Mantle Convection: Current Status and Future Challenges

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Outline

1) Introduction.

- *What is mantle convection and why it is interesting/important.*

2) Current Status.

- *Numerical methods & codes.*
- *Example problems.*

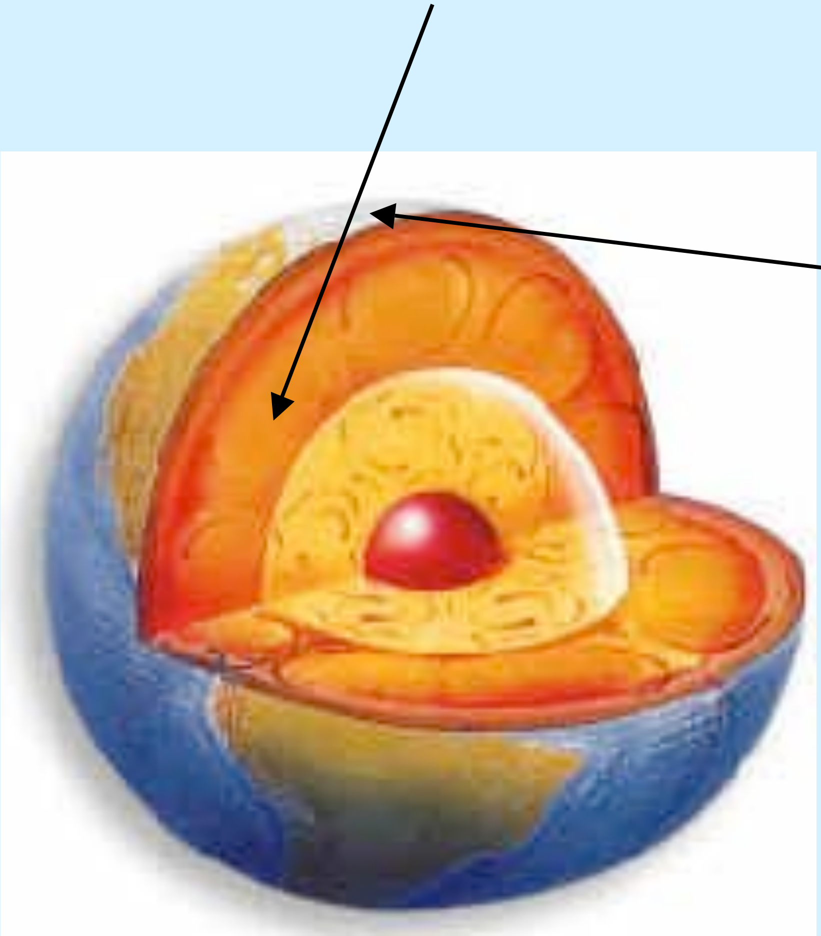
3) Unique challenges.

- *Multi-scale physics and non-linear and highly variable rheology.*
- *Ongoing and future developments*

What is mantle convection?

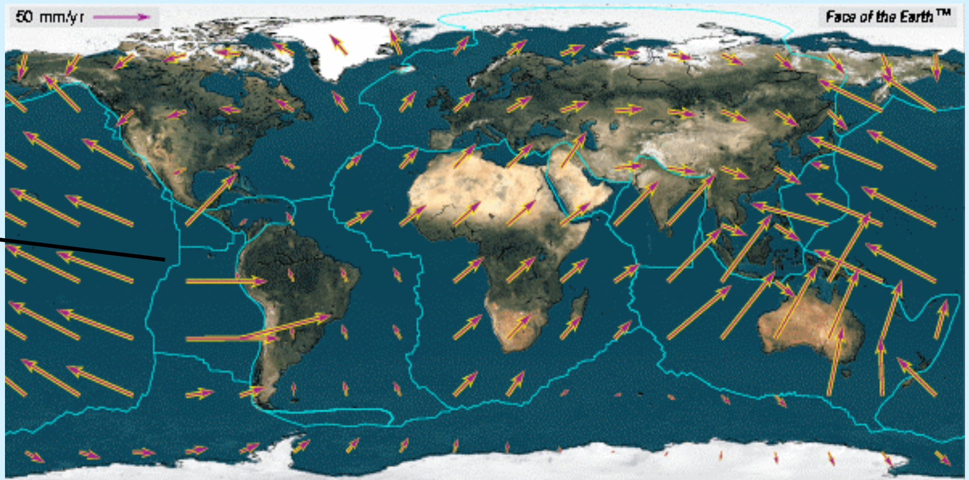
- **<u>Objectives:</u> Understand the long-term heat and mass *transfer* and structure *formation* in the mantle and their *implications* for surface geological, geochemical and geophysical observables and for evolution of the Earth and other planets (terrestrial planets and icy satellites.**
- **<u>Physical basis:</u> Treat the mantle as viscous flow. Conservation of *MASS*, *ENERGY*, and *MOMENTUM* + *VISCOUS* rheological equation.**

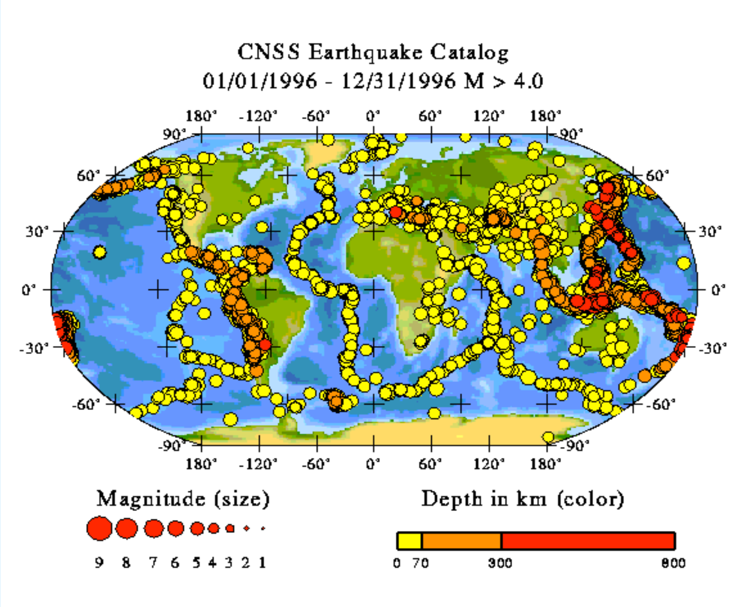
The Mantle – The Earth's "Engine"



To remove radiogenic heating & primordial heating

Present-day surface plate motion





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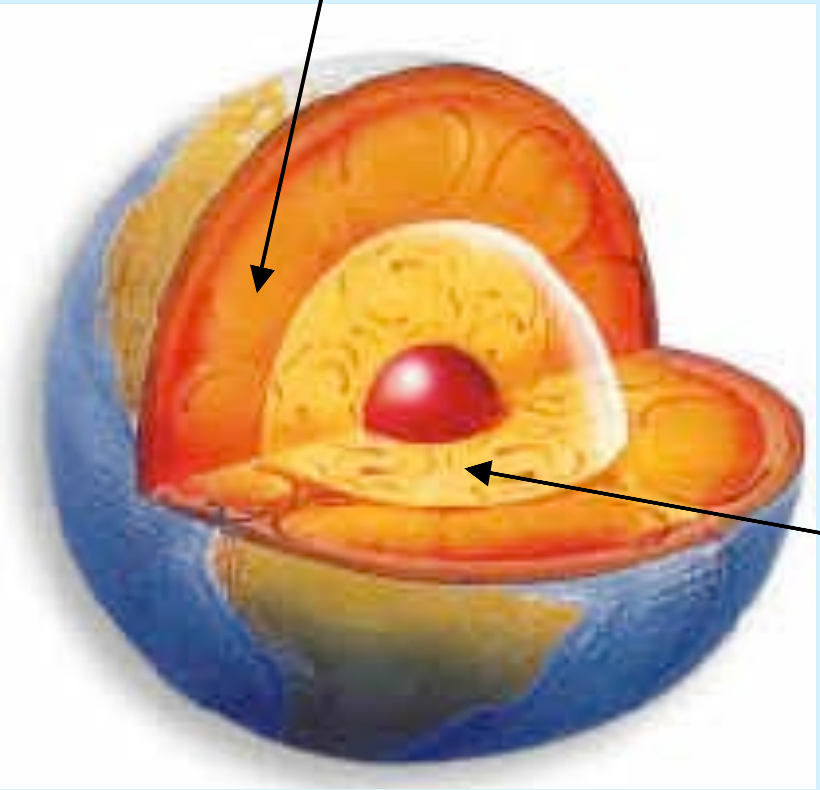
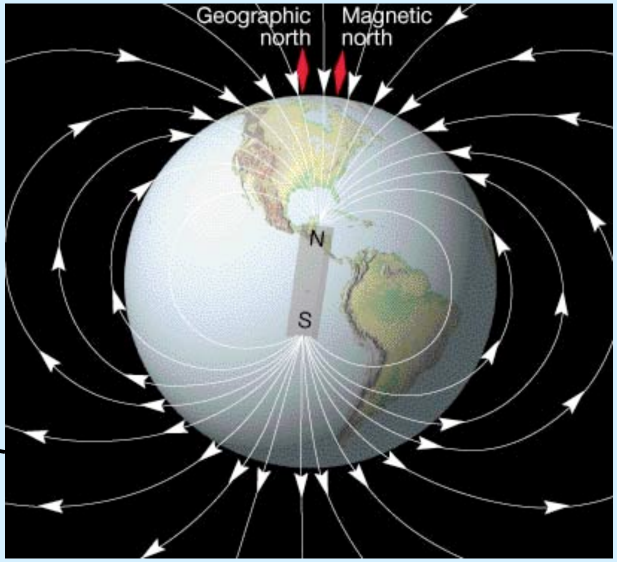
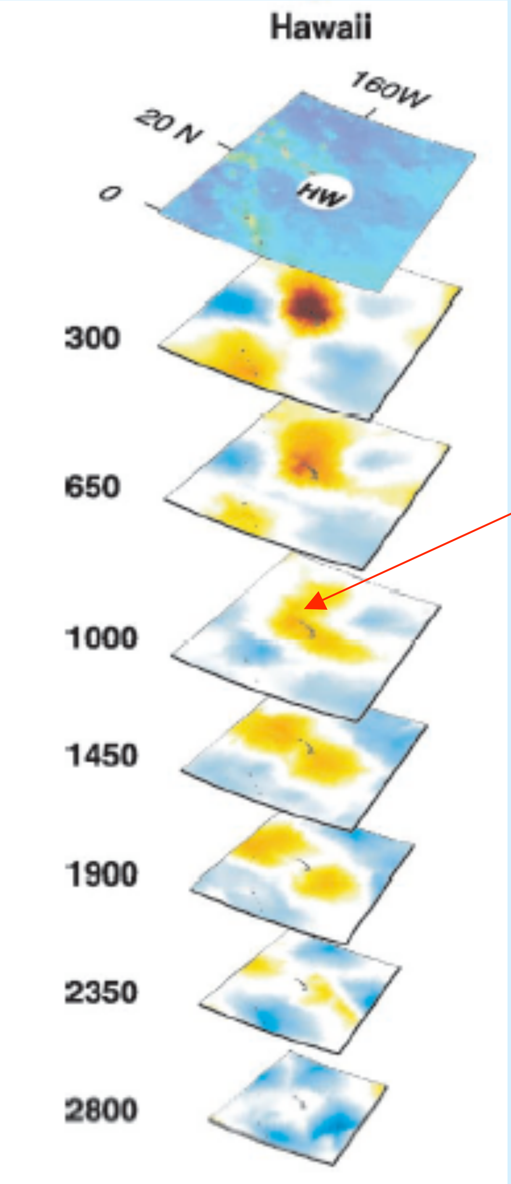
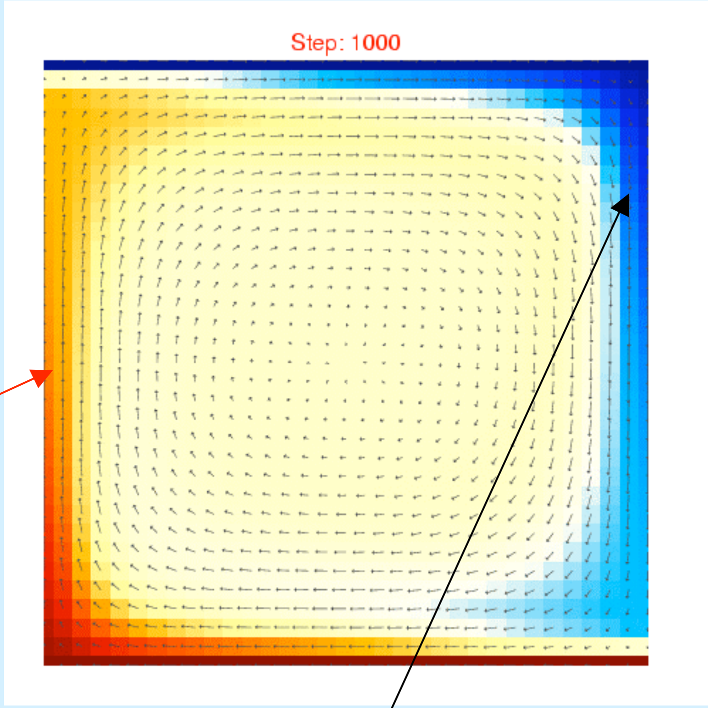


