A model for ductile shear initiated by shear fracture: Application to short term and secular fault slip.

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What? Where? How?

- INTRO: Modeling Large Deformation.
- GEOLOGY: The Semi-Brittle Transition, Ductile Shear Zones.
- MODEL: Modeling the YSE with the Brittle Ductile Transition.
- PHYSICS: Analytic formulation and modeling of fracture events for Brittle-Ductile materials.
- TEST: Slow Slip Events.

The problem of deformation

- It can be stated with two equations:

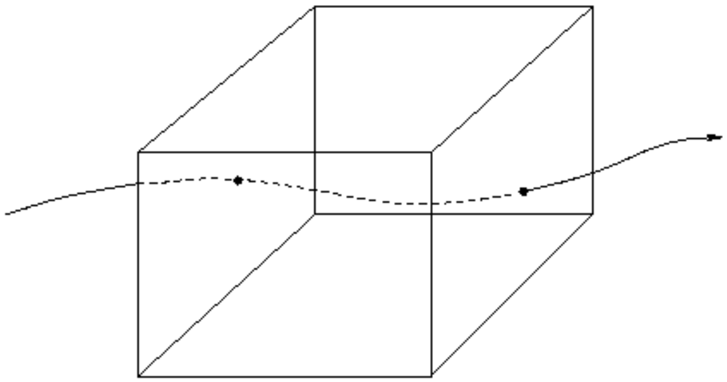
  - A force balance:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i = 0,$$

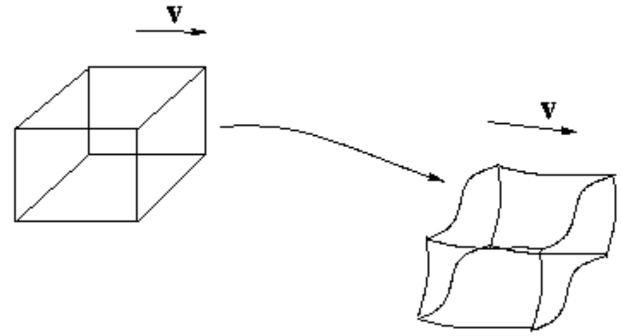
  - A constitutive relationship:

$$\sigma_{ij} = f(\sigma_{ij}, \varepsilon_{ij}, \dot{\varepsilon}_{ij}, \ldots).$$

  which defines a relationship between the strain and stress.



Eulerian vs. Lagrangian



- Viscoplastic approach that uses Stokes flow in an Eulerian grid that is adjusted for large deformation (Arbitrary Eulerian Lagrangian (ALE), Particle In Cells (PIC) are also suitable).

- Elastoplastic and viscoelastic that uses the quasi-static formulation of the equation of motion in a Lagrangian grid. This is the traditional mechanical engineering approach.

Lagrangian Large deformation

- Remeshing needed every time the mesh is too distorted: Innacurate!!

- The coupling with fluids in the Lagrangian mesh is not conservative (remeshing + distortion).

- Explicit vs. Implicit.

Eulerian Large deformation (visco-plastic, no true elasticity + rigid body) (GALE, etc…)

- Uses the equilibrium Equation:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i = 0$$

- With viscous Incompressible Flow:

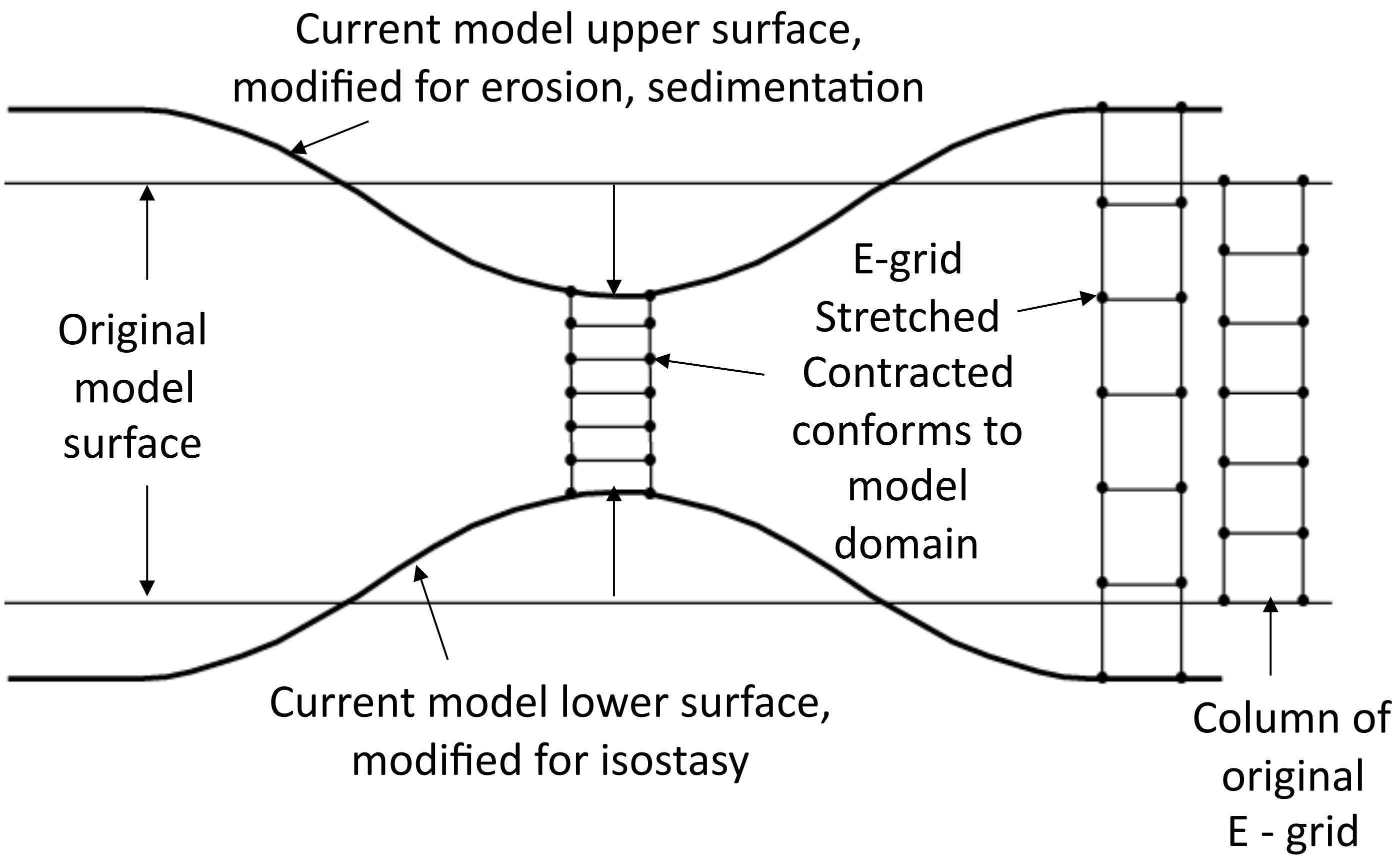
$$\sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\varepsilon}_{ij} \text{ and using } \dot{\varepsilon}_{ij} = \tfrac{1}{2}\left(\partial v_i / \partial x_j + \partial v_j / \partial x_i\right)$$

- It leads to Stokes Equation:

$$-\frac{\partial p}{\partial x_j} + \eta \frac{\partial}{\partial x_i}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \rho g_j = 0 \qquad j = 1,2$$

Ritske Huismans

E – Remeshing to Conform to Model Domain



Finite element problem is solved on the E- grid.
The E – grid is stretched/contracted vertically to conform to the material domain.

Ritske Huismans

Possible: Eulerian formulation of an elastic solid (Duddu, Lavier, Calo).

Momentum balance:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = \rho\,\mathbf{f} + \nabla\cdot\boldsymbol{\sigma}.$$

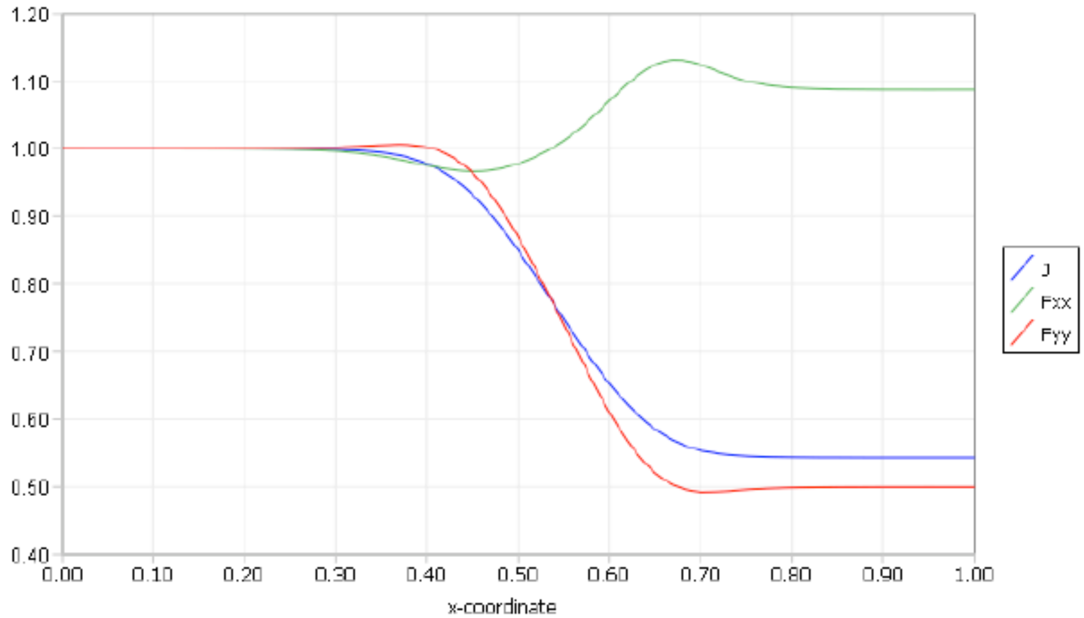
Mass balance:

$$\frac{\partial J}{\partial t} + \mathbf{v}\cdot\nabla J - J\,\nabla\cdot\mathbf{v} = 0,$$

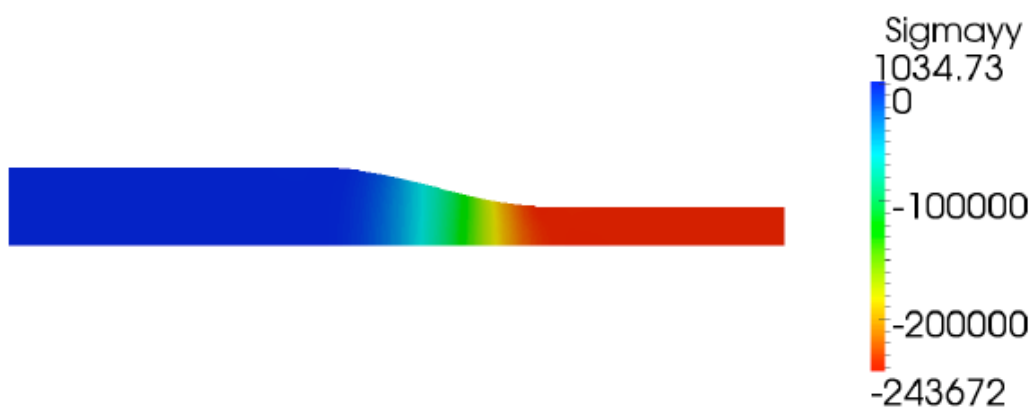
Transport of deformation gradient:

$$\frac{\partial \boldsymbol{F}}{\partial t} + \mathbf{v}\cdot\nabla\boldsymbol{F} = \boldsymbol{l}\cdot\boldsymbol{F},$$

Infancy: stability analysis



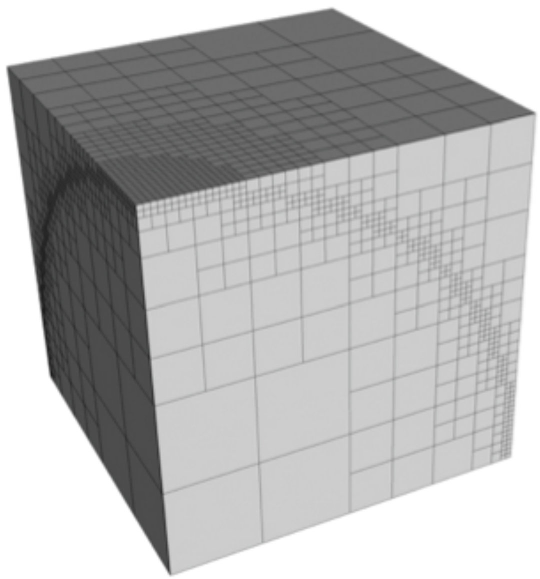
(a) Data along the line joining (0,0,0) and (1,0,0)

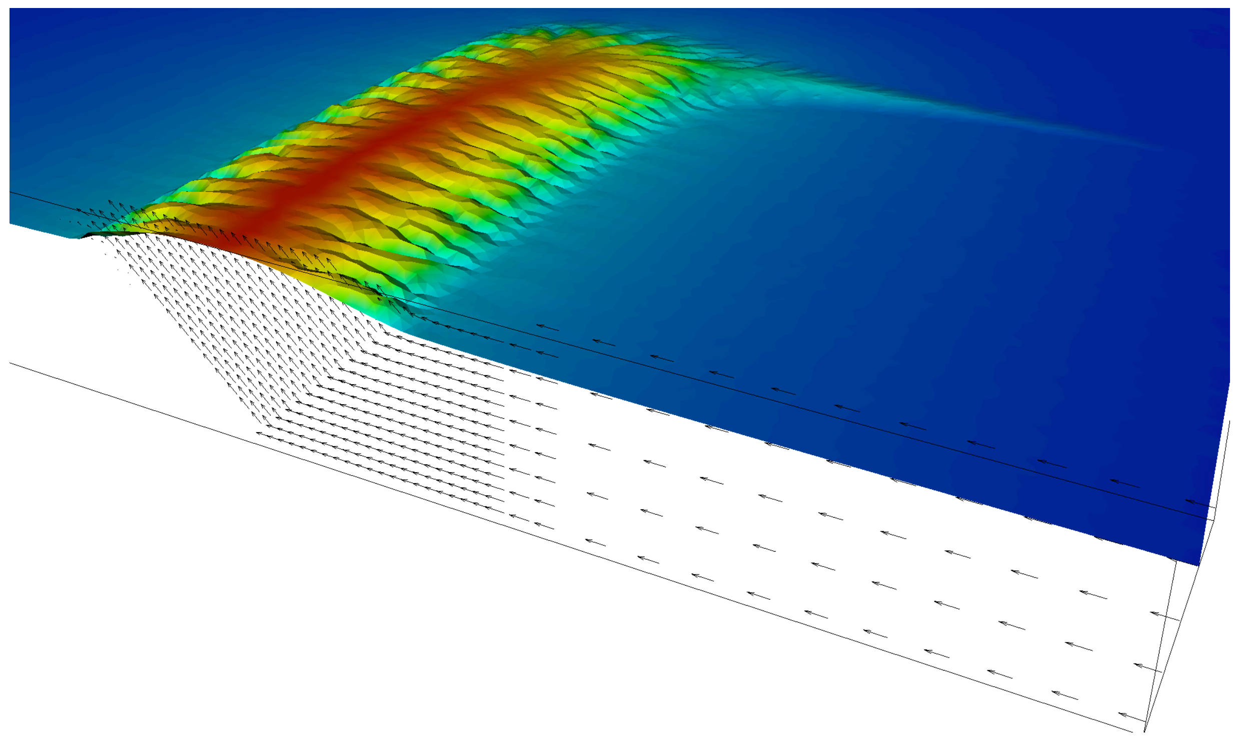


(b) Cauchy stress, σ_{yy} in MPa

Duddu et al., 2010

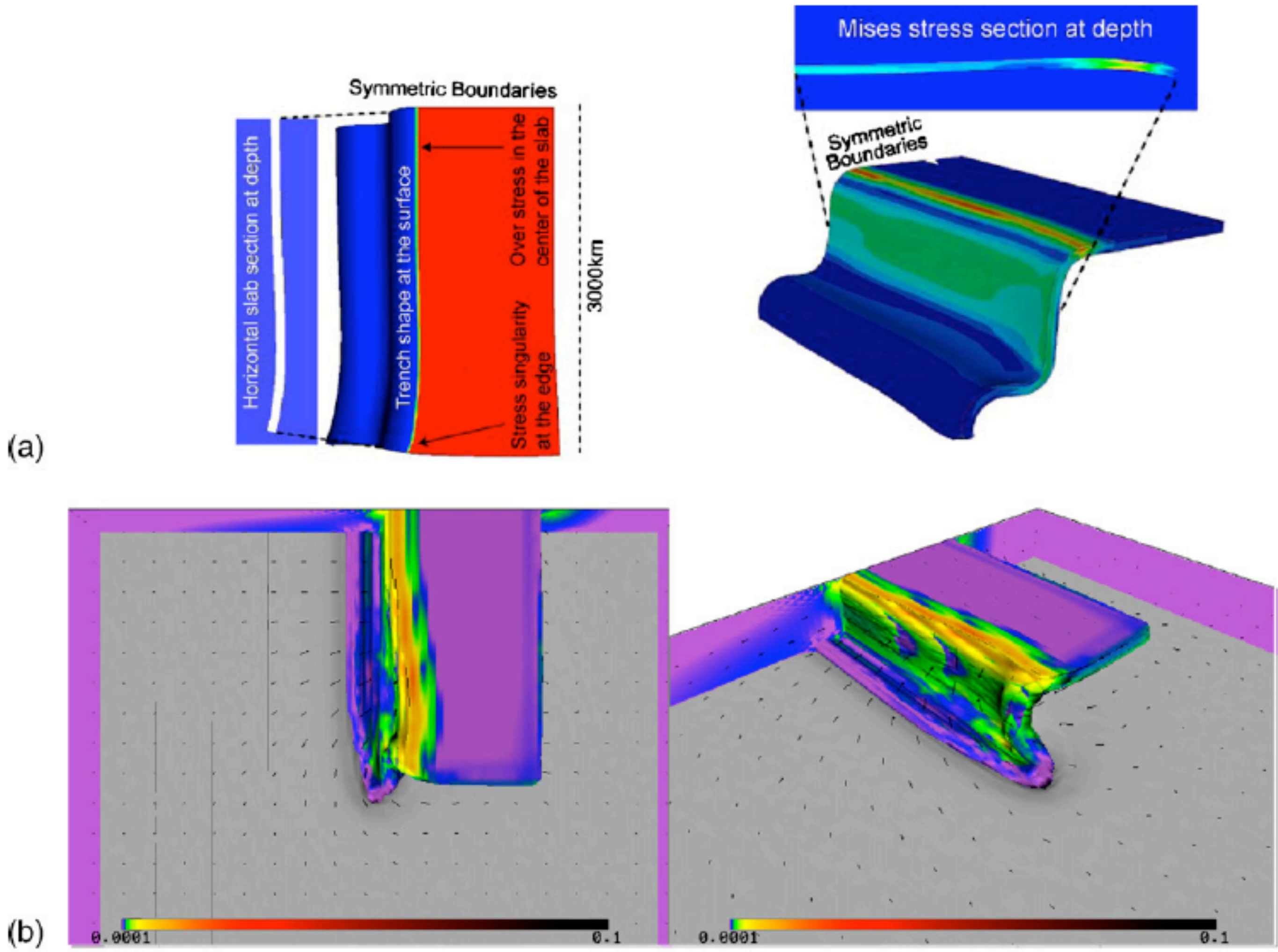
DOUAR (Braun et al., 2008)





