Rapid CAD and tetrahedral mesh generation for dynamic rupture problems

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SeisSol - Project overview

Coordination, Host, Physics, Numerics, Algorithm, Pre- and Postprocessing, Application, User support

Technical development, HPC, Optimization, Visualization, Design

Consulting, Scaling, BlueGene/Q adaption

Visualization, parallel I/O

Automated CAD generation

...and others ...
Support, Guidance, Experience sharing, Consulting, ...
Goal

Complete seismic wave propagation package including solutions for

- dynamic rupture simulations
- exploration industry
- Seismology

with complex geometry and heterogeneous medium.


Requirements for solver

What do we need for this?

- Accurate numerical methods for reliable results (num. errors, boundary-, initial conditions)
- Proper geometry representation (topography, material interfaces)
- Use of acoustic, elastic, viscoelastic, and anisotropic material to approximate realistic geological subsurface properties
- Scalability on HPC architecture to tackle big problems with high frequency
Advantages of the ADER-DG Method

- Enables use of unstructured meshes – low velocity basins, curved or kinked faults, branching, surface rupture, fault interaction
- Mesh coarsening – adjustment of resolution
- High-order accurate simulation of the wave propagation including heterogeneous media and topography
Advantages of the ADER-DG Method

- ADER high-order time integration with local time stepping
- High-accurate results of the rupture process: Oscillation free dynamic rupture

**SEM vs ADER-DG**
Mathematical Model

Elastic Wave Equation as a Linear Hyperbolic System:

Vector-matrix notation:
\[
\frac{\partial Q_p}{\partial t} + A_{pq} \frac{\partial Q_q}{\partial x} + B_{pq} \frac{\partial Q_q}{\partial y} + C_{pq} \frac{\partial Q_q}{\partial z} = S_p
\]

Velocity-stress formulation:
3D: \[
Q = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, u, v, w)^T
\]
Mathematical Model

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Numerical Approximation of the solution

\[
\left( Q_h^{(m)} \right)_p (\xi, \eta, \zeta, t) = \hat{Q}^{(m)}_{pl}(t) \Phi_I(\xi, \eta, \zeta)
\]

- $\Phi_I$ are orthogonal basis functions
- the mass matrix is diagonal
Discontinuous Galerkin Approach – Flux computation

Flux computation

Exact Riemann solver is used to compute the state at the interfaces by upwinding:

\[
F_p^h = \frac{1}{2} T_{pq} \left( \begin{array}{c} A_{qr}^{(m)} + \left| A_{qr}^{(m)} \right| \\ A_{qr}^{(m)} - \left| A_{qr}^{(m)} \right| \end{array} \right) (T_{rs})^{-1} \hat{Q}_{sl}^{(m)} \Phi_l^{(m)} + \frac{1}{2} T_{pq} \left( \begin{array}{c} A_{qr}^{(m)} + \left| A_{qr}^{(m)} \right| \\ A_{qr}^{(m)} - \left| A_{qr}^{(m)} \right| \end{array} \right) (T_{rs})^{-1} \hat{Q}_{sl}^{(m)} \Phi_l^{(m)}
\]

Computation of the line integrals:

- Pre-computed analytically
- Gauss-Legendre integration

Opens up new possibilities:
- non-conforming meshes, dynamic rupture source type

Locality of the computations:
- only directly neighboring elements are required to exchange data, which leads to small communication times for parallel calculations
Suitability for large scale HPC infrastructure

Efficiency on the BlueGene/P machine Shaheen at KAUST

- 7.7 Mio. Elements
- Order of accuracy in space and time: O5
- Pure MPI parallelization – code is openMP hybrid now
- Metis partitioning

http://glaros.dtc.umn.edu/gkhome/metis/metis/overview
**Dynamic Earthquake rupture**

Incorporate source process

- To understand earthquake faulting
- Support physics-based ground motion prediction

Treat dynamic rupture as an interior time-dependent 'boundary condition' using the flux term!

- Impose new traction following the failure criterion
- Impose fault parallel velocities in opposite directions

Example: 1994 Northridge by A. Gabriel


**Ingredients**

- **Initial Shear Traction** $\tau_0$

- **Geologic Structure** (Fault Geometry & Material Properties)

- **Failure Criterion**

- **Computer Program that Simulates Earthquakes as Spontaneous Ruptures**

- **Ground Shaking (Seismograms), Fault Slip, etc.**

(Brietzke et al. (2009))

Maps of the Southern California fault structure. Dots are earthquake centers. (Shaw et al. for SCEC, 2003)

(friction: non linear relation between fault stress and slip)

(Harris et al. (2009))
Failure criterion:

Coulomb friction model

\[ |\sigma_{xy}| \leq \mu_f \sigma \]

traction fault strength

\[(|\sigma_{xy}| - \mu_f \sigma) \Delta v = 0\]

\(\sigma_{xy}\) traction
\(\mu_f\) friction coefficient
\(\sigma\) normal stress
\(\Delta v\) slip rate
Failure criterion:

Coulomb friction model

$$|\sigma_{xy}| \leq \mu_f \sigma$$

traction        fault strength

$$(|\sigma_{xy}| - \mu_f \sigma) \Delta v = 0$$

$\sigma_{xy}$ traction

$\mu_f$ friction coefficient

$\sigma$ normal stress

$\Delta v$ slip rate

$\Delta d$ slip

$D_c$ critical slip distance

Linear Slip Weakening friction law
(laboratory experiments – rate-and-state also implemented)

