Slow and fast slip: coupled friction-elasticity-dilatancy with pore-fluid and thermal transport

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With a little help from my friends: A. Rubin, J.R. Rice, A.M. Bradley, S. Schmitt, T. Matsuzawa
Fig. 4. A synoptic shear zone model, illustrating the major geological and seismological features.

Scholz, 1988
Temperature Dependence of Frictional Properties

\[ a-b = \frac{df_{ss}}{d \log(v)} \]

Blanpied et al, 1995

a-b mapped to depth via San Andreas isotherm
Depth dependence of a-b

Blanpied et al, 1991
Model Earthquakes

- Slow interseismic slip at depth.
- Nucleation at base of seismogenic zone.
- Elastodynamic rupture.
- Postseismic slip.

- \( d_c >> \) lab values.
- Assumed effective stress

Lapusta and Rice, 2000;2003
What’s missing from the Standard Model?

- Dilatancy/compaction, cementation.
- Dynamic weakening mechanisms:
  - Thermal pressurization.
  - Flash heating.
  - Thermally induced dehydration.
- New observations of slow slip and non-volcanic tremor

Dilatancy: stabilizing

\[
\frac{\partial p}{\partial T} \approx 1 \text{ MPa/K}
\]

Thermal pressurization: de-stabilizing

thermal expansion of water \(\gg\) pores w/ no permeability:

\[
\frac{\partial p}{\partial T} \sim 1 \text{ MPa/K}
\]
Episodic Tremor and Slip

Dragert et al, 2001
Rogers and Dragert, 2003

Obara, 2002

[Maps and graphs showing distribution and patterns of tremors and slip events]
1999 Cascadia Slow Slip Event

McGuire and Segall, GJI, 2003
Tokai Slip-rate Distribution

Miyazaki et al, 2006
Kilauea, Hawaii Slow Slip Events
Slow Slip Observations

- Dimension of slip zones ~ 10’s km.
- Slip ~ 1 cm -> Stress drops < 0.1 Mpa.
- Average slip-rate ~ 10 - 100 x plate-rate.
- Periods of order 1 yr (0.5 to 6 yr).
- Propagation speeds ~ 10 km/day.
- Tremor triggered by teleseisms.

Szeliga et al, 2007
Possible Mechanisms for Slow Slip

• Change in frictional behavior at high slip speed [e.g., Shibazaki and Iio, 2003; Shibazaki and Shimamoto, 2007].


• Stable friction perturbed from steady-state [Perfittini and Ampuero, 2009].

Rate-state friction near neutral stability

\( h^* = \text{critical fault dimension for nucleation of transient slip} \)

\[ h^* \approx \frac{Gd_c}{(\sigma - p^\infty)(b - a)} \]

Behavior varies dramatically with width of fault, \( W/h^* \)

Liu and Rice, 2007
Limited Range of Fault Widths

\[ a/b = 0.9 \]

Rubin, 2008

Diagram: V_{\text{max}} / V_{\text{plate}} vs. W / h^*

- Slip Law
- Instability
- Aging Law
- Aperiodic
- Periodic

Width of transition zone / Width of nucleation zone

Rubin, 2008
Slip Law Consistent with Velocity Step Tests

[Bayart et al, 2006]
A Constitutive Law for Dilatancy

Rate state friction Ruina [1983]; Dieterich [1979]; Potentially unstable for $a - b < 0$.

$$\tau = f(\sigma - p) = (\sigma - p)[f_0 + a \log \frac{v}{v_0} + b \log \frac{\theta v_0}{d_c}]$$

$$\frac{d\theta}{dt} = 1 - \frac{\theta v}{d_c} \quad \text{or} \quad \frac{d\theta}{dt} = -\frac{\theta v}{d_c} \ln \left( \frac{\theta v}{d_c} \right)$$

Constitutive Law for Dilatancy Segall and Rice [1995]

$$\delta \phi = -\epsilon \log \left( \frac{v_0 \theta}{d_c} \right), \quad \delta \phi_{ss} = \epsilon \log \left( \frac{v}{v_0} \right).$$

$\phi$: porosity, $\epsilon \sim 10^{-4}$, based on fits to Marone [1990] lab data.