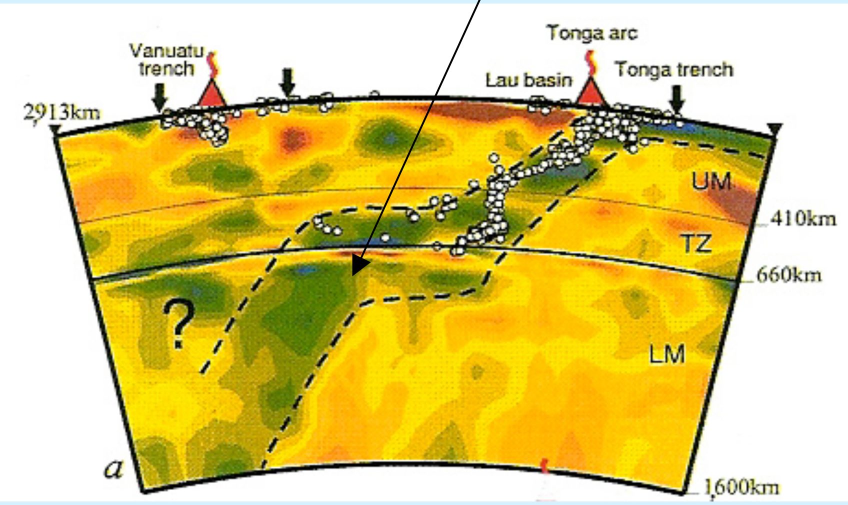


A Simple Picture of Mantle Convection: Boundary Layers



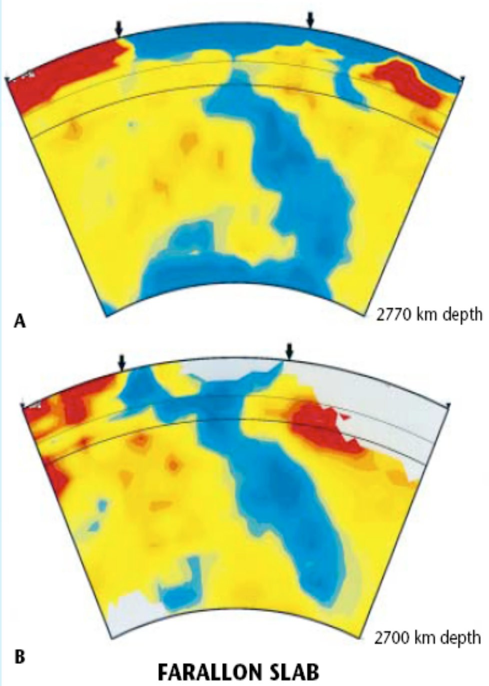
Montelli et al. [2004]

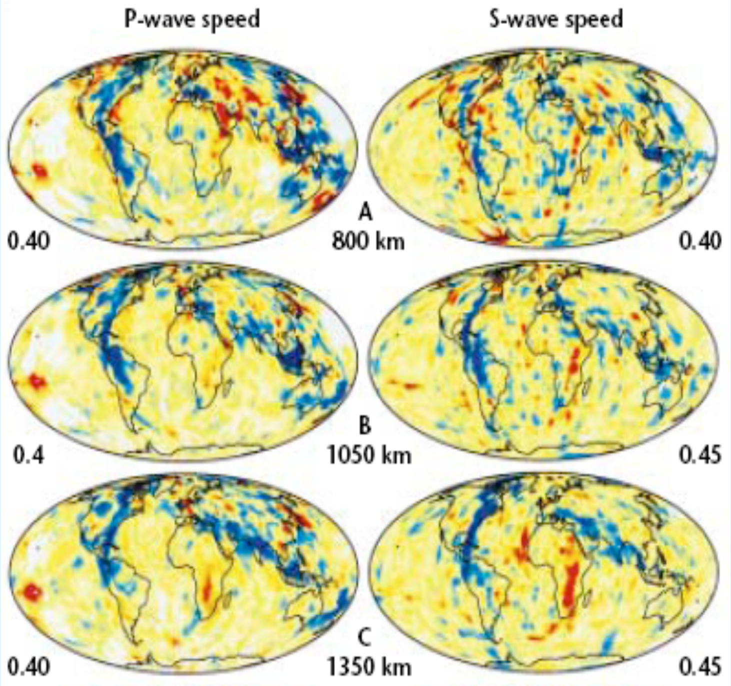




van der Hilst [1995]

More seismic images of mantle convection





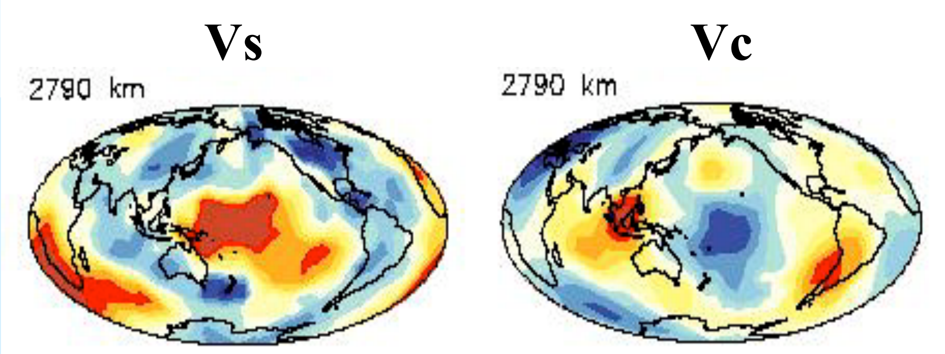
Grand, van der Hilst, & Widiyantoro [1997]

van der Hilst et al. [1992; 1997], Grand [1994], and early models by Dziewonski's group, Masters' group, and Romanowicz's group.

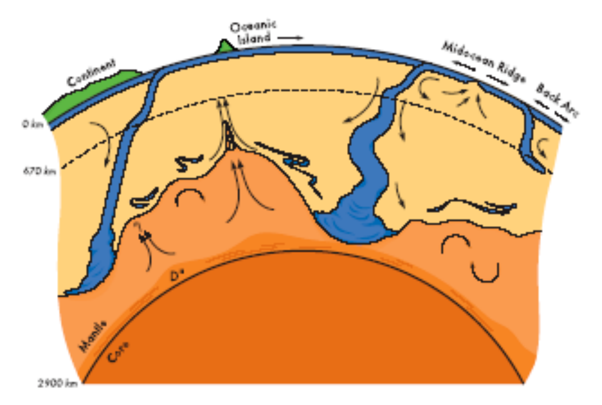
Compositionally Distinct Mantle Reservoirs

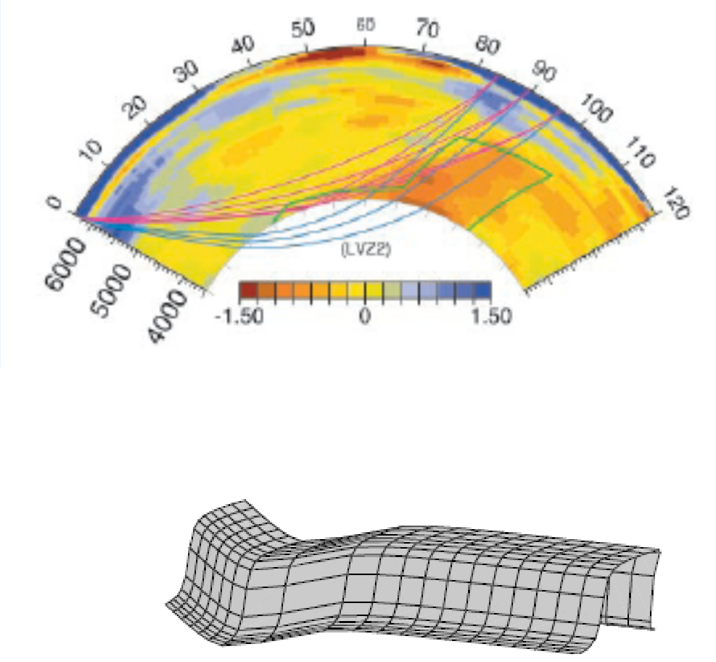
A lot of geochemical evidences.