Provides:

- initial rupture
- arrest of sliding
- reactivation of slip
Verification – TPV3 SCEC Test Case

(Harris et al., 2004)

- spontaneous rupture propagation on a straight fault
- homogeneous fullspace
- linear slip weakening friction

Comparison between
ADER-DG method order 4 and 200m triangles at the fault (larger tetrahedrons in bulk)
and
DFM - Finite Difference staggered-grid split node order 2 with 50m grid interval
and
MDSBI - Multidimensional spectral boundary integral with 50m grid interval
DFM data provided by Luis Dalguer. MDSBI data computed with the code of E. Dunham (version 3.9.10).
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Verification – TPV3 SCEC Test Case
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Workflow

From CAD to seismogram...

- Get geometry and model data
- Assemble CAD model
- Create mesh
- Partitioning
- Set model parameters
- Solve physical equation
- Analysis of output

“Time to solution!”
Automated CAD generation

Current bottleneck: CAD generation can easily consume *weeks to month*

Difficulties:

- Surface reconstruction of different types of initial raw data
- Undulating 3D surfaces that merge under shallow angles, intersect
- Remove non-physical features
- Clip too small features depending on the desired mesh size
- Representation by splines as typically used by (commercial) CAD/mesh software unfortunate for geological data
- Watertight model
- Seamless integration into meshing software (avoid format conversion)
Automated CAD generation – preliminary workflow

1. Download topography/bathymetry, e.g. from NOAA's ETOPO data collection
2. Define bounding box: rectangular or spherical
3. Material interfaces: structured grids of points
4. Faults: structured grids of points, gOcad's TS format
5. Check projection
6. (Triangulated) surface generation: Poisson surface reconstruction (MeshLab)
7. Assemble model: apply union, intersection, trimming operations with Simmetrix discrete modeling tools
Customized problem definition and mesh generation interface for SeisSol by RPI/SCOREC/Simmetrix (C. Smith, M. Shephard)

- Accepts e.g. Parasolid, ACIS and STL input
- Trims automatically geometry and creates a watertight model
- Meshes with millions of elements in seconds/minutes
- Mesh coarsening/refining
- Handling complex geometries (no violation)
- User-friendly interface
- Quality metrics
- Exports SeisSol format
- Non-manifold geometry required

Two faces. At the intersection there are two edges overlapping. = assembly

Two faces. At the intersection there is one shared edge. = non-manifold
Gambit vs SimModeler
THex approach

Work by Surendra Nadh Somala and Jean-Paul Ampuero
Compare final slip and slip rate from homogeneous dynamic rupture simulation on planar dipping fault with rate-and-state friction (Olsen et al., 1998)

Example – The Mw 6.7 1994 Northridge earthquake

Work by A. Gabriel
Conclusion & Outlook

- ADER-DG solver ready, functional and benchmarked
- Bring all features into production version (under construction)
- Combine dynamic rupture with local time stepping

- Current bottleneck CAD generation (under construction)
- Use CAD for quality control

- Open Source (soon), already available through [http://verce.eu/](http://verce.eu/)
  [http://seissol.geophysik.uni-muenchen.de/](http://seissol.geophysik.uni-muenchen.de/)
Failure criterion

Implementation of rate-and-state friction

- Updating scheme includes Newton-Raphson search for slip rate and two iterations for state variable (Kaneko et al., 2008)

\[
\mu_f = \mu_0 + a \ln \frac{v}{v_0} + b \ln \frac{v_0 \theta}{D_c}
\]

\[
\dot{\theta} = 1 - \frac{v \theta}{D_c}
\]

\(\theta\) state variable
\(a\) direct effect
\(b\) evolution effect
\(v_0\) steady-state reference velocity
\(\mu_0\) steady-state reference friction
Dipping fault geometry
(SCEC Test Cases TPV10 and TPV11)

- 60 degree dipping normal fault geometry
- Initial stress linearly depth dependent
- Subshear / supershear rupture conditions

Rupture time – contour plot (each 0.5 s)

Mesh geometry, computational domain and particle velocity on the fault plane after ~9.6 s

Off-fault station
(body 1.0 km, strike 0.0 km, depth 0.0 km)
Heterogeneous background stress
(SCEC Test Cases TPV16 and TPV17)

- Vertical strike-slip fault
- Randomly-generated heterogeneous initial stress conditions
- Trilinear interpolation to map background values on irregular distributed integration points

Rupture time – contour plot (each 0.5 s)

Initial shear stress on the fault plane

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hansel (Michael Hansel – Finite Element – FaultSim)
pelties (Christian Pelties – Discontinuous Galerkin (GF wise))

On-fault station
(strike -9.0 km, dip 9.0 km)
Fault branching geometry
(2D SCEC Test Cases TPV14 and TPV15)

- Left-lateral, vertical, strike-slip fault with a rightward branch forming a 30 degree angle
- Slightly stress-heterogeneous
- High resolution required
Tohoku

- CAD generation difficult
- Extremely shallow angle at trench
- Skewed elements