CIG Geodynamo Workshop

- Science Goals -

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Science Goals for Numerical Models

1. "Realistic" dynamics of outer core
   - low viscosity liquid metal

2. Link to dynamics of inner core
   - a record of geological history?

3. Complexity of core-mantle interactions
   - transfer of heat, mass, momentum, charge
Buoyancy Sources

Interplay between convection and solidification is probably important.
Computational Challenges

1. Momentum equation \((ma = f)\)

\[
\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} + 2\Omega \times \mathbf{V} = -\nabla P + \frac{\Delta \rho}{\rho} \mathbf{g} + \frac{1}{\mu \rho} \mathbf{B} \cdot \nabla \mathbf{B} + \nu \nabla^2 \mathbf{V}
\]

Coriolis | buoyancy | magnetic | viscous

Character of Flow

\[
\frac{\text{viscous}}{\text{Coriolis}} \approx \frac{\nu}{2\Omega L^2} \approx 10^{-15} \quad \frac{\text{inertia}}{\text{Coriolis}} \approx \frac{V}{2\Omega L} \approx 10^{-7}
\]
Vertical Vorticity

$E = 5 \times 10^{-5}$

from Hiroaki Matsui
Spherical Couette Flow

Azimuthal Flow

Ro = 0.6

(Matsui et al. 2011)
Computational Challenges

2. Induction Equation

\[ \frac{\partial B}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B} \]

\[ \frac{\text{induction}}{\text{diffusion}} \approx \frac{VL}{\eta} \approx 10^3 \]

3. Prandtl Numbers

\[ Pr = \frac{\nu}{\kappa} \approx 0.1 - 100 \]

\[ Pm = \frac{\nu}{\eta} \approx 10^{-6} \]
Vertical Field

$P_m = 0.5$
Scale Separation

Velocity (radial)

Magnetic Field (radial)

\[ E = 3 \times 10^{-6} \quad Pm = 0.1 \]

(Sakuraba and Roberts, 2009)
Small-Scale Flow

Convection can occur at scales as small as 10s of meters.
Just Say No to Eddy Viscosity

Spatial Correlation for $\tau_{zz}$

Reynolds stress $\tau_{ij}$

$$\tau_{ij} = \overline{v_i v_j} - \overline{v_i} \overline{v_j}$$

where

$v_i$ is true velocity

$\overline{v_i}$ is resolved velocity

eddy viscosity

scale similarity
A few remarks about complexity

Persistent non-axial-dipole field is attributed to core-mantle interaction

implementing these interactions should influence model design
Summary

How do we achieve the scientific objections?

1. General vs. specific application?
   - buoyancy and mechanical forcing?

2. One approach or many?
   - same mesh and method for all fields?

3. Will brute force succeed?
Are Numerical Models Realistic?

physical properties
Solid Inner Core

Growth rate ~ 1 mm / year