Masters et al. [2000].



Kellogg et al. [1999].

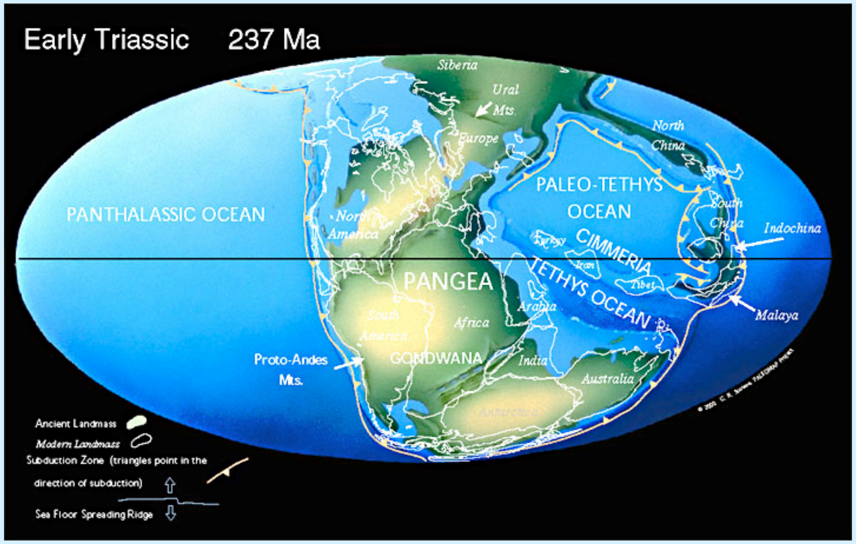


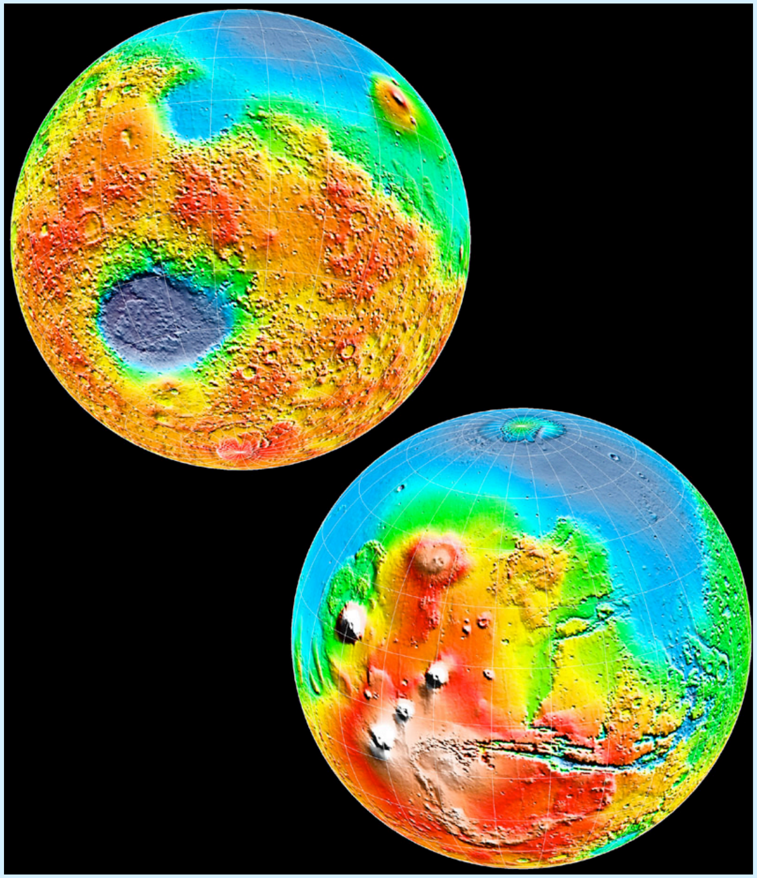


Ni et al. [2002], Ni & Helmberger [2003], Wen et al. [2001]

Important implications for the cooling of the core and geodynamo!

Back in time or out into the space …





Governing Equations (a simplified form)

Mass conservation:

$$\nabla \cdot \boldsymbol{u} = 0,$$

Momentum conservation:

$$-\nabla p + \nabla \cdot [\eta(\nabla \boldsymbol{u} + \nabla^T \boldsymbol{u})] - \delta\rho g_0 \boldsymbol{e}_r = 0,$$

Stokes' flow

Energy conservation:

$$\rho_0 C_p \left(\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T\right) = k\nabla^2 T + \rho_0 H,$$

Equation of state:

$$\delta\rho = -\rho_0 \alpha (T - T_0) + \Delta\rho C,$$

Composition conservation:

$$\frac{\partial C}{\partial t} + \boldsymbol{u} \cdot \nabla C = 0,$$

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3) *Unique challenges.*

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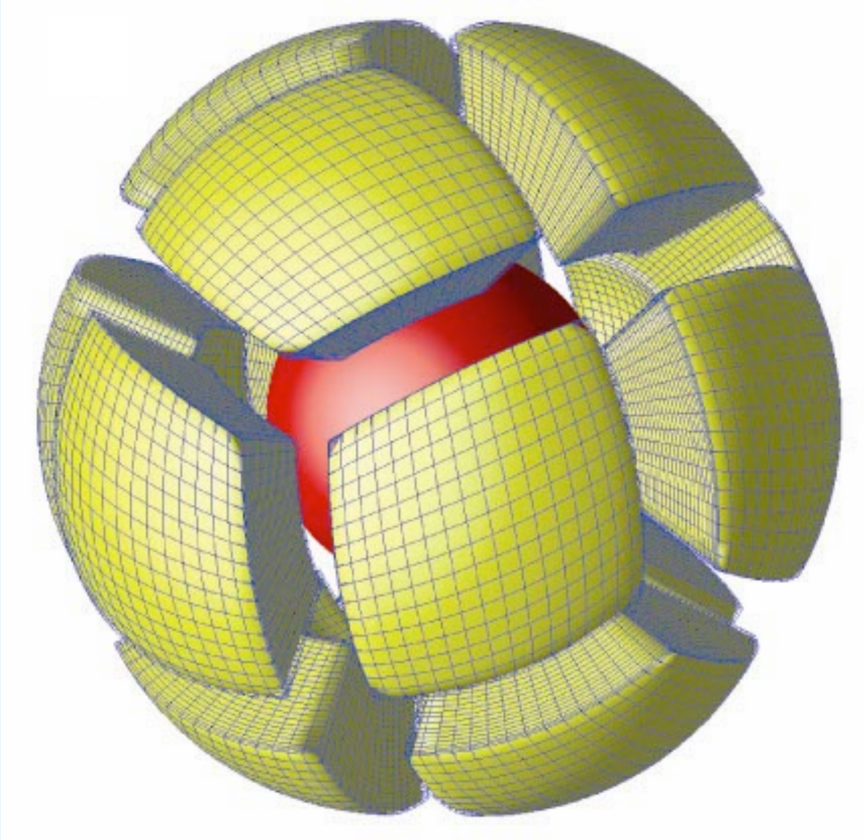
Various Methods

1) ***Spectral methods*** **[Glatzmaier et al., 1990; Zhang & Christensen, 1993; Zhang & Yuen, 1995].**
2) ***Finite volume methods*** **[Ratcliff et al., 1997; Harder & Hansen, 2005; Tackley, 2008].**
3) ***Finite elements*** **[Baumgardner, 1985; King et al., 1990; Moresi & Gurnis, 1996; Zhong et al., 2000].**

Some General Features in CitcomS



Zhong, Zuber, Moresi & Gurnis [2000]

Derived from the 3D Cartesian code Citcom [Moresi & Gurnis, 1996].

Grid: First divide a spherical shell into 12 caps, and then further divide each cap into elements with roughly uniform size. Each cap has the same number of elements.

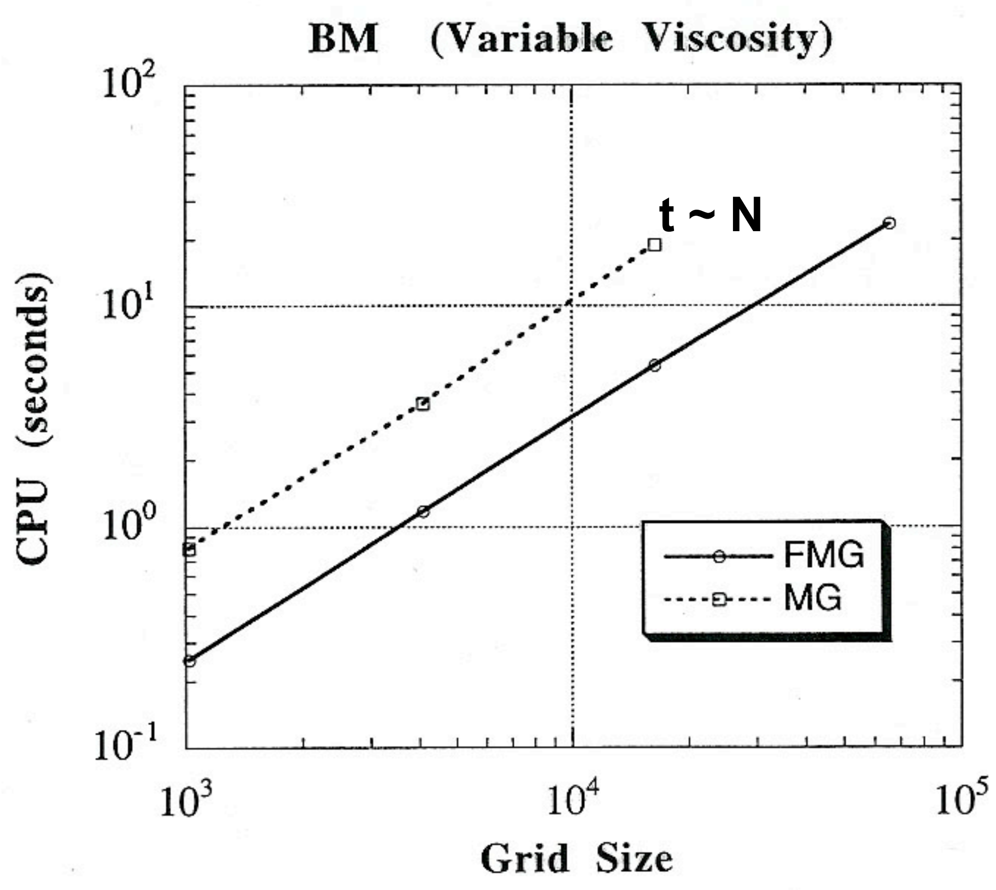
All the equations are explicitly written in spherical geometry and then coded up in stiffness matrix, force,...