UNDERWORLD (L. Moresi)

www.underworldproject.org



OzBench et al., 2008.

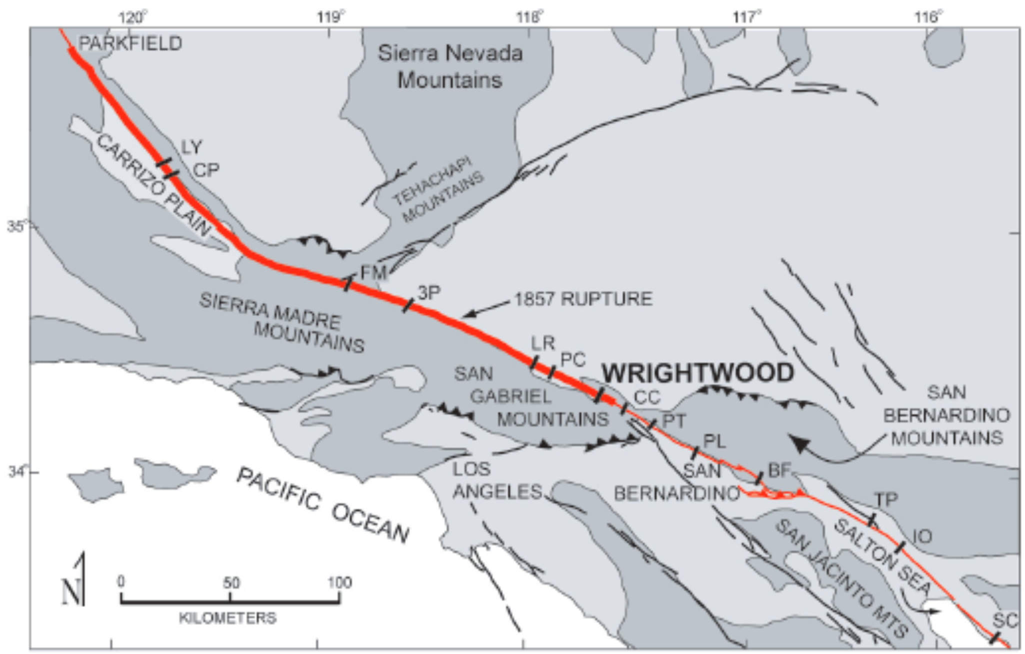
What constitutive update?

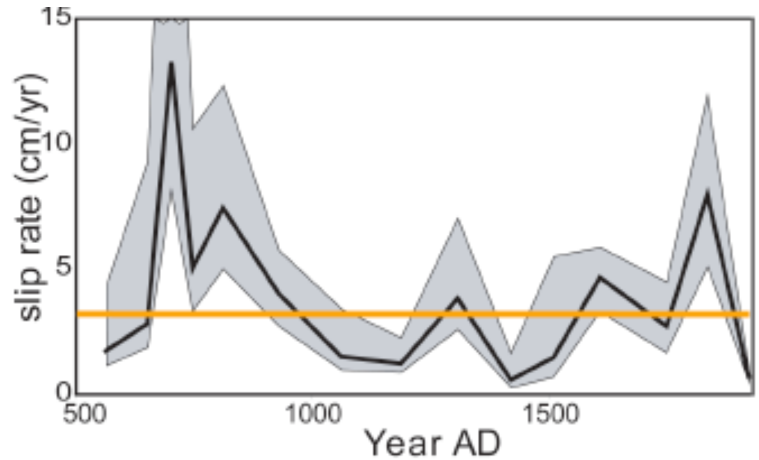
- If we want to understand deformation over 100s of seismic cycle we need to add anelastic behavior with true elastic behavior.

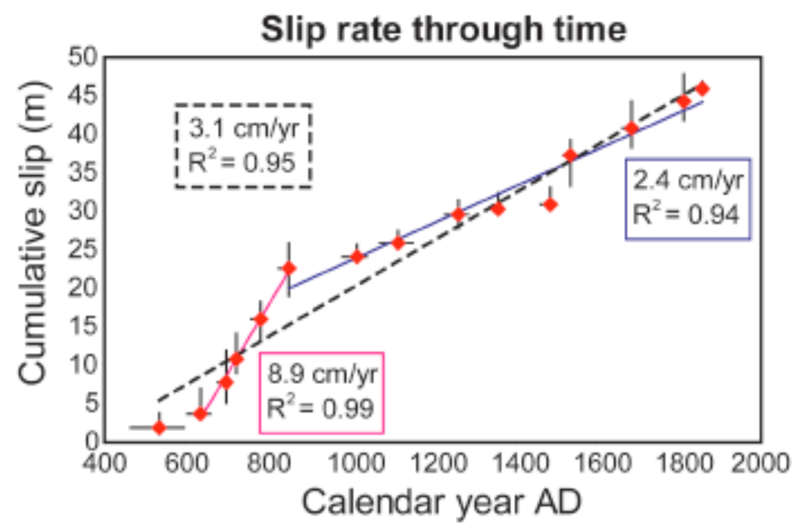
- Why not just "visco-elastic"?
  - Ubiquitous occurrence of *plastic deformation* in crust

- Why not just "elasto-plastic"?
  - Observation of *viscous relaxation* of stress in the crust
  - *Localization* is possible, but *rate-dependence* is desired.

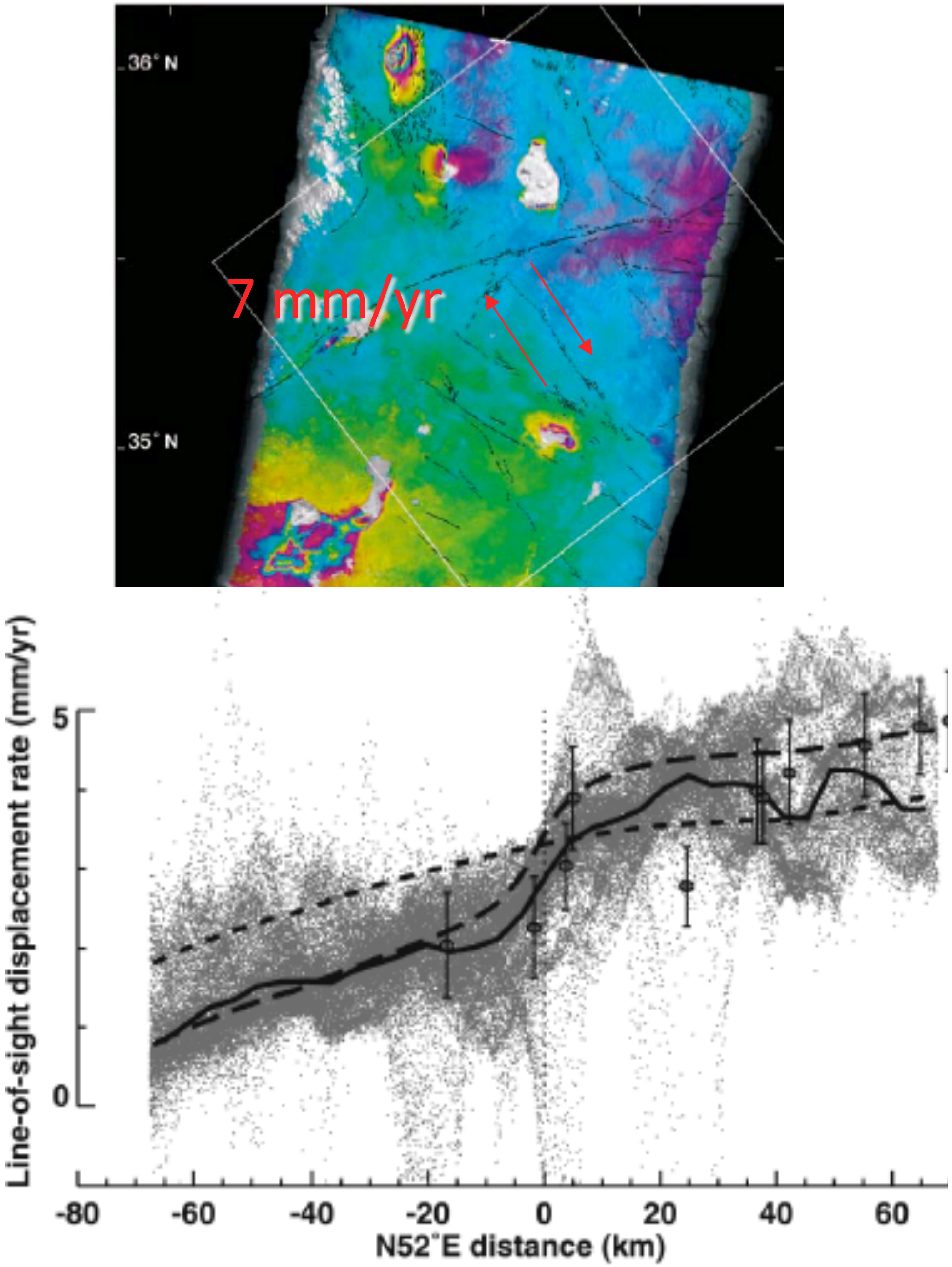
Possible secular oscillations in slip rate on fault zones.



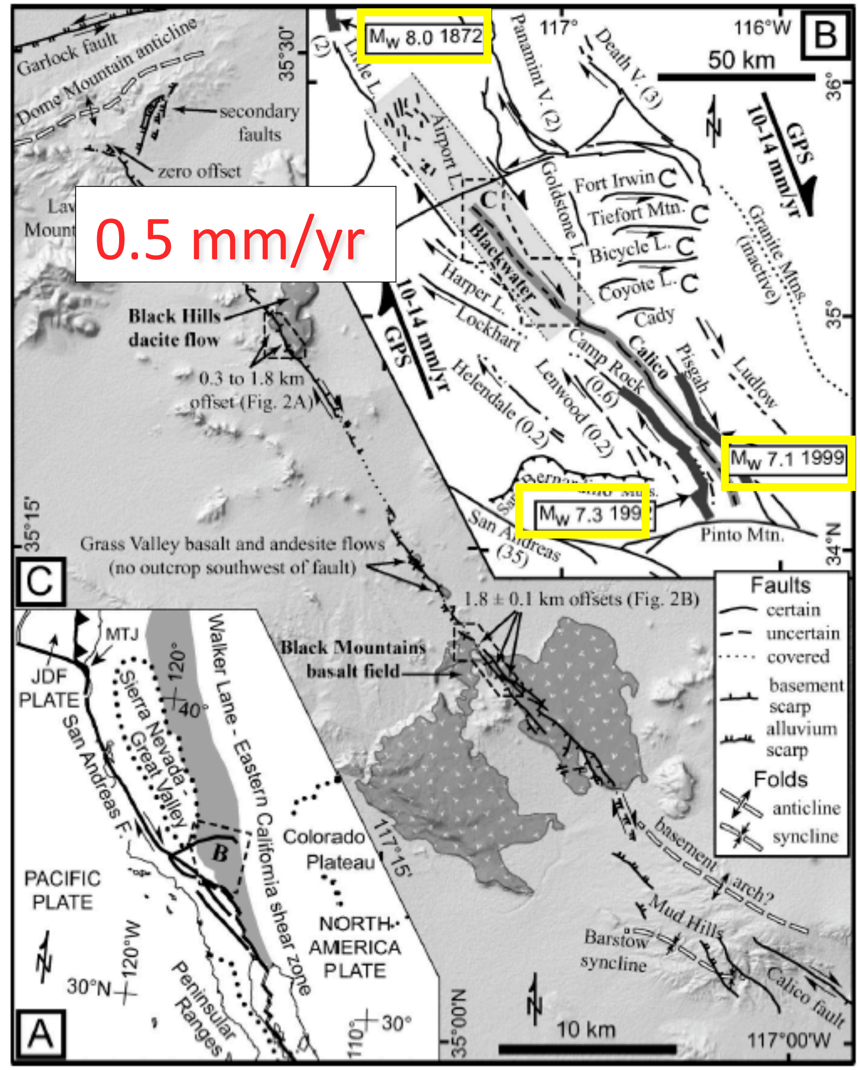




Weldon, 2004.

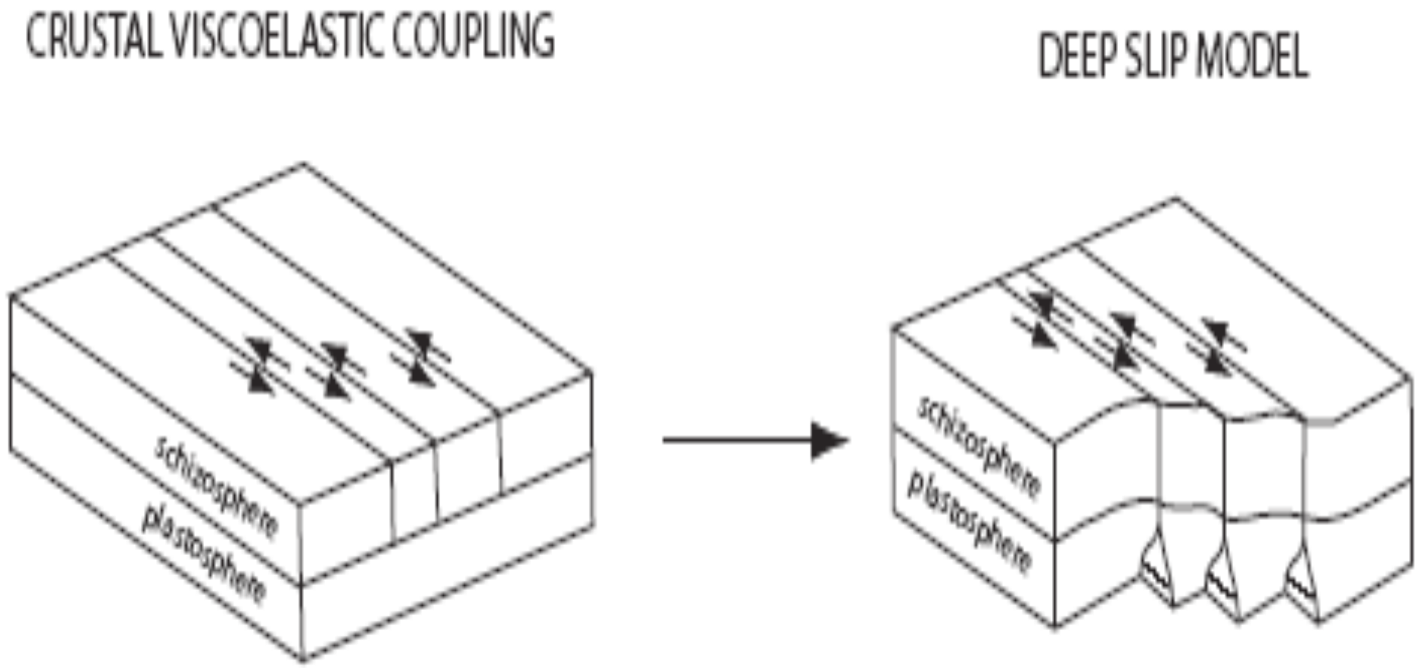


Peltzer et al., 2001



Oskin et al., 2004

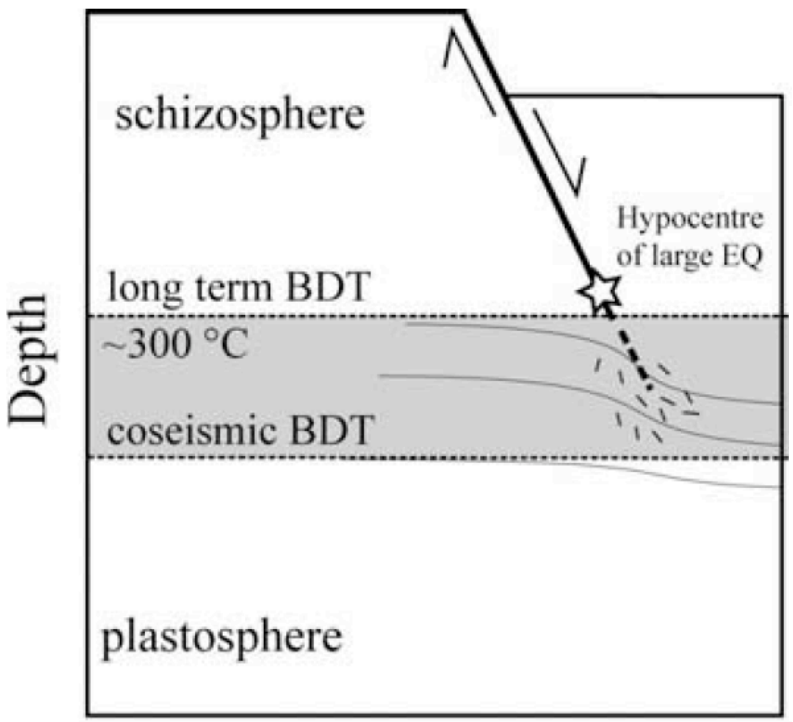
Time dependent behavior.

