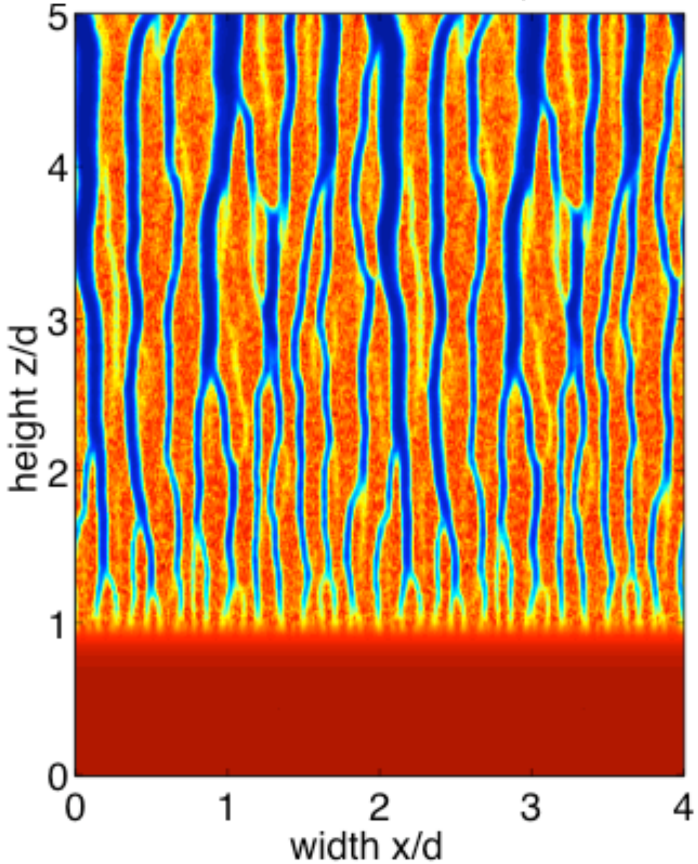
Plumbing the Depths:
Towards an integration of magma dynamics and global mantle convection

Marc Spiegelman & Richard Katz

(DAPAM/DEES Columbia University)



with:

Peter Kelemen (DEES/LDEO)

Matt Knepley (Argonne Natl. Labs)

Einat Aharonov (Weizmann)

Pegasus Fang (DAPAM)

Ben Holtzmann (LDEO)

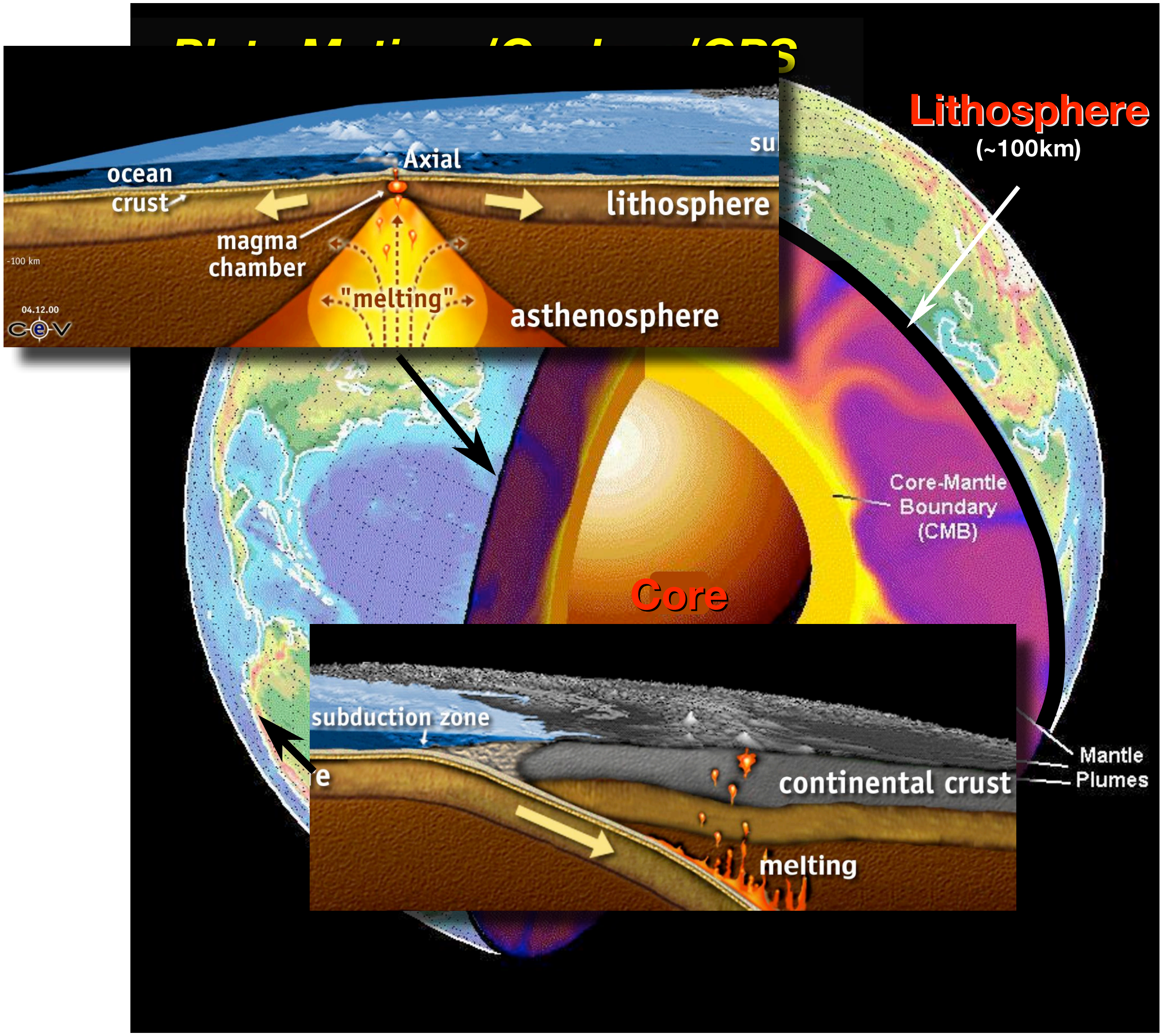
Conclusions

- Magma Dynamics is not hard!
- Magma Dynamics is a natural extension of mantle convection
- Magma Dynamics is important for understanding geochemistry (and rheology)
- Adding explicit fluid flow can change our inferences on how the Earth works.
- CIG will make all of this more accessible…

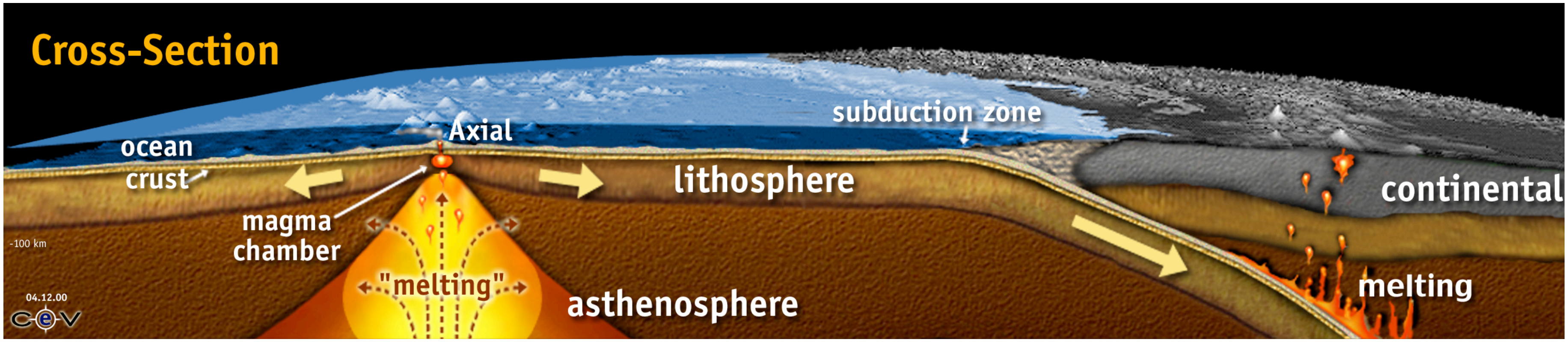
Outline

- The Big Picture
- Model Formulation
- Numerical methods and computational issues
- Example Problems: localization phenomena
  - Reactive Flow and chemical transport
  - Mechanical instability and laboratory validation
- Future directions & Software needs

The Big Picture



Ingredients for a theory of magma dynamics



- At least two phases (solid & liquid)
- Significant mass-transfer between phases (melting/reactions)
- Solid must be permeable at some scale
- Solid must be *deformable*: consistency with mantle convection (and lithospheric deformation)

Model Formulation

(McKenzie, 1984, J. Petrol; Spiegelman, 1993; JFM, 2001, JGR)

Conservation of Mass: Fluid

$$\frac{\partial(\rho_f \phi)}{\partial t} + \nabla \cdot (\rho_f \phi \mathbf{v}) = \Gamma$$

Conservation of Mass: Solid

$$\frac{\partial[\rho_s(1-\phi)]}{\partial t} + \nabla \cdot [\rho_s(1-\phi)\mathbf{V}] = -\Gamma$$

Conservation of Momentum for fluid: Darcy's Law

$$\phi(\mathbf{v} - \mathbf{V}) = \frac{-k_\phi}{\mu}\left[\nabla P - \rho_f \mathbf{g}\right]$$