Data from Marone et al [1990]
A Constitutive Law for Dilatancy

Constitutive Law for Dilatancy Segall and Rice [1995]

\[ \delta \phi = -\epsilon \log \left( \frac{v_0 \theta}{d_c} \right), \quad \delta \phi_{ss} = \epsilon \log \left( \frac{v}{v_0} \right). \]

\( \phi \) : porosity, \( \epsilon \approx 10^{-4} \), based on fits to Marone [1990] lab data.
\( h \) : shear zone thickness. \( \lim_{h \to 0} h d\phi/dt = (1 - \phi) dh/dt. \)

Data from Samuelson, Elsworth, Marone, in press
"Isothermal Membrane Diffusion" Model

\[
V = \begin{cases} 
V_{\text{plate}} & \text{W} \text{ Friction + Dilatancy} \\
0 & \text{V}_{\text{plate}}
\end{cases}
\]

\[
\frac{G}{(1 - \nu)} \mathcal{H}[\delta'] - f(v, \theta)(\sigma - p) = \frac{G}{2v_s} v \\
\mathcal{H}[\delta'] = \frac{1}{2\pi} \int_{-\infty}^\infty \frac{\partial \delta}{\partial \xi} \frac{1}{\xi - x} d\xi
\]

"Membrane Diffusion"

\[
\frac{\partial p}{\partial t} = \frac{p^\infty - p}{t_f} - \frac{1}{\beta} \frac{d\phi}{dt}
\]
Normalized Equations

\[ \ddot{x} \equiv \frac{x}{h^*} = \frac{x(\sigma - p^\infty)(b - a)}{G(1 - \nu)dc} \quad \ddot{t} \equiv \frac{tv^\infty}{dc} \quad \ddot{v} \equiv \frac{v}{v^\infty} \]

\[
\left( \frac{b - a}{b} \right) H[\dot{v}'] - \frac{(\sigma - p)}{(\sigma - p^\infty)} \left[ \frac{a}{b} \frac{\dot{v}}{\overline{\tau}} + \frac{\dot{\theta}}{\overline{\theta}} \right] + \frac{f(\dot{\theta}, \dot{v})f_0}{b(\sigma - p^\infty)} \frac{\partial p}{\partial t} = \frac{Gb}{2(\sigma - p^\infty)} \frac{v^\infty}{v_s} \frac{\partial \dot{v}}{\partial t}
\]

\[
\frac{f_0}{b(\sigma - p^\infty)} \frac{\partial p}{\partial t} = \frac{f_0}{b} \left( p^\infty - p \right) \left( \frac{dc}{v^\infty tf} \right) + \frac{f_0\epsilon}{\beta b(\sigma - p^\infty)} \frac{\dot{\theta}}{\overline{\theta}}
\]

\[
\frac{\partial \dot{\theta}}{\partial t} = -\dot{\theta} \dot{v} \ln(\dot{\theta} \dot{v})
\]

Dimensionless Parameters

\[ \mathcal{E} \equiv \frac{f_0\epsilon}{\beta b(\sigma - p^\infty)} \quad \mathcal{U} \equiv \frac{v^\infty tf}{dc} \quad \frac{a}{b} \quad \frac{W}{h^*} \quad \frac{f_0}{b} \sim 30 \]
Dilatancy dramatically expands range for slow slip

\[ E=1; \quad U=1 \]
Model Slow Slip Event

\[ E = 1; \ a/b = 0.833; \ \nu^\infty \ t/d_c = 1 \]

**Slip Speed (m/s)**

**Propagation**

**\( \tau/ (\sigma - \rho^\infty) \)**

**\( (\rho - \rho^\infty) / (\sigma - \rho^\infty) \)**

**Along Fault Distance**
Slow vs Fast Slip

\[ E = 0.32; \ a/b = 0.7; \ \frac{\nu}{t/d_c} = 0.1; \ W/h^* = 8 \]

\[ E = 0.25; \ a/b = 0.7; \ \frac{\nu}{t/d_c} = 0.1; \ W/h^* = 8 \]
Critical Stiffness in Single Degree of Freedom System

\[ k_{crit} = (\sigma - p^\infty) \frac{(b - a)}{d_c} - \frac{f_0 \varepsilon}{\beta d_c} F(\varepsilon, U, a/b) \]

\[ \tilde{K}_{crit} = \frac{k_{crit} d_c}{(\sigma - p^\infty)(b - a)} = 1 - \varepsilon \left( \frac{b}{b - a} \right) F(\varepsilon, U, a/b) \]

