Flexible Multi-Physics Solvers for Magma Dynamics

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1 Introduction:

Magma dynamics, the flow of partially molten rock in the Earth’s interior, is an essential feature of global plate tectonics and may be critical for understanding the structure of mantle plumes. Melting and melt transport play a fundamental role in the evolution of the planet, for all of these reasons it is important to understand the dynamics and observable consequences of magma dynamics as the result of crustal interest to geophysical studies.

From a computational standpoint point, magma dynamics relies on a range of multi-physics problems in the solid Earth sciences. In this work, I describe a model for magma dynamics that couples mean-field models of Darcy flow of the fluid with Stokes flow for the solid matrix. More complex formulations etc. can be constructed for more complex problems but keep in mind that the solid matrix remains a key part of these models.

Considerable work has been done to explore and understand models of magma dynamics, and has demonstrated a variety of examples of magma dynamics. In this paper, we explore the effects of filling of inclusions into new magma and new magma instabilities (see history). The model has a single parameter, the dimensionless compaction length, which is a key parameter of interest. The inclusions considered in this work are of particular interest because they can cause significant changes in physical behavior which may require concomitant changes in computational methods (such as the choice of inclusion size). The inclusion size, however, is made to be small and hardwired into these codes making the codes and the simplifications that go with them unwieldy. An alternative approach is to discretize the entire system as a non-linear problem which can be difficult to both compute and solve. Nevertheless, the underlying “near linear” system can be linearized and solved for the entire system. For traditional codes, many of the computational “bets” (choice of constitutive relationships) can cause significant changes in physical behavior which may require concomitant changes in computational methods (such as the choice of inclusion size). The inclusion size, however, is made to be small and hardwired into these codes making the codes and the simplifications that go with them unwieldy. An alternative approach is to discretize the entire system as a non-linear problem which can be difficult to both compute and solve. Nevertheless, the underlying “near linear” system can be linearized and solved for the entire system.

2 PDE’s for Magma Migration

2.1 Formulation

There are several formulations available for magma migration. In 1D, 2D, and 3D models, see [1]. The interplay of melt and solid is important. We have recently established a robust coupling of the Darcy flow and Stokes flow that leads to a more natural and more readily integrable both computational methods for mantle convection. The key is to split the fluid flow into three components: a viscous component, a convective component, and a solid body component.

\[ \frac{\partial \eta}{\partial t} = \nabla \cdot (\nabla P), \]

where \( \eta \) is the effective viscosity of the magma, and \( P \) is the pressure.

2.2 Basic behavior of the magma dynamics equations

Coupled Darcy flow for the fluid and Stokes flow for the solid give rise to a much richer range of behavior compared to the simplifications that arise from considering the solid matrix and fluid separately. For example, the coupling of Darcy flow of the fluid with Stokes flow for the solid matrix is a key component of these models.

3 Magneto-Flow Waves

In the limit of no solid deformation or melting, small porosity \( \alpha \ll 1 \) and the simplification of (2)–(3) is often made in computational models for magma migration. With these simplifications the equation reduces to the standard Stokes equations for the solid.

\[ \nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot (-\mathbf{u} + \kappa \nabla \phi) = 0, \]

where \( \mathbf{u} \) is the velocity field, \( \kappa \) is the permeability, \( \phi \) is the porosity, and \( \nabla \cdot \mathbf{u} = 0 \) is the divergence of the velocity field.

4 Solitary waves exist for fluctuation in melt flux that occur on scales larger than the compaction length. How do the structures interact when large scales of inhomogeneity are present?

5 Structure/Solution strategy for multi-physics problems

Equations (2)–(3) are typical of the system of PDE’s arising in solid earth dynamics, in each of which the interaction of solutions is complicated. The system of equations (2)–(3) is coupled to other systems of equations with similar structure.

6 Computational ingredients for multi-linear multi-physics solvers

Flexible multi-Physics solvers for magma dynamics

A parameterized finite element space with a well-posedness and computational model of the system. The key is to split the fluid flow into three components: a viscous component, a convective component, and a solid body component.

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