Conservation of Momentum for Solid (viscous rheology)

$$\nabla P = \nabla \cdot \left[\eta(\nabla \mathbf{V} + \nabla \mathbf{V}^T)\right] + \nabla[(\zeta - \frac{2}{3}\eta)\nabla \cdot \mathbf{V}] + \bar{\rho}\mathbf{g}$$

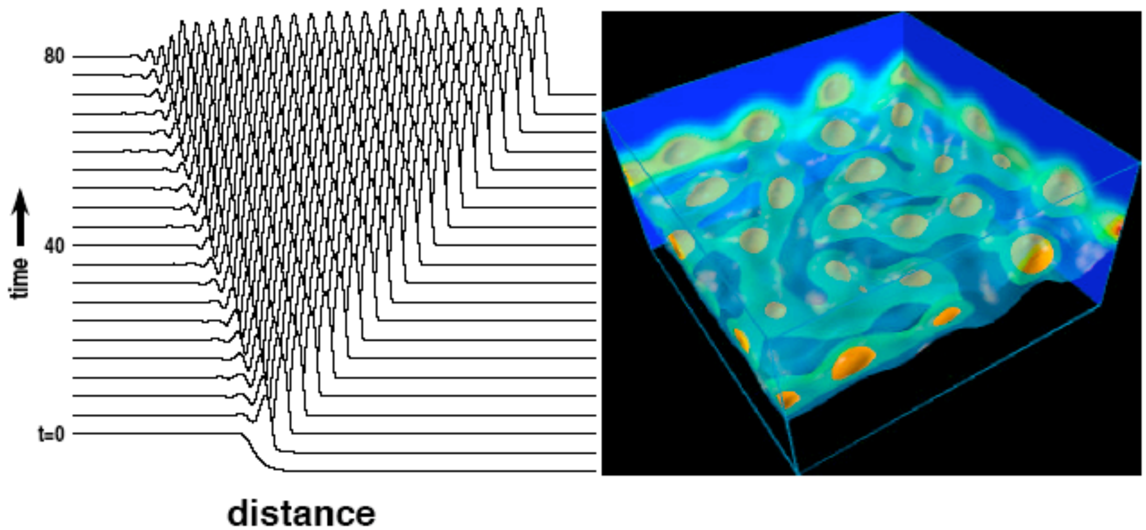
Permeability

$$k_\phi \sim \frac{a^2 \phi^n}{b}$$

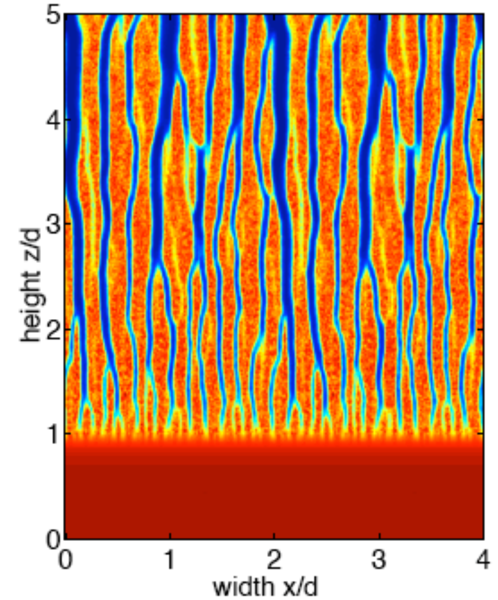
Physics/Computational Issues

- Non-linear Coupled System:
  - Fluid flow sensitive to dynamic pressure (requires accurate pressure *gradients)*
  - Pressure depends on rheology and deformation which are affected by fluids
- Intrinsic length and velocity scales
  - Compaction length $\delta = \sqrt{k_\phi(\zeta + 4\eta/3)/\mu} \sim 10^2 - 10^4\text{m}$
  - Separation velocity $w_0 = k_\phi \Delta\rho g/\mu \sim 10 - 100 W_0$
  - Accuracy requires resolving δ and melt velocity time scale