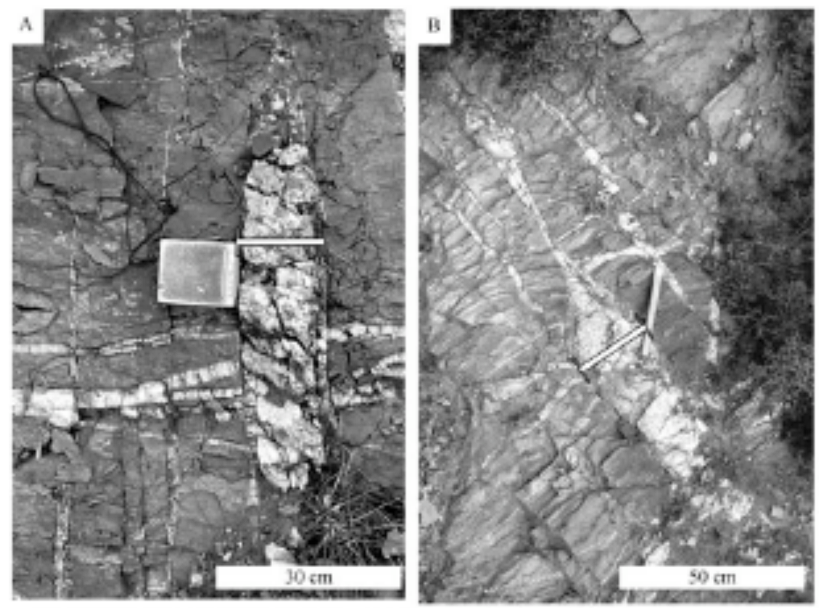


(Savage et al, 1999)

(Scholz, 2002)

Semi-brittle fractures





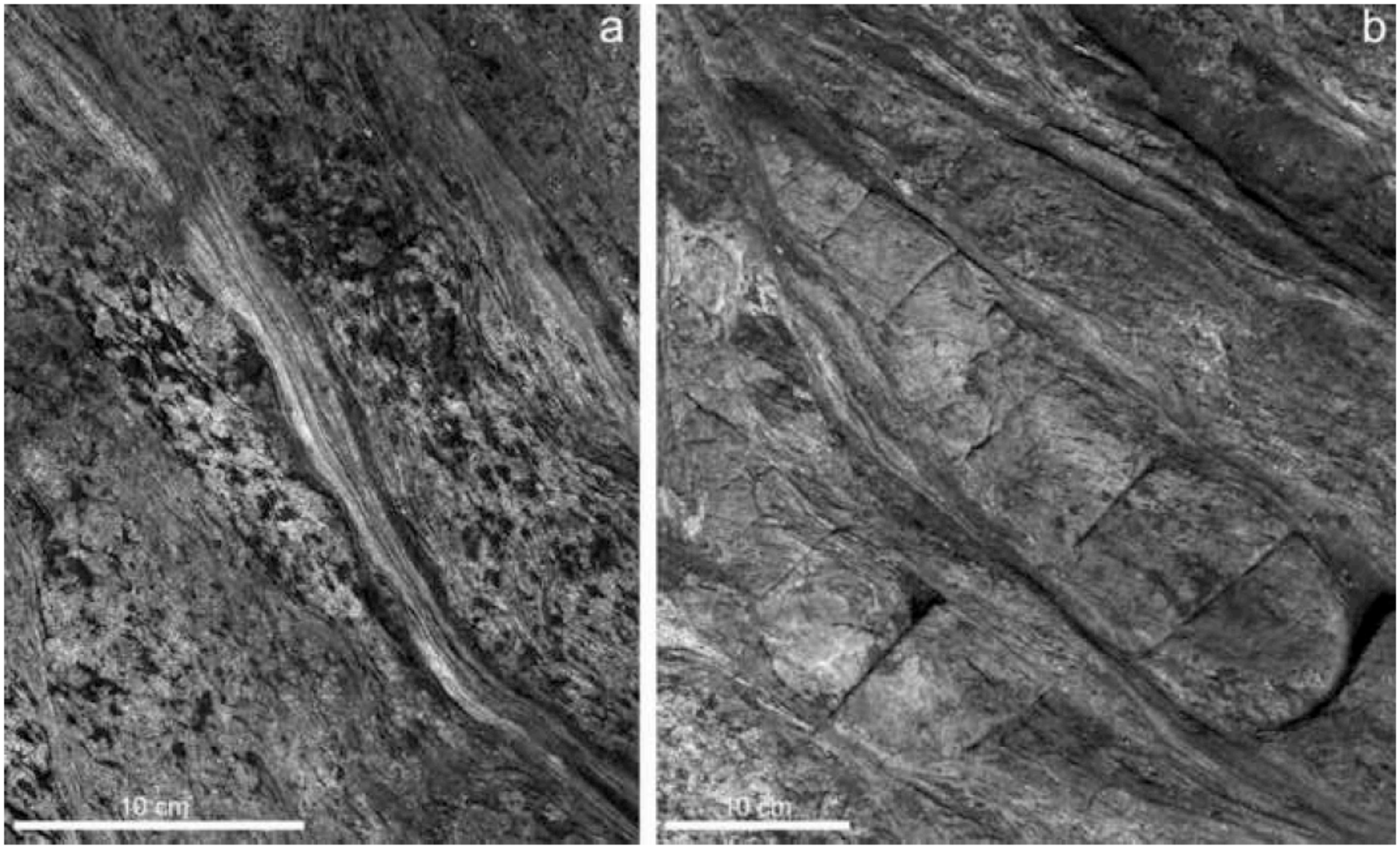
Nütscher and Stöckhert, 2008.

Localization in Greenschist (300°C to 450°C) to amphibole facies (450°C to 600°C).

- Localization in ductile shear zone is initiated by shear fractures (dilation). Fluids aid the localization (i.e. dynamic recrystallization, dissolution, etc) and form fluid filled fractures that can accommodate slip (Manktelow, Pennachioni, Handy, Carreras, etc…). It is a strain dependent phenomena.

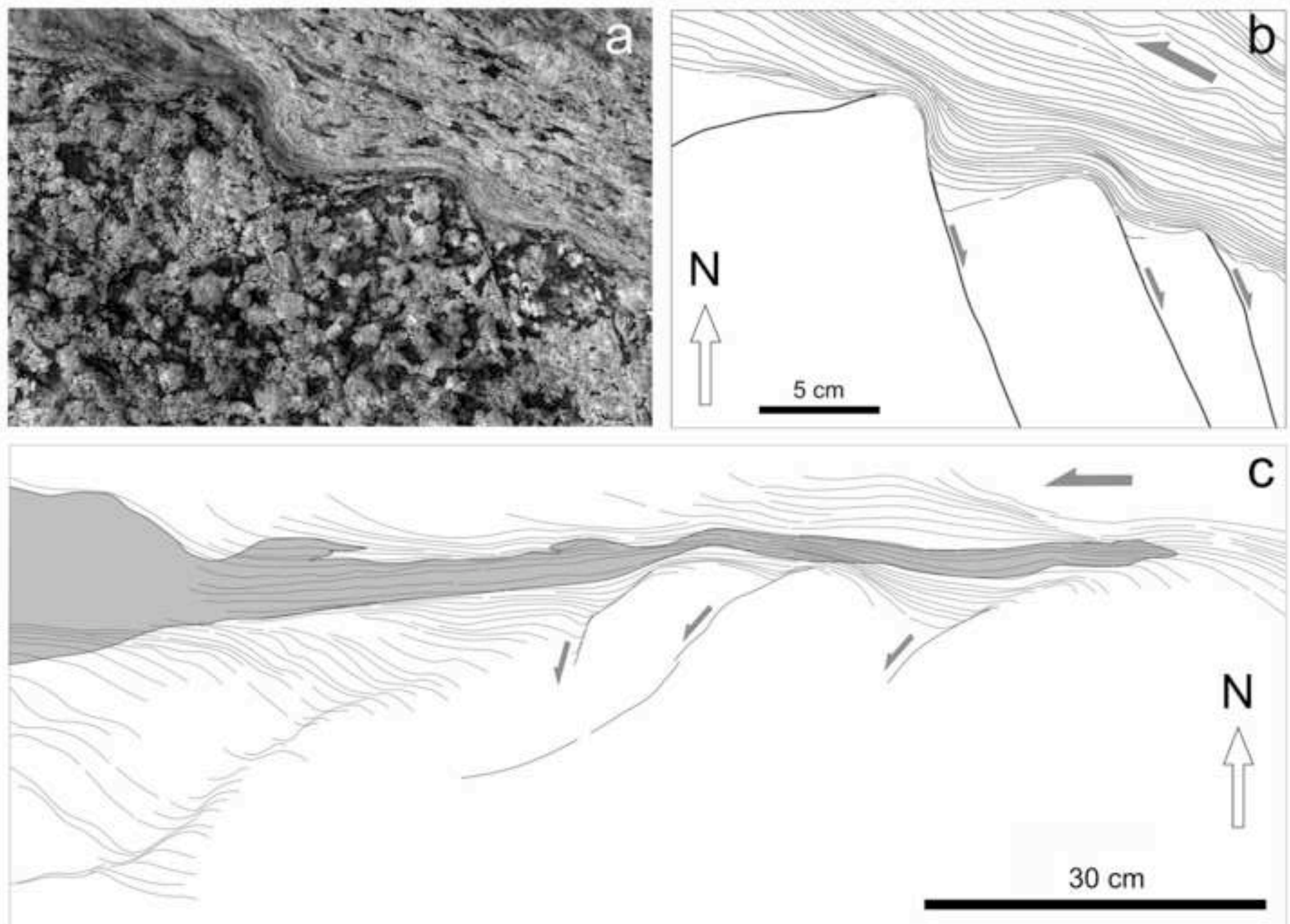
- Localization is a stain rate dependent phenomena. Phenomena such as dynamic recrystallization provide the bifurcation necessary for localization. (Braun, Chery, Hirth, etc…).

