A one dimensional model of Earth structure in the western United States from GPS observation of ocean tidal load

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- Earth tide
  (body tide and OTL [Ocean Tidal Load])
- Earth tide observation using GPS in Japan
- OTL response using GPS in the western United States
- Method
- Results
Introduction

- The Earth tidal data are limited such as gravity, strain meters etc.
- These instruments are very expensive, and observation coverage is sparse.
- GPS provides relatively good coverage.
Earth tide
Tidal forcing
Body tide

- **Wavelength is about 40,000km.**
  (depend 2 or 3 degrees)
  - same sensitivity of subsurface structure for any tidal constituents.
  - no sensitivity for local variation.

- **High accuracy prediction.**
  (about 3 micron depend on Earth model)
  - well predictable body tide

- **Well known the period of each tidal constitutes.**
  (Better than 0.01 sec)
The cycle of tidal deformation

<table>
<thead>
<tr>
<th>Tidal constitutes</th>
<th>Cycle (hours)</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M2</td>
<td>12.42060</td>
<td>Principal lunar, semidiurnal</td>
</tr>
<tr>
<td>K1</td>
<td>23.93447</td>
<td>Sun-Moon angle, diurnal</td>
</tr>
<tr>
<td>S2</td>
<td>12.00000</td>
<td>Principal solar, semidiurnal</td>
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<tr>
<td>O1</td>
<td>25.81934</td>
<td>Principal lunar declinational</td>
</tr>
<tr>
<td>P1</td>
<td>24.06589</td>
<td>Principal solar declinational</td>
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<tr>
<td>N2</td>
<td>12.65835</td>
<td>Principal lunar elliptic, semidiurnal</td>
</tr>
<tr>
<td>K2</td>
<td>11.96723</td>
<td>Sun-Moon angle, semidiurnal</td>
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<tr>
<td>MF</td>
<td>327.85899</td>
<td></td>
</tr>
<tr>
<td>Q1</td>
<td>26.86836</td>
<td></td>
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<td>MM</td>
<td>661.30919</td>
<td>Lunar evectional constituent</td>
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<tr>
<td>2N2</td>
<td>12.90537</td>
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<td>MTM</td>
<td>219.19039</td>
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<tr>
<td>S1</td>
<td>24.00000</td>
<td>Principal solar, diurnal</td>
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<tr>
<td>M4</td>
<td>6.21030</td>
<td>Principal lunar, quatdiurnal</td>
</tr>
<tr>
<td>MSQM</td>
<td>170.29902</td>
<td></td>
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</table>
Ocean tide

- Period of each constituent are well known (same as body tides).
- Ocean tide can be predicted using global ocean tide models.
The global ocean tides model

\[ Z_j(r') = u_j(r') + iv_j(r') \]
**Ocean Tidal Loading (OTL)**

\[ L_j(r) = \int_{\Omega} \rho_{\text{sea}} G_j(|r - r'|) Z_j(r') \, d\Omega \]

- \( L_j(r) \): OTL response of \( j \)th constituent at \( r \)
- \( \rho_{\text{sea}} \): density of sea water (1,035 kg/m³)
- \( G_j(|r - r'|) \): load Green’s function of \( j \)th constituent
- \( Z_j(r') \): the tidal height variations at \( r' \) (complex number)
Japan

- GPS: 1200 sites
- Sampling rate: 30 sec
- GPS software: Gps Tools [ver. 0.6.3] developed by Takasu and Kasai [2005]

- Secular velocity ➔
Earth tide observation using GPS (1/3)

- **KPPP GPS**
  (30 sec sampling)
- **Tajimi site**
  (center of Japan)
- **Period May, 2006.**
- **Green:** Predicted
  (Body tide + OTL)
- **Red:** GPS

After Ito et al. (2009)
Earth tide observation using GPS (2/3)

- **Spectral Analysis**

![Spectral Analysis Graphs]

After Ito et al. (2009)
Earth tide observation using GPS (3/3)

- Total tidal signal (body + ocean)
- Hourly snapshots

After Ito et al. (2009)
Compare GPS with super-conducting gravimeter

- **S2**: a cycle of daily orbit discontinuities
- **K1**: the orbital repeat period of the GPS satellite

### Table 1

Summary of tidal coefficients at Tajimi GPS station and super-conducting gravimetry at Inuyama. Each second row indicates the estimated error.