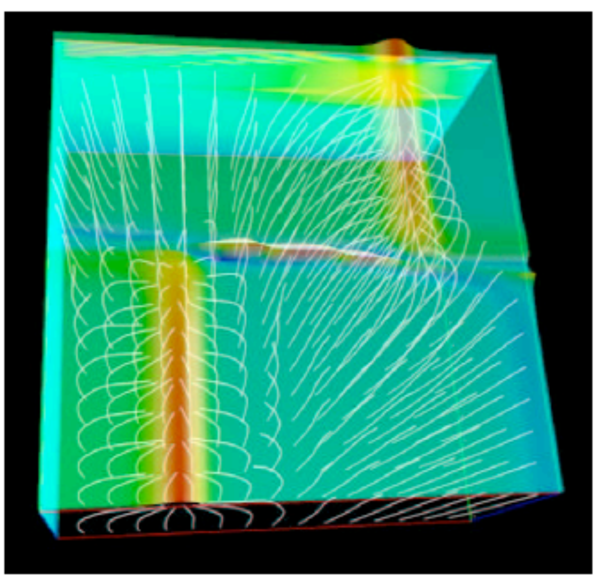
Non-linear Porosity waves



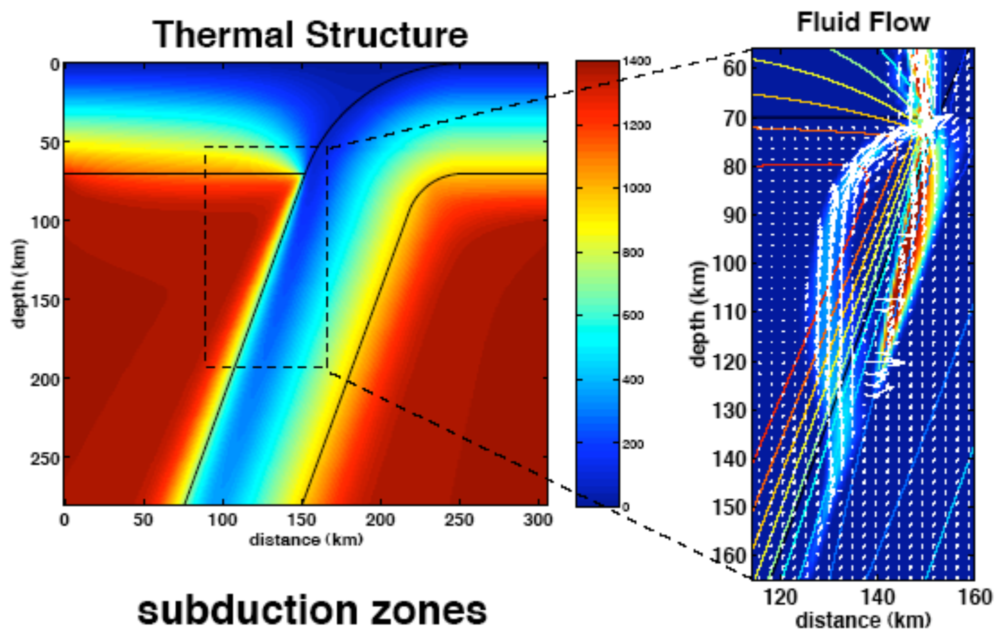
Reactive Flow Localization



Earth Science Applications



mid-ocean ridges

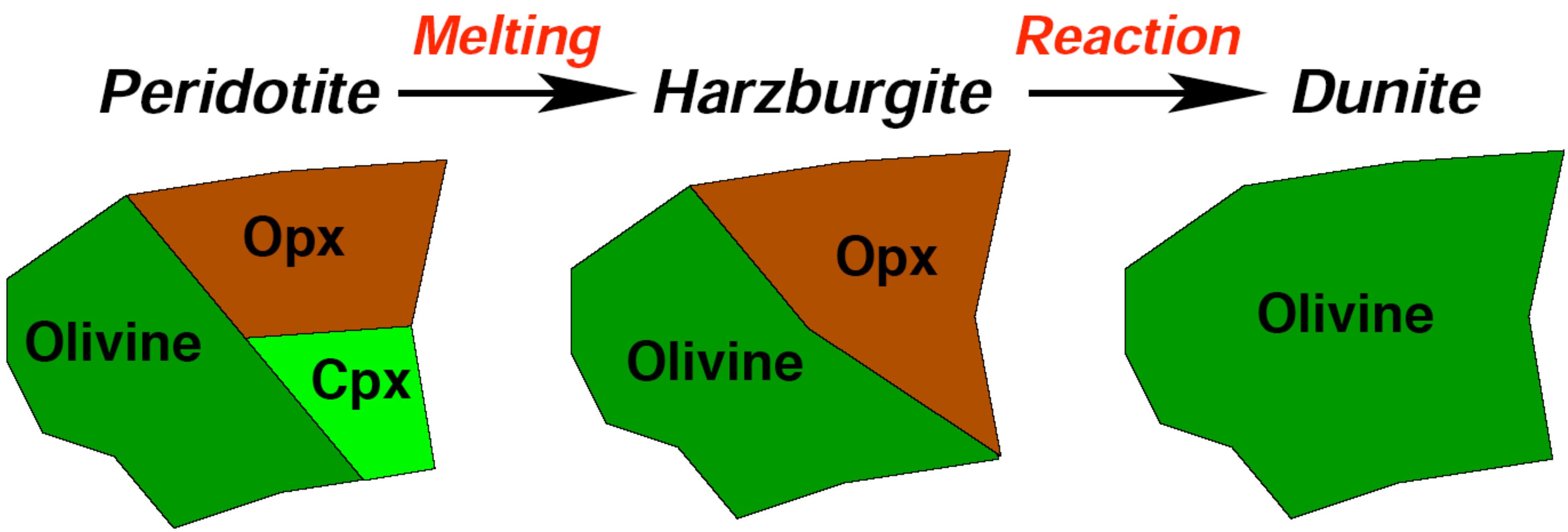


subduction zones

Field Observations: Oman Ophiolite

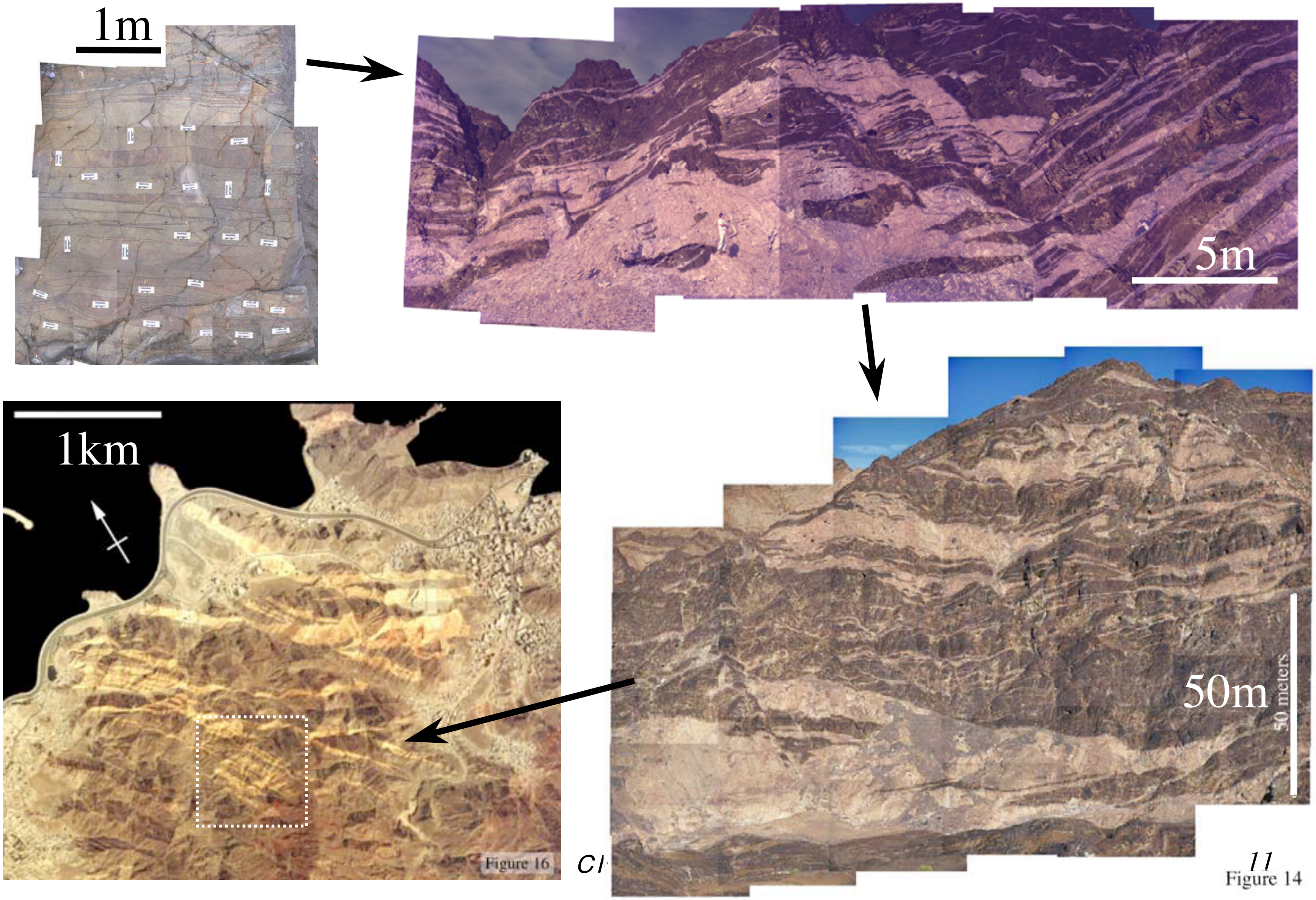
(Kelemen et. al, 1995; Nature)

Petrology for Geophysicists



Scales of Residual Dunites

(Braun & Kelemen, 2003; G^3)



Flow in reactive deformable media

(Spiegelman et al., 2001 JGR, 2003 G[3])

$$\mathcal{C} = \nabla\cdot \mathbf{V}$$

$$k_\phi \propto \phi^n$$

$$(\zeta + 4\eta/3) = f(\phi, \mathcal{C})$$

$$\frac{D_s \phi}{Dt} = \mathcal{C} + \Gamma$$

$$-\boldsymbol{\nabla}\cdot k_\phi \boldsymbol{\nabla}(\zeta + 4\eta/3)\mathcal{C} + \mathcal{C} = -\boldsymbol{\nabla}\cdot k_\phi \mathbf{k} + \frac{\Delta\rho}{\rho_f}\Gamma$$

$$\Gamma = \mathrm{Da}(c^f_{eq} - c^f)$$

$$\phi\frac{D_f c^f}{Dt} = \frac{1}{\mathrm{Pe}}\boldsymbol{\nabla}\cdot\phi\boldsymbol{\nabla}c^f + (c^f_R - c^f)\Gamma$$

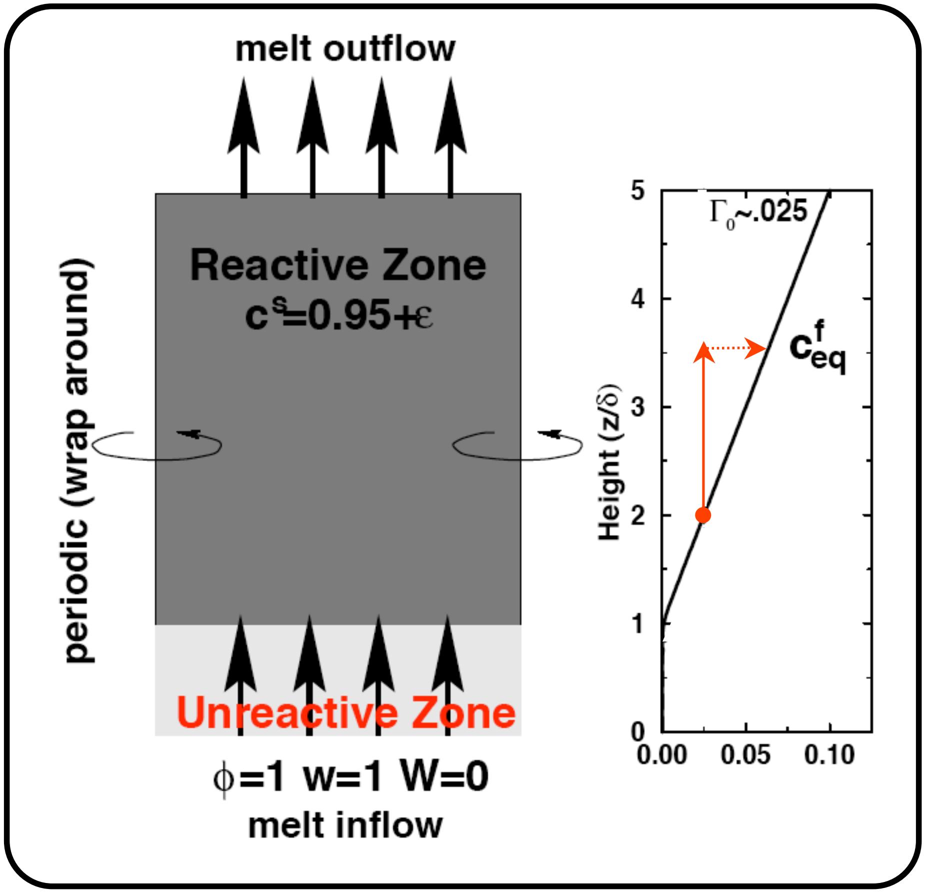
$$(1-\phi_0\phi)\frac{D_s c^s}{Dt} = -(c^f_R - c^s)\Gamma$$

Numerical Methods

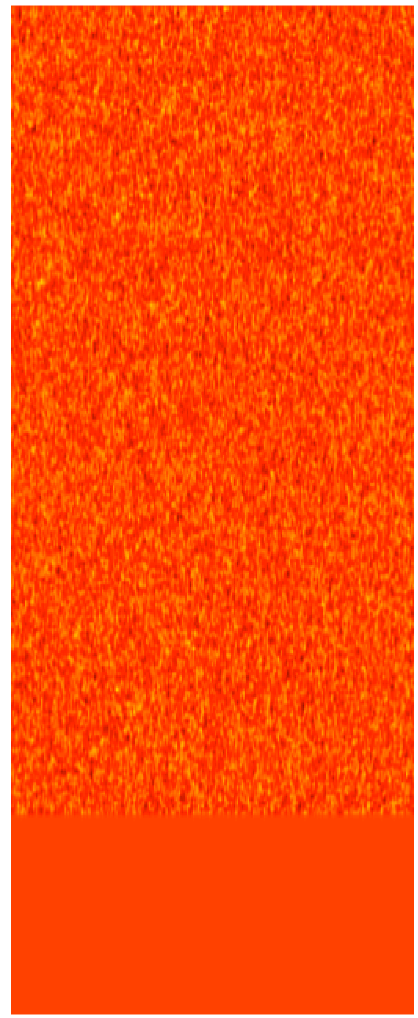
- Old codes: (Serial…but fast)
  - Hyperbolic equations: semi-lagrangian characteristic based (Staniforth and Coates, 1991)
  - Elliptic: geometric multi-grid V(2,1), hand-rolled/mudpack
  - Parabolic/reaction: Semi-lagrangian Crank-Nicholson (SL+MG, semi-implicit)
  - Non-linearity by Picard-iteration (convergence factor ~.1)
  - O(N) solvers, 2nd-order in time with no CFL
- New Codes: (parallel, more robust…but not so fast)
  - PETSc (DMMG-SNES, Newton-Krylov)
  - Currently adding Semi-Lagrangian to PETSc
  - Need to extend DMMG to staggered meshes

