Modeling the geodynamo: Successes and challenges

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Successes

• Earth-like numerical dynamos
• Agreement between laboratory and numerical flows
• Polarity reversals
  – Earth-like time-series
• Angular coupling between inner - outer core & mantle
• Scaling toward realistic core properties
• Using observable field to invert for core flow

Challenges

• Understanding the role of turbulence
• Scaling the viscous vs. electromagnetic dissipation (want low $E_k$ & $P_m$)
• Laboratory dynamo
• Heat flow in the core
  – What fraction of total Earth heat flow comes from inner, outer core
  – How old is the inner core?
• Interplanetary comparisons
## Some known planetary dynamos

<table>
<thead>
<tr>
<th>Planet / satellite</th>
<th>Surface field (in T, approx.)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>$2 \times 10^{-7}$</td>
<td>Likely dynamo. Thin shell? MESSENGER, BP</td>
</tr>
<tr>
<td>Venus</td>
<td>$&lt;10^{-8}$</td>
<td>No dynamo. Small remanence</td>
</tr>
<tr>
<td><strong>Earth</strong></td>
<td>$5 \times 10^{-5}$</td>
<td>Core dynamo, dipolar, reversals</td>
</tr>
<tr>
<td>Moon</td>
<td>$10^{-9} - 10^{-7}$ (patchy)</td>
<td>Impact-origin? Ancient dynamo?</td>
</tr>
<tr>
<td>Mars</td>
<td>$10^{-9} - 10^{-4}$ (patchy)</td>
<td>Ancient dynamo, magnetic stripes</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$4.2 \times 10^{-4}$</td>
<td>Dynamo (extends to near surface)</td>
</tr>
<tr>
<td>Io</td>
<td>$&lt; 10^{-6}$ ?</td>
<td>Complex (Embedded in Jovian field)</td>
</tr>
<tr>
<td>Europa</td>
<td>$10^{-7}$</td>
<td>Induction response (salty ocean)</td>
</tr>
<tr>
<td>Ganymede</td>
<td>$2 \times 10^{-6}$</td>
<td>Dynamo likely</td>
</tr>
<tr>
<td>Callisto</td>
<td>$4 \times 10^{-9}$</td>
<td>Induction response (salty ocean)</td>
</tr>
<tr>
<td>Saturn</td>
<td>$2 \times 10^{-5}$</td>
<td>Dynamo (Axial symmetry, deep down)</td>
</tr>
<tr>
<td>Titan</td>
<td>$&lt; 10^{-7}$</td>
<td>Need more data</td>
</tr>
<tr>
<td>Uranus</td>
<td>$2 \times 10^{-5}$</td>
<td>Dynamo (Quad/Octupolar, uncertain depth)</td>
</tr>
<tr>
<td>Neptune</td>
<td>$2 \times 10^{-5}$</td>
<td>Dynamo (Quad/Octupolar, uncertain depth)</td>
</tr>
</tbody>
</table>

Based upon D. Stevenson, EPSL, 2003
Governing Equations
(Boussinesq approximation)

Navier-Stokes Equation
\[
\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \nabla \mathbf{U} = -\nabla P - 2\hat{\mathbf{z}} \times \mathbf{U} + Ra^* \frac{r}{r_0} \mathbf{T} + (\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{E} \, \nabla^2 \mathbf{U}
\]

Buoyancy Equation
\[
\frac{\partial T}{\partial t} = \mathbf{U} \cdot \nabla T + \frac{E}{Pr} \, \nabla^2 T + q
\]

Dynamo Equation
\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}) + \frac{E}{Pm} \, \nabla^2 \mathbf{B}
\]

Continuity Equations
\[
\nabla \cdot \mathbf{U} = 0 \quad \nabla \cdot \mathbf{B} = 0
\]
Thermal Conditions