Rainy Lake Zone, Oblique strike-slip, Greenschist to Amphibole facies



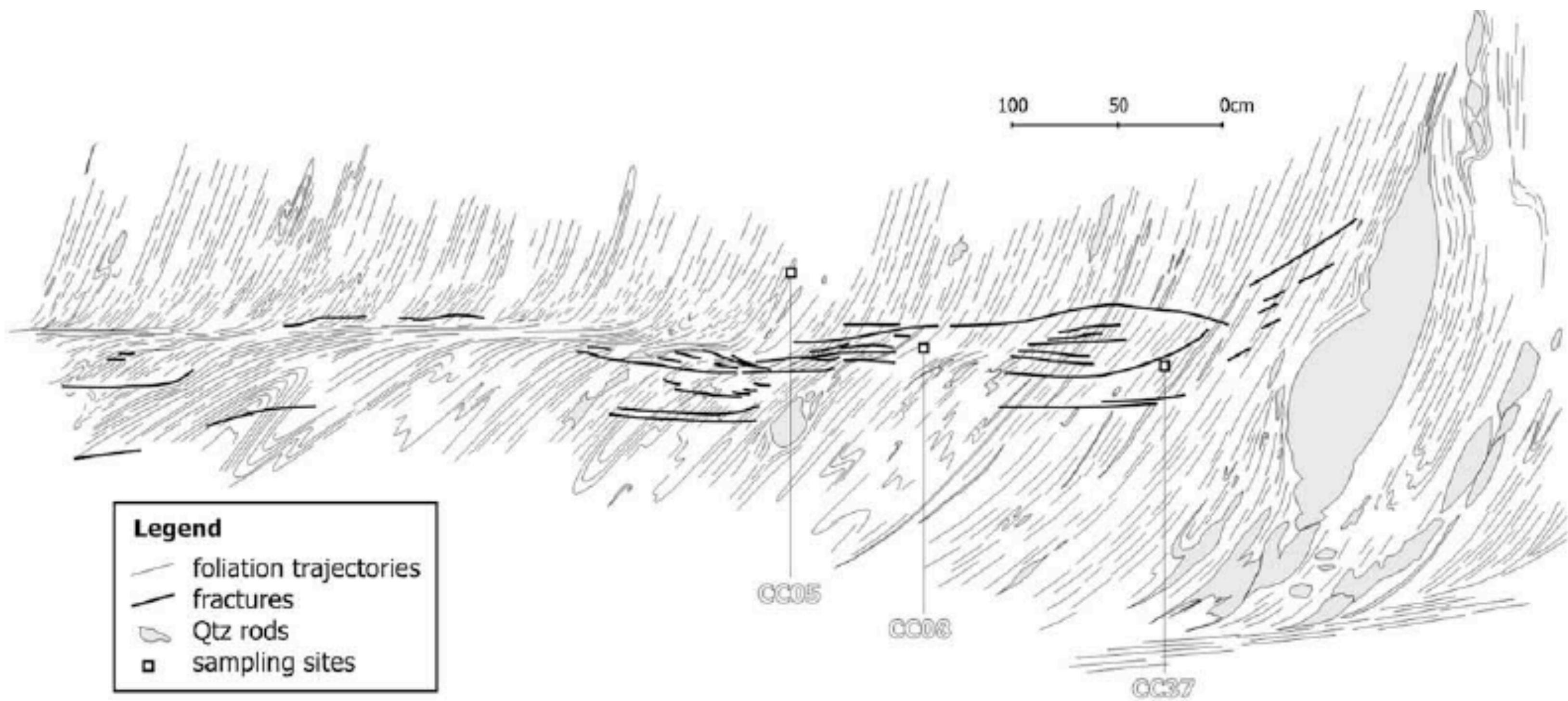
Carreras et al., 2010

Rainy Lake Zone, Oblique strike-slip, Greenschist to Amphibole facies



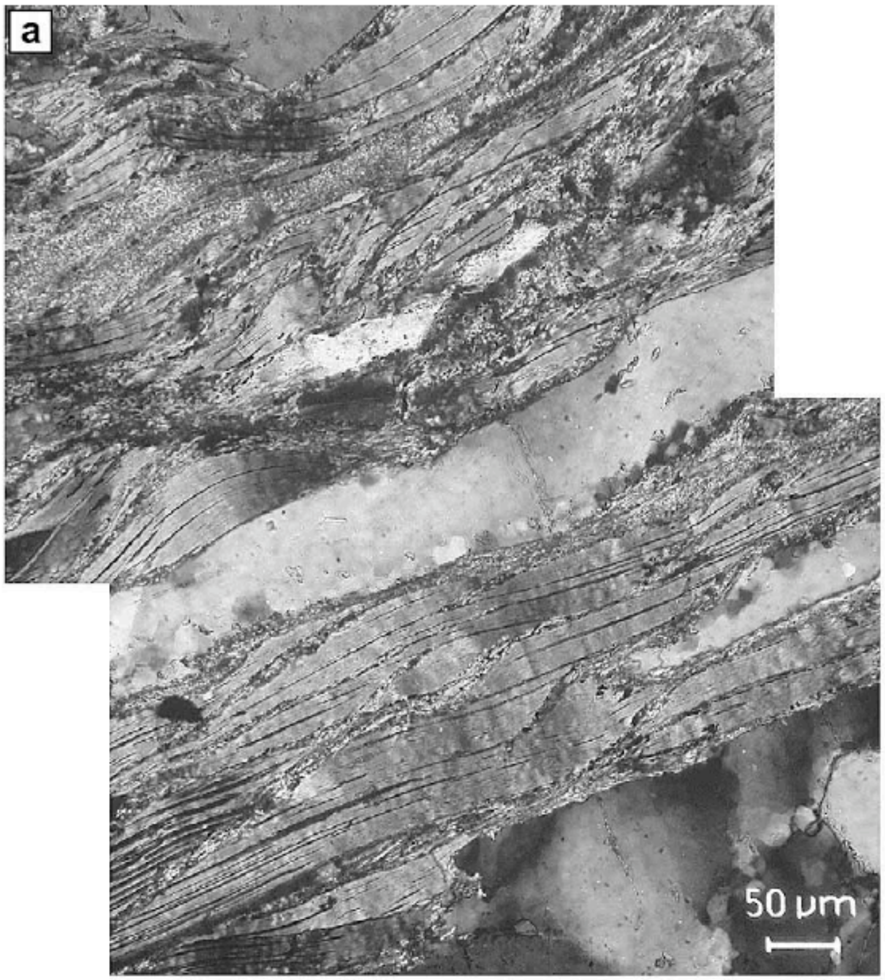
Carreras et al., 2010

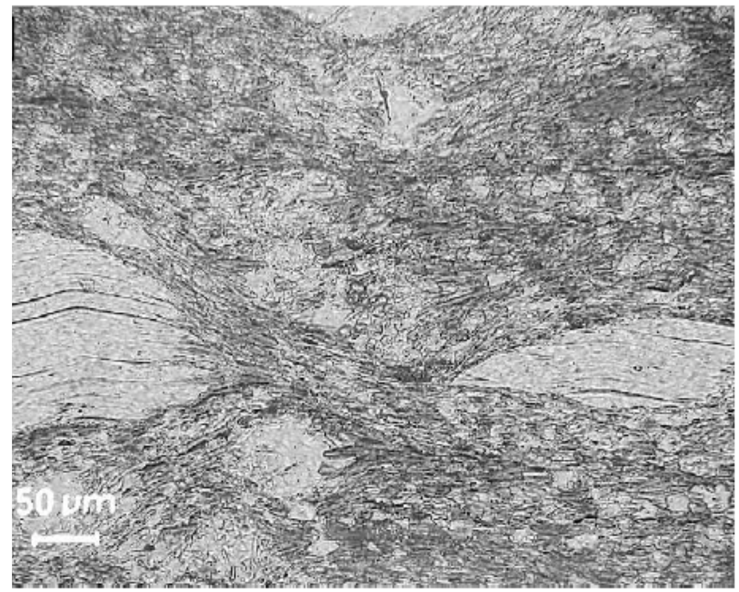
Cap de Creus Peninsula, Greensshict facies



Fusseis and Handy, 2008

Cap de Creus Peninsula, Greensshict facies

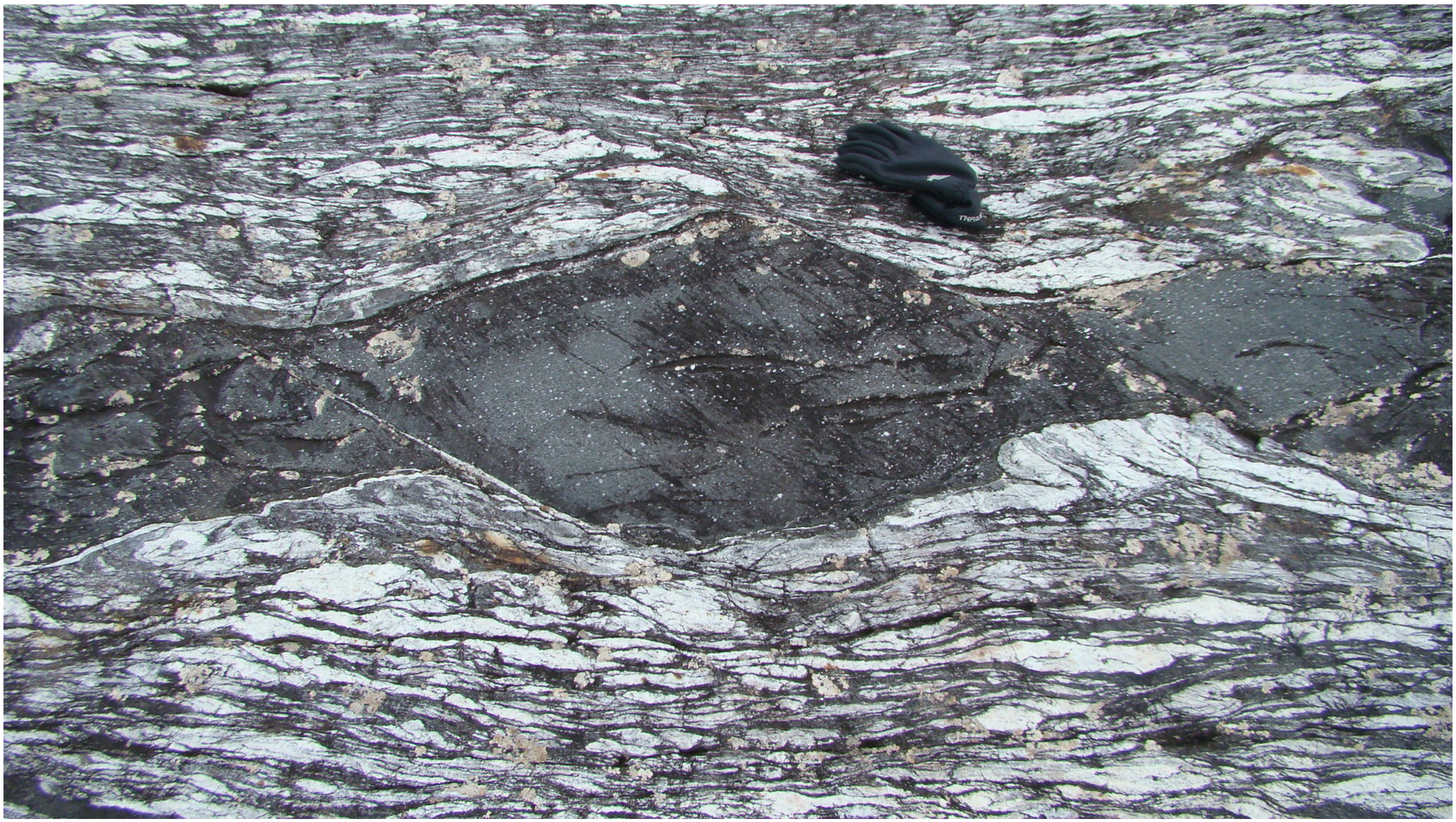




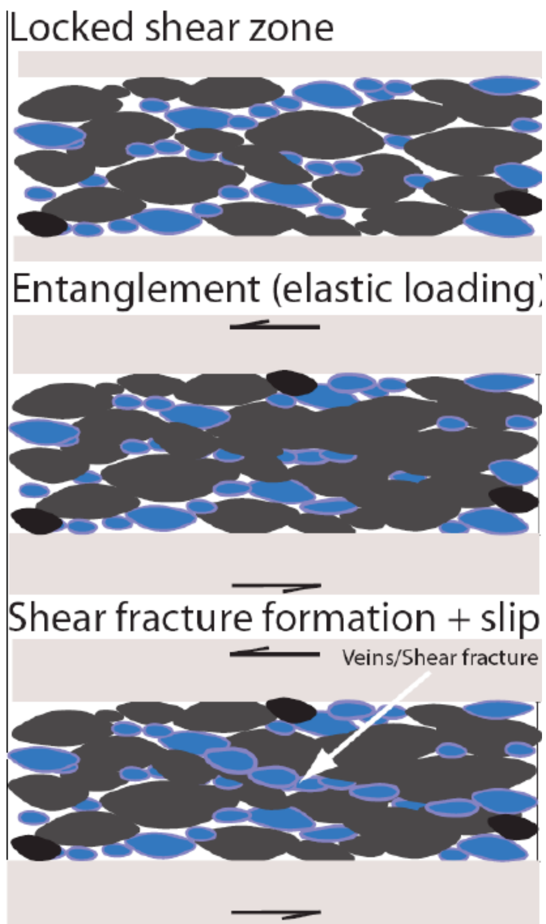
Fusseis and Handy, 2008

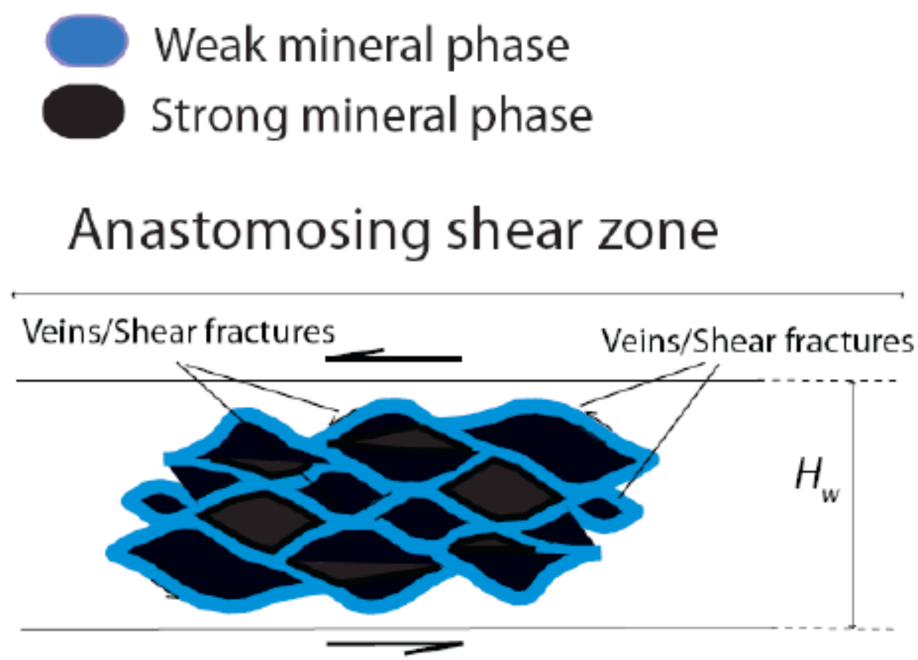
Cordillera Darwin (with Nick Hayman and Ian Dialzel)

Mafic dikes in silicic matrix, Amphibole facies.



Physical Model: 1D (continued)





Lavier and Bennett, 2010

Semi-brittle failure

- As strain accumulate by Mohr Coulomb failure acqueous fluid facilitate weakening:

  - By accumulation of brittle deformation.

  - By accumulation of plastic creep, likely dislocation creep→Diffusion creep

Failure criterion formulation (.

$$\int_0^{\varepsilon} \sigma \, d\varepsilon = C$$

We assume that the Mises Stress is constant.

We chose C consistent with ductile failure at the plastic transition for quartz.

$$\sigma \int_0^{\varepsilon} d\varepsilon = 4.10^6 J$$

$$\sigma \varepsilon_c = 4.10^6 J$$

Strain energy density

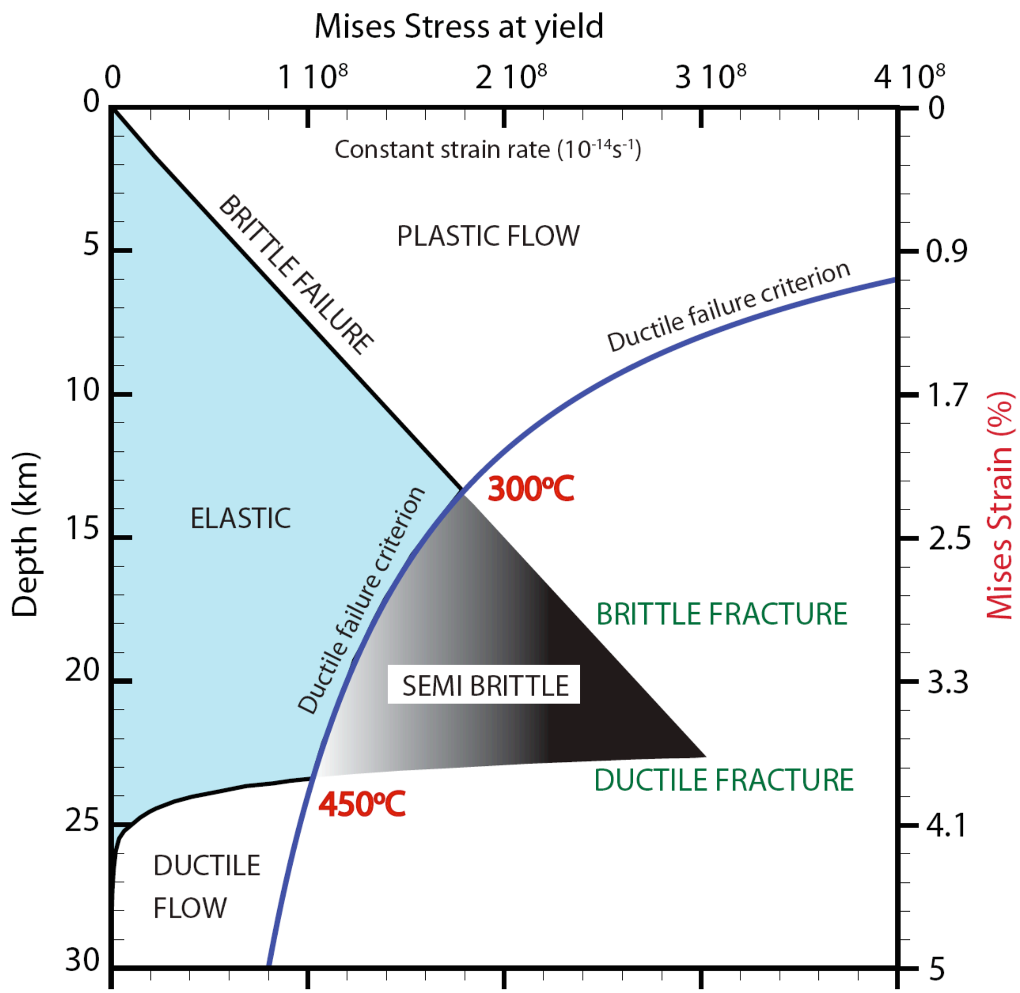
$$\int_0^{\varepsilon_c} \sigma^{II} d\varepsilon = C$$

$$\varepsilon_c = \varepsilon_b^e + \varepsilon_d^p$$

Elastic strain at failure

$\varepsilon_b^e \approx \frac{\sigma_{Mohr}}{E}$ where σ_{Mohr} is the stress at the Mohr-Coulomb failure criteria.

$\varepsilon_d^p \approx \frac{C}{\sigma_{creep}}$ where σ_{creep} is the dislocation creep stress.


Mises Stress at yield
0
1 10^8
2 10^8
3 10^8
4 10^8
Constant strain rate (10^-14s^-1)
PLASTIC FLOW
BRITTLE FAILURE
Ductile failure criterion
ELASTIC
300ºC
Ductile failure criterion
BRITTLE FRACTURE
SEMI BRITTLE
DUCTILE FRACTURE
450ºC
DUCTILE
FLOW
Depth (km)
0
5
10
15
20
25
30
Mises Strain (%)
0
0.9
1.7
2.5
3.3
4.1
5

Forced localization: Parameterization of Rheology.

Mohr-Coulomb Elasto-Plastic.

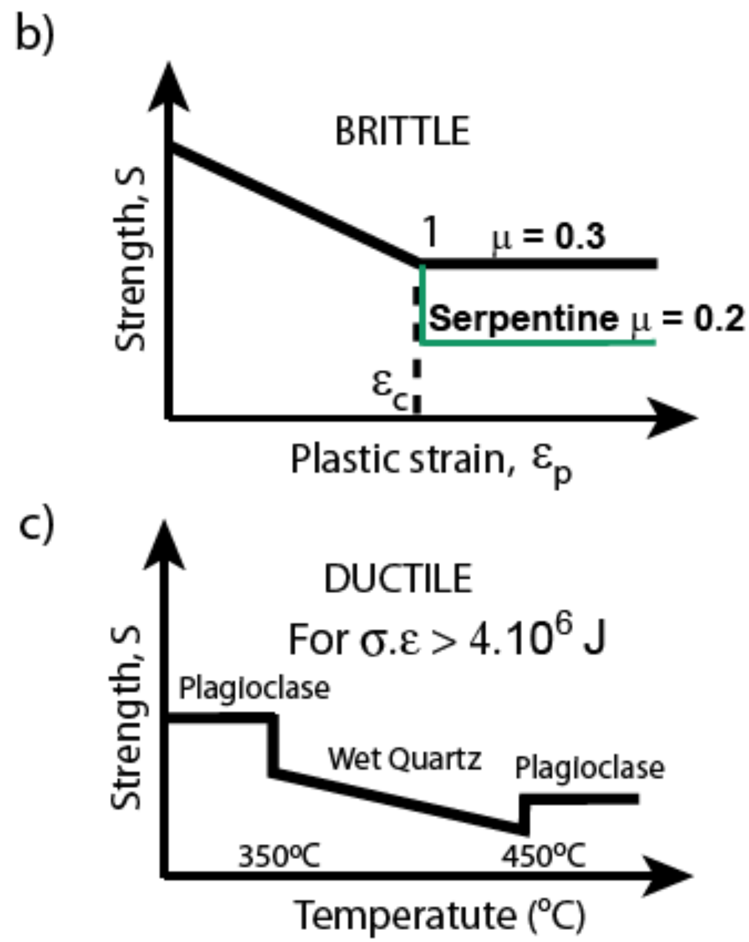
Cohesional and frictional.

