FEMs of dislocations within a subduction zone
Simulating complex structures with both forward and inverse models

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Virtual field trips via FEMs

Deformation of big subduction quakes

\[ \mu \nabla^2 u_i + \mu \left( \frac{\mu}{1-2\nu} \right) \left[ \frac{\partial^2 u_k}{\partial x_i \partial x_k} \right] = \alpha \frac{\partial P}{\partial x_k} - F_i \]

\[ \alpha \frac{\partial \varepsilon_{kk}}{\partial t} + S_e \frac{\partial P}{\partial t} = \frac{k}{\mu} \nabla^2 P + Q \]

**Old:** 1995 M8
Jalisco, Mexico quake

**New:** 2004 M9
Sumatra-Andaman quake
Study Site

West-central Mexico

North America

Pacific Ocean

North American Plate

1995 M8 Jalisco quake

Rivera Plate

4 cm/yr

Trench
Study Site

1995 Jalisco earthquake

1995 M8 Jalisco quake

1995 Feb, GPS sites installed
Oct 9, M=8

North America

Pacific Ocean

Rivera Plate

North American Plate

Trench

4 cm/yr

0.5 m

1995 M8 Jalisco quake
Deformation models

observed deformation ↔ causes of deformation

Deformation
what we see (InSAR, GPS)

Fault
what we want to know!

Slip along a fault
inverse models

Forward models
deformation

Displacement

1995 M8 Jalisco quake
Forward Model

Predict deformation caused by dislocation

Analytical solution

\[ G \cdot s = d \]

Slip \rightarrow \text{vertical displacement}

Unit impulse response function \rightarrow \text{geometry}

\[ G = f(D, W, L, \theta, x, y, z) \]

e.g., Okada [1992]

Elasto-static behavior
Inverse Model

Estimate dislocation to account for observed deformation

forward model

\[ G s = d \]

slip

(displacement
(vector))

inverse model

\[
\hat{s} = \left( G^T G \right)^{-1} G^T d
\]
Complicated Deformation Pattern

Array of dislocation patches

d_{ij} = s_i G_{ij}

- displacement
- slip
- unit impulse response function (geometry)

1995 M8 Jalisco quake
Matrix Assembly

Array of dislocation patches

\[ G_{ij} s_i = d_j \]

system of linear equations

\[
\begin{align*}
G_{11} s_1 + \ldots + G_{n1} s_n &= d_1 \\
G_{12} s_1 + \ldots + G_{n2} s_n &= d_2 \\
&\vdots \\
G_{1m} s_1 + \ldots + G_{nm} s_n &= d_m
\end{align*}
\]

\[
\begin{bmatrix}
G_{11} & G_{21} & G_{31} & \ldots & G_{n1} \\
G_{12} & G_{22} & G_{32} & \ldots & G_{n2} \\
G_{13} & G_{23} & G_{33} & \ldots & G_{n3} \\
& & & \ddots & \\
G_{1m} & G_{2m} & G_{3m} & \ldots & G_{nm}
\end{bmatrix}
\begin{bmatrix}
s_1 \\
\vdots \\
s_n
\end{bmatrix}
= 
\begin{bmatrix}
d_1 \\
\vdots \\
d_m
\end{bmatrix}
\]

\[
G \mathbf{s} = \mathbf{d}
\]

unknown

- we can solve for \( \mathbf{s} \) using
  linear matrix inverse methods
Solution Constraints

Laplacian operator $\rightarrow$ smoothing

Matrix expression: $G s = d$

Unknown

Additional constraints: avoid large variations between neighboring dislocations

1995 M8 Jalisco quake
Solution Constraints

Laplacian operator $\rightarrow$ smoothing

matrix expression: $Gs = d$

unknown

additional constraints: avoid large variations between neighboring dislocations

1995 M8 Jalisco quake
Solution Constraints

Laplacian operator $\rightarrow$ smoothing

**matrix expression:** $G \mathbf{s} = \mathbf{d}$

**unknown**

**additional constraints:** avoid large variations between neighboring dislocations

1995 M8 Jalisco quake
Solution Constraints

Laplacian operator $\rightarrow$ smoothing

**Matrix expression:** $G \mathbf{s} = d$

**Unknown**

**Additional constraints:** avoid large variations between neighboring sources

$\nabla^2 \mathbf{L} \mathbf{s} = 0$

$\beta$ scalar, controls amount of smoothing

$\begin{pmatrix} G \\ \beta \mathbf{L} \end{pmatrix} \mathbf{s} = \begin{pmatrix} d \\ 0 \end{pmatrix}$