Segall and Rice (1995)
\[ \tilde{K}_{crit} = \frac{k_{crit}d_c}{(\sigma - p^\infty)(b - a)} = 1 - \mathcal{E} \left( \frac{b}{b - a} \right) F(\mathcal{E}, \mathcal{U}, a/b) \]

Drained Limit  \[ \lim_{\mathcal{U} \to 0} \tilde{K}_{crit} \to 1. \]

Undrained Limit  \[ \lim_{\mathcal{U} \to \infty} \tilde{K}_{crit} \to 0. \]

Segall and Rice (1995)
\[
\frac{L_c}{h^*} = \tilde{K}^{-1}_{crit}
\]
Thus, for \( \mathcal{E} \geq 1 - \frac{a}{b} \), 
\[
\lim_{u \to \infty} \frac{L_c}{h^*} \to \infty.
\]

Thus, if \( \mathcal{E} \geq 1 - a/b \), fast (undrained) slip is stable for all \( W/h^* \).
For $a/b = 0.9$:
- $E < 1 - a/b$ results in the region being unstable.
- $E > 1 - a/b$ results in the region being stable.
Stability Boundary

\[ E = 1 - \frac{a}{b} \]
**Dilatant Stabilization vs Thermal Weakening**

Heat & Pore-Fluid Transport for Thin Shear Zone \([\text{after Rice, 2006}]\)

\[
\begin{align*}
\frac{\partial T}{\partial t} &= c_{th} \frac{\partial^2 T}{\partial y^2} \\
\frac{\partial p}{\partial t} &= c_{\text{hyd}} \frac{\partial^2 p}{\partial y^2} + \Lambda \frac{\partial T}{\partial t} \\
\frac{\partial T}{\partial y} \bigg|_{y=0} &= -\frac{\tau v}{2 \rho c_p c_{th}} \\
\frac{\partial p}{\partial y} \bigg|_{y=0} &= \frac{h \dot{\phi}}{2 \beta c_{\text{hyd}}}
\end{align*}
\]

- \(c_{th}\): thermal diffusivity,
- \(\rho\): density,
- \(c_p\): heat capacity,
- \(\Lambda \approx 1 \text{ MPa/}^\circ\text{C.}\)

\[
\begin{align*}
\frac{\partial \hat{p}}{\partial y} \bigg|_{y=0} &= -E_p \frac{\hat{\theta}}{\hat{\theta}} \\
E_p &= \frac{\epsilon}{2 \beta (\sigma - p^\infty)} \sqrt{\left[ \frac{h^2\upsilon\infty}{c_{\text{hyd}}d_{c}} \right]},
\end{align*}
\]

\[
\begin{align*}
\frac{\partial \hat{T}}{\partial y} \bigg|_{y=0} &= -E_T \frac{\hat{v}}{\hat{v}} \\
E_T &= \frac{f_0 (\sigma - p^\infty)}{2 \rho c_p T_0} \sqrt{\frac{c_{\text{hyd}}d_{c} v^\infty}{\epsilon_{th}^2}}.
\end{align*}
\]

Dilatancy to Shear Heating Efficiency:

\[
\frac{\epsilon \rho c_p T_0}{\beta f_0 (\sigma - p^\infty)^2 d_c c_{\text{hyd}}} h c_{th}
\]

Slow slip favored by dilatancy (large \(\epsilon h\)), and low \((\sigma - p^\infty)\) & friction \(f_0\).
Coupled Friction / Dilatancy - Diffusion

\[ V = \begin{cases} V_{\text{plate}} \\ 0 \end{cases} \quad \text{Friction + Dilatancy} \]

\[
\frac{G}{(1-\nu)} \mathcal{H}[\delta'] - f(v, \theta)(\sigma - p) = \frac{G}{2v_s}v \\
\mathcal{H}[\delta'] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\partial \delta}{\partial \xi} \frac{d\xi}{\xi - x}
\]

Isothermal Homogenous Diffusion for Thin Shear Zone

\[
\frac{\partial p}{\partial t} = c_{\text{hyd}} \frac{\partial^2 p}{\partial y^2} \\
\left. \frac{\partial p}{\partial y} \right|_{y=0} = \frac{h \dot{\phi}}{2\beta c_{\text{hyd}}}
\]

\( c_{\text{hyd}} \): hydraulic diffusivity
\( \beta \): fluid + pore compressibility.