***Stokes flow solver*: Uzawa algorithm with two level iterations [Ramage and Wathen, 1994].**

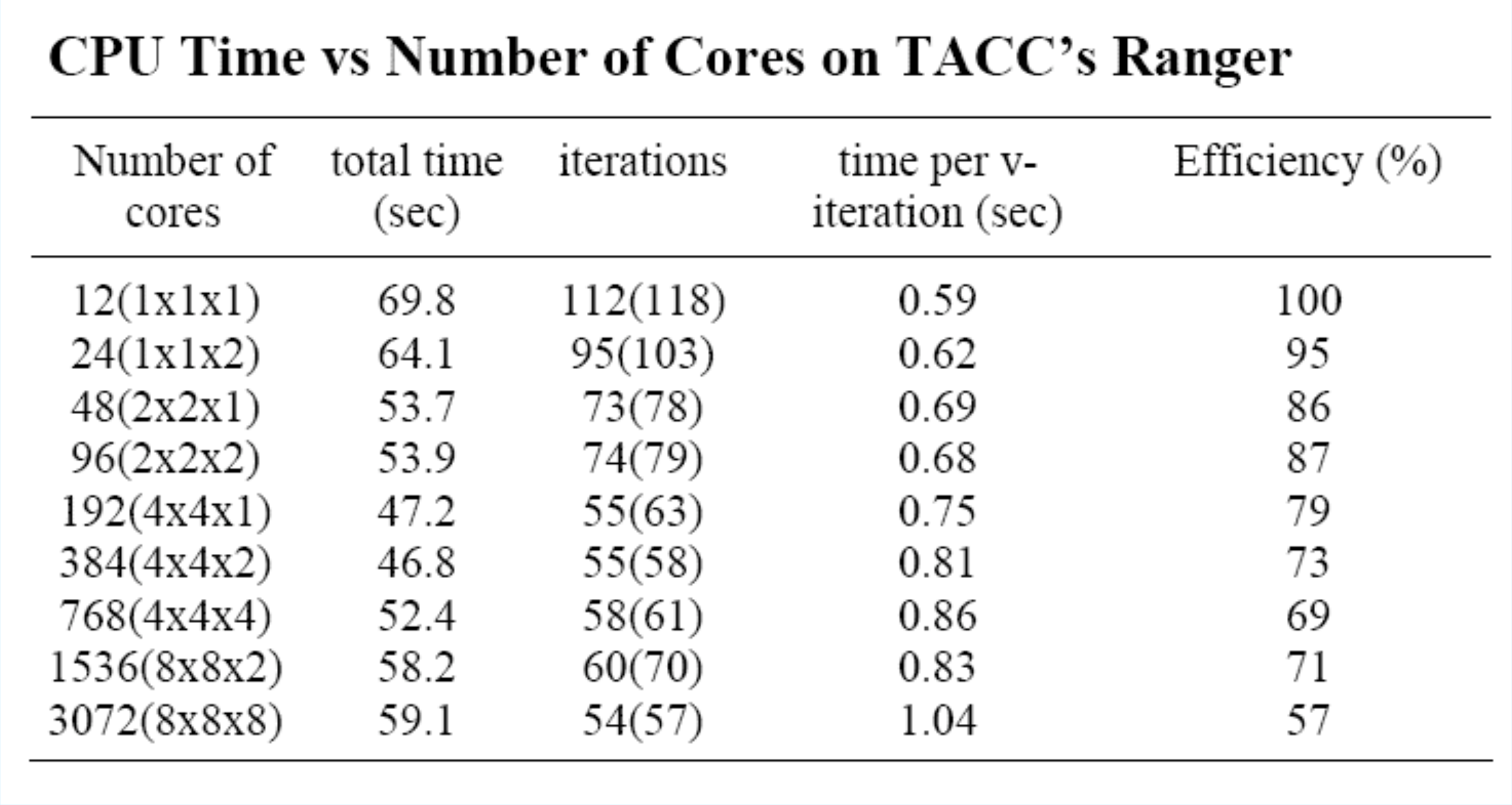
***Energy equation solver*: Streamline Upwind Petrov Galerkin formulation [Brooks, 1981]**

Elements: Brick element (8 velocity nodes and constant pressure per element).

Geometric Multigrid for the Inner Loop Solver in the Uzawa



CitcomS' Parallel Efficiency

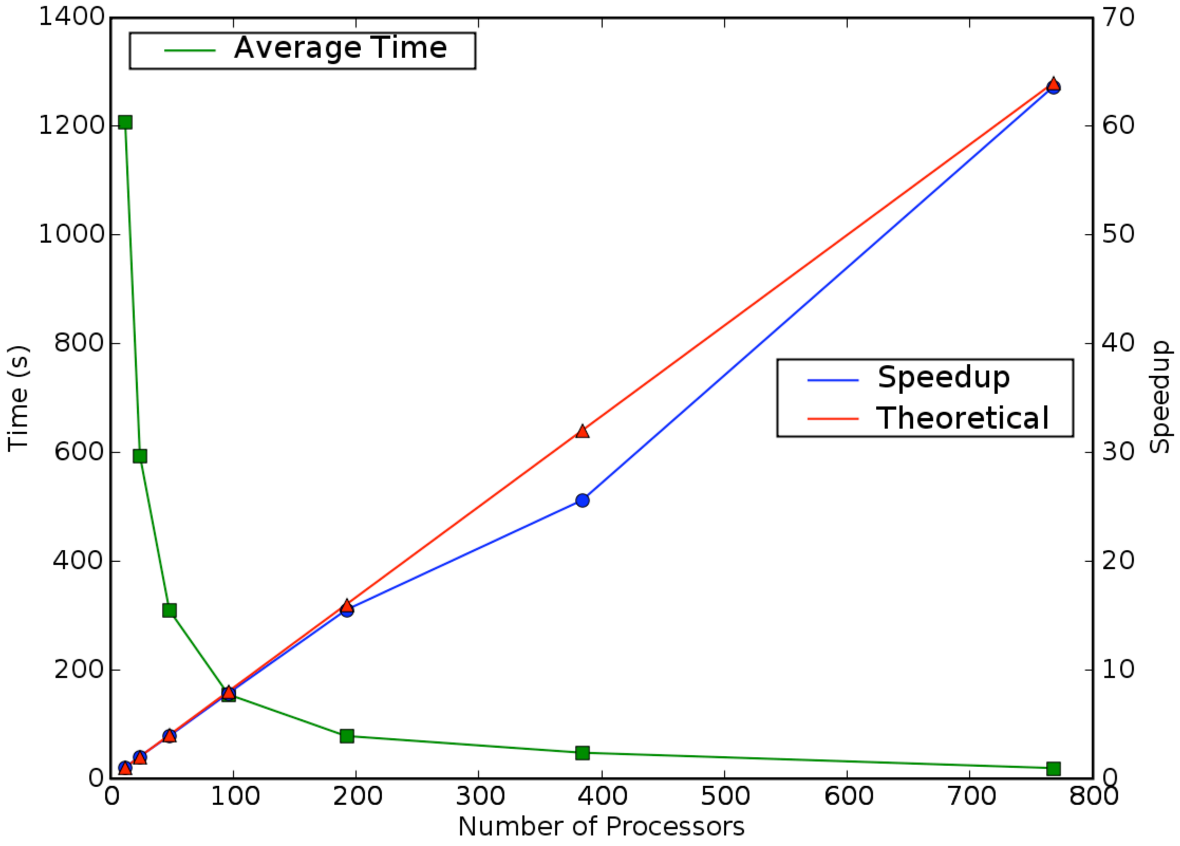


CPU Time vs Number of Cores on TACC's Ranger

Number of cores	total time (sec)	iterations	time per v-iteration (sec)	Efficiency (%)
12(1x1x1)	69.8	112(118)	0.59	100
24(1x1x2)	64.1	95(103)	0.62	95
48(2x2x1)	53.7	73(78)	0.69	86
96(2x2x2)	53.9	74(79)	0.68	87
192(4x4x1)	47.2	55(63)	0.75	79
384(4x4x2)	46.8	55(58)	0.81	73
768(4x4x4)	52.4	58(61)	0.86	69
1536(8x8x2)	58.2	60(70)	0.83	71
3072(8x8x8)	59.1	54(57)	1.04	57

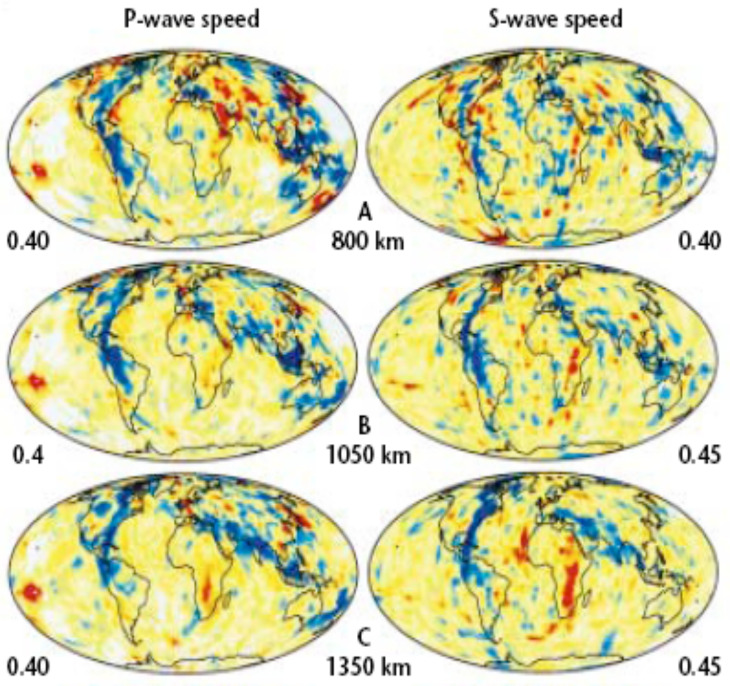
Zhong et al. [2008, G^3, in press]

CitcomS' Parallel Efficiency on BlueGen (only conj-grad)