Matrix expression: forward solution

$\mathbf{G} \mathbf{s} = \mathbf{d}$

Unknown

1995 M8 Jalisco quake
Inverse Solution

Damped least-squares

\[
\hat{G} \hat{s} = \hat{d}
\]

matrix expression
– forward solution

\[
\text{s}^\text{est.} = \left(\hat{G}^T \hat{G}\right)^{-1} \hat{G}^T \hat{d}
\]

matrix expression
– inverse solution

\[
\begin{pmatrix} G \\ \beta L \end{pmatrix} s = \begin{pmatrix} d \\ 0 \end{pmatrix}
\]

recall…

trade-off: misfit vs roughness

1995 M8 Jalisco quake
Inverse Solution

Over-smoothing

matrix expression
– forward solution

\[ \hat{G} \hat{s} = \hat{d} \]

unknown

matrix expression
– inverse solution

\[ \hat{s}^{\text{est.}} = \left( \hat{G}^T \hat{G} \right)^{-1} \hat{G}^T \hat{d} \]

recall...

\[ \begin{pmatrix} G \\ \beta L \end{pmatrix} s = \begin{pmatrix} d \\ 0 \end{pmatrix} \]

trade-off: misfit vs roughness

\[ \beta \rightarrow \infty \]

\[ \beta \rightarrow 0 \]

poor fit to data

smooth solution

1995 M8 Jalisco quake
Inverse Solution

Under-smoothing

matrix expression  
– forward solution
\[ \hat{\mathbf{G}} \hat{\mathbf{s}} = \hat{\mathbf{d}} \]
unknown

matrix expression  
– inverse solution
\[ \mathbf{s}^{\text{est.}} = \left( \mathbf{G}^T \mathbf{G} \right)^{-1} \mathbf{G}^T \hat{\mathbf{d}} \]

\[ \begin{bmatrix} \mathbf{G} \\ \beta \mathbf{L} \end{bmatrix} \mathbf{s} = \begin{bmatrix} \mathbf{d} \\ 0 \end{bmatrix} \]

recall…

trade-off: misfit vs roughness

1995 M8 Jalisco quake

\( \beta \rightarrow \infty \)

\( \beta \rightarrow 0 \)

good fit to data

rough solution
Inverse Solution

Balance of fit and smoothing

Matrix expression – forward solution
\[ \hat{G} \hat{s} = \hat{d} \]

Unknown

Matrix expression – inverse solution
\[ \hat{s}^{\text{est.}} = \left( \hat{G}^T \hat{G} \right)^{-1} \hat{G}^T \hat{d} \]

Recall...
\[ \begin{pmatrix} G \\ \beta L \end{pmatrix} s = \begin{pmatrix} d \\ 0 \end{pmatrix} \]

Trade-off: misfit vs roughness

Prediction misfit vs solution roughness

\[ \beta \rightarrow \infty \]

\[ \sim \checkmark \text{ good fit to data} \]

\[ \sim \checkmark \text{ smooth solution} \]

1995 M8 Jalisco quake
“Fit” Complicated Deformation Patterns

Geometric distribution of dislocation patches

deformation surface

dislocation (s')

null substantial

1995 M8 Jalisco quake
Wait a minute…
How well does the model represent the natural system?

defformation surface

analytical solution

\( n \) required assumptions:
- homogeneous
- isotropic
- Poisson-solid
- half-space

\( 1995 \ M8 \ Jalisco \ quake \)
Wait a minute…

How well does the model represent the natural system?

analytical solution

required assumptions:
- **homogeneous**
- **isotropic**
- Poisson-solid
- half-space

1995 M8 Jalisco quake
Wait a minute…

How well does the model represent the natural system?

required assumptions:
- homogeneous
- isotropic
- Poisson-solid
- half-space

1995 M8 Jalisco quake
Wait a minute…

How well does the model represent the natural system?

\[ \frac{V_p}{V_s} \rightarrow 0.25 \leq \nu \leq 0.30 \]

\textit{lower limit}

\[ \nu < \nu_u \]

\[ \nu = \frac{3 \nu_u - \alpha \beta (1 + \nu_u)}{3 - 2 \alpha \beta (1 + \nu_u)} \]

\[ \nu_u = \frac{3 \nu - \alpha \beta (1 - 2 \nu)}{3 - \alpha \beta (1 - 2 \nu)} \]

\textit{required assumptions:}
- homogeneous
- isotropic
- Poisson-solid
- half-space

1995 M8 Jalisco quake
Wait a minute…

How well does the model represent the natural system?

relief: 1000’s of meters

required assumptions:
• homogeneous
• isotropic
• Poisson-solid
• half-space
Test sensitivity to HIPSHS assumptions
Systematically relax and isolate each assumption