Non-dimensional Parameters

\[
W/h^*, \quad a/b, \quad \text{and} \quad E_P = \frac{\epsilon}{2\beta(\sigma - p^\infty)} \sqrt{\frac{h^2 v^\infty}{c_{\text{hyd}} d_c}}.
\]
Numerical Methods

Diffusion equation solved with Finite Difference on logarithmic mesh.

Numerical Procedure: The system of PDEs has the form

\[ \dot{v} = \tilde{f}(v, \theta, \dot{\theta}, p, \dot{p}) \quad \dot{\theta} = h(v, \theta) \quad \text{on } y = 0 \]  

(1)

\[ \dot{p} = c_{hyd} \frac{\partial^2 p}{\partial y^2} \quad \text{on } y > 0 \]  

(2)

\[ \frac{\partial p}{\partial y} \bigg|_{y=0} = g(\theta, v) \quad \lim_{y \to \infty} p = p_\infty. \]  

(3)

Let \( u = [v, \theta, \dot{\theta}] \), such that (1-3) become:

\[ \dot{u} = f(u, p, \dot{p}) \quad \text{on } y = 0 \]  

(4)

\[ \dot{p} = c_{hyd} \frac{\partial^2 p}{\partial y^2} \quad \text{on } y > 0 \]  

(5)

\[ \frac{\partial p}{\partial y} \bigg|_{y=0} = g(u) \quad \lim_{y \to \infty} p = p_\infty. \]  

(6)

- Explicit time steps limited by CFL condition: \( \Delta t < \Delta x^2/c \).
- Fully implicit scheme requires solution of large linear system.
Implicit-Explicit Time Stepping. Let $p_{km}^n$ be the value of $p$ at the $k$th point in the $y$ direction and the $m$th point in the $x$ direction, at $n$th time step. For simplicity illustrate with Euler’s method, where (4-6) are discretized as:

\[
\frac{u_{m}^{n+1} - u_{m}^{n}}{\Delta t} = f \left( u_{m}^{n}, p_{0m}^{n}, \frac{p_{0m}^{n} - p_{0m}^{n-1}}{\Delta t} \right) \quad m = 1, \ldots M
\]

\[
\frac{p_{km}^{n+1} - p_{km}^{n}}{\Delta t} = c_{hyd} \left( \frac{p_{(k-1)m}^{n+1} - 2p_{km}^{n+1} + p_{(k+1)m}^{n+1}}{\Delta y^2} \right) \quad k = 1, \ldots K
\]

\[
\frac{p_{1m}^{n+1} - p_{(-1)m}^{n+1}}{2\Delta y} = g(u_{m}^{n+1}) \quad \text{and} \quad p_{Km}^{n+1} = p_{\infty}.
\]

- First equation is explicit.
- Second and third equations are implicit, but depend only on $u_{m}^{n+1}$. Equations in $y$ decouple, such that $M$ small systems of equations (with $K$ elements) are solved at each time step, rather than one large system.
Dilatancy greatly expands permissible range of transient stable slip

\[ a/b = 0.9 \]
\[ a/b = 0.8 \]

Rubin, 2008

Normalized Fault Width
Dependence on $E_p$

Increasing Dilatancy and/or Pore Pressure; Decreasing Diffusivity
Along Fault Distance

Tremor between ETS events?

Slow phase precedes faster event.
Propagating Creep Occurs Between Model Slow Slip Events

- Creep during slow phase driven by down-dip displacement - no stress drop.
- Stress released during fast phase.
- Fast phase accommodates ~1/2 interseismic slip ~1/4 of plate motion.
Evidence for high pore-pressure

- Low Stress Drop
- Triggered tremor
- Large Fault Width $\rightarrow$ large $h^*$
- Thermal modeling.

\[ h^* \sim \frac{Gd_c}{(\sigma - p^\infty)(b - a)} \]

\[ E_p = \frac{\varepsilon}{2\beta(\sigma - p^\infty)} \sqrt{\frac{h^2}{v_\infty}} \]

What’s missing from the Standard Model?

- Dilatancy, compaction, cementation.
- Dynamic weakening mechanisms:
  - Thermal pressurization.
  - Flash heating.
  - Thermally induced dehydration.
- New observations of slow slip and non-volcanic tremor

Dilatancy: stabilizing

\[ \frac{\partial p}{\partial T} \approx 1 \text{ MPa/K} \]

Thermal pressurization: de-stabilizing

thermal expansion of water >> pores

w/ no permeability : \( \frac{\partial p}{\partial T} \approx 1 \text{ MPa/K} \)
Thermal Pressurization

Narrow shear zones and low permeability lead to strong thermal pressurization.