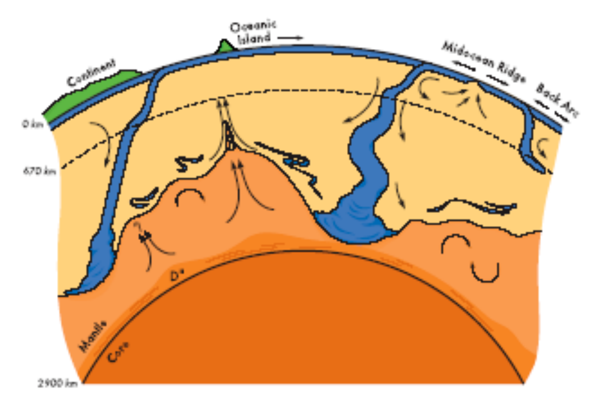
Provided by Scott King

More seismic images of mantle convection



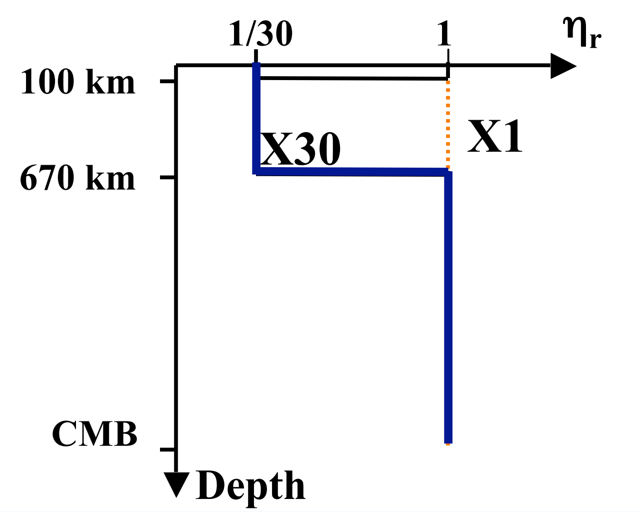
Grand, van der Hilst, & Widiyantoro [1997]

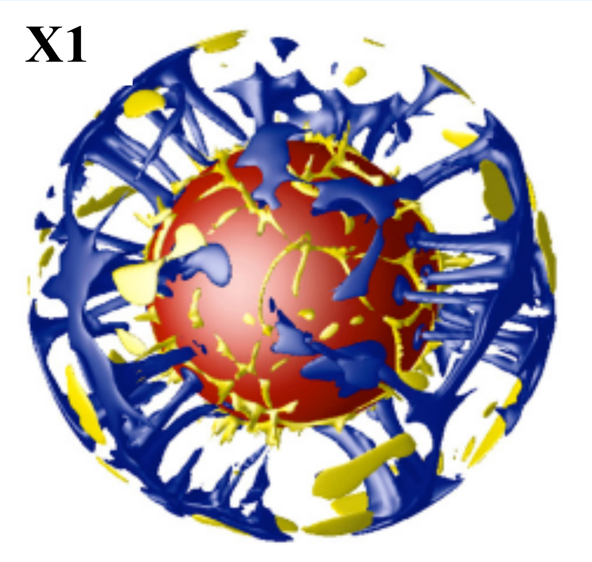
Kellogg et al. [1999].

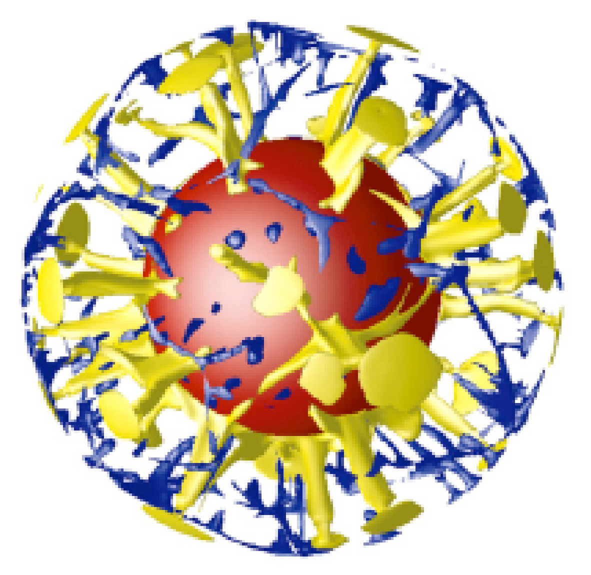


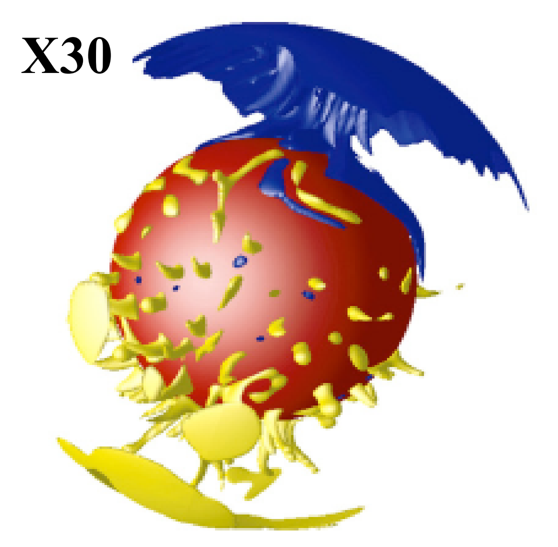
Example 1: Structure Formation [Zhong et al., 2007]

