PyLith 1.5
Friction, Small Strains, and Elastoplasticity

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Computational Infrastructure for Geodynamics

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Outline

Major new features in PyLith 1.5 and important issues

- Insertion of cohesive cells and fault edges

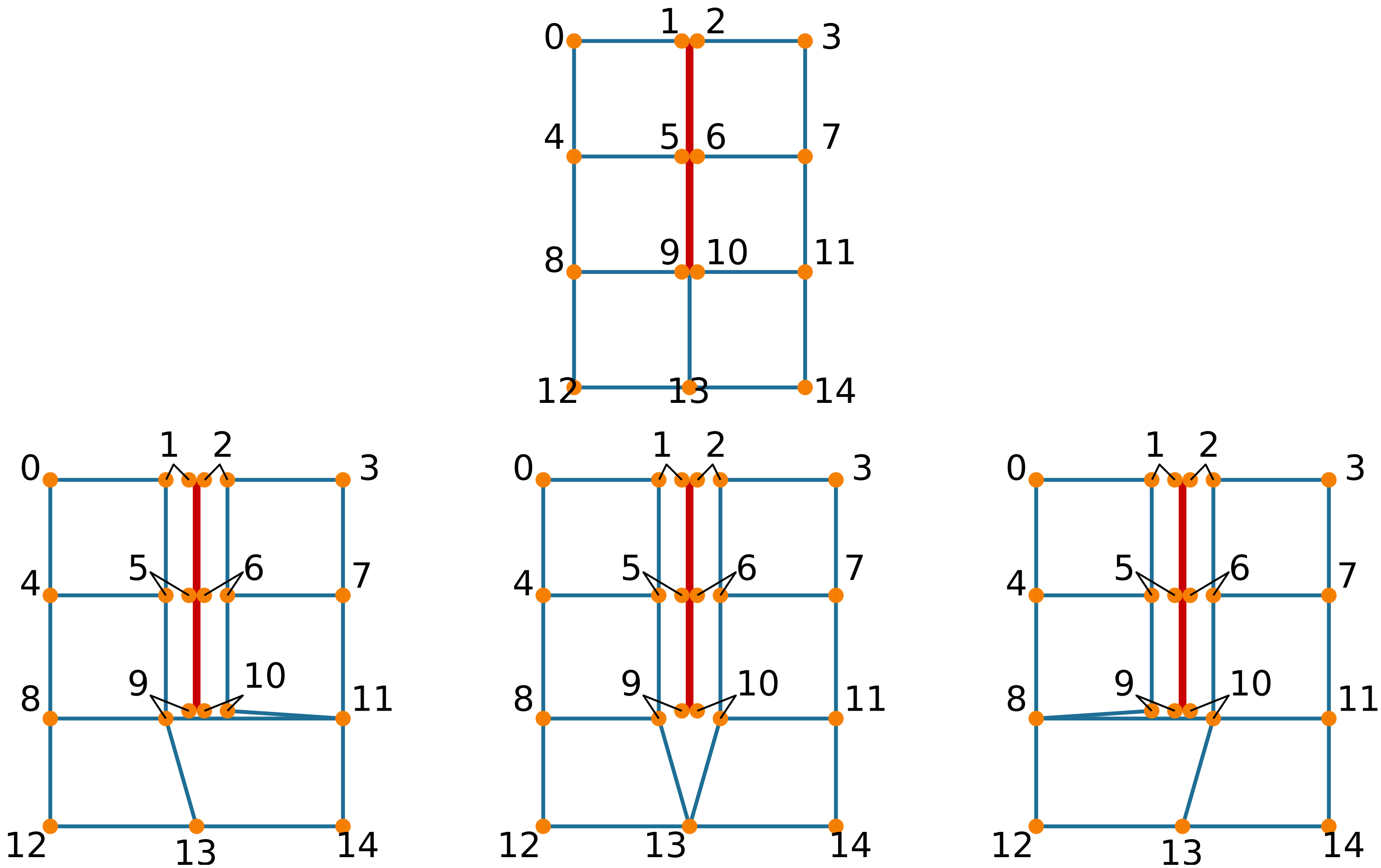
- Dynamic fault ruptures (frictional interfaces)
  - Implementation
  - Fault constitutive models
    - Static friction
    - Linear slip-weakening
    - Rate and state friction w/ageing law