**definition**

**Finite Element Model (FEM):** numerical model; predicts deformation, stress, and pore pressure

\[
\mu \nabla^2 u_i + \frac{\mu}{(1-2\nu)} \left[ \frac{\partial^2 u_k}{\partial x_i \partial x_k} \right] = \alpha \frac{\partial P}{\partial x_k} - F_i
\]

\[
\alpha \frac{\partial \varepsilon_{kk}}{\partial t} + S_k \frac{\partial P}{\partial t} = \frac{k}{\mu} \nabla^2 P + Q
\]

△ - GPS station

1995 M8 Jalisco quake
Dislocation distribution for non-HIPSHS

FEM-generated unit impulse response functions

\[ \text{node pair} \]
relative displacement via kinematic constraint equations

[Smith, 1974]

\( \Delta \) - GPS station

1995 M8 Jalisco quake
Dislocation distributions

Non-HIPSHS configurations

non-HIPSHS deviations

1995 M8 Jalisco quake
Sensitivity to HIPSHS assumptions

Loading a non-HIPSHS FEMs with HIPSHS loads

\[ \Delta = G^H s^H - G s^H \]

\( \Delta \rightarrow 0 \), not sensitive
\( \Delta \rightarrow \infty \), sensitive

1995 M8 Jalisco quake
Sensitivity to HIPSHS assumptions

Loading a non-HIPSHS FEMs with HIPSHS loads

$$\Delta = G^H s^H - G s^H$$

<table>
<thead>
<tr>
<th>assumption</th>
<th>sensitivity</th>
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<tbody>
<tr>
<td>half-space</td>
<td>0</td>
</tr>
<tr>
<td>Poisson-solid</td>
<td>100</td>
</tr>
<tr>
<td>isotropy</td>
<td>200</td>
</tr>
<tr>
<td>homogeneity</td>
<td>combined</td>
</tr>
</tbody>
</table>

$$\Delta \rightarrow 0$$, not sensitive

$$\Delta \rightarrow \infty$$, sensitive

Ellipses are 1σ uncertainty, centered on GPS site locations

1995 M8 Jalisco quake

Surface projection of rupture
down-dip

Mexico

5 cm

Kilometers
Why worry about the dislocation distribution?

Dislocation drives postseismic processes

- Coseismic dislocation
- Poroelastic deformation
- Viscoelastic deformation
- Coulomb Stress

1995 M8 Jalisco quake
Rupture & Deformation

2004 M9 Sumatra-Andaman earthquake & tsunami

Indo-Australian Plate

Eurasian Plate

Dec 26
2004

6 cm/yr

up
down

2004 M9 Sumatra-Andaman quake
Predicting seafloor deformation

Changes overlying water column

Indo-Aust Plate

Eurasian Plate

2004 M9 Sumatra-Andaman quake
Predicting seafloor deformation

Changes overlying water column

Indo-Aust Plate

Eurasian Plate

2004 M9 Sumatra-Andaman quake
Predicting seafloor deformation
Drives tsunami propagation models

India and Sri Lanka
wave propagation
Sumatra & Thailand

earthquake deformation
seafloor

cross-section

2004 M9 Sumatra-Andaman quake
Predicting Seafloor Deformation
The source of the tsunami

typical deformation model

required assumptions:
• homogeneous
• isotropic
• Poisson-solid
• half-space

2004 M9 Sumatra-Andaman quake
Predicting Seafloor Deformation
Preliminary model: 2D FEM

2004 M9 Sumatra-Andaman quake
Predicting Seafloor Deformation

Heterogeneity & displacement

Vertical displacement above sea level

Mantle
Crust
Continental crust
Accretionary wedge
Sediments

Horizontal displacement (landward positive)

Distance along profile (km)

2004 M9 Sumatra-Andaman quake
Coming soon:

3D virtual Sumatra-Andaman subduction zone

Indo-Australian Plate

Eurasian Plate

6 cm/yr

Dec 26 2004

Mar 28 2005

9.2

8.7

2004 M9 Sumatra-Andaman quake
ABAQUS: Comprehensive FEM environment

**Capabilities**

**preprocessor**
- complex geometry
  - axisym., 2D, 3D

**mechanics simulated** (partial list)
- elastic
- poroelastic
- viscoelastic (linear & nonlinear)
- thermoelastic
- plastic

**scripts → inverse models**
- linear GF’s
- automated geometric perturbations & remeshing non-linear inverse methods

**postprocessor**
- slicing
- time-series extractions
- animations
- VRML for geowall export
Thanks to numerous colleagues and organizations for generous financial and professional support.