New Computational Approach to Nonplanar Elastodynamic Ruptures

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1. Abstract

We present a new approach to modeling dynamic ruptures on nonplanar faults. A fundamental challenge for modeling rupture dynamics on complicated fault networks using current techniques is dealing with the computational mesh. Generation of a mesh that is both faithful to the underlying fault structure and suitable for efficient computation is an open problem. Here, we test the possibility of using an extended finite element method, XFEM[3], for problems of repeated rupture.

This method is mesh-free – the fault need not lie on mesh edges – drastically reducing the requirements for suitable computational meshes. The XFEM handles spatial discretization, so suitable temporal discretizations are added. Friction and failure on the fault is characterized by a mixed boundary condition; some regions are sticking via a Dirichlet boundary condition, where others are slipping via a Neumann traction boundary condtion. We derive and present weak conditions for sticking and slipping under the XFEM, and present a solution strategy for alternating quasistatic loading and dynamic rupture. To demonstrate the feasibility of this approach, we present results for long time-series of ruptures on complicated, two-dimensional fault networks.

Using this strategy, sequences of dynamic ruptures on networks of faults, including branching, are generated. Distributions of event rupture length, magnitude, epicenter location, and other statistical measures are presented and compared as a function of geometry. While the problems and geometries we solve are feasible with existing methods, this demonstration indicates that XFEM should prove useful for the solution of problems limited by mesh generation, especially three-dimensional, fault system-level problems.

2. Dynamic Repeated Rupture

We solve elastodynamic rupture in both Mode III and mixed Mode I/II loading under plain stress assumptions in two dimensions.

$$\rho\frac{\partial^2 \mathbf{u}}{\partial t^2} = \nabla \cdot E\left(\mathbf{x}\right) \nabla \mathbf{u} + f\left(\mathbf{x}\right)$$

The system is loaded via a background basal loading, ν, at depth H on the fault. ν is determined similarly to the ideas of Cooke and Marshall[2] of applying zero friction on the fault and a far-field loading.

2.1 Quasistatic Loading

- ▷ First term is ignored as acceleration is negligible.
- ▷ Entire fault is stuck, i.e. $[\![\frac{\partial \mathbf{u}}{\partial t}]\!] = 0$
- ▷ Boundary conditions in the far-field given by constant rate, $\mathcal{V}t$
- ▷ Failure is given by a vector-valued failure criteria. Given solution u to the above, calculate $\mathcal{T}_f$, the tractions of friction required to keep the fault stuck.
- ▷ Determine the minimum time such that $\mathcal{T}_f$ at this new time will equal the interface strength, Φ. This indicates rupture initiation.

2.2 Dynamic Rupture

- ▷ Friction is specified by a weakening interface strength where slipping.
- ▷ As a region ruptures, it loads neighboring regions, which must be checked for failure.
- ▷ A region becomes stuck when $[\![\frac{\partial \mathbf{u}}{\partial t}]\!] \to 0$.

During rupture, a region's friction is determined by whether it is sticking or slipping:

$$\begin{aligned} [\![\tfrac{\partial \mathbf{u}}{\partial t}]\!] &= 0 && \text{where stuck} \\ \mathcal{T} &= -\phi\left(\tfrac{\partial S}{\partial t}, \tfrac{\partial \hat{S}}{\partial t}, t\right) + [\![\tfrac{\nu t - \mathbf{u}}{\delta}]\!] && \text{otherwise} \end{aligned}$$

where νt is the loading term calculated above, and ϕ is given by a friction that weakens with slip:

$$\phi\left(\frac{\partial S}{\partial t}, \hat{\mathcal{T}}, t\right) = \Phi\left(\hat{\mathcal{T}}\right) - \left(\frac{\alpha Q}{1-\alpha Q} + \sigma_0 \frac{t - t_s}{t_0}\right)\hat{\Phi}$$

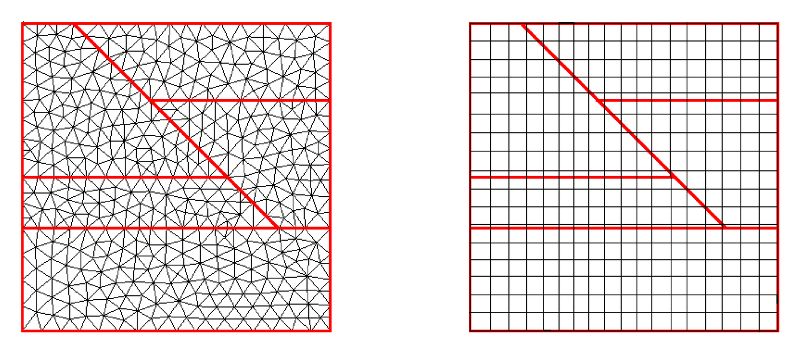
Equations are closed by radiating boundary conditions on the exterior.