Cohesion and friction loss.

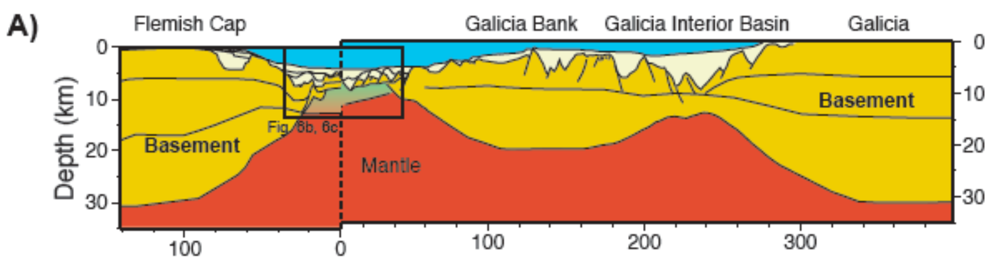
Power law creep.

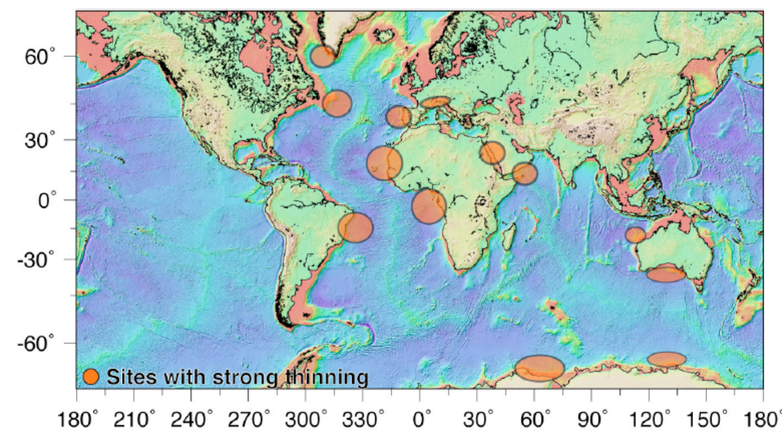
Bi-mineralic viscosity weakening.
(Plagioclase →→→→→wet quartz)

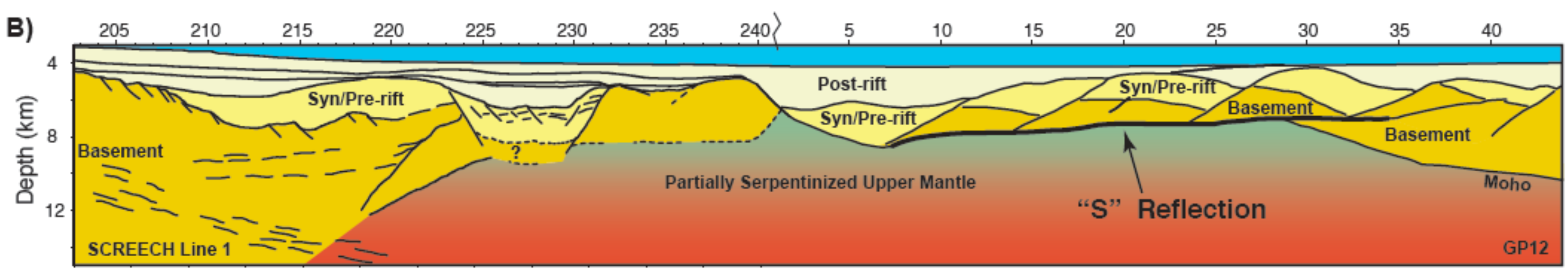
Parameterization of shear localization.



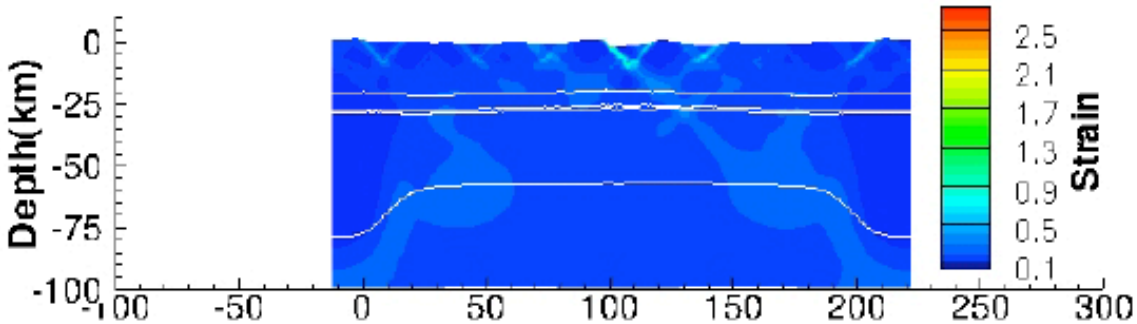
Extreme thinning with little faulting



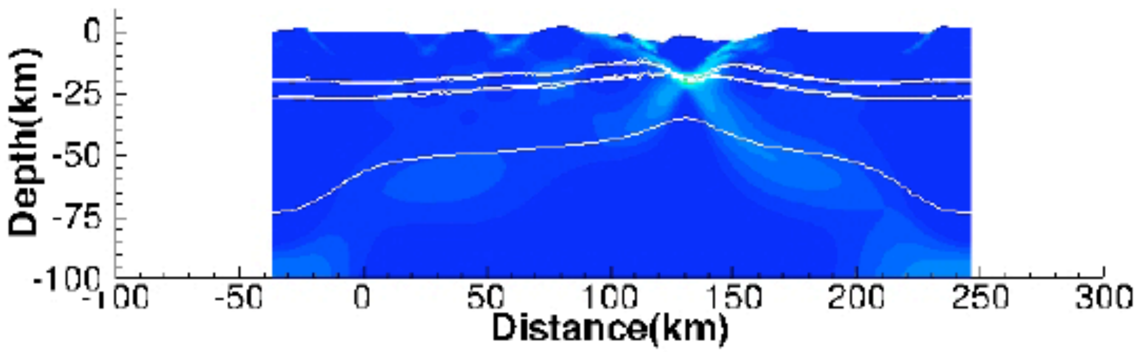




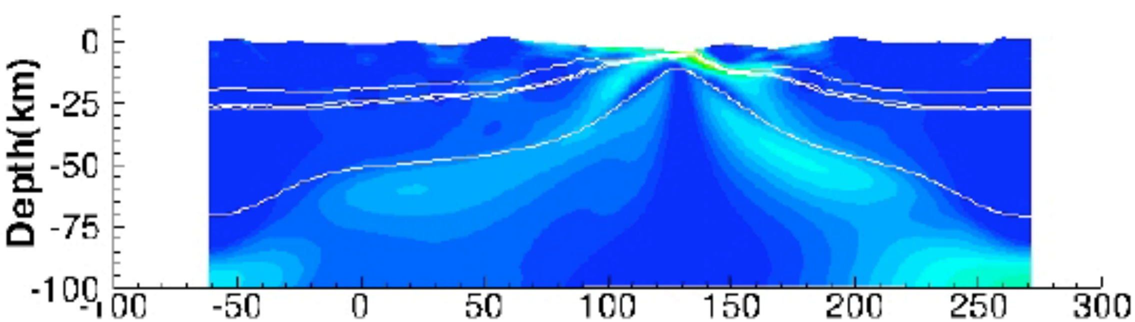
Hopper et al, 2007



STRETCHING



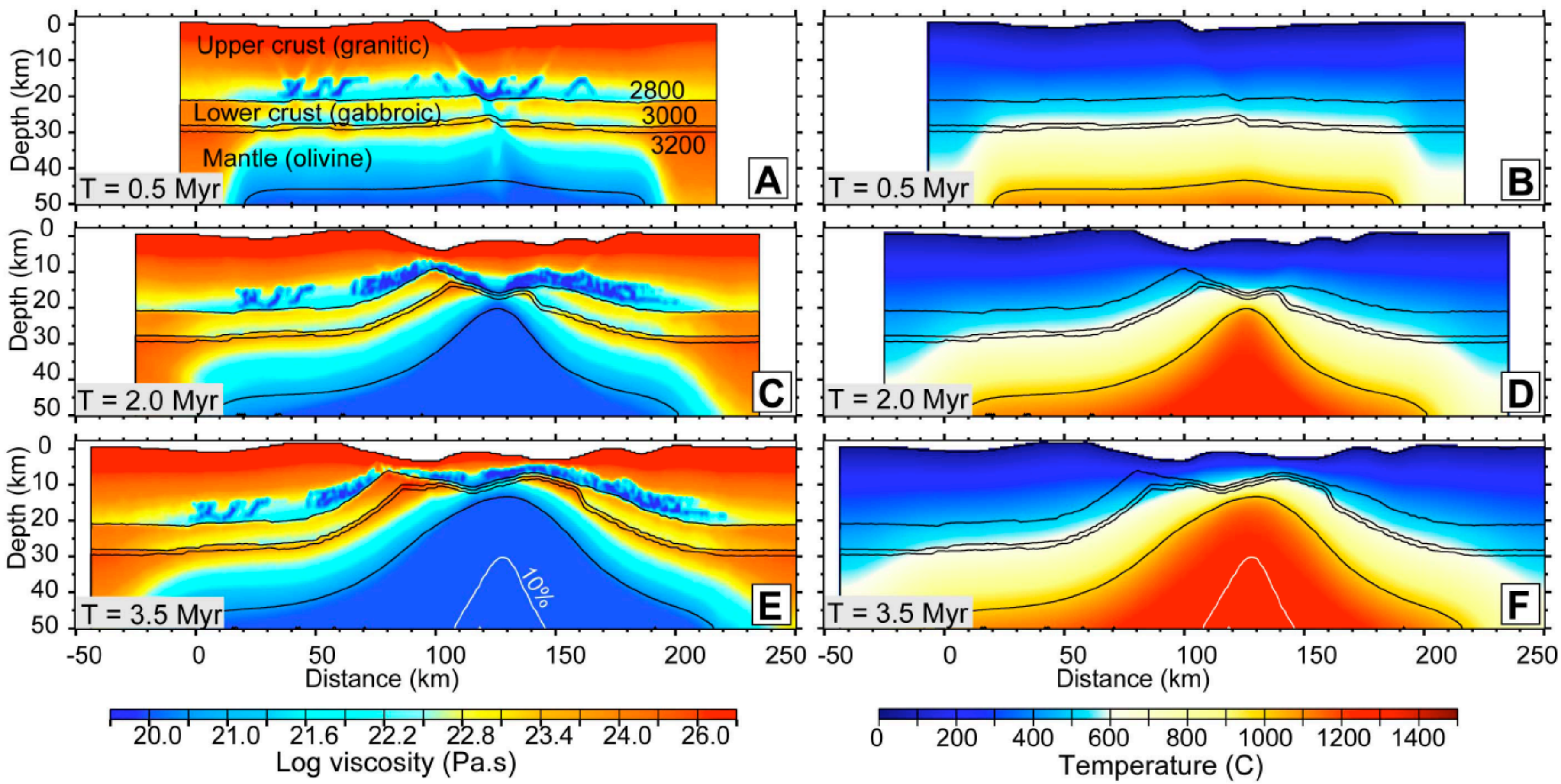
THINNING



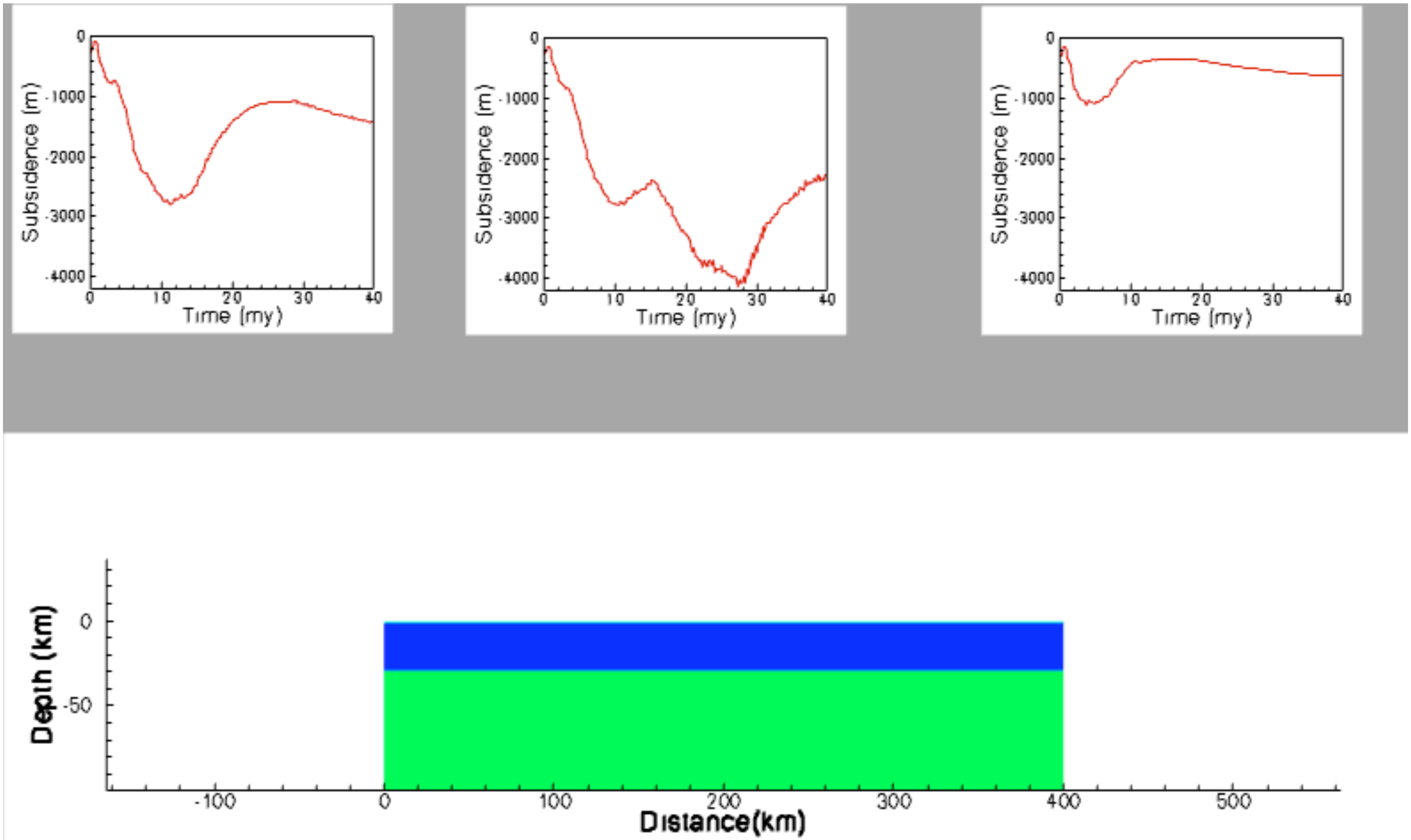
EXHUMATION

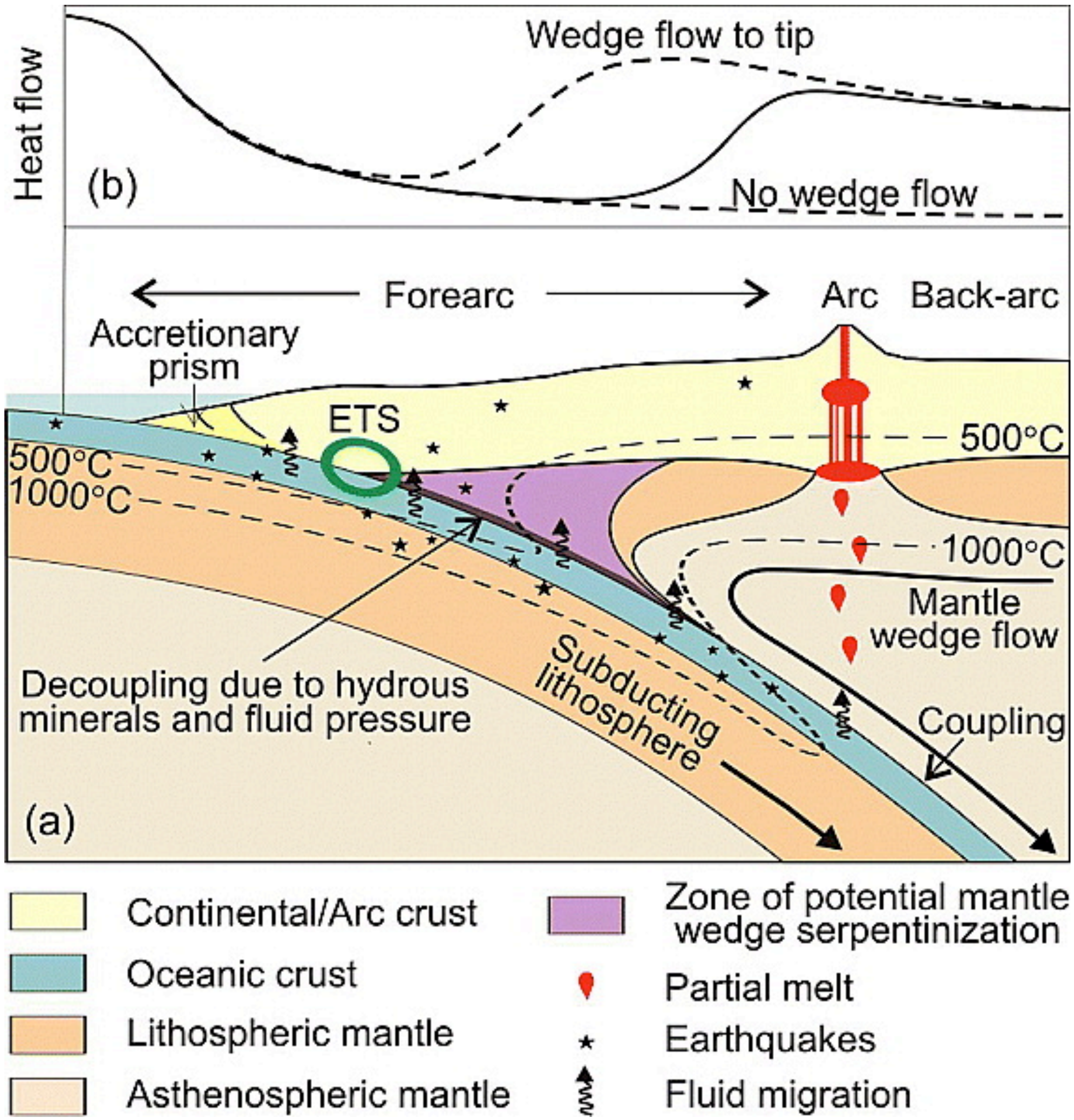
Modeling rifting with pervasive weakening in the middle crust

