$\frac{dU}{dt} = \frac{d}{dt}(F + TS) = \frac{1}{\rho} \sigma_{ij} e_{ij} - \nabla_i J_i$$

**Entropy balance**

$$\frac{dS}{dt} = -\nabla_i \left( \frac{J_i}{T} \right) + \Gamma$$

**Gibbs equation**

$$dF = -SdT + \frac{\partial F}{\partial \varepsilon_{ij}} d\varepsilon_{ij} + \frac{\partial F}{\partial \alpha} d\alpha$$

The internal entropy production rate per unit mass, $\Gamma$, is:

$$\Gamma = -\frac{J_i}{\rho T^2} \nabla_i T + \frac{1}{T} \sigma_{ij} e_{ij} - \frac{1}{T} \frac{\partial F}{\partial \alpha} \frac{d\alpha}{dt} \geq 0$$
\[ \frac{d\alpha}{dt} = C_d \cdot I_2 (\xi - \xi_0) \]

\[ \xi = \frac{I_1}{\sqrt{I_2}} \]

Strain invariant ratio

\[ I_1 = \varepsilon_{kk} \]
\[ I_2 = \varepsilon_{ij} \varepsilon_{ij} \]

\[ \frac{d\alpha}{dt} = C_1 \cdot \exp\left(\frac{\alpha}{C_2}\right) \cdot I_2 (\xi - \xi_0) \]

Shear Stress \( \tau \)

Normal Stress \( \sigma_n \)

- \( \frac{d\alpha}{dt} > 0 \)
  - \( \xi > \xi_0 \)
  - Weakening (degradation)

- \( \frac{d\alpha}{dt} < 0 \)
  - \( \xi < \xi_0 \)
  - Healing (strengthening)

\[ \tau = \tan (\varphi) \sigma_n \]
\[ \xi = \xi_0 \]
Non-linear Continuum Damage Rheology (3) Dynamic aspects associated with stable and unstable failures

1. Gradual accumulation of an irreversible strain at the weakening stage following the first yielding
2. Brittle instability and stress drop at the ultimate stress (associated with loss of convexity of the strain energy function)
Fracturing and AE experiments constrain parameters $C_d$, $\xi_0$, $C_v$.

Berea sandstone under 50 MPa confining pressure

\[ \eta = \frac{1}{C_v \dot{\alpha}}, \quad \dot{\alpha} > 0 \]

$\mu_0 = 1.4 \times 10^{10} \, \text{Pa}$,
$C_v = 10^{-10} \, \text{Pa}^{-1}$,
$R = 1.4$

Data from Lockner lab. USGS
Model from Hamiel et al., 2004
Rate- and state-dependent friction experiments constrain parameters $c_1$ and $c_2$.

Lyakhovsky et al. (GJI, 2005)
Evolving structure of a newly created strike-slip fault
Lyakhovsky and Ben-Zion (2008)

After a few m of displacement

After a few tens of m of displacement

Erdogan & Sih (1963)

\(\beta = 15^\circ\)
\(\phi = 65^\circ\)

\(\beta = 45^\circ\)
\(\phi = 53^\circ\)

\(\beta = 60^\circ\)
\(\phi = 43^\circ\)

\(\beta = 80^\circ\)
\(\phi = 19^\circ\)
In places with persisting stepovers significant damage extends throughout the seismogenic zone.

The damage generation is suppressed with increasing normal stress (depth), leading to a flower structure.

This is a general result of other models.

Finzi, Lyakhovsky, Ben-Zion & Hearn (2008)
We fix all the large scale parameters (e.g., dimensions, background elastic properties, viscosity) using data associated with the San Andreas fault.

The evolving results depend on the ratio of time scale for damage healing $\tau_H$ to time scale for tectonic loading $\tau_L$. 

**Coupled evolution of earthquakes and faults in a regional lithospheric model**
slow effective healing

Weak upper crust, low friction angle

Random initial damage distribution

0 y.

1000 y.

Strong upper crust, high friction angle

Damage self organization

Note the different geometry and rate of damage zone formation between the strong (right) and weak (left) upper crust

Characteristic earthquakes

Power-law statistics
The results are compatible with other theoretical frameworks (e.g., Damhen et al., 1998; Zöller et al., 2004) and observations (e.g., Stirling et al., 1996; Marco et al., 1996).
Seismicity in discrete fault zones in elastic half-space

Ben-Zion and Rice, 1993, 1995; Ben-Zion, 1996; Fisher et al., 1997; Dahmen et al., 1998; Mehta et al., 06; Zöller et al., 05, 07; Dahmen and Ben-Zion, 08; Bailey and Ben-Zion, 08)