Solve until everything is stuck, then repeat.

3. the XFEM

Mesh-free methods such as the partition of unity FEM (PUFEM[4]) provide a mechanism to discretize faults without placing the fault on mesh edges. For complicated geometries, especially in three dimensions, forming a mesh that conforms to all faults is an open problem. Therefore, mesh-free methods present definite potential for rupture on complex fault systems, especially in three dimensions.

For example, compare a standard FEM mesh and a typical XFEM mesh.



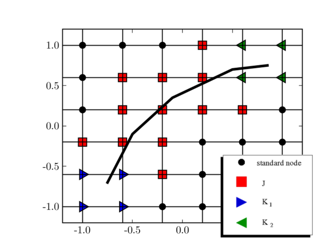
In the standard FEM, elements must be chosen to conform to the boundaries. In mesh-free methods, we can start from most any mesh, including regular meshes. On this regular mesh, we form basis functions by extending standard finite element spaces with appropriate functions via the eXtended Finite Element Method (XFEM[3]).

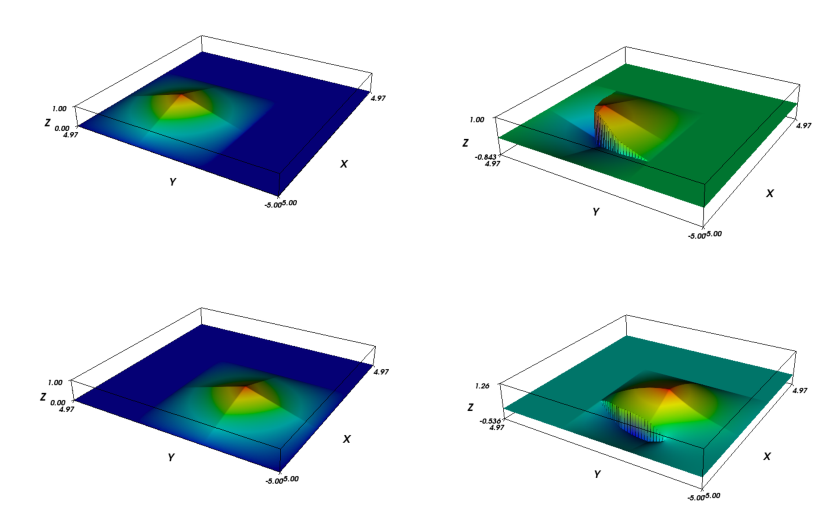
- ▷ Basis functions are built from standard finite elements and extensions.
- ▷ Extensions are chosen to respect the faults, including heaviside functions, tip functions, and branch functions.
- ▷ Instead of defining split nodes or other element based discontinuities, extensions trace the fault through the element interior.

The FEM space is then given by:

$$\mathcal{P} \equiv \sum_i \mathcal{P}^i \psi_i = \left\{ \sum_i v_i \psi_i \mid v_i \in \mathcal{P}^i \right\} \subset H^1 \text{ on } \Omega - \Gamma_C$$

where v_i form a partition of unity (standard basis function) on element i and ψ_i are extension functions that match the geometry across that element.





Sample basis function and corresponding extension function for cracks and tips

4. Weak Formulation for Slip Weakening

In the definition of the XFEM, it is important to remember that no concept may be considered strongly (point-wise) on the fault, making split nodes impossible. In the strong form, stickslip friction enforces different types of boundary conditions on different points of the fault. This is not possible weakly. Instead:

- ▷ Each vector-valued degree of freedom ψ_i is considered stuck or unstuck.
- ▷ Each fault section, $\Gamma_C \cap \Omega_E$, is considered unstuck if any ψ_i whose support contains that section is unstuck. It is stuck only if all such ψ_i are stuck.