$\eta = \eta_r \exp[E(0.5 - T)]$



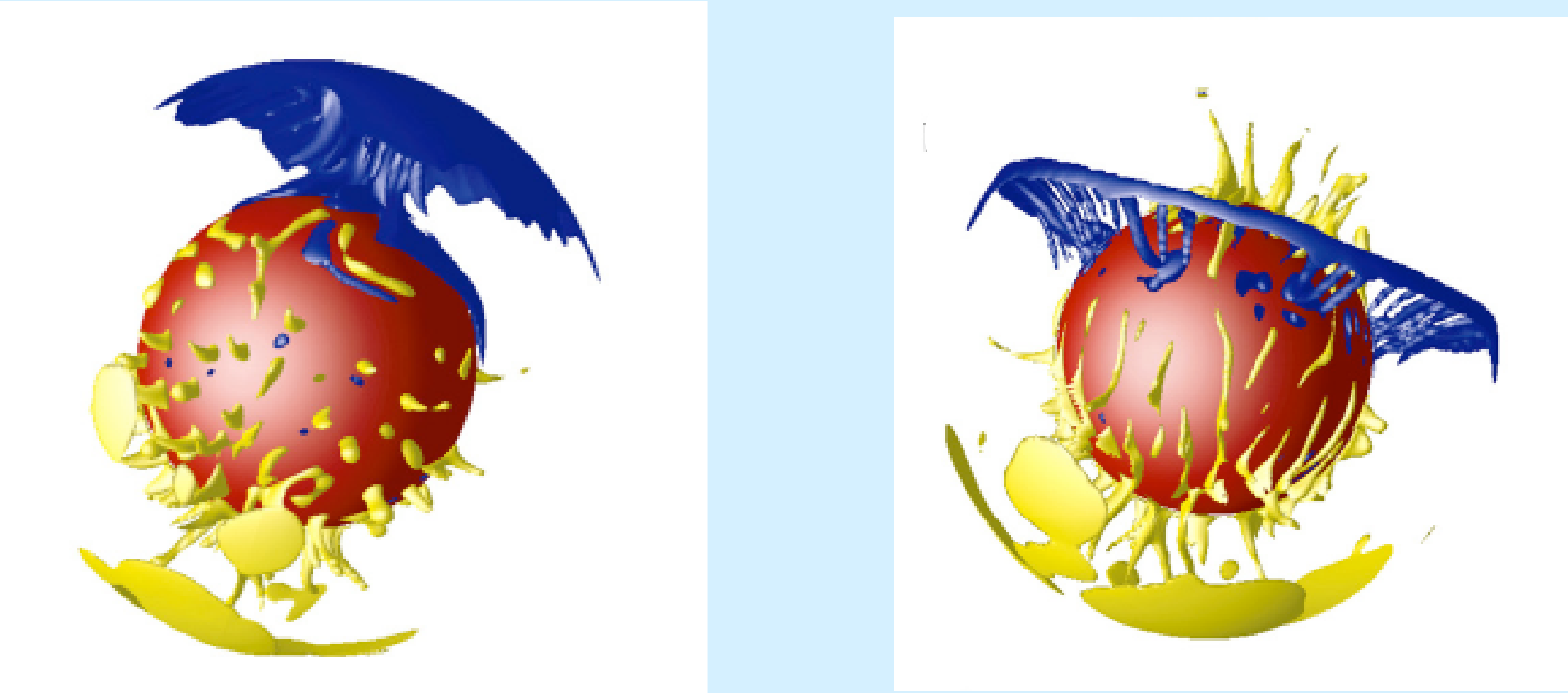




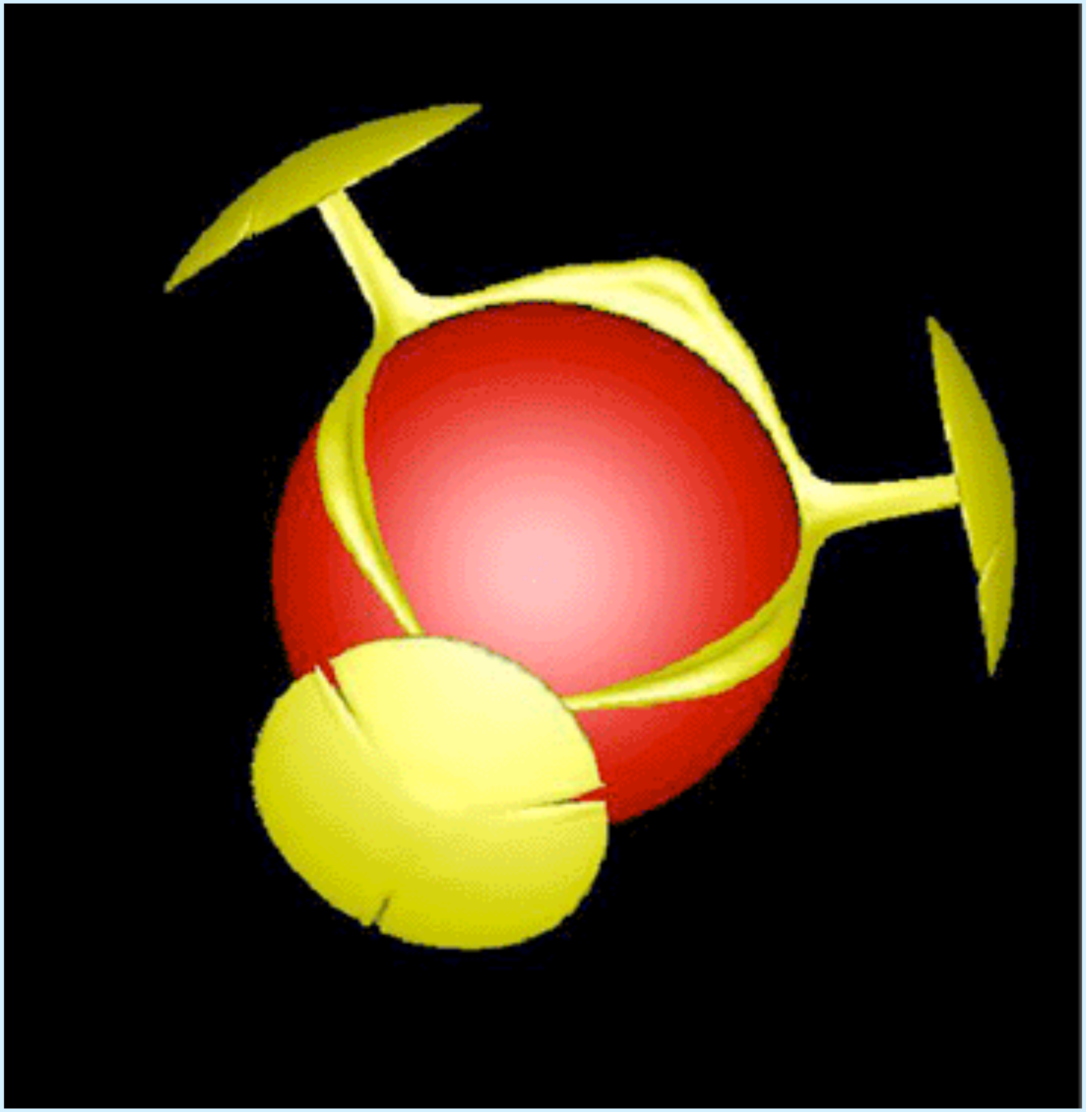


Effects of a supercontinent on mantle structure

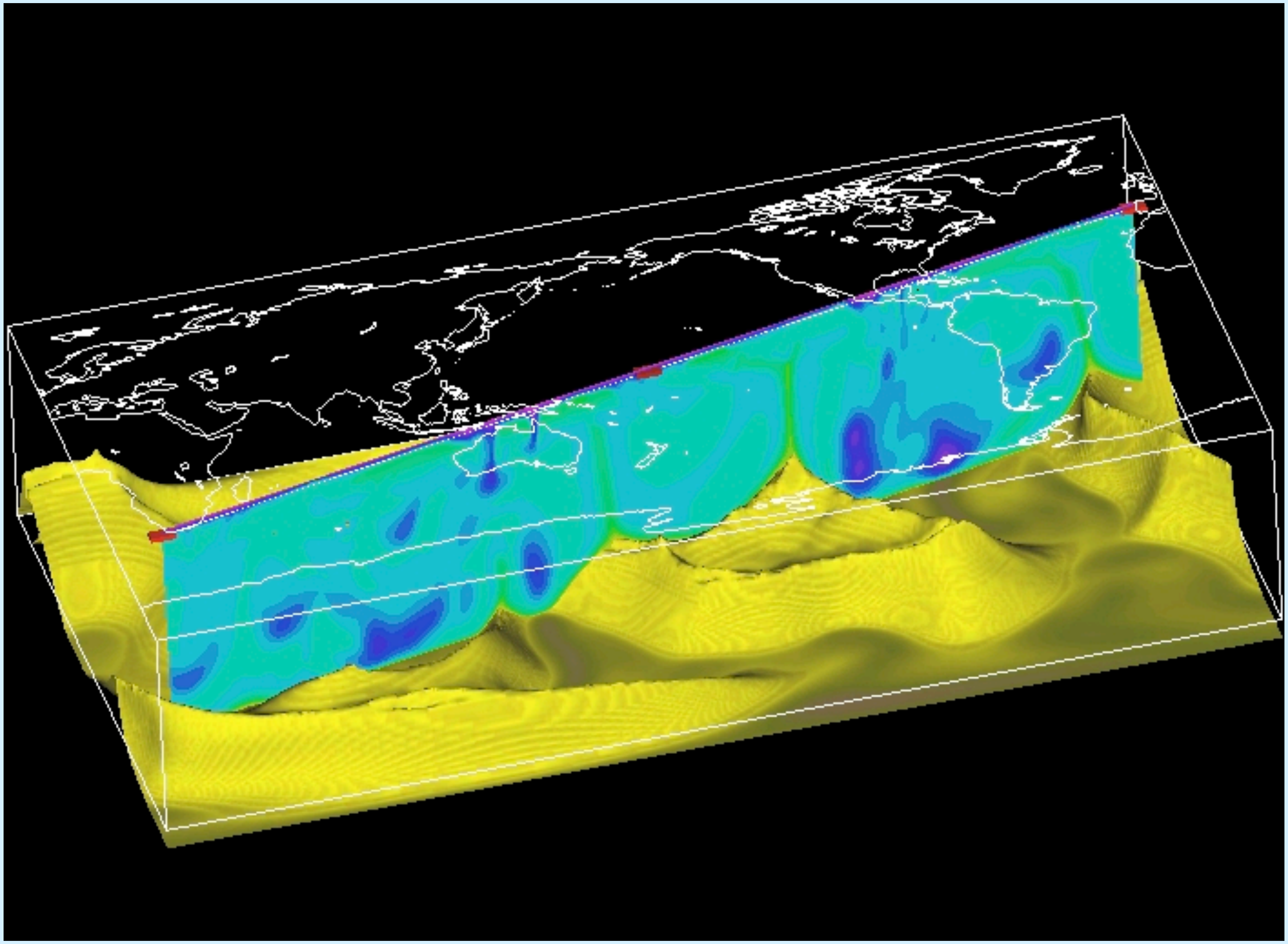
Add a supercontinent



Time evolution of mantle structure

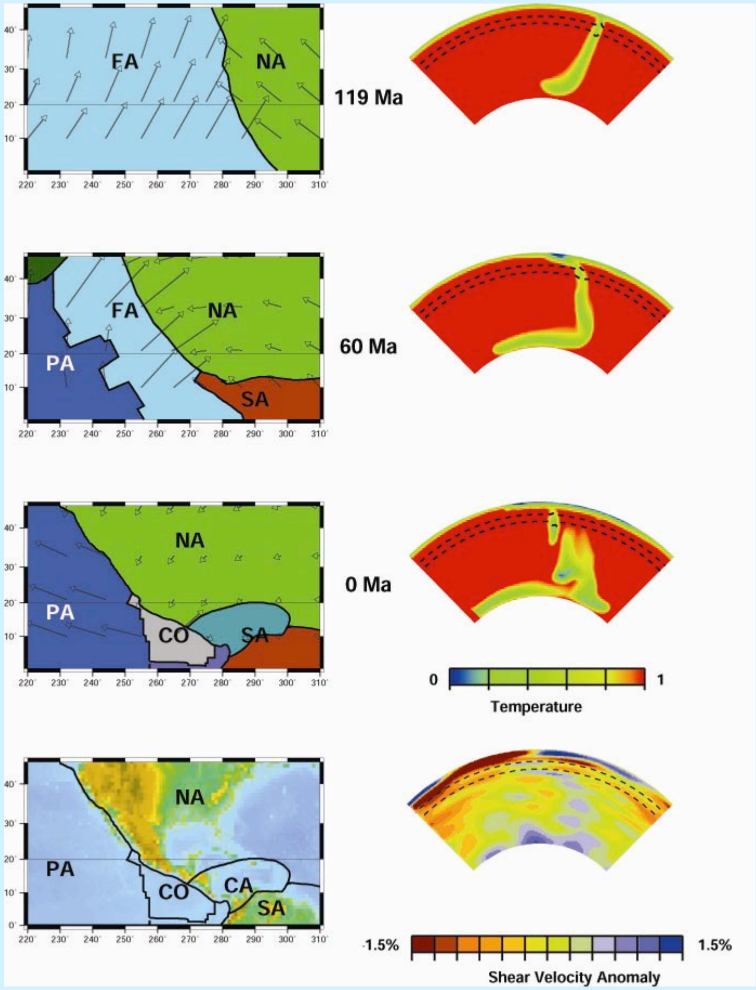


Mantle Convection with Distinct Mantle Reservoirs



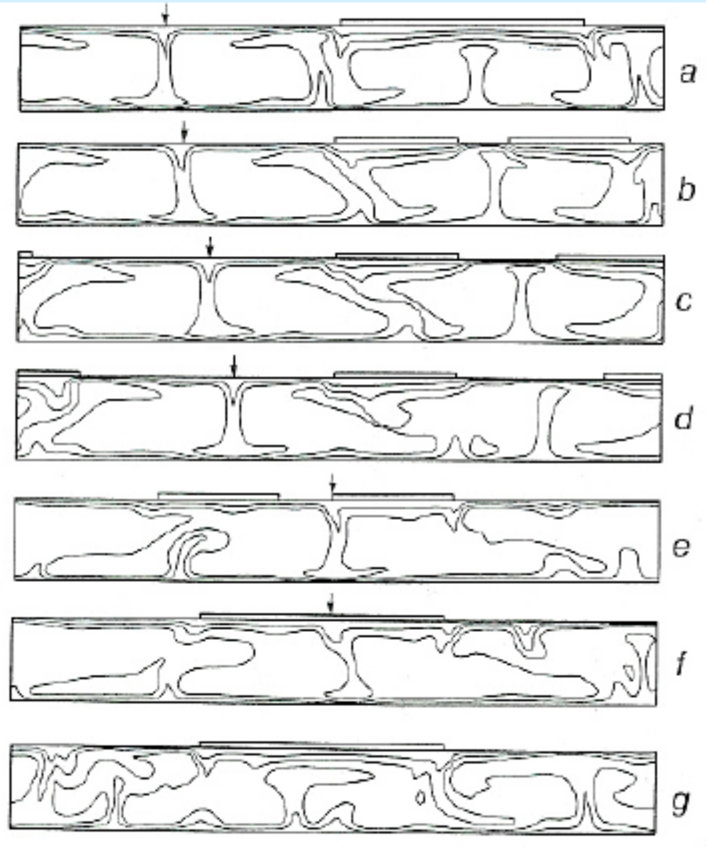
McNamara and Zhong [2005]