- Drucker-Prager elastoplastic bulk rheology

Insertion of Cohesive Cells

Topology of fault edge is ambiguous

Constrain slip to be zero at fault edge to remove ambiguity in behavior.



Dynamic Fault Ruptures

Frictional interface with fault constitutive model

$$\begin{pmatrix} A & C^T \\ C & 0 \end{pmatrix} \begin{pmatrix} u \\ l \end{pmatrix} = \begin{pmatrix} b \\ d \end{pmatrix} \quad (1)$$

Nonlinear solve:

- Begin each iteration usng current estimates of slip and fault tractions (Lagrange multipliers)

- Iteration algorithm

1. Compute allowable fault traction using fault constitutive model
2. If fault tractions (Lagrange multipliers) do not exceed values allowed by friction
   (a) No change to Lagrange multipliers
   (b) Current estimate of slip is correct
3. If fault tractions (Lagrange multipliers) exceed values allowed by friction
   (a) Reduce Lagrange multipliers to be compatible with friction
   (b) Update slip estimate based on change in Lagrange multipliers

Dynamic Fault Ruptures: Frictional Interfaces

Updating slip according to friction

Equations for conventional DOF:

$$\underline{A}\vec{u} + \underline{C}^T\vec{l} = \vec{b} \tag{2}$$

Variation in displacement field for variation in Lagrange multiplier values:

$$\underline{A}\partial\vec{u} = -\underline{C}^T\partial\vec{l} \tag{3}$$

Solve for $\partial\vec{u}$ using portion of A associated w/fault DOF
Example PETSc setting for friction solve: `friction_pc_type = asm`

- Slip estimate is exact if all other DOF are fixed.
- Slip estimate is good approx. (fast convergence) if deformation for fault slip decays rapidly w/distance from fault
- Slip estimate is poor approx. (slow convergence) if deformation for fault slip is nearly uniform (see examples)

Dynamic Fault Ruptures: Frictional Interfaces

- Tractions driving slip, superposition of
  - Constant initial values imposed directly on the fault surface
  - Computed from deformation
- Must use nonlinear solver with sparse system Jacobian matrix
- Parameters

  db_initial_tractions Spatial database with initial tractions
  friction Fault constitutive model

Fault Constitutive Models

$$T_f = \begin{cases} T_c - \mu_f T_n & T_n \leq 0 \\ 0 & T_n > 0 \end{cases} \tag{4}$$

- Static friction

$$\mu_f = \mu_s \tag{5}$$

- Linear slip-weakening friction

$$\mu_f = \begin{cases} \mu_s - \frac{d(t)}{d_0}(\mu_s - \mu_d) & d(t) \leq d_0 \\ \mu_d & d(t) > d_0 \end{cases} \tag{6}$$

- Rate and state friction

$$\mu_f = \mu_s + a \ln\left(\frac{V}{V_0}\right) + b \ln\left(\frac{V_0 \theta}{L}\right) \tag{7}$$