Reactive Channel Formation

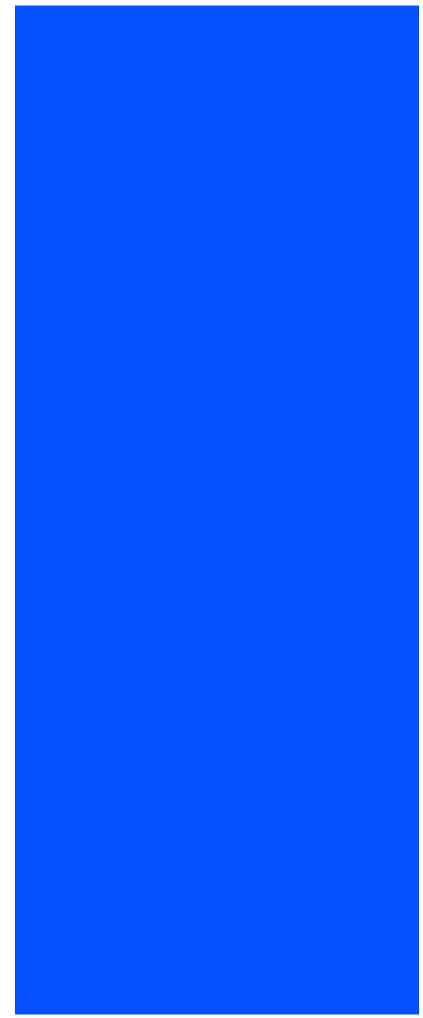
(Spiegelman et al, 2001; JGR)



Model geometry



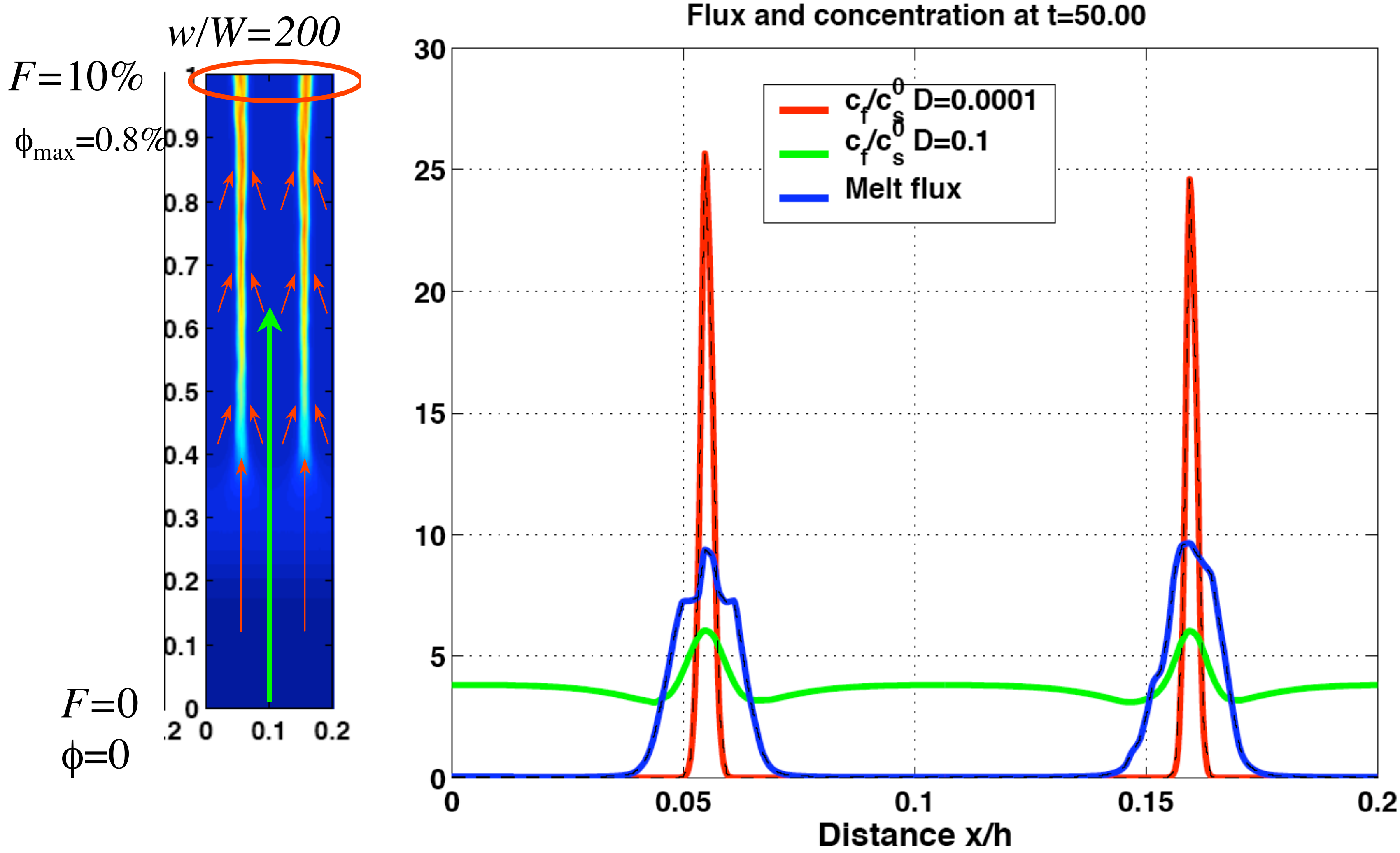
residue



porosity

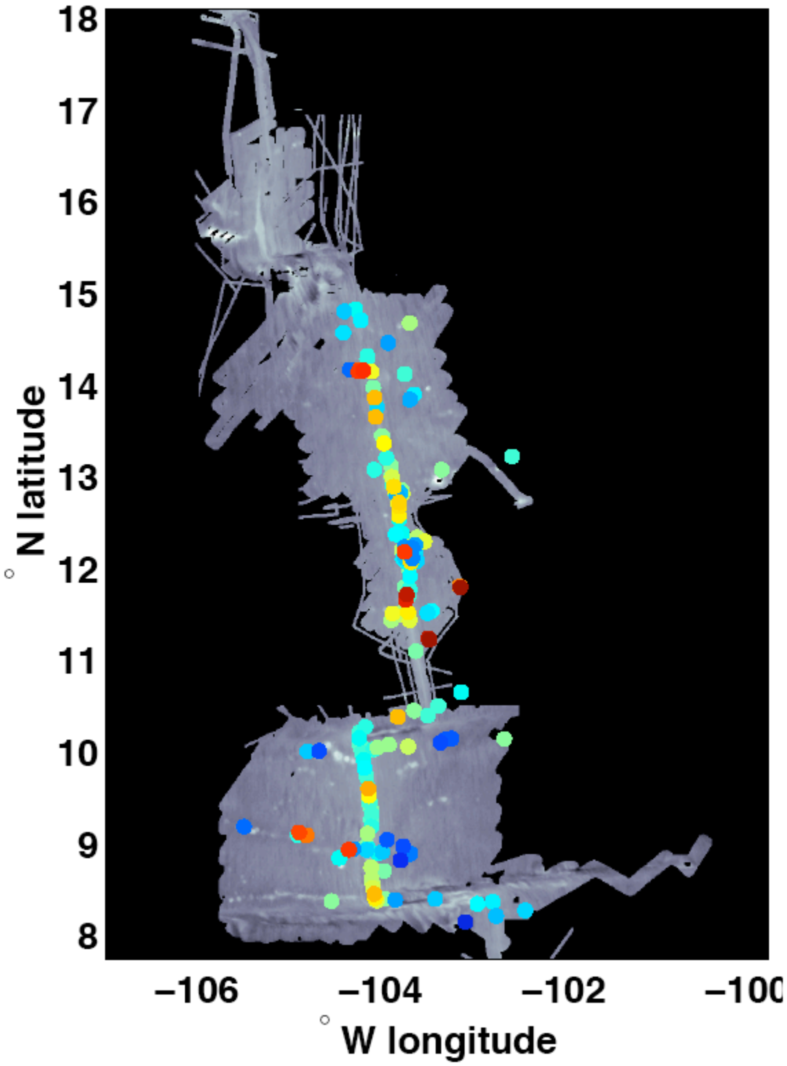
Chemical Consequences of Melt Channeling