Using these concepts, weak conditions for event rupture and termination are derived.

4.1 Initiation of Slip

- ▷ Given a solution $\mathbf{u}$ under stuck conditions, determine the (weak) tractions $\mathcal{T}$ required to keep degree of freedom ψ_i stuck:

$$\mathbb{T}_f \equiv \int_{\Gamma_C} \psi_i \mathcal{T}_f = \mathbb{M}\mathbf{u}_{tt} + \mathbb{K}\mathbf{u} - \int_{\Gamma_C} \psi_i(\mathcal{T} - \mathcal{T}_f) \tag{1}$$

- ▷ At every point on the fault, we have a failure criteria that is a function of direction. Integrate this to find a weak failure criteria (that is still a function of direction):

$$\mathbb{F}_i(\theta, t) \equiv \int_{\Gamma_C} \psi_i \phi\left(\theta, t\right)$$

- ▷ We now have a weak required frictional traction and a weak failure criteria. Compare these to determine if the basis function ψ_i should become unstuck; ψ_i becomes unstuck when:

$$|\mathbb{T}_i| > \mathbb{F}_i\left(\hat{\mathbb{T}}_i, t\right)$$

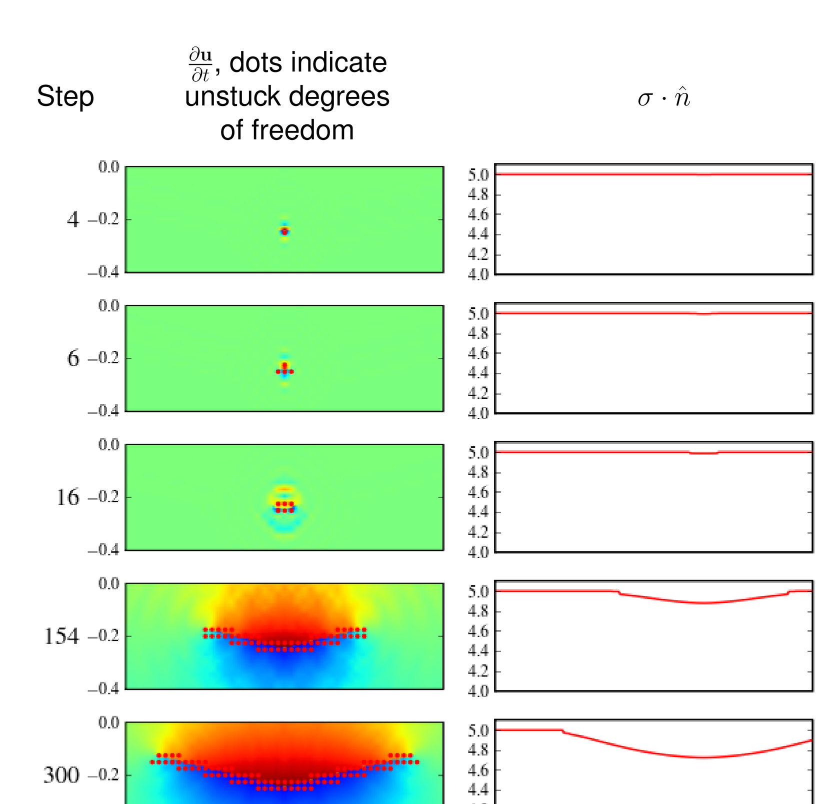
Update state variables on the portion of the crack contained in the support of ψ_i.

4.2 Termination of Slip

- ▷ Individual degrees of freedom do not become stuck.
- ▷ Instead, fault section E becomes stuck to prevent backslip:

$$\int_{\Gamma_C \cap \Omega_E} \frac{\partial S}{\partial t} < 0$$

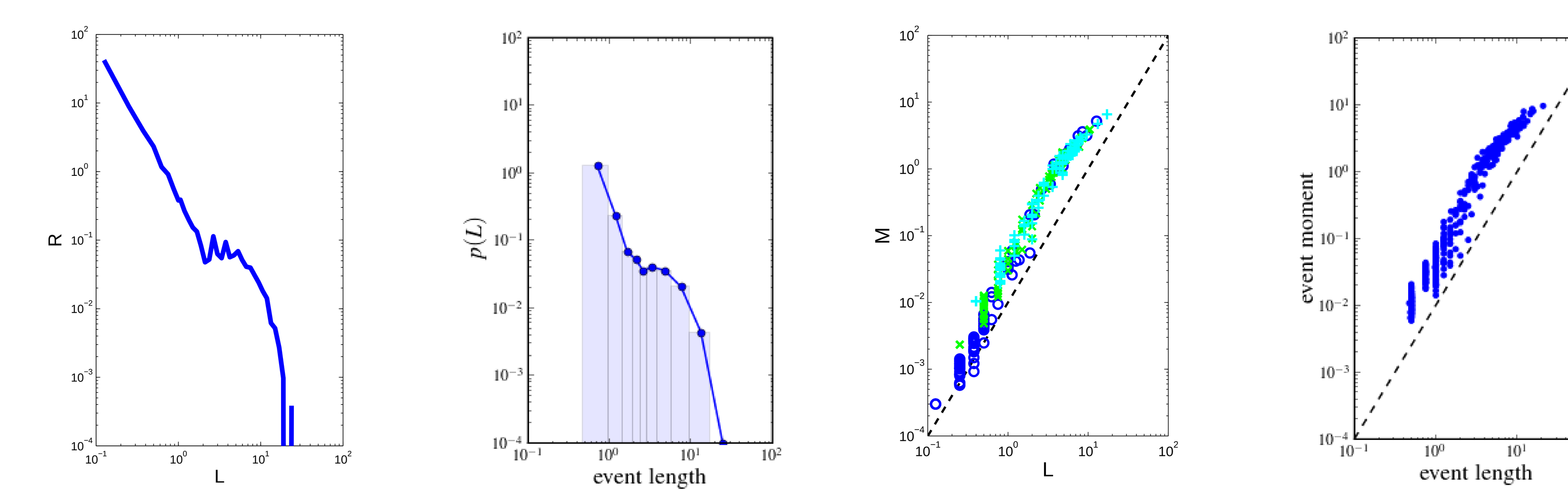
- ▷ When the section becomes stuck, all ψ_i whose support contains Ω_E become stuck, and the Dirichlet boundary condition is enforced.



5. Method Validation and Verification

To verify our use of the XFEM, we compare results to previously published solutions generated using finite differences on a single, infinitely long planar geometry.

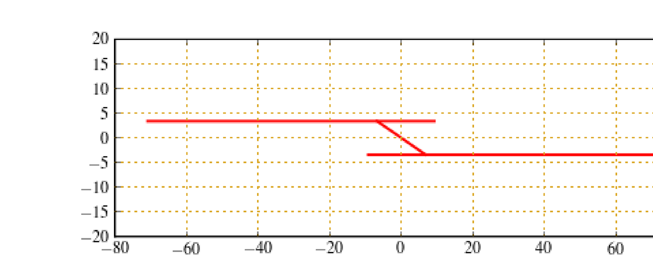
Due to the highly chaotic nature of the dynamic system, we do not expect exact simulation replication, but instead compare statistics of event lengths.