FAST EXTENSION = 2.5 cm.yr-1

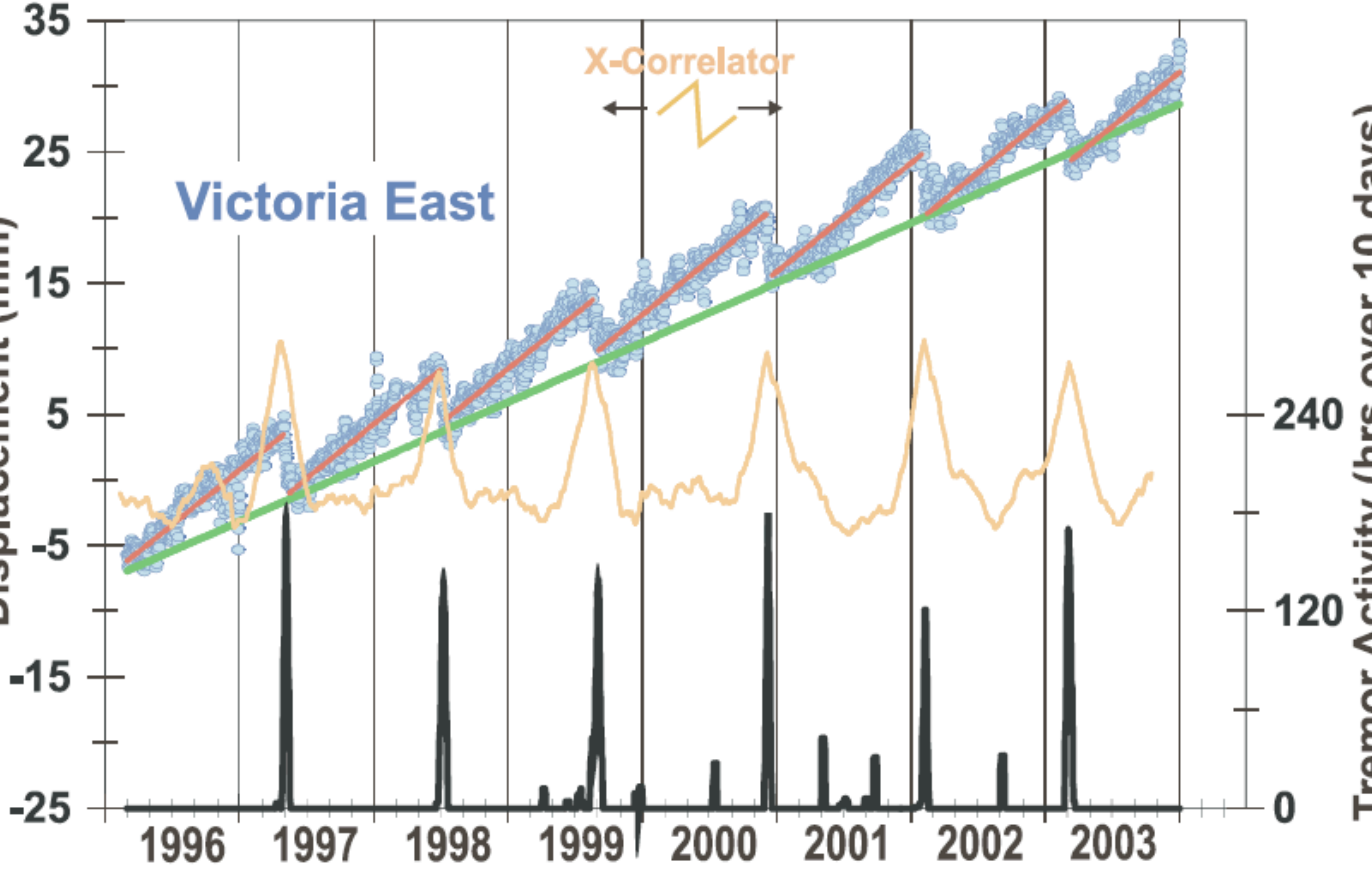


Boudinage at lithospheric scale





Heat flow
(b)
Wedge flow to tip
No wedge flow
Forearc
Arc
Back-arc
Accretionary prism
ETS
500°C
1000°C
500°C
1000°C
Mantle wedge flow
Decoupling due to hydrous minerals and fluid pressure
Subducting lithosphere
Coupling
(a)
Continental/Arc crust
Oceanic crust
Lithospheric mantle
Asthenospheric mantle
Zone of potential mantle wedge serpentinization
Partial melt
Earthquakes
Fluid migration

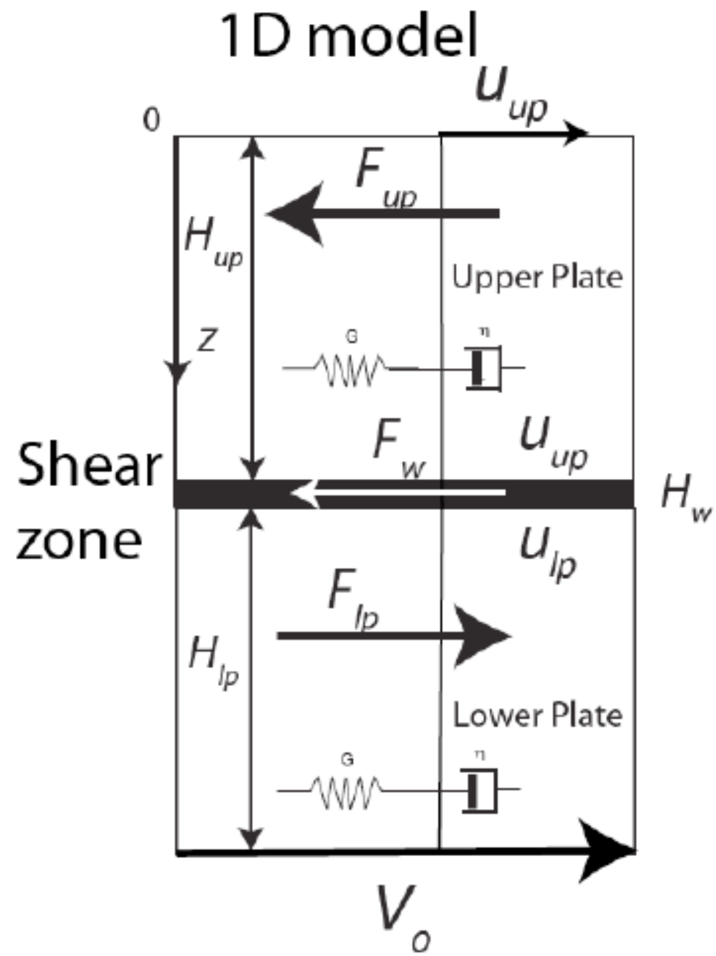

Victoria East
X-Correlator
35
25
15
5
-5
-15
-25
240
120
0
1996
1997
1998
1999
2000
2001
2002
2003

Physical Model: 1D (continued)

$$\ddot{S} + D\dot{S} + \omega_o^2 \left(S + S_c\right) = 0$$

$$D = G\left(\overline{\eta}_L + \overline{\eta}_w\right) / \overline{\eta}_L \overline{\eta}_w$$

$$\omega_o^2 = G\upsilon' / S_c \overline{\eta}_L \overline{\eta}_w$$



Model

- Force balance

$$\frac{1}{H_{up}}\int_{0}^{H_{up}}\tau_{up}dz+\frac{1}{H_w}\int_{H_{up}}^{H_{up}+H_w}\tau_w dz-\frac{1}{H_{lp}}\int_{H_{up}+H_w}^{H_{up}+H_w+H_{lp}}\tau_{lp}dz=0$$

or

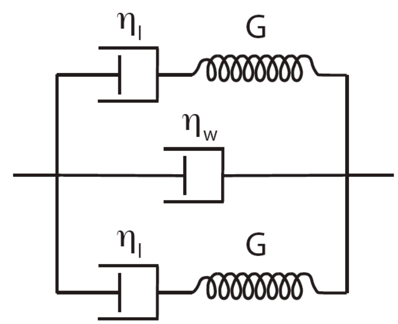
$$\overline{\tau}_{up}+\overline{\tau}_w-\overline{\tau}_{lp}=0$$

- Rheology

$$T_i\frac{d\overline{\tau}_i}{dt}=-\left[\overline{\tau}_i-\overline{\eta}_i\overline{u}_i\right]$$

- Bi-viscous Model

$$\ddot{S}+G\left(\frac{1}{\overline{\eta}_w}+\frac{1}{\overline{\eta}_L}\right)\dot{S}=0$$



- Dynamic form

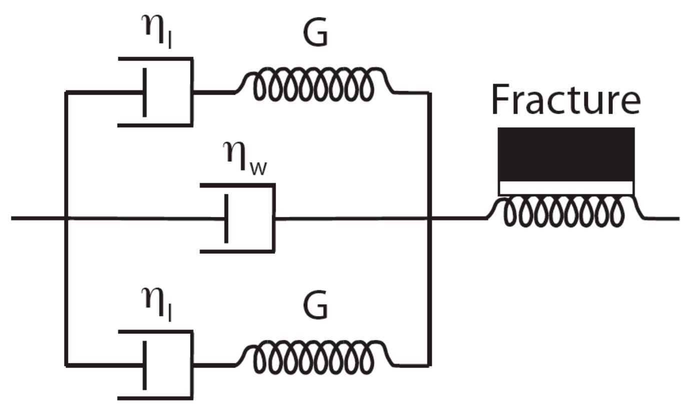
$$\frac{\overline{\eta}_L\overline{\eta}_w}{G}\ddot{S}=-\overline{\tau}_w-\overline{\eta}_L\dot{S}$$

$$\frac{MLT^{-2}}{L^2}=\sum\frac{F}{L^2}$$

Slip oscillator

Perturbation is a circular fracture of stiffness, *v'* that corresponds to an asperity.

$$\bar{\tau}_f = \frac{7\pi\upsilon}{12S_c}\left(S + S_c\right)$$

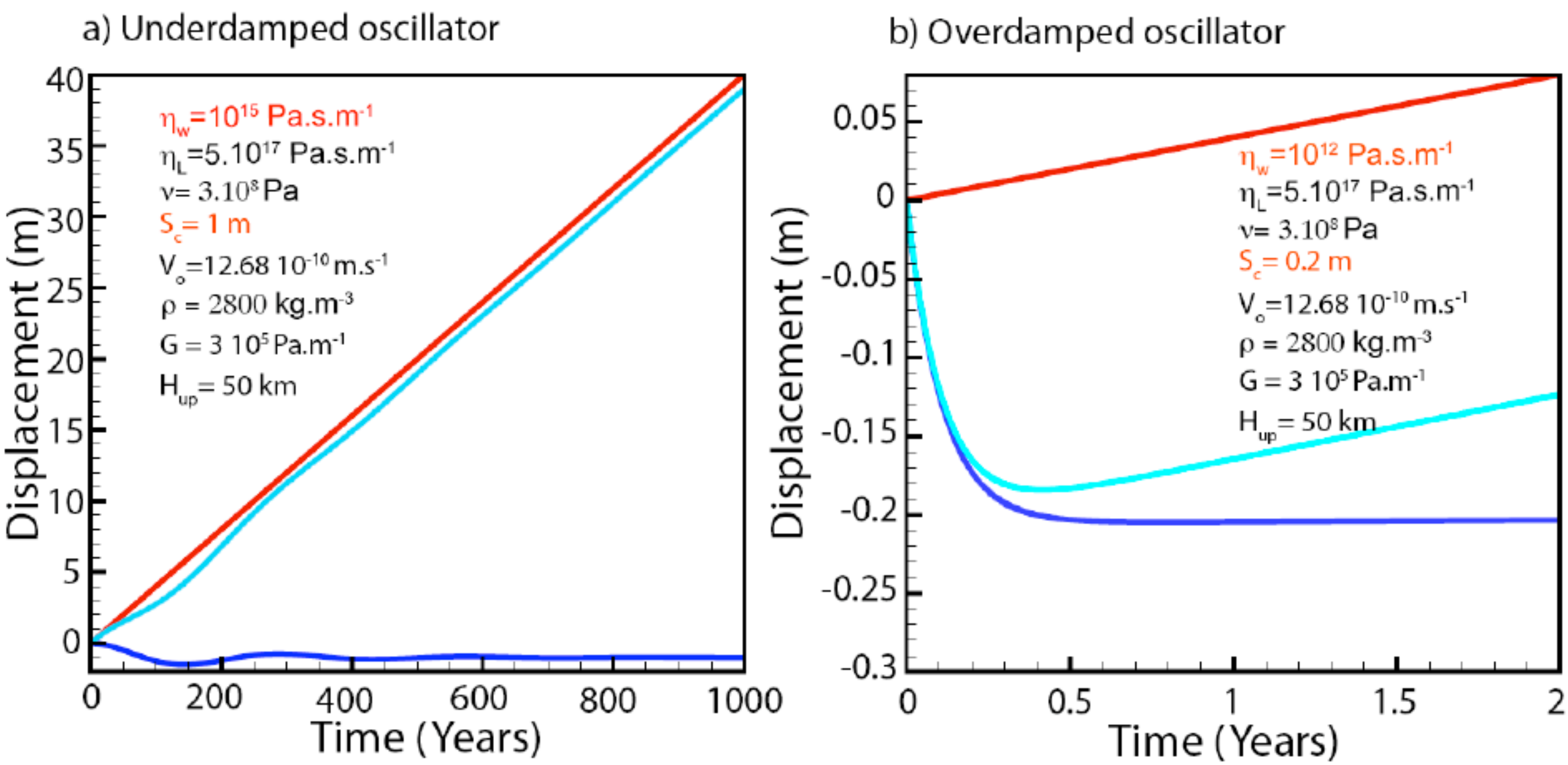


$$\frac{\bar{\eta}_w \bar{\eta}_L}{G}\ddot{S} + \left(\bar{\eta}_L + \bar{\eta}_w\right)\dot{S} + \frac{\upsilon'}{S_c}\left(S + S_c\right) = 0$$

