Problems in solid Earth deformation: crust and upper mantle

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Overview:

• Data-driven models
• Quest for “realistic” constitutive relationships for lithosphere, including upper (brittle) crust, lower crust, and upper mantle
• Data require material heterogeneity, non-linear rheologies, localization
  – models need to be sufficiently flexible to resolve multiple spatial and temporal scales
  – sufficient flexibility/efficiency in generating many realizations for inverse modeling
• Are FEM an ultimate answer?
\( \varepsilon(\sigma) \)?
\[ \varepsilon(t) = F(\sigma, E, t, D, \ldots) \]
Calico fault

seismic tomography
Time-dependent deformation following earthquakes: Common suspects

• Localized slip on or below the seismic rupture ("afterslip")
• Visco-elastic relaxation (lower crust/upper mantle; various stress-strain relationships)
• Poro-elastic rebound (incapable of large horizontal displacements; mostly vertical deformation)
• ...or a combination of the above
\[ \sigma_{ij} = 2G\epsilon_{ij} + \left( K - \frac{2G}{3} \right) \delta_{ij}\epsilon_{kk} - \alpha\delta_{ij} p \]

\[ \frac{k}{\alpha\eta_f} \nabla^2 p = \frac{\partial}{\partial t} \left( \epsilon_{kk} + \frac{\alpha}{K_u - K} p \right) \]
Landers rupture
Stack of ERS-1/2 data
1992-1999
39 interferograms
Post-seismic deformation due to the M7.3 Landers earthquake
"Thin viscous sheet vs Fault-block"

"Jelly Sandwich vs Crème Brule"

Savage and Burford, 1970
Thatcher, 1983
Elsasser, 1969
Savage and Prescott, 1978
$V = 4 \text{ cm/yr}$

t = 200 yrs

$D = 8 \text{ m}$
\varepsilon = A \sigma^{3.5}
Thermo-mechanical coupling

\[ \dot{\varepsilon} = C \sigma^n \exp \left( - \frac{Q}{RT} \right) \]

\[ T \propto \dot{\varepsilon} \sigma \]

Yuen et al., 1978; Fleitout and Frodivaux, 1980; Turcotte and Schubert, 2002
Post-Landers CGPS data

- Rapid initial transient followed by a more gradual decay

- Difficult to fit assuming exponential dependence (not consistent with linear Maxwell viscoelastic behavior)

- Possible explanations:
  
  Bi- (or multi-) viscous rheology
  
  Power-law rheology
  
  Rate-and-state friction (or some other form of non-linear localized creep)
Equivalence between dislocations and body force couples (point-source solution)

Potency Tensor (Eigenstrain)
\[ \varepsilon^i(x,y) = \frac{1}{2}(\hat{n} \otimes s + s \otimes \hat{n}) \delta(x - y) \]

Moment Density Tensor
\[ m(x,y) = C : \varepsilon^i = C : s \otimes \hat{n} \delta(x - y) \]

Equivalent Body Forces
\[ f(x,y) = -\nabla \cdot m = -\nabla \cdot (C : \varepsilon^i) \]

Equivalent Body Forces Are Linear Combination of 6 Double Couples
Finite Fault Source

Potency Tensor (Eigenstrain)

$$\epsilon^i(x) = \int_\Sigma \epsilon^i(x,y) \, dy$$

Moment Density Tensor

$$m(x) = \int_\Sigma C : \epsilon^i(x,y) \, dy$$

Example for Uniform Rectangular Fault

$$\epsilon^i(x) = \frac{1}{2} (\hat{e}_1 \otimes \hat{e}_2 + \hat{e}_2 \otimes \hat{e}_1) \, \Pi \left( \frac{x_1 - y_1}{L} \right) \delta(x_2 - y_2) \, \Pi \left( \frac{x_3 - y_3}{W} \right)$$

$$f(x) = -2\mu \, \nabla \cdot \epsilon^i = -\mu \left[ \Pi \left( \frac{x_1 - y_1}{L} \right) \frac{\partial}{\partial x_2} \delta(x_2 - y_2) \, \Pi \left( \frac{x_3 - y_3}{W} \right) \right]$$