<table>
<thead>
<tr>
<th>Comp.</th>
<th>Coef.</th>
<th>Lag</th>
<th>Std.</th>
<th>M2</th>
<th>S2</th>
<th>K1</th>
<th>O1</th>
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<tbody>
<tr>
<td></td>
<td>Amp.</td>
<td>Phase</td>
<td>Amp.</td>
<td>Phase</td>
<td>Amp.</td>
<td>Phase</td>
<td>Amp.</td>
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<tr>
<td>N-S</td>
<td>0.61</td>
<td>146</td>
<td>2.13</td>
<td>1.068</td>
<td>1.44</td>
<td>1.534</td>
<td>–2.91</td>
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<tr>
<td></td>
<td>0.0006</td>
<td>0.023</td>
<td>0.0033</td>
<td>0.070</td>
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<tr>
<td>E-W</td>
<td>0.76</td>
<td>971</td>
<td>2.09</td>
<td>0.984</td>
<td>0.26</td>
<td>0.737</td>
<td>–32.45</td>
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<tr>
<td></td>
<td>0.0006</td>
<td>0.026</td>
<td>0.0003</td>
<td>0.001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>0.94</td>
<td>–363</td>
<td>4.17</td>
<td>1.010</td>
<td>0.19</td>
<td>1.195</td>
<td>–0.91</td>
</tr>
<tr>
<td></td>
<td>0.0001</td>
<td>0.005</td>
<td>0.0005</td>
<td>0.015</td>
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<tr>
<td>Grav.</td>
<td>0.99</td>
<td>4</td>
<td>1.00</td>
<td>1.009</td>
<td>0.10</td>
<td>1.013</td>
<td>0.31</td>
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<tr>
<td></td>
<td>0.0002</td>
<td>0.002</td>
<td>0.0007</td>
<td>0.005</td>
<td></td>
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</tr>
</tbody>
</table>

After Ito et al. (2009)
The spatial perturbation of the Earth tidal field

- Variation relate to subsurface structure
- The average phase lag is 0.11 degrees of angle.
- The average amplitude ratio is 1.007
- Average structure is fine, but regionally coherent variations are apparent

After Ito et al. (2009)
Conclusion (Part 1)

- We generated a high resolution map of the regional Earth tides response using KPPP GPS observations of the Japanese islands.
- Comparisons of the KPPP GPS results with SG observations confirmed the validity of the KPPP GPS analyses.
- The M2 tidal constituent derived from the vertical component of the displacement is the most robust and accurate observation.
Observation map

- 702 GPS observation (from PBO)
- Observation periods: 1/1/2008-12/31/2008
- Sampling rate: 5 minutes
- GPS software: Gps Tools [ver. 0.6.4]
- Ocean tidal load (OTL)
Observation = **Body tide** + OTL disp. + err

**Body tide** is theoretical estimate. Use complex (attenuation) Love numbers with 500 tidal constituents and frequency dependence.
Observation = **Body tide** + OTL disp. + err

**Body tide** is theoretical estimate.

Use complex (attenuation) Love numbers with 500 tidal constituents and frequency dependence

Red: Predicted body tide
Black: GPS observation
Body tide on a 3-D elastic earth

- Sensitivity of earth elastic structure to body tides is too small for detection by GPS.
- Although it may be possible to estimate the Earth inner structure.

After Latychev et al. (2009)

The maximum of the absolute value of the perturbation in the radial displacement body tide response computed using a 3-D models and PREM.
Observation - Body tide = OTL disp. + err

OTL disp. is estimated from observation.

Applied Sidereal filter (remove multipath).

\[
O(t) = \sum_{j=1}^{15} \{ A_j^{OBS} \cos(\psi_j t) + B_j^{OBS} \sin(\psi_j t) \} + \sum_{n=3}^{\infty} \{ C_n \cos(n\psi_{sidereal} t) + D_n \sin(n\psi_{sidereal} t) \} + \epsilon(t)
\]

Green: GPS observation – Predicted Body tide
Blue: Estimated OTL response
Spatial distribution of the OTL response

- 3 components of M2 OTL
Observation error

- Estimated error [STD]
  - North: 1.1 cm
  - East: 1.3 cm
  - Vertical: 2.7 cm
M2 North-South

Constituent: M2
Component: North-South
Variance Reduction: 93.4%
Total Power: 2.52 m

Model: FES2004+PREM
O1 Vertical

Constituent: O1
Component: Vertical
Variance Reduction: 93.4%
Total Power: 7.41 m

Model: FES2004+PREM
K1 East-West

Constituent: K1
Component: East-West
Variance Reduction: 50.6%
Total Power: 3.10m