Estimated fracture energy consistent with seismological observations [Rice, 2006].
Thermal Pressurization During Nucleation

- Causes nucleation zone to contract, making detection even more difficult.
Thermal weakening dominates rate-state friction prior to seismic radiation.

However, thermal weakening not dominant at slip speeds characteristic of average rates during slow slip events.
Conclusions

• Fluid-thermal-dilatancy effects can not be neglected.
• Dilatancy can allow slip to nucleate at low speeds but limits fast slip.
• High pore-pressure mitigates against frictional and thermal weakening and favors slow slip (consistent with some observations).
• Thermal pressurization significant during aseismic nucleation, but not at average slow slip speeds.
• Whether slip is ultimately slow or fast may depend on whether dilatancy or thermal pressurization dominates.
Where do we go from here?

- Combine thermal/dilatant processes.
- Model depth dependent variations over full megathrust cycle (many slow slip events).
- Spatial variations in permeability.
- Self-consistent geodetic inversions *between and during* ETS.
- Precise locations of tremor *between and during* ETS – relate to strain signals (?)
- Imaging properties of locked megathrust.

Audet, 2009
Where do we go from here?

- Pore-pressure, temperature, strain in near-field of nucleating events (SAFOD, mines), and after large earthquakes.
- In situ fault structure, permeability,…
- Lab data: friction, dilatancy, dynamic weakening mechanisms, wave speeds vs effective stress.
- Need *global* observations to understand what controls ETS and triggered seismicity.
Analytical Approximation

For step change in slip speed: \( v(t) = v H(t) \);

\[
\theta = \theta_f \left( \frac{\theta_1}{\theta_f} \right)^{\exp(-\delta/d_c)} \Rightarrow \log \left( \frac{v \theta}{d} \right) = \log \left( \frac{v \theta}{d_c} \right) e^{-vt/d_c} \quad \text{and} \quad vt = \delta
\]

Friction:
\[
f = f_0 + (a - b) \log \left( \frac{v}{v_0} \right) + b \log \left( \frac{v \theta}{d_c} \right) e^{-\delta/d_c}
\]

Pore Pressure:
\[
p(y = 0, t) = -\frac{2E_p}{\sqrt{\pi}} (\sigma - \rho^\infty) \sqrt{\frac{v}{v^\infty}} \log \left( \frac{v \theta}{d_c} \right) D \left( \sqrt{\frac{\delta}{d_c}} \right)
\]

"Dawson’s Integral":
\[
D(z) = e^{-z^2} \int_0^z e^{t^2} dt \quad D(z) \sim \frac{1}{2z} \quad z \gg 1
\]
Fracture Energy

\[ G_c \equiv \int_0^{\delta^*} [\tau(\delta) - \tau(\delta^*)] \, d\delta \]

\[ G_c \approx G_c^f + G_c^p = (\sigma - p^\infty) \int_0^{\delta^*} [f(\delta) - f(\delta^*)] \, d\delta - f_0 \int_0^{\delta^*} [p(\delta) - p(\delta^*)] \, d\delta \]

Friction

Dribanacy

\[ G_c^f(v) = (\sigma - p^\infty)bd_c \log \left( \frac{V_{\theta_1}}{d_c} \right) \]

\[ G_c^p(v) = \frac{2E_p}{\sqrt{\pi}} (\sigma - p^\infty)d_c \sqrt{\frac{V}{\nu}} \log \left( \frac{V_{\theta_1}}{d_c} \right) I(\delta^*) \text{ where } I(\delta^*) = \int_0^{\delta^*} D \left( \sqrt{\frac{\delta}{d_c}} \right) \, d\delta \]

Can fracture energy arguments predict maximum slip rates?
Simulation

\[ a/b = 0.9 \quad E_p = 0.00029669 \quad W/h^* = 10 \]

- \( \tau \)
- \( f_0 \rho \)
- \( f_0(\sigma-p_0) \)

Normalized Stress vs. \( \delta/d_c \)

Normalized Stress vs. Time yrs

\( x_{(pos)} = 10\% \) from locked end
Seismic Swarm Lags Slow Slip

Cumulative number of earthquakes

2005 Slow Slip Event

Boso Peninsula Slow Slip Events, Japan

1996  2002

Ozawa et al, GRL 2003