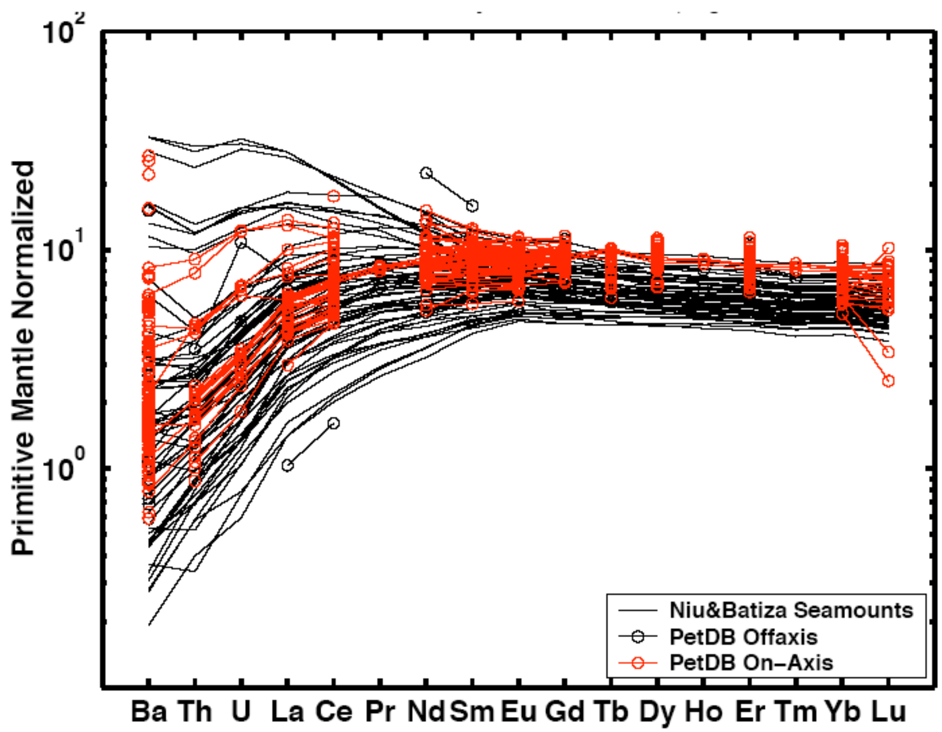
(reactive 1-D upwelling column, Spiegelman et al. 2003, G^3)



Scales of Chemical Variability

(1000 km scale)

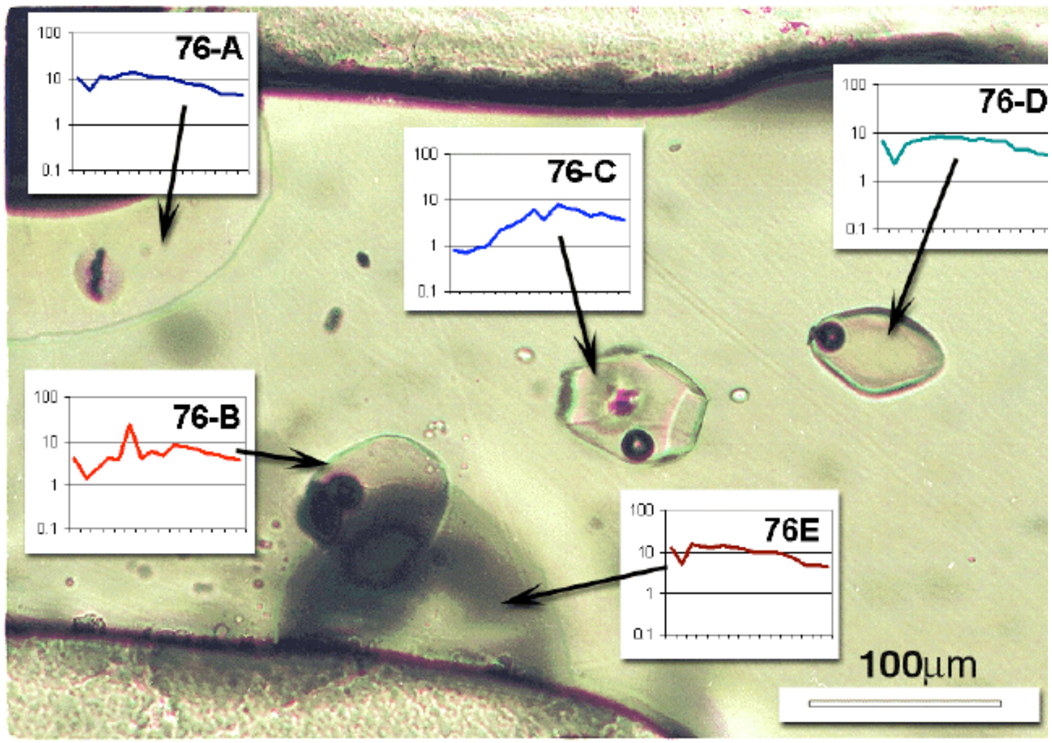




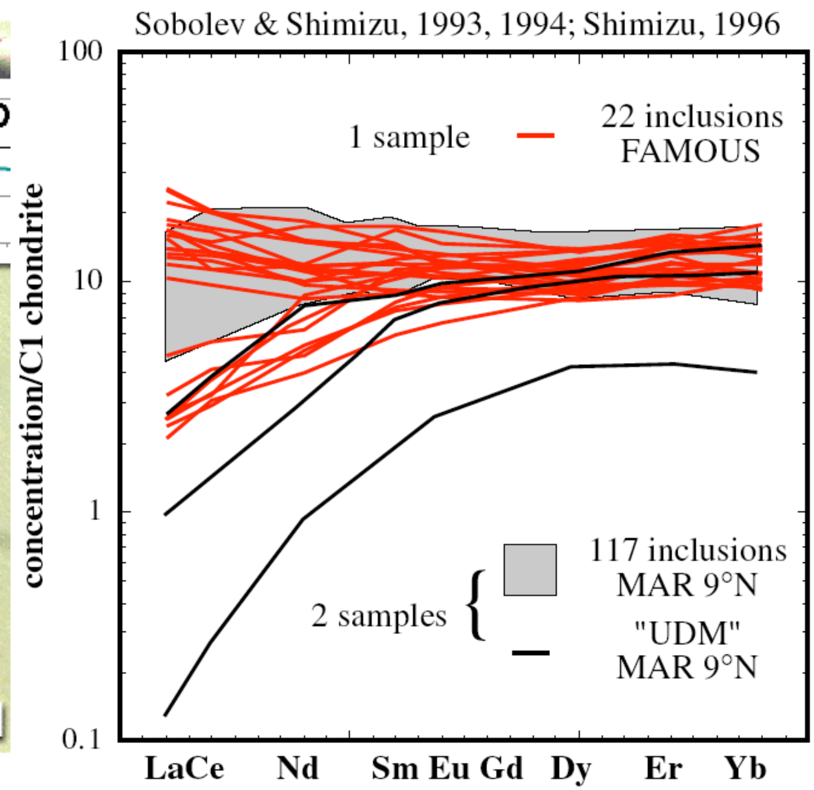
Trace Element Variability

8–18° N EPR, MgO>7%, 387 Samples

Scales of chemical Variability: Melt Inclusions

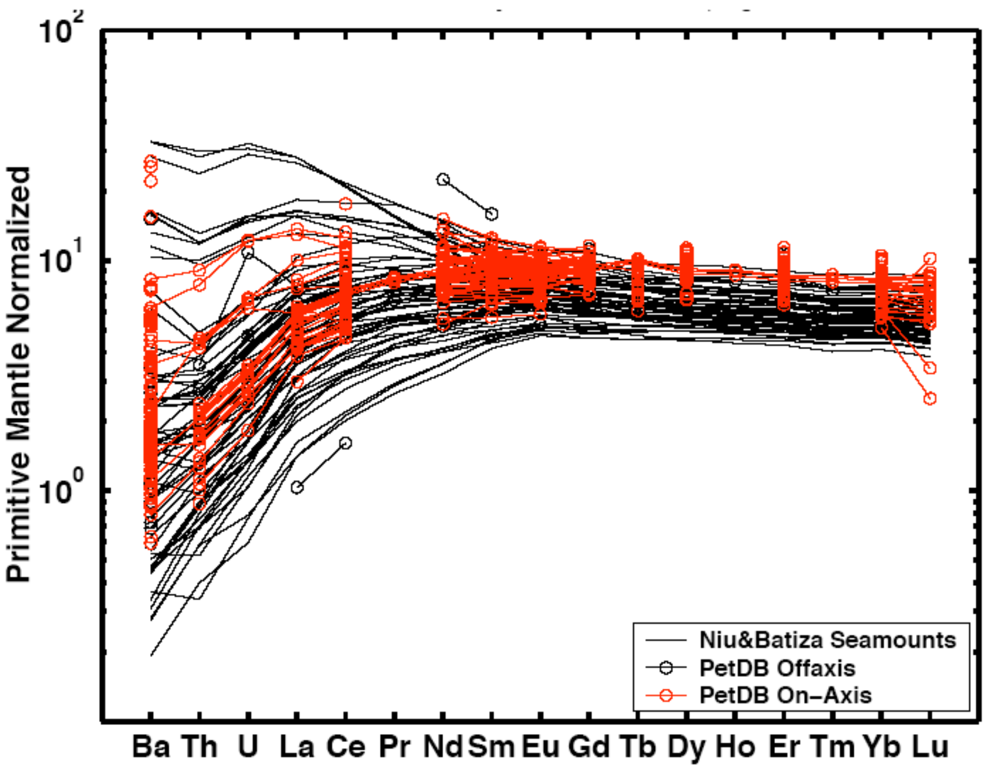


(Sobolev et al., Nature 2000)