Model: FES2004+PREM
# Variance reduction

<table>
<thead>
<tr>
<th>Wave</th>
<th>Period</th>
<th>North VR[%]</th>
<th>North Amp.[m]</th>
<th>East VR[%]</th>
<th>East Amp.[m]</th>
<th>Vertical VR[%]</th>
<th>Vertical Amp.[m]</th>
<th>Total VR[%]</th>
<th>Total Amp.[m]</th>
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<tbody>
<tr>
<td>M2</td>
<td>12.42</td>
<td>93.4 (2.52)</td>
<td>74.7 (1.30)</td>
<td>61.6 (3.72)</td>
<td>74.5 (7.54)</td>
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<tr>
<td>K1</td>
<td>23.93</td>
<td>61.0 (1.62)</td>
<td>50.6 (3.10)</td>
<td>83.1 (11.67)</td>
<td>74.8 (16.39)</td>
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<tr>
<td>S2</td>
<td>12.00</td>
<td>76.8 (1.09)</td>
<td>38.6 (0.56)</td>
<td>59.7 (2.49)</td>
<td>61.3 (4.14)</td>
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<td>O1</td>
<td>25.82</td>
<td>73.4 (1.03)</td>
<td>89.8 (2.01)</td>
<td>93.4 (7.41)</td>
<td>90.7 (10.45)</td>
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<tr>
<td>P1</td>
<td>24.07</td>
<td>48.9 (0.53)</td>
<td>60.9 (1.02)</td>
<td>74.0 (3.85)</td>
<td>69.1 (5.40)</td>
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<tr>
<td>N2</td>
<td>12.66</td>
<td>81.2 (0.57)</td>
<td>43.9 (0.22)</td>
<td>44.5 (0.62)</td>
<td>59.2 (1.41)</td>
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<tr>
<td>K2</td>
<td>11.97</td>
<td>0.3 (0.02)</td>
<td>0.3 (0.02)</td>
<td>2.3 (0.15)</td>
<td>1.9 (0.19)</td>
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<tr>
<td>Mf</td>
<td>327.86</td>
<td>18.3 (0.05)</td>
<td>31.3 (0.11)</td>
<td>32.4 (0.21)</td>
<td>30.1 (0.37)</td>
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<tr>
<td>Q1</td>
<td>26.87</td>
<td>57.1 (0.20)</td>
<td>67.3 (0.37)</td>
<td>76.1 (1.35)</td>
<td>72.4 (1.92)</td>
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<tr>
<td>Mm</td>
<td>661.31</td>
<td>7.2 (0.02)</td>
<td>9.9 (0.04)</td>
<td>13.5 (0.10)</td>
<td>11.8 (0.16)</td>
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<tr>
<td>2N2</td>
<td>12.91</td>
<td>43.3 (0.08)</td>
<td>11.7 (0.03)</td>
<td>21.9 (0.13)</td>
<td>27.8 (0.24)</td>
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<td>Mtm</td>
<td>219.19</td>
<td>3.7 (0.01)</td>
<td>8.0 (0.03)</td>
<td>9.0 (0.05)</td>
<td>8.1 (0.09)</td>
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</tr>
<tr>
<td>S1</td>
<td>24.00</td>
<td>4.7 (0.04)</td>
<td>4.1 (0.06)</td>
<td>6.2 (0.16)</td>
<td>5.5 (0.26)</td>
<td></td>
<td></td>
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<tr>
<td>M4</td>
<td>6.21</td>
<td>2.2 (&lt;0.01)</td>
<td>1.6 (&lt;0.01)</td>
<td>3.5 (&lt;0.01)</td>
<td>2.4 (&lt;0.01)</td>
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<td>Msqm</td>
<td>170.30</td>
<td>0.6 (&lt;0.01)</td>
<td>1.1 (&lt;0.01)</td>
<td>1.2 (&lt;0.01)</td>
<td>1.0 (&lt;0.01)</td>
<td></td>
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</table>
Using OTL to estimate elastic structure: Inversion method (1/2)

- Non-linear observation equation
  \[ d = g(m) \]

- Unknown parameters are elastic moduli and density vs. depth.
  \[ m = \{ \log \frac{\lambda}{\lambda_0}, \log \frac{\mu}{\mu_0}, \log \frac{\rho}{\rho_0} \} \]

- Inverse linearized approach based on Levenberg-Marquardt algorithm

  Total unknown parameters: 249
Inversion method (2/2)

- Constrain
  Total mass of the Earth
dynamical moment of inertia of the Earth \([I/\text{MR}^2]\)