- **Temperature boundary conditions**
  - Constant Temperature inner & outer
    - Plume formation dominantly at inner boundary (ICB)
    - Spatial temperature heterogeneity
  - Constant flux or mixed conditions
    - Longer thermal equilibration time

- **Internal heat source**
  - Secular cooling or active heating by radiogenic decay (K?)
  - Enhanced plume formation at outer boundary (CMB)

- **Internal heat sink**
  - Proxy for compositional convection
  - Enhanced plume formation at inner boundary
Velocity & electromagnetic boundary conditions

• No-slip velocity boundary condition
  – Physical
  – Inhibits zonal flow

• Free-slip velocity boundary
  – Longer spin-up time
  – Promotes zonal flow
    • More “realistic”? (compensates for low E)

• Electrical insulator outside working fluid
  – Potential field outside \( j=0, B_{tor} = 0 \)

• Semiconducting mantle, conducting inner core
  – Continuous magnetic field across boundary
  – Continuous radial current
## Dimensionless numbers

<table>
<thead>
<tr>
<th>Number</th>
<th>Force balance</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ekman number</td>
<td>viscosity/Coriolis</td>
<td>$E = \frac{\nu}{\Omega d^2}$</td>
</tr>
<tr>
<td>Rayleigh number</td>
<td>buoyancy/diffusion</td>
<td>$Ra = \frac{\alpha g_0 \Delta T d^3}{\kappa \nu}$</td>
</tr>
<tr>
<td>Magnetic Prandtl number</td>
<td>diffusivity ratio</td>
<td>$Pm = \frac{\nu}{\eta}$</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>diffusivity ratio</td>
<td>$Pr = \frac{\nu}{\kappa}$</td>
</tr>
<tr>
<td>Modified Rayleigh number</td>
<td>buoyancy/Coriolis</td>
<td>$Ra^* = \frac{\alpha g_0 \Delta T}{\Omega^2 d}$</td>
</tr>
<tr>
<td>(convective Rossby)</td>
<td></td>
<td>$= Ra E^2/Pr$</td>
</tr>
<tr>
<td><strong>Output</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reynolds number</td>
<td>advection/viscosity</td>
<td>$Re = \frac{u D}{\nu}$</td>
</tr>
<tr>
<td>Magnetic Reynolds number</td>
<td>advection/mag diff.</td>
<td>$Rm = \frac{u D}{\eta}$</td>
</tr>
<tr>
<td>Rossby number</td>
<td>inertia/Coriolis</td>
<td>$Ro = \frac{u}{\Omega d}$</td>
</tr>
<tr>
<td>Elsasser number</td>
<td>Lorentz/Coriolis</td>
<td>$\Lambda = \frac{B^2}{\rho \mu \eta \Omega}$</td>
</tr>
</tbody>
</table>
Core and numerical models: nondimensional numbers

<table>
<thead>
<tr>
<th>Input</th>
<th>Control Parameters</th>
<th>Characteristic numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Ra^*$</td>
<td>$E$</td>
</tr>
<tr>
<td>Core</td>
<td>$\sim 10^{-7}$</td>
<td>$10^{-15} - 10^{-14}$</td>
</tr>
<tr>
<td>Simple Models</td>
<td>0.01 - 0.3</td>
<td>$&gt; 10^{-4}$</td>
</tr>
<tr>
<td>Advanced Models</td>
<td>$10^{-4} - 1$</td>
<td>$10^{-6} - 10^{-4}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Output</th>
<th>$Rm$</th>
<th>$Re$</th>
<th>$Ro$</th>
<th>$\Lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core</td>
<td>$10^2 - 10^3$</td>
<td>$10^8 - 10^9$</td>
<td>$\approx 10^{-7}$</td>
<td>0.1-10</td>
</tr>
<tr>
<td>Simple Models</td>
<td>40-100</td>
<td>$&lt; 30$</td>
<td>$10^{-2} - 10^{-1}$</td>
<td>0.3-3</td>
</tr>
<tr>
<td>Advanced Models</td>
<td>$10^2 - 10^3$</td>
<td>$&lt; 500$</td>
<td>$3 \times 10^{-4} - 10^{-2}$</td>
<td>0.1-100</td>
</tr>
</tbody>
</table>

Based on Table from Christensen & Wicht, submitted
Numerical Solution

- Poloidal-toroidal decomposition of B and U (Boussinesq approx) enforces non-divergence & decreases 3-vector fields to 2 scalar fields.