Focus:
- Heterogeneities spanning different ranges of size scale
- Different values of $\varepsilon$

$\tau_d - \tau_a \equiv$ dynamic overshoot

$\varepsilon_D = (\tau_s - \tau_d) / \tau_s =$ dynamic weakening

Dahmen and Ben-Zion 2008
Frequency-size statistics for relatively-regular fault zones with Narrow Range of Size Scales \((\varepsilon > 0; \tau_d < \tau_s)\)
Frequency-size statistics for disordered fault zones with Wide Range of Size Scales ($\varepsilon > 0$)
Mode-switching behavior

Time Series of Earthquake Sizes and Configurational Entropy

Earthquake Size

Entropy

Time $t/T$
Relations to observations?
Wesnousky 1994; Stirling et al., 1996

Relatively regular

Highly irregular

NROSS

WROSS
Statistics for highly irregular structure with WROSS are compatible with the Gutenberg-Richter distribution.
Statistics for relatively regular structure with NROSS are compatible with the Characteristic earthquake distribution.
Statistics for relatively regular structure with NROSS are compatible with the Characteristic Earthquake distribution.

Discrete statistics for seismicity along the Parkfield section of the SAF (Ben-Zion and Rice, 1993)

Discrete statistics for seismicity preceding the 1980 eruption of Mount St Helens (Main, 1992)
Marco et al., JGR, 1996
Long-term earthquake clustering: A 50,000-year paleoearthquake record in the Dead Sea Graben

Shmuel Marco, Mordechai Steina, and Amotz Agnon
Institute of Earth Sciences, Hebrew University, Jerusalem, Israel

Hagai Ron
Institute for Petroleum Research and Geophysics, Holon, Israel

Abstract. The temporal distribution of earthquakes in the Dead Sea Graben is studied through a 50,000-year paleoearthquake record recovered in laminated sediments of the Late Pleistocene Lake Lisan (palaeo-Dead Sea). The Lisan represents more than 10 times the 4000 years of historical earthquake records. It is the longest and most complete paleoearthquake record along the Dead Sea Transform and possibly the longest continuous record on Earth. It includes unique exposures of seismites beds (earthquake-induced structures) associated with slip events on syndepositional faults. The seismites are layers consisting of mixtures of fragmented and pulverized laminae. The places where the seismites and syndepositional faults are interpreted as evidence for their formation at the sediment-water interface during slip events on these faults. Thicker sediment accumulation above the seismites in the downthrown blocks indicates that a seismites formed at the water-sediment interface on both sides of the fault scarp. Modern analogs and the association with surface ruptures suggest that each seismites formed during a $M>5.5$ earthquake. The $^{260}$Th-$^{234}$U ages of all sections, obtained by thermal ionization mass spectrometry, give a mean recurrence time of $\sim$1600 years of $M>5.5$ earthquakes in the Dead Sea Graben. The earthquakes cluster in $\sim$10,000-year periods separated by quiet periods of similar length. This distribution implies that a long-term behavior of the Dead Sea Transform should be represented by a mean recurrence of at least 50,000 year record. This observation has ramifications for seismic hazard assessment based on shorter records.

Number of earthquakes
5 kyr, section PZ1

Figure 10. (left) The distribution of mixed layers (i.e., earthquakes) along section PZ1; open diamonds show the individual layers. Crosses and open circles are the number of earthquakes per 5 kyr sliding window, shifted by 2-kyr increments leaving 3-kyr overlap. The crosses show the distribution when the ages of the mixed layers are calculated by a single linear regression, whereas the circles show the distribution when the ages are calculated in three segments (Fig. 3). The distribution shows two clusters of frequent events. A cycle about 20,000 years long includes a cluster period and a quiet intercluster period. The pattern of clustering is independent of our choice of calculated sedimentation rates. (right) The distribution of individual mixed layers along sections PZ1, PZ2, and M1. The top of the Lisan is used as a datum. All three sections show clusters of mixed layers separated by quiescent intervals.
Other examples compatible with mode-switching behavior:

50 kyr paleoseismic record along the Arava segment of the DST south of the study area of Marco et al. [Leonard et al., 1998; Amit et al., 2002]

Changes in the character of activity along several faults in the basin and range, western US, province [Wallace, 87],

Episodic clustering of activity in the last 10 kyr years along fault segments in the Eastern CA Shear Zone [Rockwell et al., 2000]

Changes in the character of accumulation and release of seismic energy on the San Miguel fault, Mexico [Hirabayashi et al., 96]

Several widely separated periods with and without large earthquakes in the new Madrid, eastern US, seismic zone [Sexton and Jones, 86]

Seismicity on the North Anatolian fault [Sengor et al., 2005]

Seismicity on the Anza section of the San Jacinto fault [Rockwell et al., 2008]
Large earthquakes are sometimes preceded by phases of accelerated seismic release (ASR) in a broad region around the eventual rupture zone.