- Smoothing hyper-parameter

\[
\text{ABC}(\sigma^2, \alpha^2) = -2 \log[p(d|\sigma^2, \alpha^2)] + 2N_p
\]

\[
p(d|\sigma^2, \alpha^2) = \int_{-\infty}^{+\infty} p(d|m^*, \sigma^2)p(m^*|\alpha^2)dm^*
\]

\[
= (2\pi\sigma^2)^{-\frac{N}{2}}(\alpha^2)^{-\frac{M}{2}}|\Sigma_d|^{-\frac{1}{2}}|C^TC|^{\frac{1}{2}}
\]

\[
\frac{1}{\sigma^2}g(m^*)^T\Sigma_d^{-1}g(m^*) + \frac{1}{\alpha^2}C^TC|^{-\frac{1}{2}}\exp\left[-\frac{1}{2}\Psi(m^*)\right]
\]
Fine mesh

- 3 different resolutions
  - 1\textsuperscript{st} order grid [0.5° by 0.5° ]
  - 2\textsuperscript{nd} order grid [5’by 5’]
  - 3\textsuperscript{rd} order grid [1’ by 1’]

3\textsuperscript{rd} \leq [5^\circ] \leq 2\textsuperscript{nd} \leq [30^\circ] \leq 1\textsuperscript{st}
Load Green function

- **Q effects**
  
  \[
  \begin{align*}
  \mu(T) &= \mu(1) \left(1 - \frac{\ln T}{\pi} \frac{1}{Q_\mu}\right) \\
  \lambda(T) &= \lambda(1) \left[1 - \frac{\ln T}{\pi} \left(\frac{1+\gamma}{Q_\kappa} - \frac{\gamma}{Q_\mu}\right)\right]
  
  \text{where } \gamma = \frac{2}{3} \left(\frac{\mu}{\lambda}\right)
  \end{align*}
  \]

- **SNREI Earth model**
  
  the spherically symmetrical, non-rotating, perfectly elastic and isotropic Earth model

![Diagram showing frequency response and internal friction vs. frequency.](image)
Sensitivity Kernel of Load Green function

\[ K(R, S) = \frac{G(R, S + \delta S)}{G(R, S)} \]
Compare global ocean tides models

<table>
<thead>
<tr>
<th></th>
<th>NAO.99b</th>
<th>GOT99.2b</th>
<th>CSR4.0</th>
<th>Schwiderski</th>
<th>FES2004</th>
</tr>
</thead>
<tbody>
<tr>
<td>VR</td>
<td>74.01</td>
<td>74.19</td>
<td>74.23</td>
<td>68.79</td>
<td>75.48</td>
</tr>
</tbody>
</table>

NAO.99b [assimilation model]: Matsumoto et al. [2000]
GOT99.2b [empirical model]: Ray [1999]
CSR4.0 [empirical model]: Eanes and Bettadpur [1994]
Schwiderski [a purely hydrodynamical model]: Schwiderski [1980]
FES2004 [assimilation model]: Lyard et al. [2006].
1-D model

- Our 1-D model: VR: 77.13%
- PREM: VR: 75.43%

Red: our 1-D model
Blue: PREM
Green of VP: T7 (Burdick and Helmberger [1978])
Green of VS: TNA (Grand and Helmberger [1984]).
1-D model

Red: our 1-D model
Blue: PREM
Green of VP: T7 (Burdick and Helmberger [1978])
Green of VS: TNA (Grand and Helmberger [1984]).

Our 1-D model
VR: 77.13%
PREM
VR: 75.43%
Histogram of residual (1/2)

- PREM
  Mean: 0.66 mm
  STD: 1.60 mm

- Our 1-D model
  Mean: 0.42 mm
  STD: 1.53 mm

Red: our 1-D model
Blue: PREM
Histogram of residual (2/2)
Spatial distribution of residual (1/2)

- Residual from final estimated 1D model
- Spatial coherency suggests need for 3D model in the future.
Spatial distribution of residual (2/2)

North | East | Vertical

P1

N2

Q1

Unit: cm
Known Issues:

- Use of mass and moment of inertia constraint may not be appropriate for a regional study.
- Currently using a gradient search technique, and we are concerned about local minima and correlations between moduli and density.
- Coherent spatial residual suggest a need for 3D variations.
Conclusion (Part 2)

- We generate a large collection of OTL response data (M2, K1, S2, O1, P1, N2 and Q1) from GPS data.
- We have demonstrated that OTL response data from GPS can constrain independent regional variation in $\lambda$, $\mu$, and $\rho$.
- Our 1-D model are consistent with existing global travel-time models.
- The main differences are in the upper mantle where we infer a low-velocity zone from our 1-D model.