Eigenstrain characterizes:
- slip system (tensor part)
- location
- dimension

analytic expression for equivalent body forces allows:
- numerical sampling & processing
- analytic Fourier transform
- continuum representation of a discontinuous field
Greens’ Function in Fourier Domain

Navier’s Equation in Space Domain

\[ \nabla \cdot (C : \nabla \otimes u) + f = 0 \]

or, for isotropic elasticity

\[ (\lambda + \mu) \nabla \cdot u + \mu \nabla^2 u + f = 0 \]

solution is

\[ u = \int_\Omega G(x,x_0) \cdot f(x_0) \, dV \]

Navier’s Equation in Fourier Domain

\[ k \cdot (C : k \otimes u) = f/4\pi \]

or

\[ (\lambda + \mu) k \otimes k \cdot u + \mu k \cdot k \otimes u = f/4\pi \]

or simply, with full space elastic Greens’ function

\[ G^{-1}(k;\lambda,\mu) \cdot u(k) = f(k) \]

Boussinesq’s & Cerruti’s Problems: Elastic Deformation for Surface Traction

\[ t_3 (k,x_3 = 0) \]
\[ t_1 (k,x_3 = 0) \]
\[ t_2 (k,x_3 = 0) \]

arbitrary distribution of surface traction.

\[ x_1 \]
\[ x_2 \]
\[ x_3 \]

\[ u = \begin{pmatrix} -2B_1\beta^2 + \alpha \omega_1 (B_1 \omega_1 + B_2 \omega_2)(1 + \beta x_3) + \alpha i \omega_1 \beta B_3 (1 - \alpha^{-1} + \beta x_3) \\ -2B_2\beta^2 + \alpha \omega_2 (B_1 \omega_1 + B_2 \omega_2)(1 + \beta x_3) + \alpha i \omega_2 \beta B_3 (1 - \alpha^{-1} + \beta x_3) \\ \alpha \beta^2 \left( i (B_1 \omega_1 + B_2 \omega_2) x_3 - B_3 (\alpha^{-1} + \beta x_3) \right) \end{pmatrix} e^{-\beta x_3} \]

Use Boussinesq and Cerruti’s solution to remove stress at the surface.
Benchmark

case of strike-slip fault, comparison with Okada [1992] and Wang [2002]:
• less than 5% error wrt Okada
• comparable with Wang
• larger error in the near field (due to discontinuity approximation)

Numerical code implements strike-slip and dip-slip fault and opening (or closing) cracks of arbitrary orientation.
Examples

The Fourier domain method is an attractive alternative to FEM in a number of applications:
- 3-D static deformation
- nonlinear 3-D viscoelasticity
- rate-and-state fault creep
- poroelasticity
- ...

A number of mathematical issues that arise from this formulation (optimal choice of an initial “homogenized” model, convergence, existence, stability analysis, errors, etc)
User Interface Example

./static <<EOF
# grid dimension (sx1,sx2,sx3)
128 128 128
# sampling (dx1,dx2,dx3), beta
0.05 0.05 0.05 0.3
# origin position (x0,y0)
0 0
# observation plane depth
0
# output directory
output
# elastic moduli (lambda,mu)
1 1
# observation points
1
1 GPS1 0.5 0.1 0
# shear dislocations
1
# index slip   x1 x2 x3 length width strike dip rake
  1  1 -0.5  0  0      1     1      0  90    0
# tensile cracks
0
EOF

simple interface produces output in:
• prescribed points (GPS)
• map view txt file (x1,x2,u1,u2,u3)
• Generic Mapping Tools (GMT)

code is implemented in Fortran90
and uses DFT fourt

above example runs in 10s on a low-end laptop