$$\frac{d\theta}{dt} = 1 - \frac{V\theta}{L} \tag{8}$$

Fault Constitutive Model: Parameters

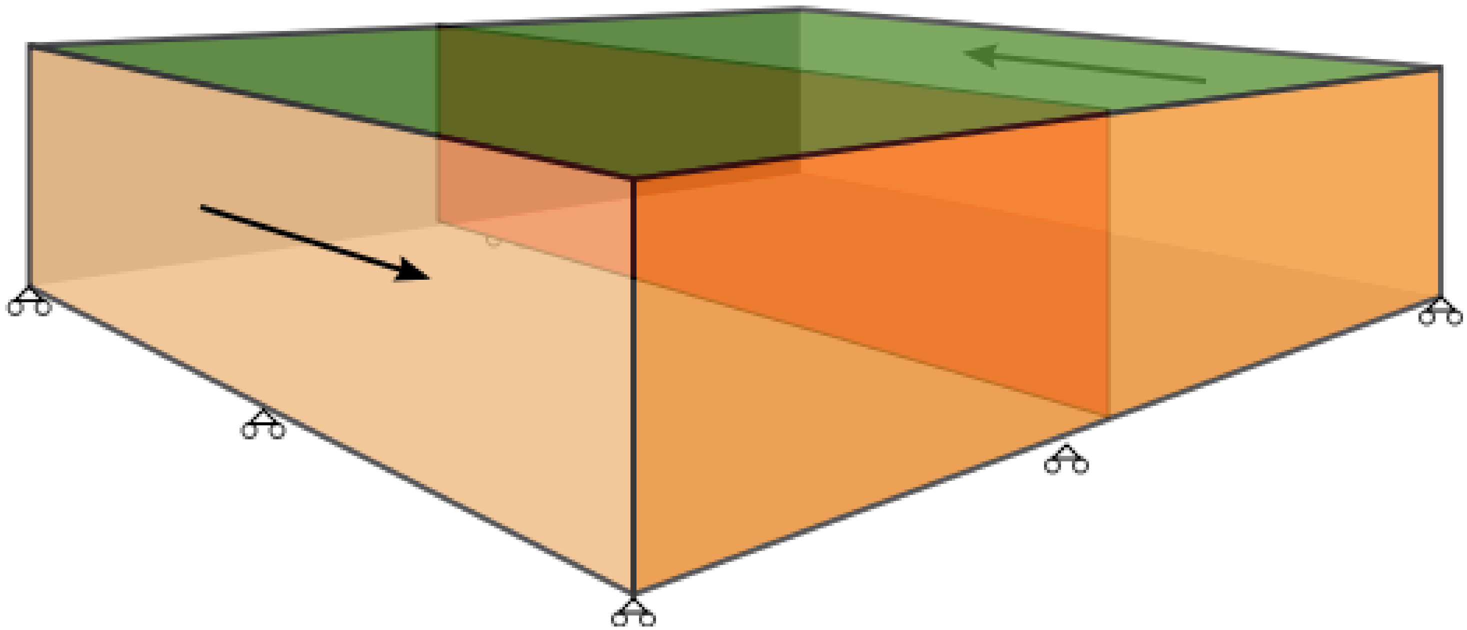
Analogous to bulk constitutive model

db_properties Spatial database with fault constitutive model parameters

db_initial_state Spatial database with initial state variables

Dynamic Fault Ruptures Example

`examples/3d/hex8 step14.cfg`



Small Strains

Total Lagrangian formulation

- Common applications
  - No strain for rigid body motion
  - Overburden pressure increases when cross-section decreases
- Stress/strain tensors

  Strain metric Green-Lagrange strain tensor

  $$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) \tag{9}$$

  Stress metric Second Piola-Kirchhoff stress tensor

  $$S_{ij} = C_{ijkl}\varepsilon_{kl} \tag{10}$$

- Nonlinear solver selected automatrically when using sparse system Jacobian matrix