Scales of Chemical Variability

(1000km vs. 10cm)



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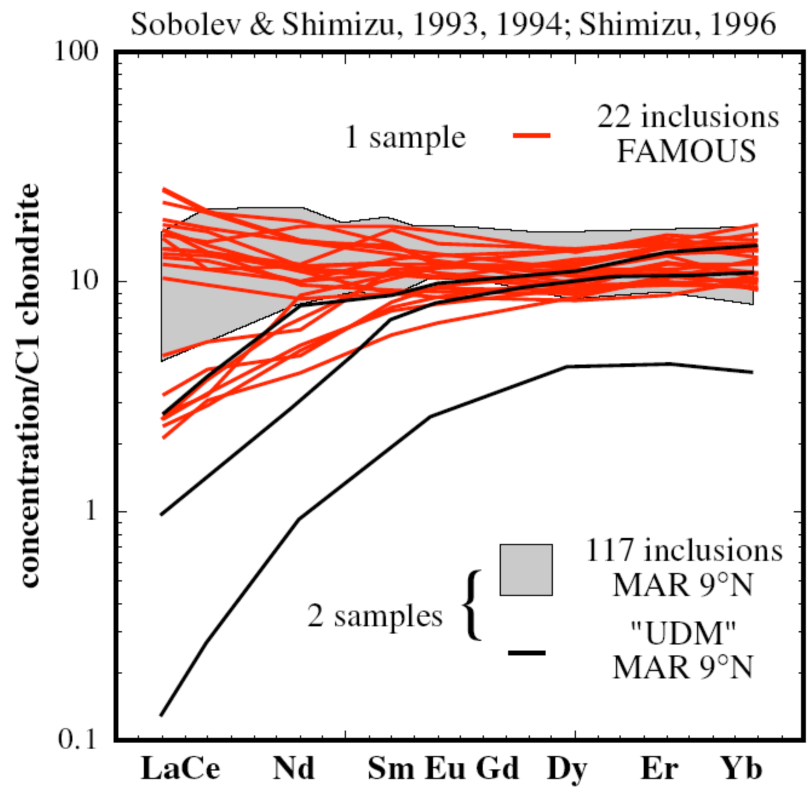
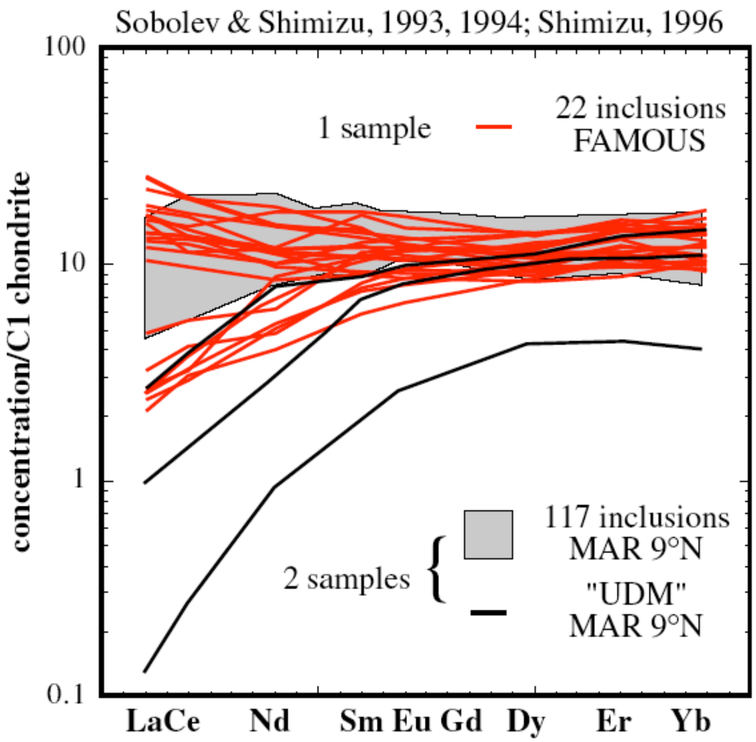
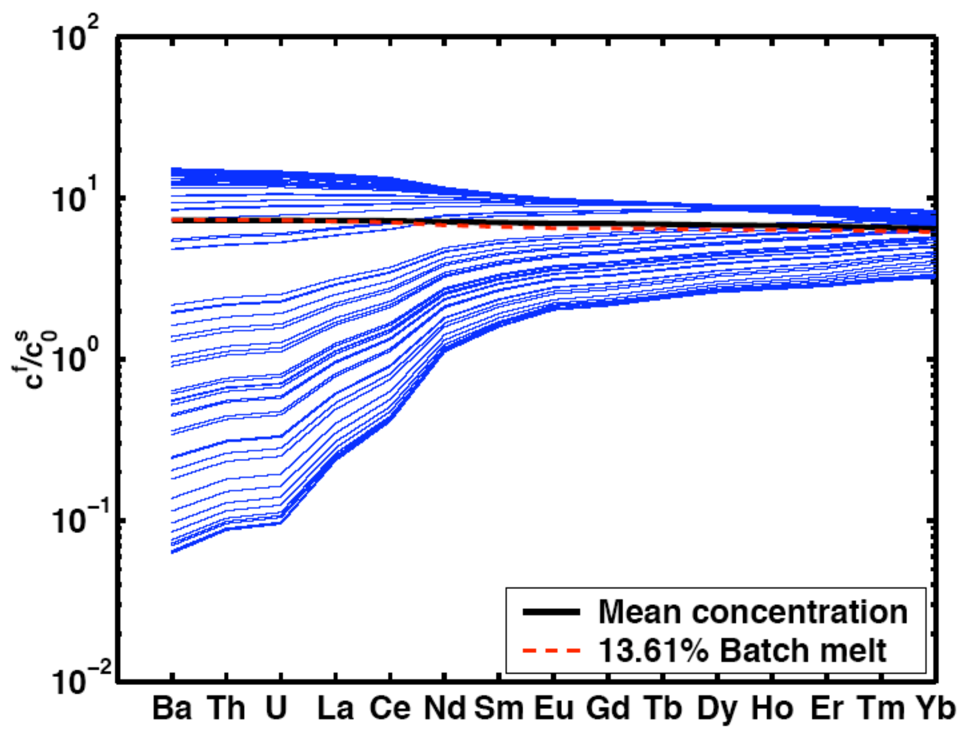


Comparison to observed chemical variability (melt inclusions)