ASR phases were characterized by cumulative Benioff strain following a power law time-to-failure relation with a term $(t_f - t)^m$ with observed values of $m$ are close to 0.3.

$$\sum M^{1/2} = A + B(t_f - t)^m$$

A 1D version of the damage model leads analytically to time-to-failure relation for strain with $m=-1/3$ and cumulative Benioff strain release with $m=1/3$. 

Jaume’ and Sykes (1990)
The line fits are based on equations (5), (8) and (9).

The results highlight the difficulty of constraining the functional form and parameters of ASR phases.
Analysis of Aftershocks

For uniform deformation and simplifying assumptions, the damage equations lead to Omori-Utsu type analytical solution for aftershocks decay rate

\[
\frac{dN}{dt} = \frac{\dot{N}_0}{2\phi R (1-\alpha_s) \dot{\bar{N}}_0} \cdot \frac{1}{t + 1/2\phi R (1-\alpha_s) \dot{\bar{N}}_0}
\]

**Omori-Utsu**: \( \frac{dN}{dt} = K(c + t)^{-p} \)

Mapping between the damage and Omori-Utsu parameters:

\[
k = \frac{1}{2\phi R (1-\alpha_s)} \quad c = \frac{1}{2\phi R (1-\alpha_s) \dot{\bar{N}}_0} = \frac{k}{\dot{\bar{N}}_0} \quad p = 1
\]

The results depend fundamentally on the non-dimensional material parameter

\[
R = \frac{\tau_d}{\tau_M} = \mu_0 C_v
\]

\( R \) is expected to increase with temperature, fluid content and sediment thickness. 

**Significantly**, \( R \) is related to the seismic coupling coefficient as

\[
\chi = 1/(1+R)
\]

From the above and observed ratios of \( k \) values in different regions, we can estimate the seismic coupling in the different regions!!!
Mainshocks: events with $4.0 \leq M' \leq 6.0$.

Aftershocks: events in subsequent 50 days, within a circular region that scales with $M'$, and with $M' \leq M \leq M'-2$.
We fit the data with a cumulative form of the Omori-Utsu law assuming $c = 0$, $p = 1$, $t_0 = 5$ day
The observed correlations between aftershock productivity, heat flow and sedimentary cover are compatible overall with the damage model predictions.
### Seismic coupling coefficients based on derived $K$ values

<table>
<thead>
<tr>
<th>Region</th>
<th>$K$</th>
<th>$\chi$</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (Imperial Valley)</td>
<td>28</td>
<td>0.72, 0.58</td>
<td>0.39, 0.72</td>
</tr>
<tr>
<td>B (Landers, HM)</td>
<td>62</td>
<td>0.85©, 0.76</td>
<td>0.18, 0.32</td>
</tr>
<tr>
<td>C (San-Bernardino)</td>
<td>39</td>
<td>0.78, 0.66</td>
<td>0.28, 0.52</td>
</tr>
<tr>
<td>D (Ventura basin)</td>
<td>33</td>
<td>0.75, 0.62</td>
<td>0.33, 0.61</td>
</tr>
<tr>
<td>E (Coso)</td>
<td>9</td>
<td>0.45, 0.31</td>
<td>1.2, 2.2</td>
</tr>
<tr>
<td>Green box in A</td>
<td>5</td>
<td>0.31, 0.20*</td>
<td>2.2, 4.0</td>
</tr>
</tbody>
</table>

$\chi_2 \approx \frac{1}{(K_1 / K_2)(1/\chi_1 - 1) + 1}$
Main Results

There are 3 general dynamic regimes:

- The first is associated with relatively mature smooth faults, FS statistics compatible with the characteristic earthquake distribution, quasi-periodic temporal occurrence of large events, and no accelerated seismic release.

- The second is associated with disordered fault structures, power law FS statistics of earthquakes, temporal clustering of intermediate and large events, and accelerated seismic release before large earthquakes.

- For a range of conditions, there is a third regime in which the response switches back and forth between the forgoing two modes of behavior over multiple large earthquake cycles.

- In the latter cases, the seismic response of the fault zone is non-stationary on time scales shorter than several mode-switching intervals (e.g., 1000-10000 yr. for large fault zones).

- Cold regions with crystalline rock have high seismic coupling and long-duration high-productivity aftershock sequences. Warm/hot regions and thick sedimentary basins have low seismic coupling and diffuse low-productivity sequences or swarms.
References