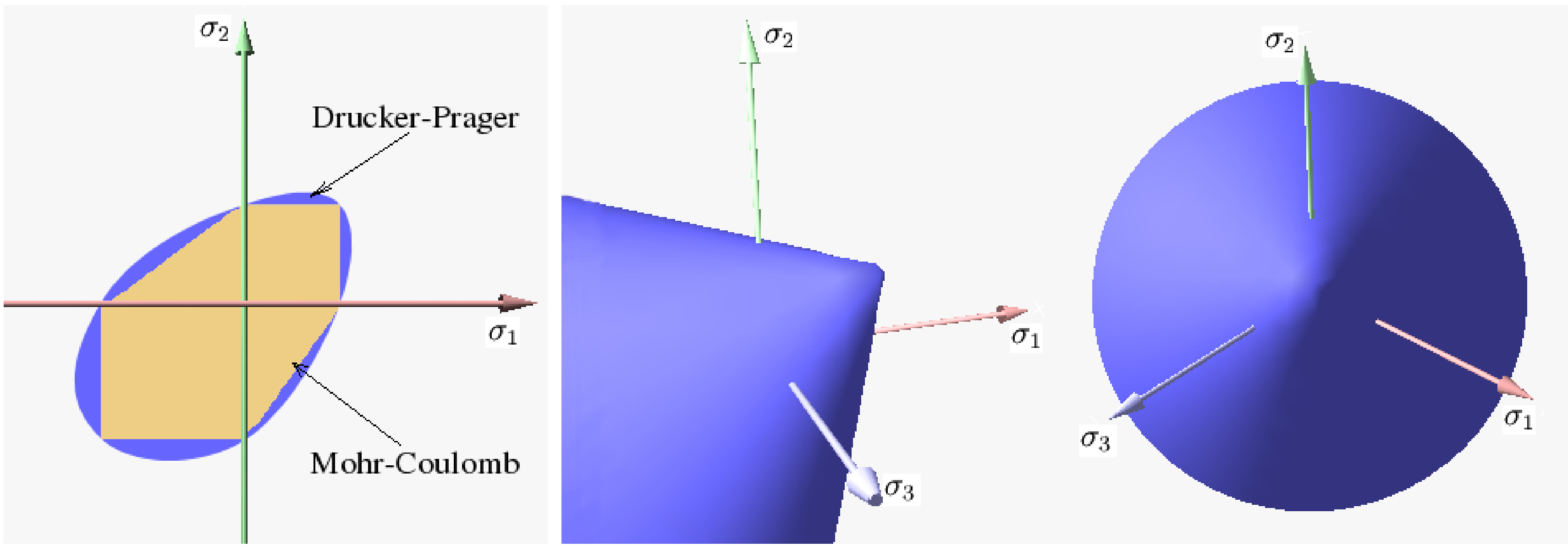
Drucker-Prager Elastoplastic Bulk Rheology

Smooth approximation to Mohr-Coulomb yield criterion

- Yield surface forms a smooth cone in principal stress space that is coincident with the outer apices of the Mohr-Coulomb yield envelope.
- Non-associated plastic flow (different yield and flow functions), which allows control over the unrealistically large amounts of dilatation that are sometimes predicted by the associated model.
- Perfectly plastic implementation, does not include hardening
- **Must use nonlinear solver with sparse system Jacobian matrix**

Drucker-Prager Elastoplastic Bulk Rheology

Smooth approximation to Mohr-Coulomb yield criterion



Drucker-Prager Elastoplastic Bulk Rheology

- Yield function

$$f(\underline{\sigma}) = \alpha_f I_1 + \sqrt{J_2'} - \beta, \tag{11}$$

$$\alpha_f = \frac{2\sin\phi}{\sqrt{3}\,(3-\sin\phi)} \tag{12}$$

$$\beta = \frac{6c\cos\phi}{\sqrt{3}\,(3-\sin\phi)} \tag{13}$$

- Flow function

$$g\,(\underline{\sigma}) = \alpha_g I_1 + \sqrt{J_2'} \tag{14}$$

$$\alpha_g = \frac{2\sin\psi}{\sqrt{3}\,(3-\sin\psi)} \tag{15}$$

First stress invariant: $I_1 = \sigma_{ii}$

Second deviatoric stress invariant: $J_2' = \frac{1}{2}\sigma_{ij}\sigma_{ij}$

Friction angle: ϕ

Cohesion: c

Dilatation angle: ψ