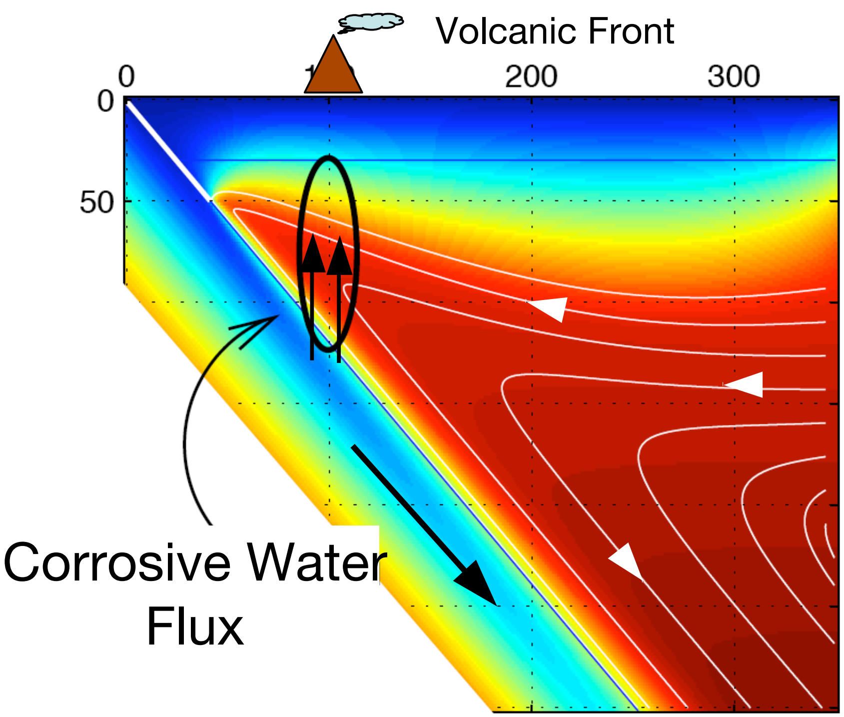
Melt Inclusion data MAR

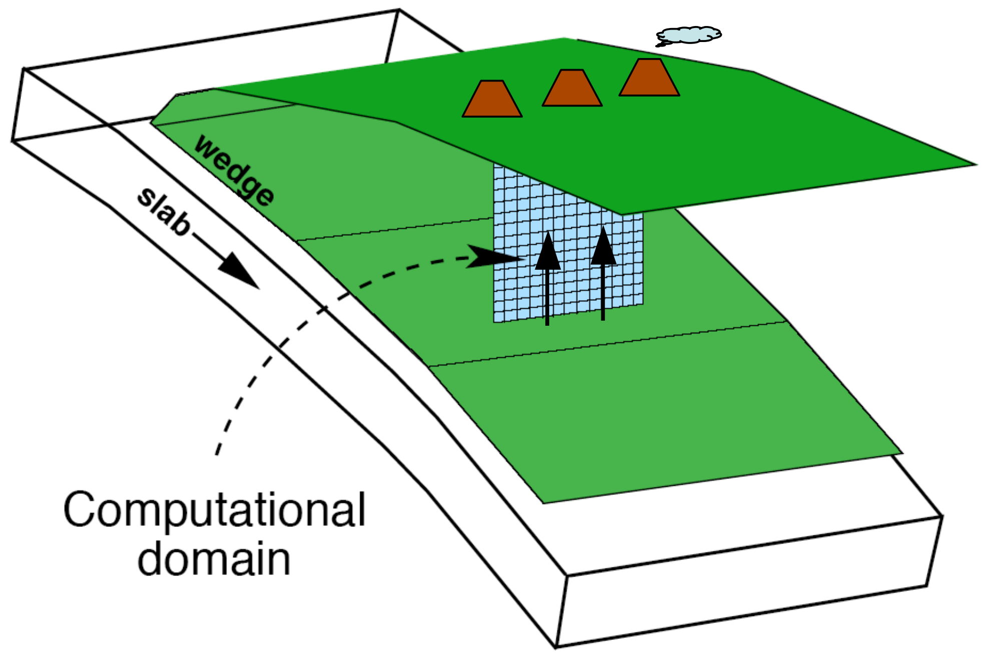


2-Level model, Gt-Sp transition at $z = .4$

Reactive flow in subduction zones

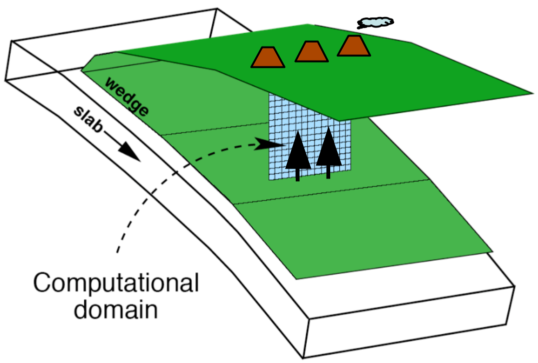
(Katz and Knepely, 2004; PETSc)

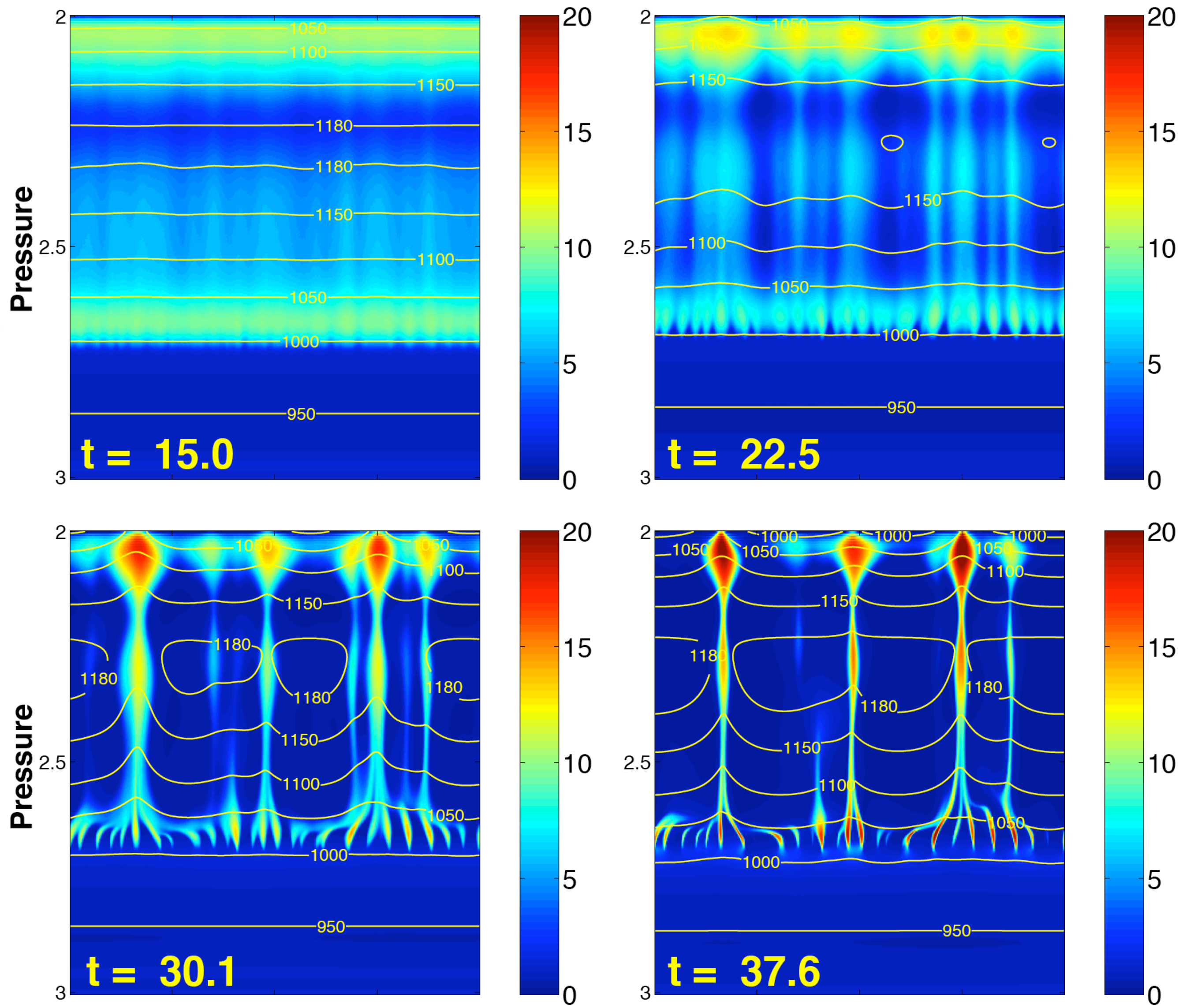




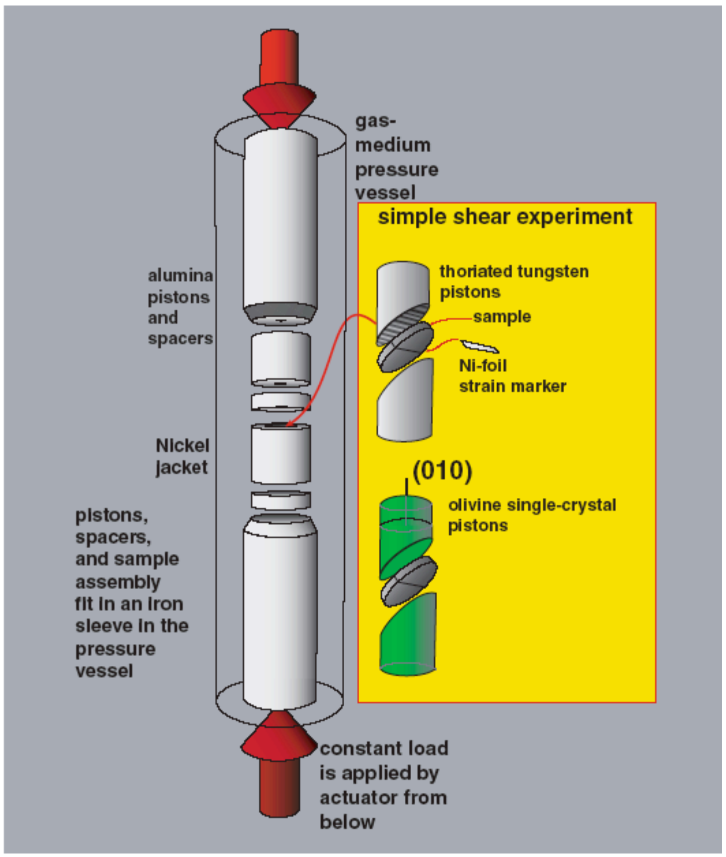
Reactive flow in subduction zones

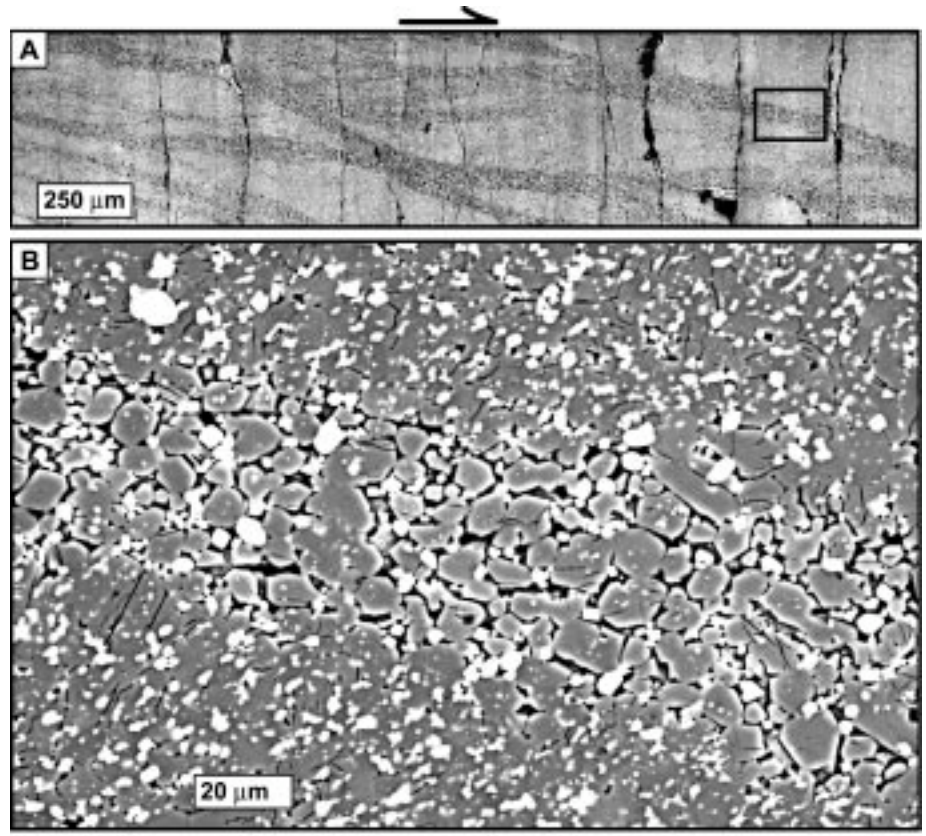
(Katz, 2005)