Example: Modeling the evolution of Farallon Subduction

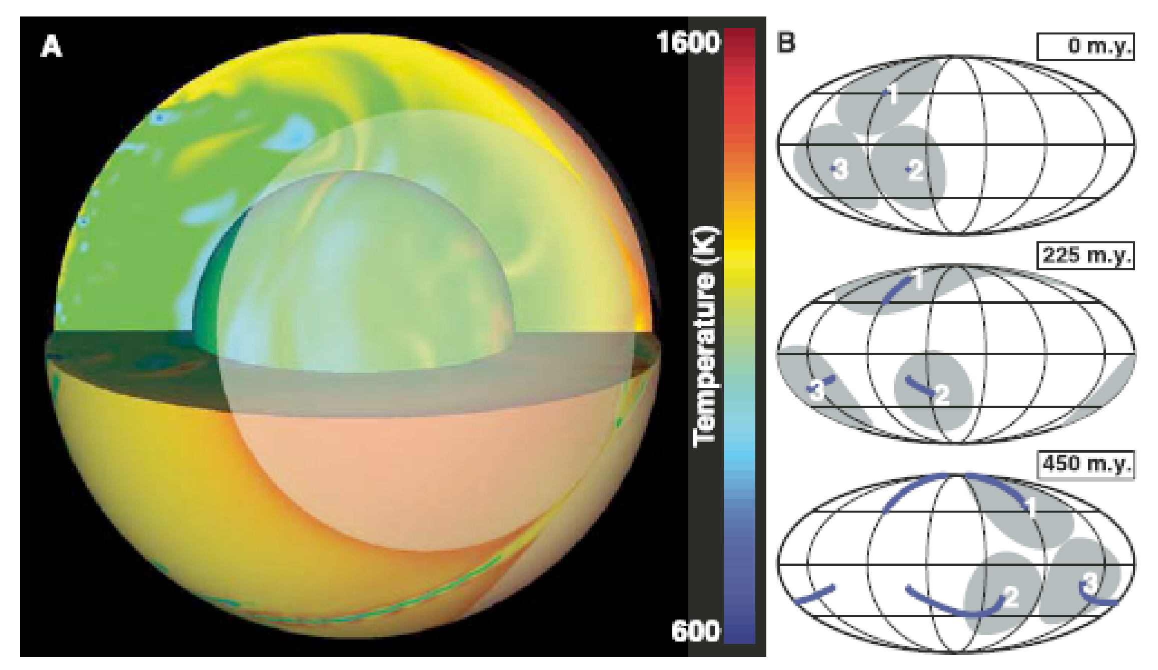


Tan et al. [2002]

Example: Time evolution of mantle structure and continents



Gurnis [1988]



Phillips & Bunge [2007]

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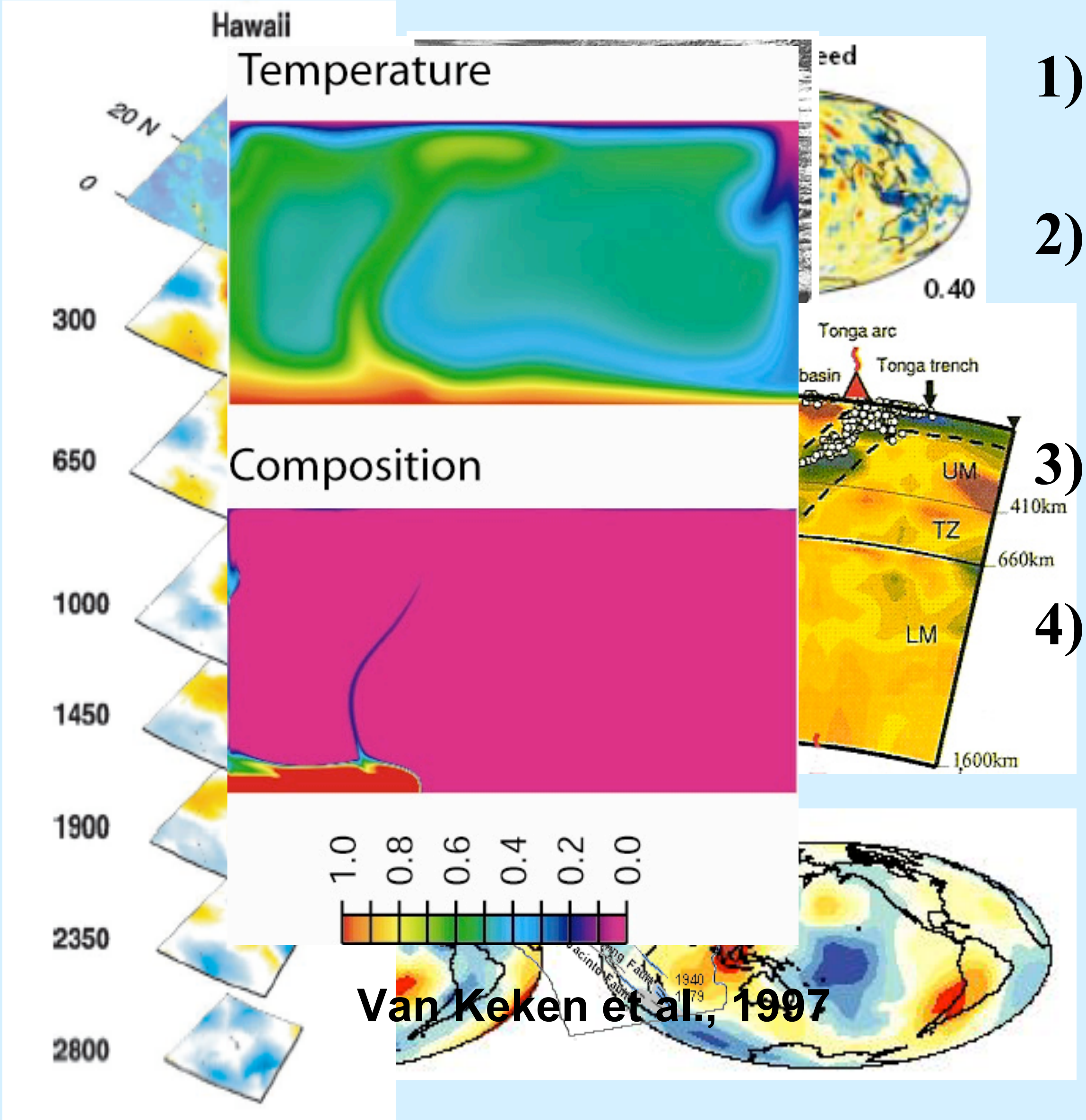
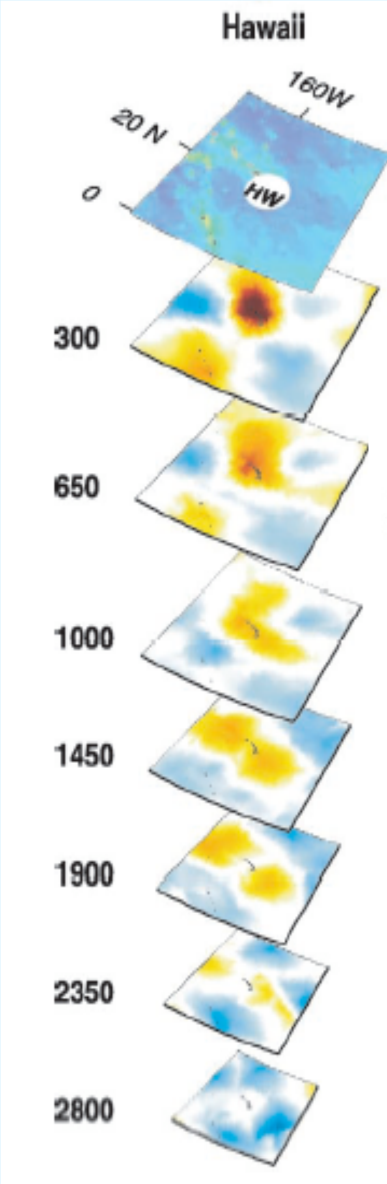
Multi-scale (from 10^0 km to 10^4 km) Physics

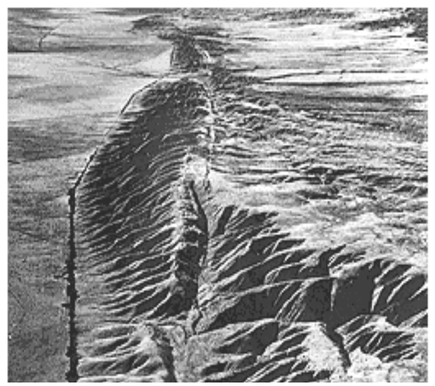


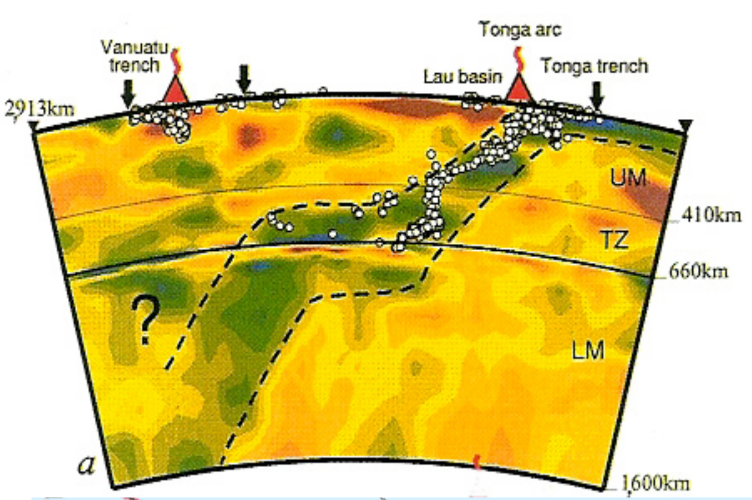
Van Keken et al., 1997

1) **Plate margins and faults: ~1- 10^2 km.**
2) **Upwelling and downwellings: ~ 10^2 km.**
3) **Chemical entrainment: ~ 1-10^2 km.**
4) **Plate scales: ~ 10^3-10^4 km.**

