New Computational Approach to Nonplanar Elastodynamic Ruptures

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1. Abstract

We present a novel approach to modeling dynamic ruptures on nonplanar faults using extended finite element methods (XFEM), for problems of repeated rupture. This method is mesh-free – the fault need not lie on mesh edges. A fundamental challenge for modeling dynamic ruptures on complicated fault networks is the loading term calculated above, and |D| otherwise.

2. Dynamic Rupture

We solve elastodynamic rupture in both Mode III and Mode I, leading under plain stress assumptions in two dimensions.

3. the XFEM

Mesh-free methods such as the partition of unity FEM (PUFEM) provide a mechanism to discretize faults without placing the fault on mesh edges. For complicated geometries, especially in three dimensions, forming a mesh that conforms to all faults is an open problem. Therefore, mesh-free methods cannot define potential for rupture on complex fault systems, especially in three dimensions. For example, compare a standard FEM mesh and a typical XFEM mesh.

In the standard FEM, elements must be chosen to conform to the boundaries. In mesh-free methods, we start from most any mesh, including regular meshes. On the regular mesh, we form basis functions by extending standard finite element spaces with appropriate functions via the extended Finite Element Method (XFEM).

The PUM space is then given by:

\[ P = \sum_{a} P_a = \sum_{a} \left( \mathbf{N}_a(\mathbf{x}) \cdot \mathbf{F}_a(\mathbf{x}) \right) \mathbf{P}_a \]

where \( \mathbf{N}_a(\mathbf{x}) \sim \) a partition of unity (standard basis function) on element and \( \mathbf{F}_a(\mathbf{x}) \) are extension functions that match the geometry across that element.

4. Weak Formulation for Slip Weakening

In the definition of the XFEM, it is important to remember that no concept may be considered strongly (point-wise) on the fault, making split nodes impossible. In the strong form, stick-slip friction enforces different types of boundary conditions on different points of the fault. This is not possible weakly. Instead:

- Each vector-valued degree of freedom \( \psi \) is considered stuck or unstuck.
- Each fault section, \( \Gamma_i(\mathbf{u}) \), is considered unstuck if any \( \psi \) whose support contains that section is unstuck. It is stuck only if all such \( \psi \) are stuck.

Using these concepts, weak conditions for event rupture and termination are derived.

4.1 Initiation of Slip

Given a solution \( \mathbf{u} \) under stick conditions, determine the (weak) tractions \( T_j(\mathbf{u}) \) required to keep degrees of freedom \( \psi \) stuck:

\[ T_j(\mathbf{u}) = \begin{cases} 
\mathbf{T}_j & \text{ where stuck} \\
\mathbf{T} - \frac{1}{\alpha} \mathbf{u} \frac{\partial \mathbf{T}}{\partial \mathbf{u}} & \text{ otherwise} 
\end{cases} \]

where \( \mathbf{T} \) is the loading term calculated above, and \( \mathbf{u} \) is given by a friction that weakens with slip:

\[ u = \left( \frac{\mathbf{T}}{\mathbf{F}} \right) \left( \frac{\mathbf{F}}{\mathbf{F}} \right) \frac{\alpha}{\mathbf{F}} \]

Equations are closed by including boundary conditions on the exterior.

Solute alone is everything, then repeat.

5. Method Validation and Verification

To verify our use of the XFEM, we compare results to previously published solutions generated using finite differences on a single, infinitely long planar geometry.

6. Sample Mode I and II Results

Finite difference solutions

XFEM solutions

Due to the highly chaotic nature of the dynamic system, we do not expect exact simulation replication, but instead compare statistics of event length.

7. Conclusions and Future Work

We also consider problems under Mode II stickslip loading, with a pseudo-third dimension. Interpretation is complicated normal/traction loading, and in this simplified approach is treated without consideration of normal stresses.

Compared to right: the tested geometry; slip on each fault section; moment \( \mathbf{M} \) of events; and distributions of lengths of events as a function of slip, the weakening parameter in friction.

\[ \psi \]

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