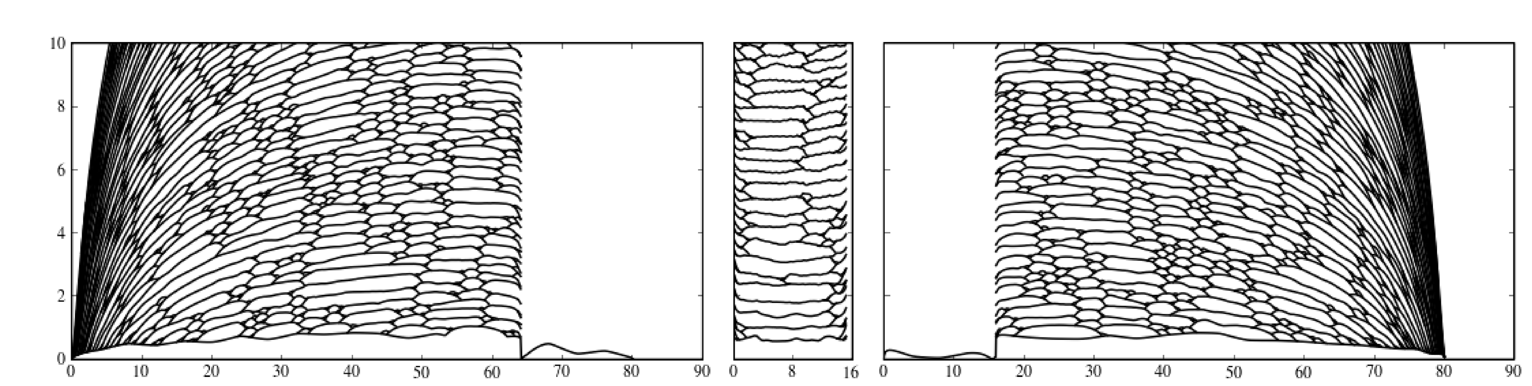


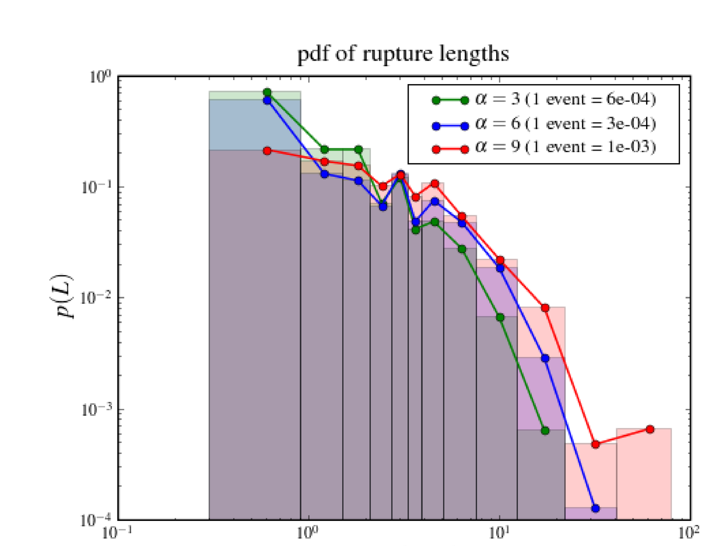
6. Sample Mode I and II Results

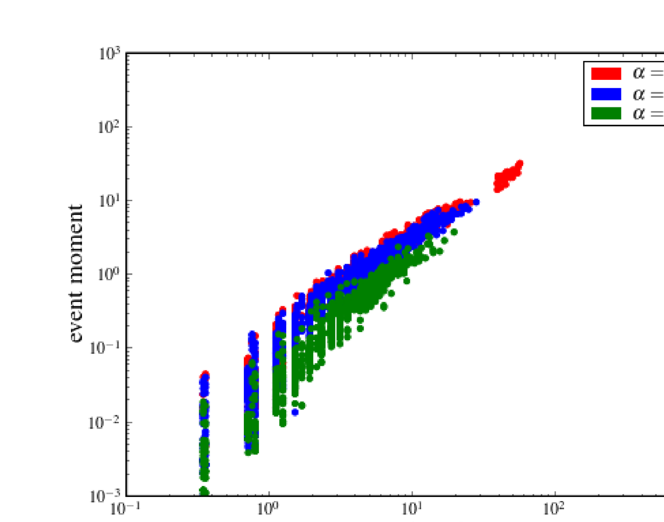
We also consider problems under Mode II strikeslip loading, with a pseudo-third dimension. Interpenetration is considered normal/thrust faulting, and (in this simplified approach) is treated without consideration of normal stresses.

Clockwise from right: the tested geometry; slip on each fault section; moment vs. length of events; and distributions of lengths of events as a function of alpha, the weakening parameter in friction.









7. Conclusions and Future Work

- ▷ XFEM allows complicated fault geometries to be easily handled in a finite element method for simulation. Without the difficulty of meshing, it allows fault dynamics to be studied as a function of varying geometry.
- ▷ With derived stick-slip weak conditions, populations of repeated ruptures on these fault network are generated.
- ▷ On planar geometries, this method is consistent with previous finite difference simulation.
- ▷ On nonplanar geometries, a pseudo-3D approach will allow for continued study of friction mechanics in repeated rupture. More realistic, normal-stress dependent frictions must be introduced.
- ▷ *This method promises an approach for problems in 3D that cannot currently be studied for realistic geometries.* Future work will leverage this ability in problems of varying or undetermined geometry.