Nonlinear and Highly Variable Rheology





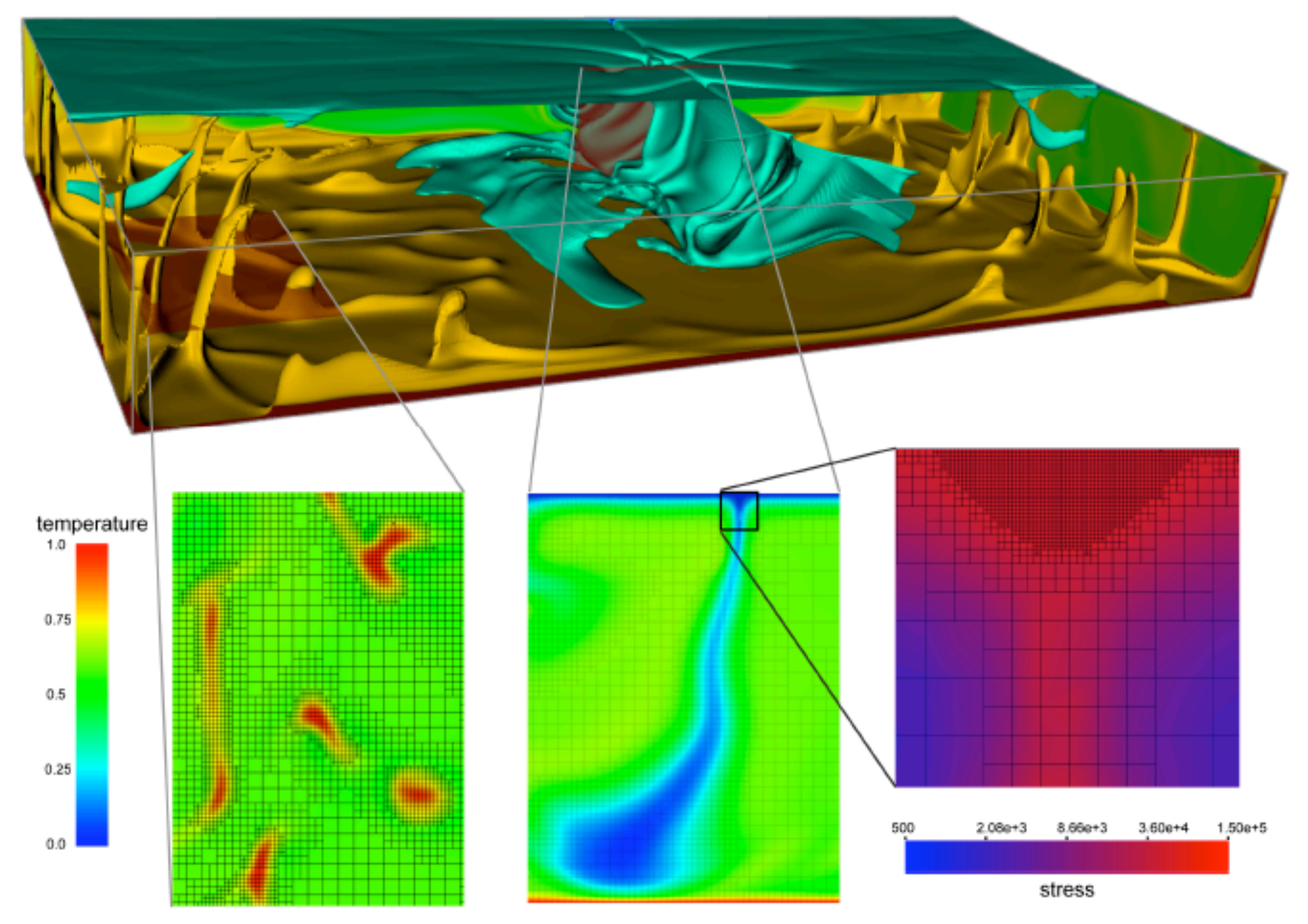


1) **Faulting and Plastic deformation: nonlinear**
2) **Highly temperature-dependent viscosity for silicate mantle (up to 3 orders of magnitude variations in plumes and downwellings).**

Ongoing and Future Developments

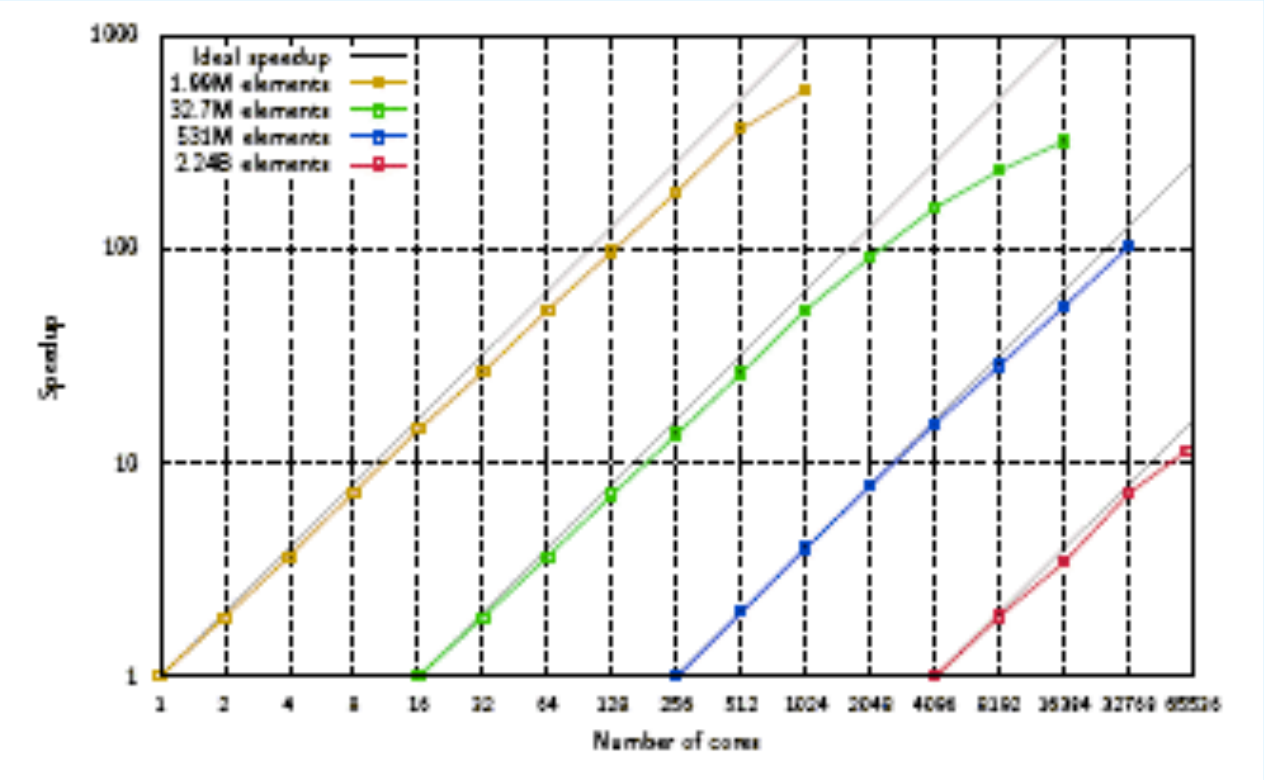
- *Better treatment of multi-scale processes (both physics and numerics).*
  - *i) Lithospheric plates and plate boundaries.*
  - *ii) Melting.*
  - *iii) Thermo-chemical convection and entrainment.*
- *Adaptive mesh refinement (AMR) [Davies & Davies, 2008 – a 2D study; Bangerth's deal-II].*
- *Efficient AMR on ~60K cores [Burstedde et al., 2008 – a collaborative effort by UT-Austin, Caltech, & UC-Boulder]*

A new code: Rhea (octree-based AMR & massively parallel; local resolution to 1.5 km!)

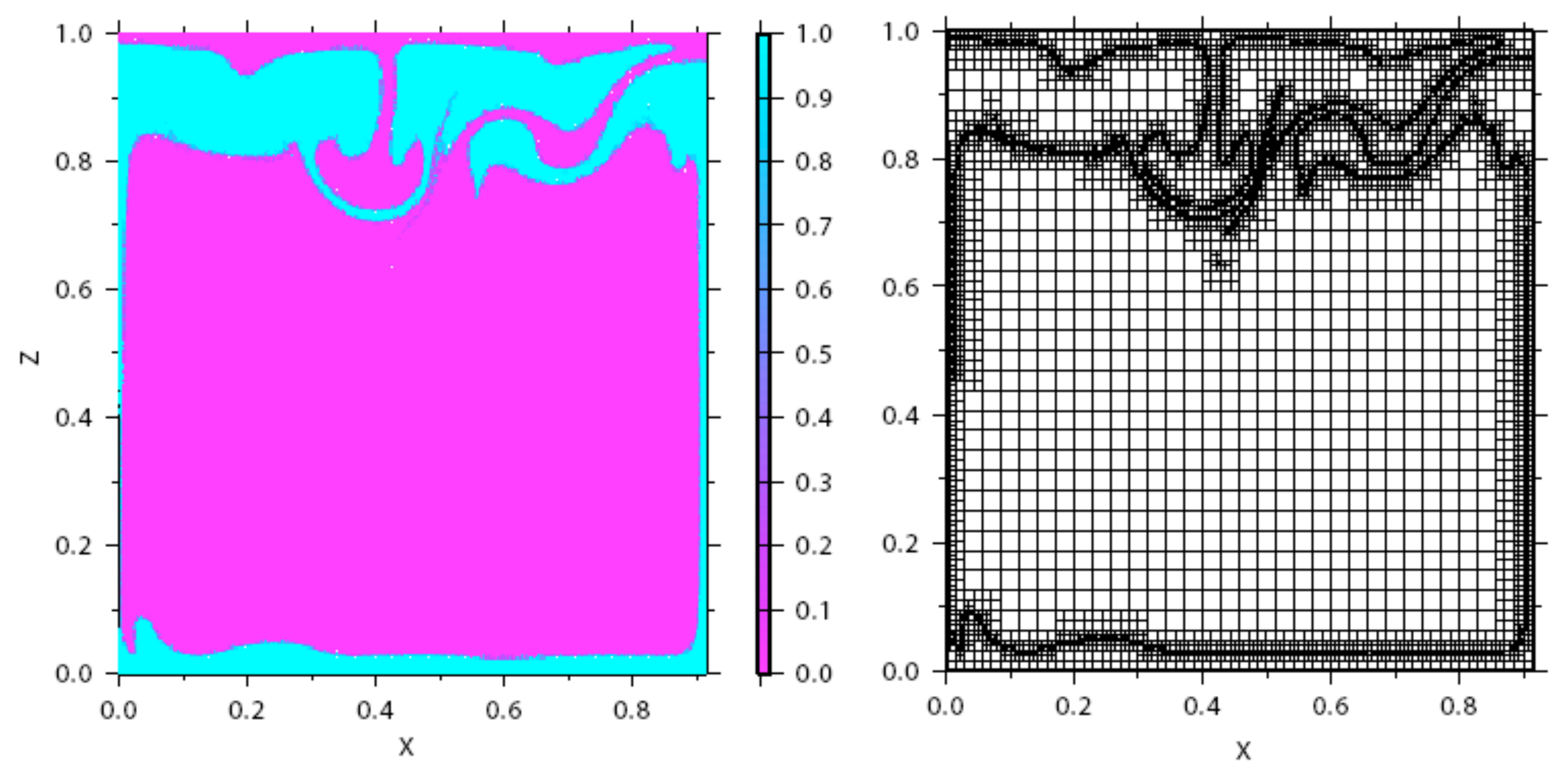


Burstedde et al. [2008]

Rhea's Parallel Efficiency (fixed problem sizes)



Applying octree-based AMR to 2-D thermo-chemical convection



Leng & Zhong [2008 Fall AGU]