- Pseudo spectral methods require transformation between grid and spectral domains
  - Easy to implement correct electromagnetic BC
  - Highly accurate
  - Problems with classical lat-lon spherical grid in polar regions
  - Difficult to develop parallel code for distributed memory clusters
    - (G. Glatzmaier has MPI, other groups working on it)

- Local methods (FEM, FV, FD …)
  - Good grid generation
  - Good parallelization
  - Not as accurate
  - Problems with E-M BC
Numerical dynamo codes

- Pseudo spectral: Glatzmaier (anelastic), Christensen & Olson, Wicht, Aubert, Zhang, Busse, Jones, Kuang & Bloxham,
- Local methods (FEM/FV/FD/SE …): Matsui, Yoshida Harder & Hansen, Hejda & Reshetnyak.
- Others: McMillan & Sarson (Pencil Code), Schaefer & Cardin (Geostrophic approximation)
Numerical dynamo driven by columnar convection

Dynamo: $E = 10^{-4}$, $Pm = 1$, $Ra = 10Ra_c$

Rotating convection, cylindrical “gravity”

Cardin and Olson (1992)

Side views
Turntable experiment & Numerical dynamo

Image: J. Aurnou, E. Spangler

Dynamo: $E = 10^{-4}$, $Pm = 1$, $Ra = 10Ra_c$

Polar Views
Recent Earth-like dynamo model

Radial B

Radial V

Sreenivasan & Jones, GJI, 2006
Comparison of Earth magnetic field at CMB with dynamo model

From Christensen, Olson and Glatzmaier GRL, 1998
Polarity reversal

Glatzmaier & Roberts, 1995, 1996, self consistent numerical dynamo, including a polarity reversal
Numerical model reversal time-series

From Johannes Wicht
Lots of dynamos
Much progress

Modified from
Christensen and Aubert, 2006
Where the field is going:
Low Pm, low E dynamos in 3D
shown here is “2.5D” geostrophic approximation

Schaeffer & Cardin, EPSL, 2006
Overdriven dynamo:
$Pm=0.04, E=3 \times 10^{-5}, Ra=3.3 \times 10^9 (Ra^*=3)$
Geodynamo modeling successes

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- Agreement between laboratory and numerical flows
- Polarity reversals
  - Earth-like time-series
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• Scaling the viscous vs. electromagnetic dissipation (want low Ek & Pm)
• Laboratory dynamo
• Heat flow in the core
  – What fraction of total Earth heat flow comes from inner, outer core?
  – How old is the inner core?
• Understanding the role of turbulence
  – Interaction with rotation
  – Large scales, Inverse cascade
• Interplanetary comparisons and beyond
  – Extra-Solar-system planets
  – Other magnetic bodies in the Universe (stars, pulsars …)
Computational wish list

• Fast Legendre Transform
  – Optimized for specific machines
  – Library function?
• MPI for spectral codes
  – Compilation stage?
• Optimal grid
• Better subgrid models (turbulence parameterization beyond hyperdiffusion)
• Advanced tools
  – Visualization
  – Data assimilation
Real time visualization: System architecture

Dynamo Program
N processors

Shared memory
Solution 1
Solution 2
Solution 3
...
Solution m

Solution server
Sim params

Solution formatter

Visualisation Workstation

Stored solution

data com

TCP/IP

Initiate Simulation
Control commands
Server status
Solution Parameters

VTK Agent

Fast Connection
Typical Display on the Client Program
U of A AccessGrid Room
VizRoom Cave