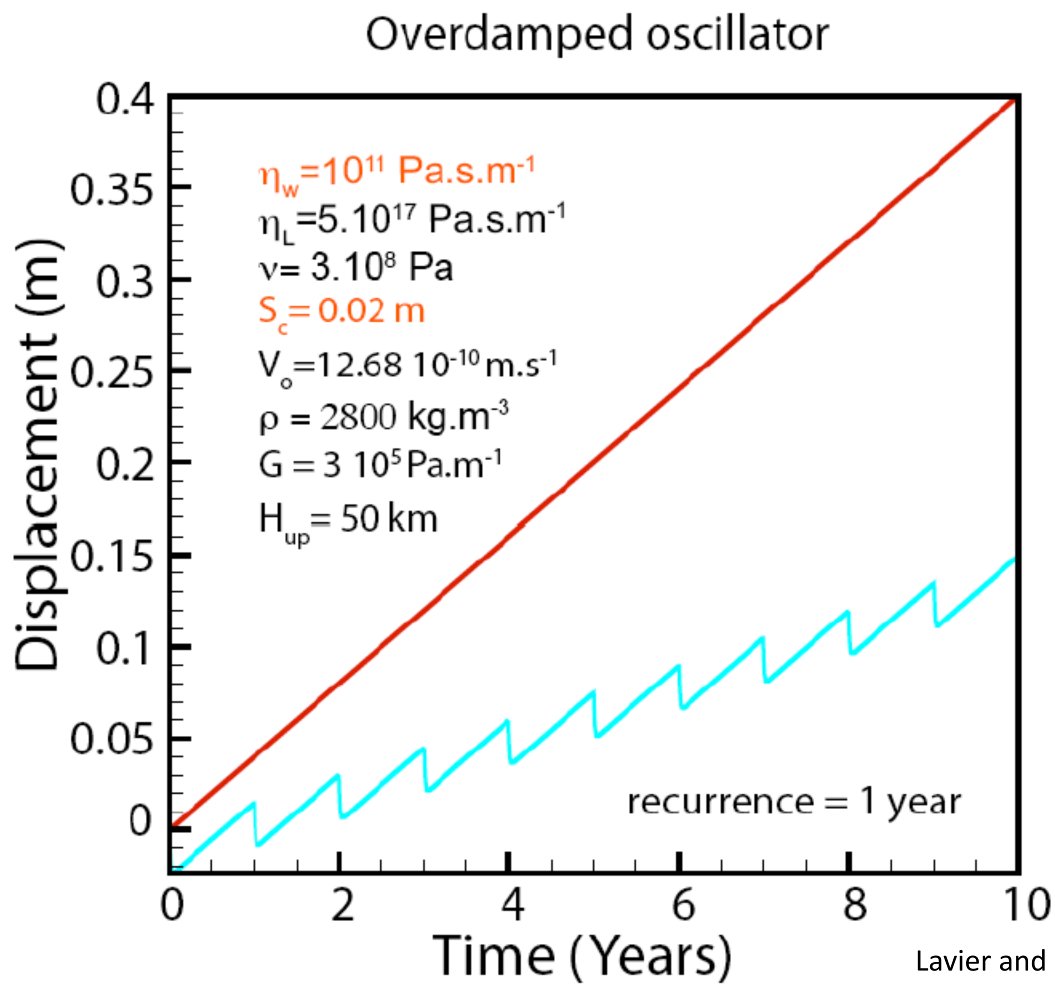
or

$$\ddot{S} + G\frac{\left(\bar{\eta}_L + \bar{\eta}_w\right)}{\bar{\eta}_w \bar{\eta}_L}\dot{S} + \frac{G\upsilon'}{\bar{\eta}_w \bar{\eta}_L S_c}\left(S + S_c\right) = 0$$

Solutions


a) Underdamped oscillator
b) Overdamped oscillator
Displacement (m)
Time (Years)
ηw=10¹⁵ Pa.s.m⁻¹
ηL=5.10¹⁷ Pa.s.m⁻¹
ν= 3.10⁸ Pa
Sc= 1 m
Vo=12.68 10⁻¹⁰ m.s⁻¹
ρ = 2800 kg.m⁻³
G = 3 10⁵ Pa.m⁻¹
Hup= 50 km
ηw=10¹² Pa.s.m⁻¹
Sc= 0.2 m

Physical Model: 1D (continued)



Lavier and Bennett, 2010

Critical behavior

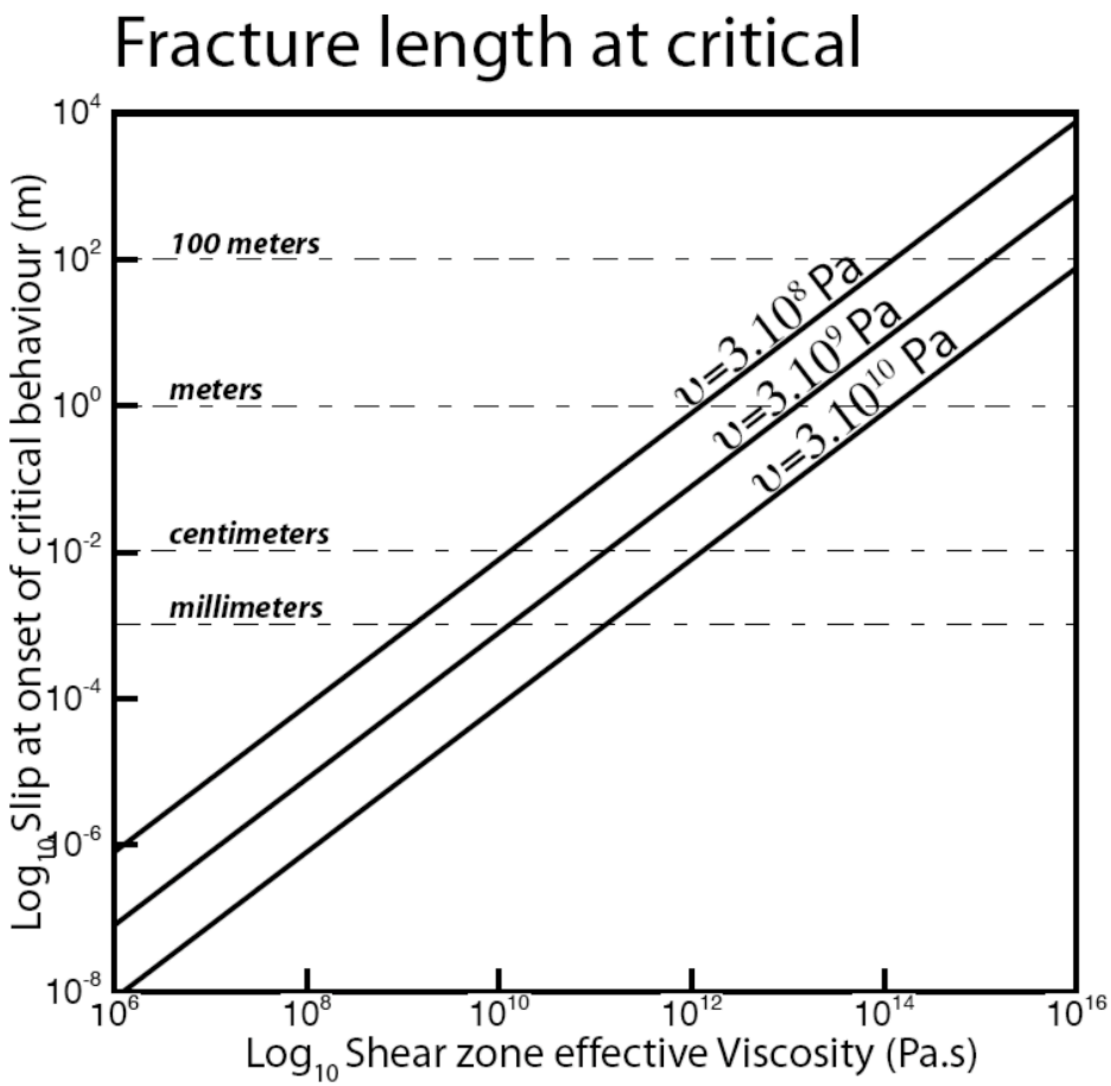
$$S_c = \frac{4\upsilon'\bar{\eta}_L\bar{\eta}_w}{G\left(\bar{\eta}_L + \bar{\eta}_w\right)^2}$$

$$E\frac{S_c}{H_w} = \frac{4\upsilon'\eta_L\eta_w H_L^2}{\left(\eta_L H_w + \eta_w H_L\right)^2}$$

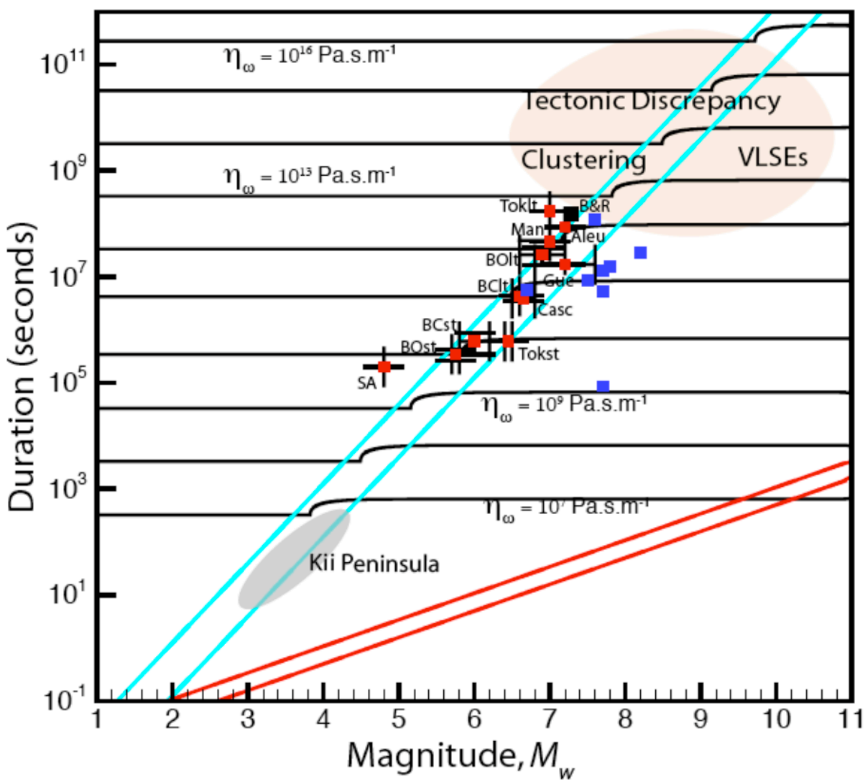
or for $\eta_L \gg \eta_w$

$$\tau_{loading} = 4\upsilon' \frac{\eta_w H_L}{\eta_L H_W}$$

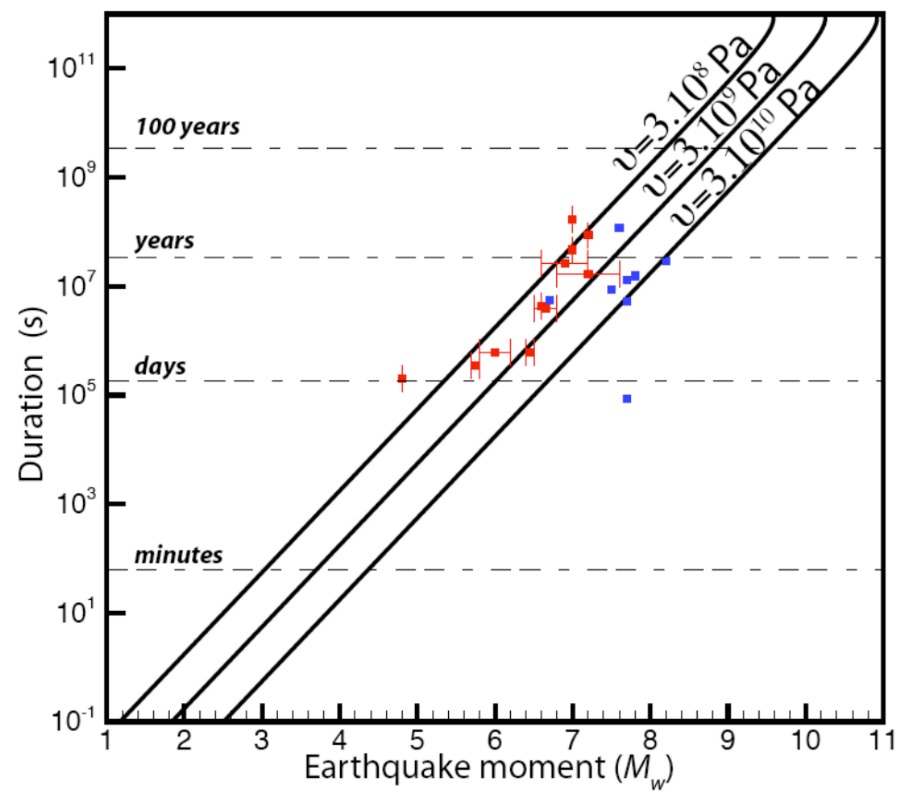
Fracture or asperity diameter



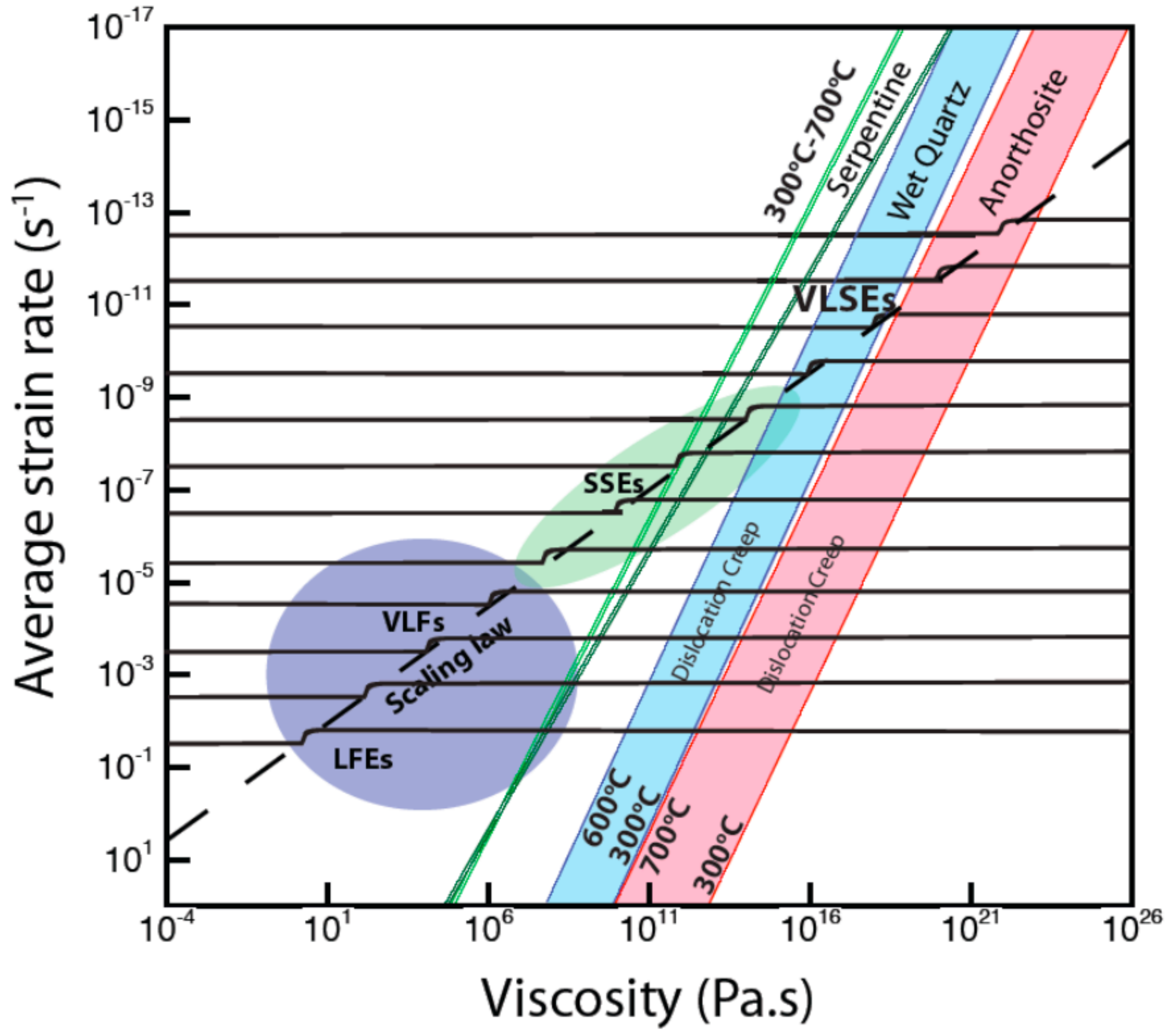
Comparison to slow slip and postseismic after slip scalings



Moment vs. duration at critical

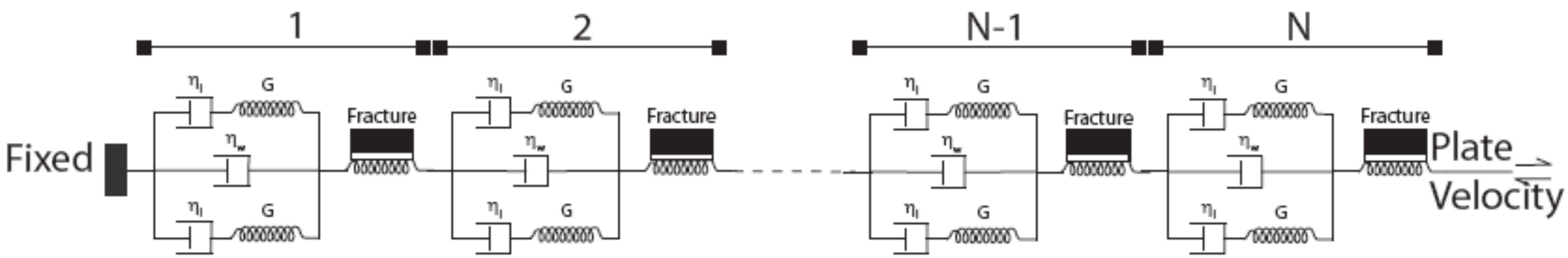


Comparison of scaling with rheology



In 2D the oscillator becomes a wave equation that propagates slip, *u(x,t)*.

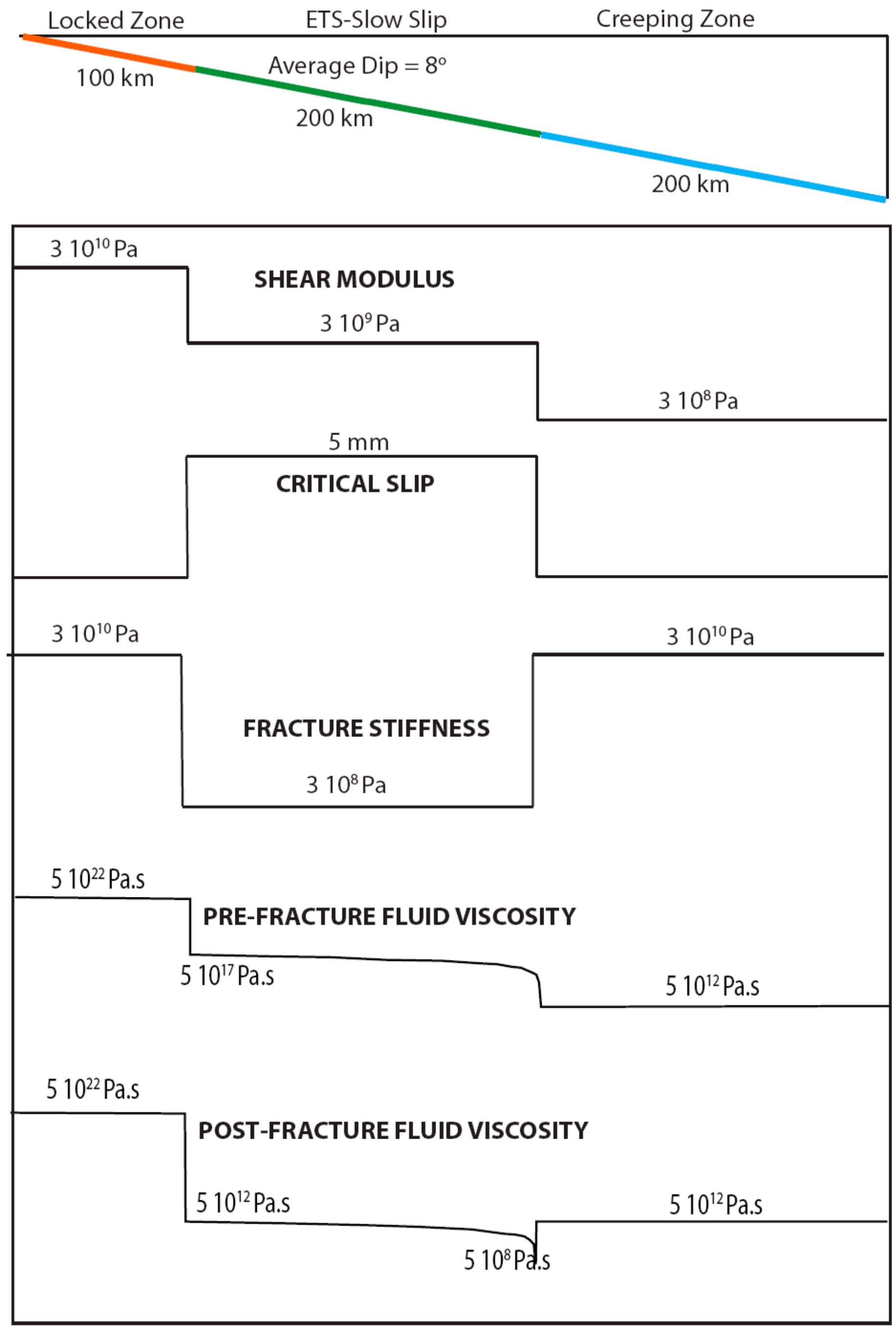
FAULT ZONE REPRESENTATION, LENGTH W



$$\frac{\partial^2 u(x,t)}{\partial t^2} + D\frac{\partial u(x,t)}{\partial t} - W^2\omega_o^2 \frac{\partial^2 u(x,t)}{\partial x^2} = 0$$

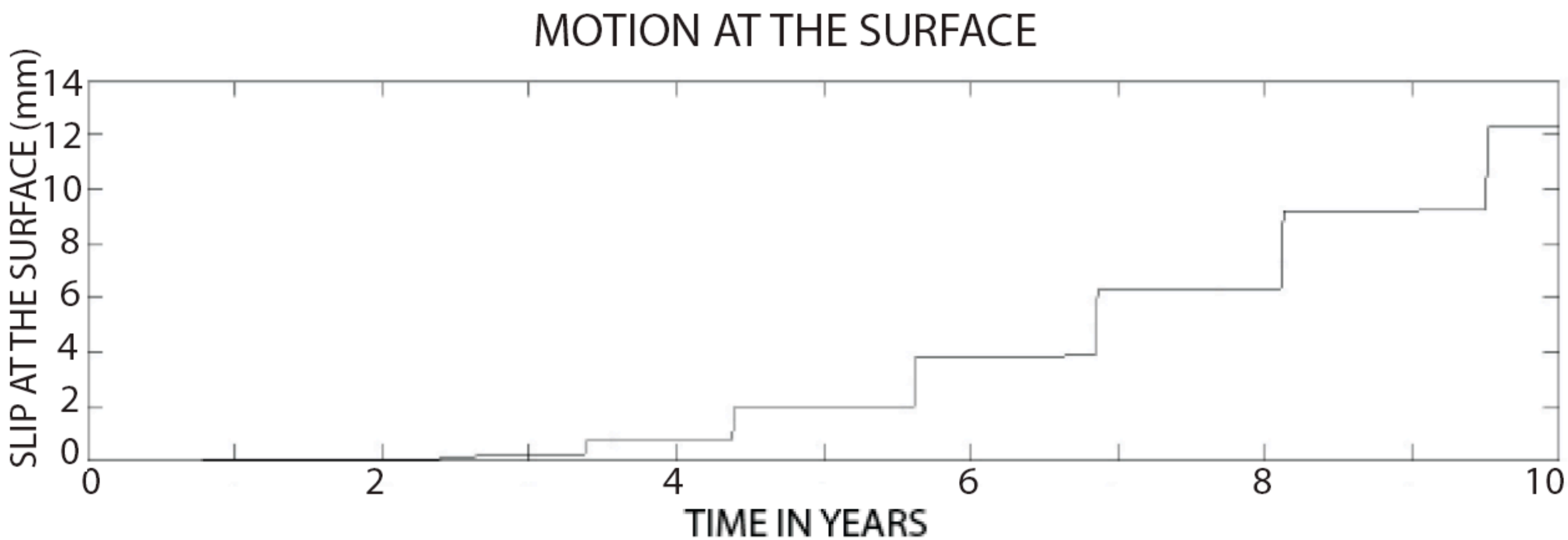
$$D = G\left(\bar{\eta}_L + \bar{\eta}_w\right) / \bar{\eta}_L \bar{\eta}_w \qquad \omega_o^2 = G\upsilon' / S_c \bar{\eta}_L \bar{\eta}_w$$

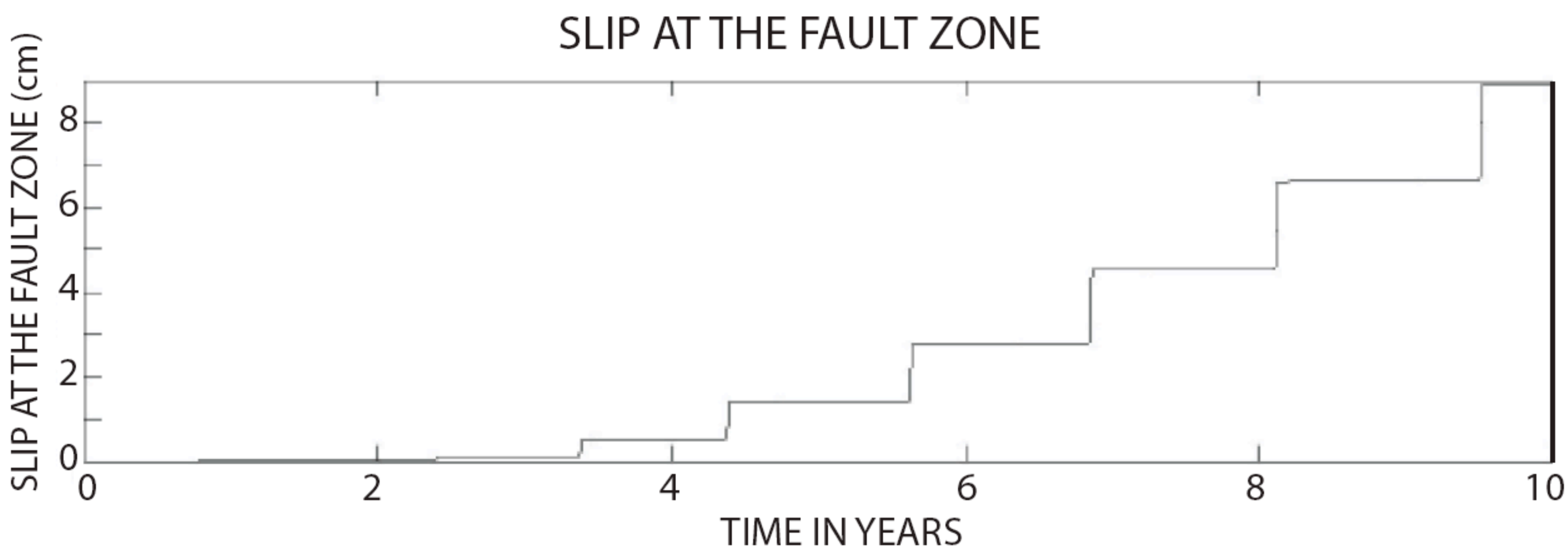
MODEL SETUP



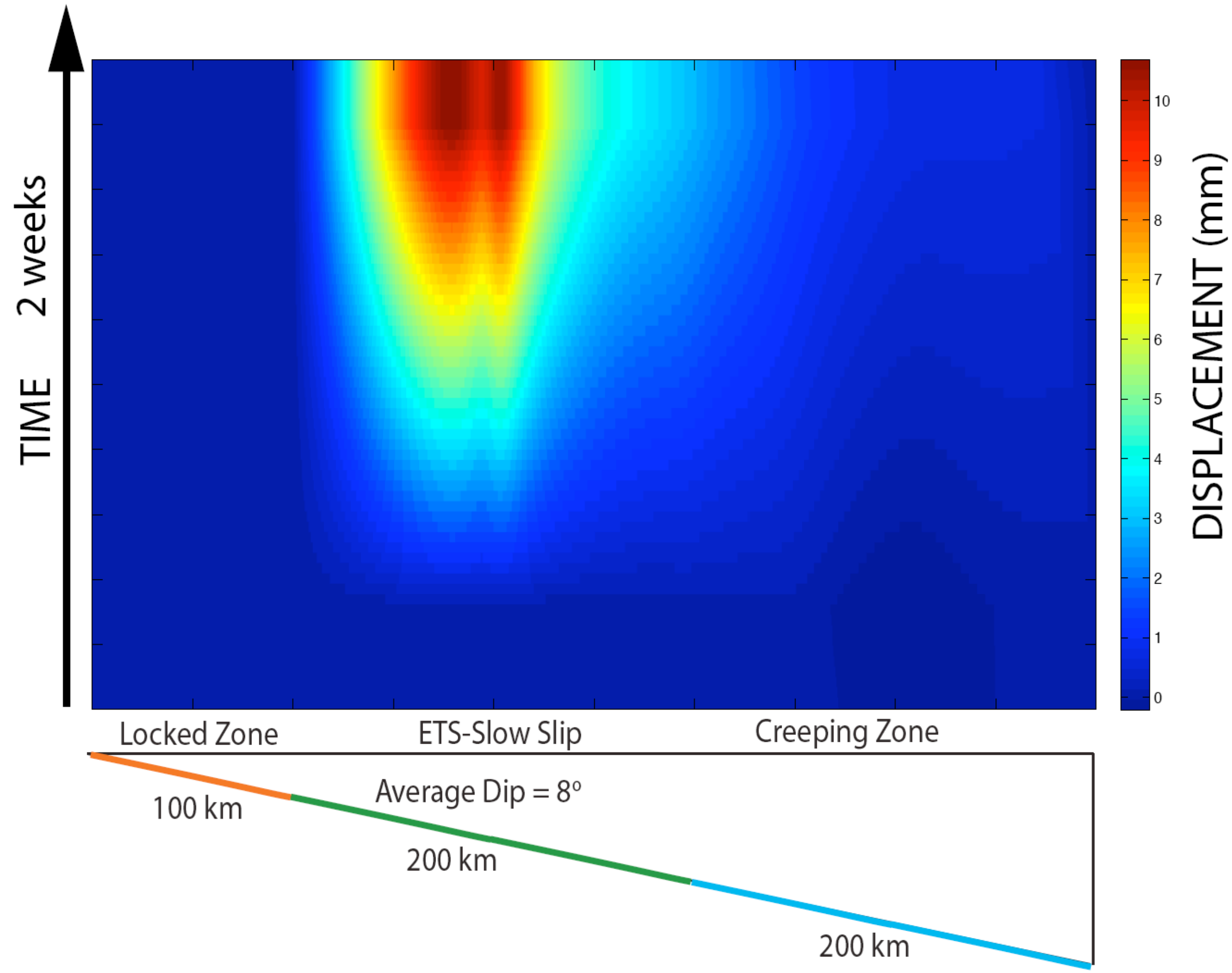
Crystal Growth in fracture to stick again after an event.

FRAME OF REFERENCE: UPPER PLATE





DISPLACEMENT AT THE SURFACE

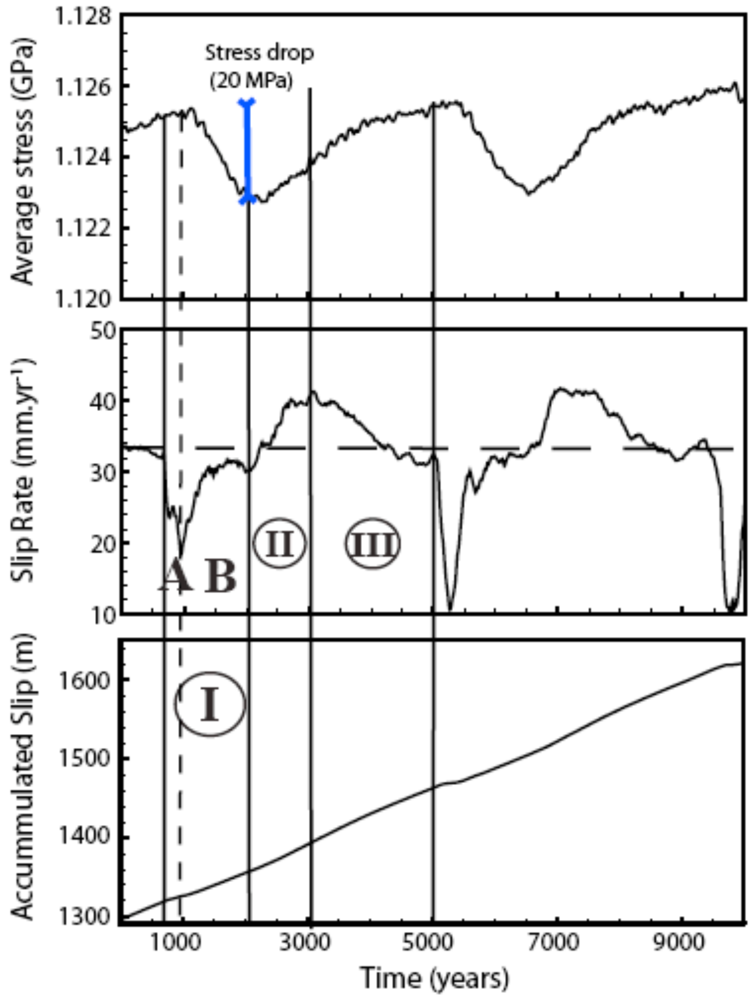


CONCLUSIONS

- A possible model for ductile shear zone creep by accumulation of event is posited.
- Acqueous fluids in the ductile shear zone are critical. (Fluid cycle in the subduction zone)
- Possible Very Large Slow Events (VLSEs) although data are still controversial.

Numerical solution

a) SAN ANDREAS FAULT TYPE
Fast far field rate 35 mm.yr[-1]



c) Ductile shear zone viscosity limited to 10^{20} Pa.s
Fast far field rate 35 mm.yr[-1]