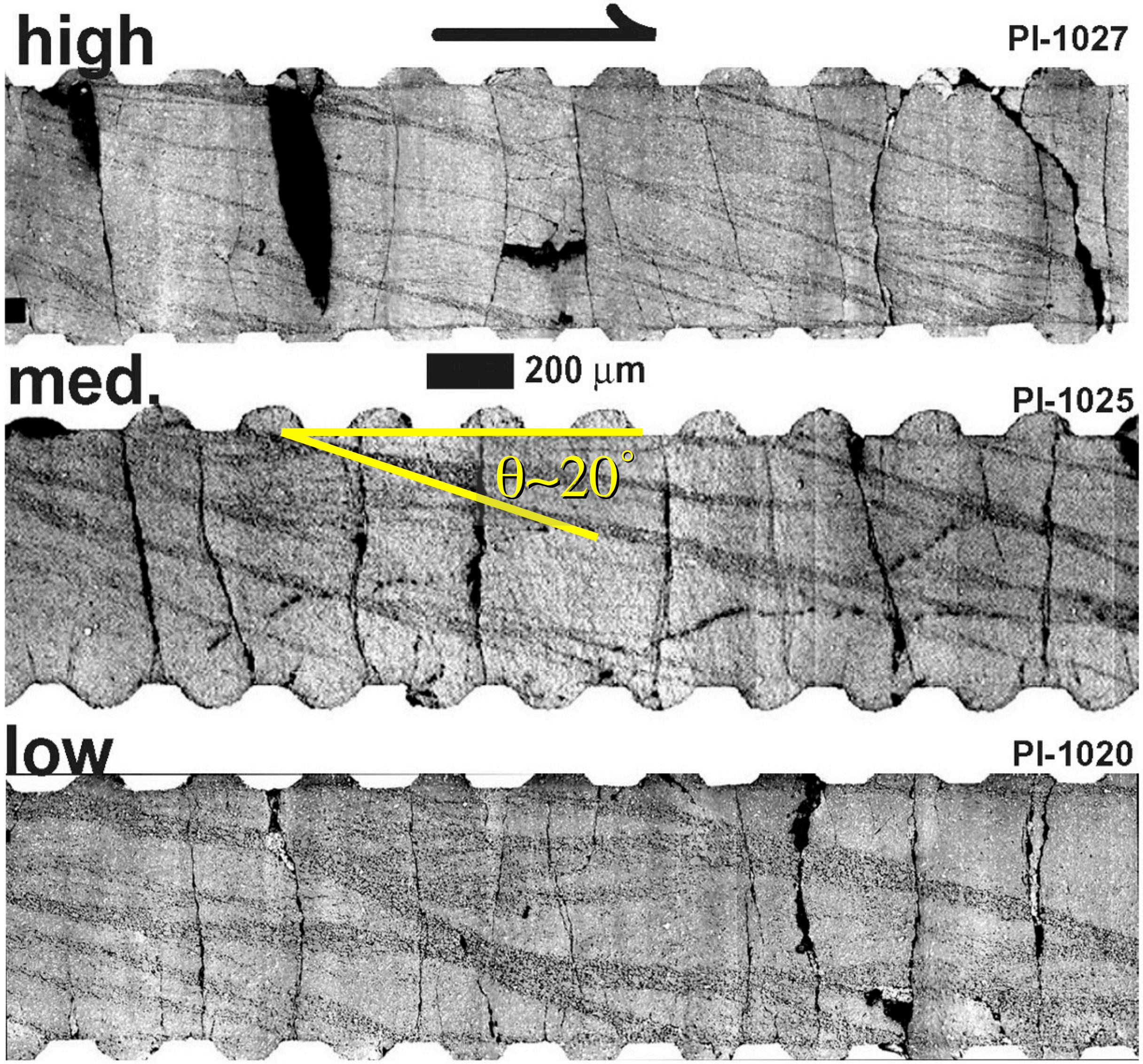
Melt Localization under shear
(Holtzmann et. al, 2003; G^3, Nature)





Melt Band Formation

(Holtzman & Kohlstedt, 2005)



Constant load series γ~3.4

Model Formulation/Numerics
(Katz et. al, 2005)

- Equations

$$\frac{D\phi}{Dt} = (1-\phi)\nabla\cdot\mathbf{V}$$

$$\nabla\cdot\mathbf{V} = \nabla\cdot\frac{k_\phi}{\mu}\nabla P$$

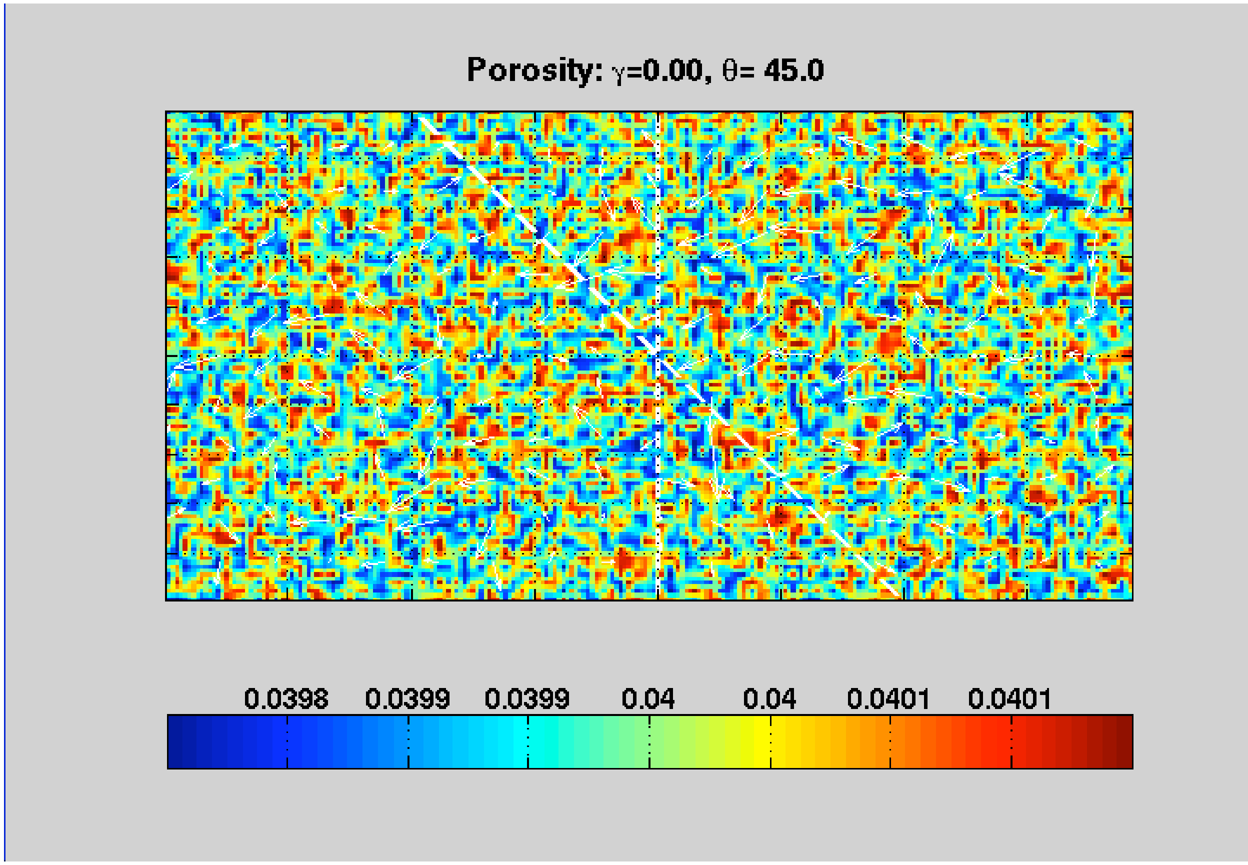
$$\nabla P = \nabla\cdot\left[\eta\left(\nabla\mathbf{V}+\nabla\mathbf{V}^T\right)\right]+\nabla\left[\left(\zeta-\frac{2\eta}{3}\right)\nabla\cdot\mathbf{V}\right]$$

$$\eta(\phi,\mathbf{V}) = \eta_0 e^{\alpha(\phi-\phi_0)}\dot{\epsilon}_{II}^{1/n-1}.$$

- PETSc, Non-linear Newton-Krylov (ILU PC-GMRES) Staggered-mesh, Fromm advection

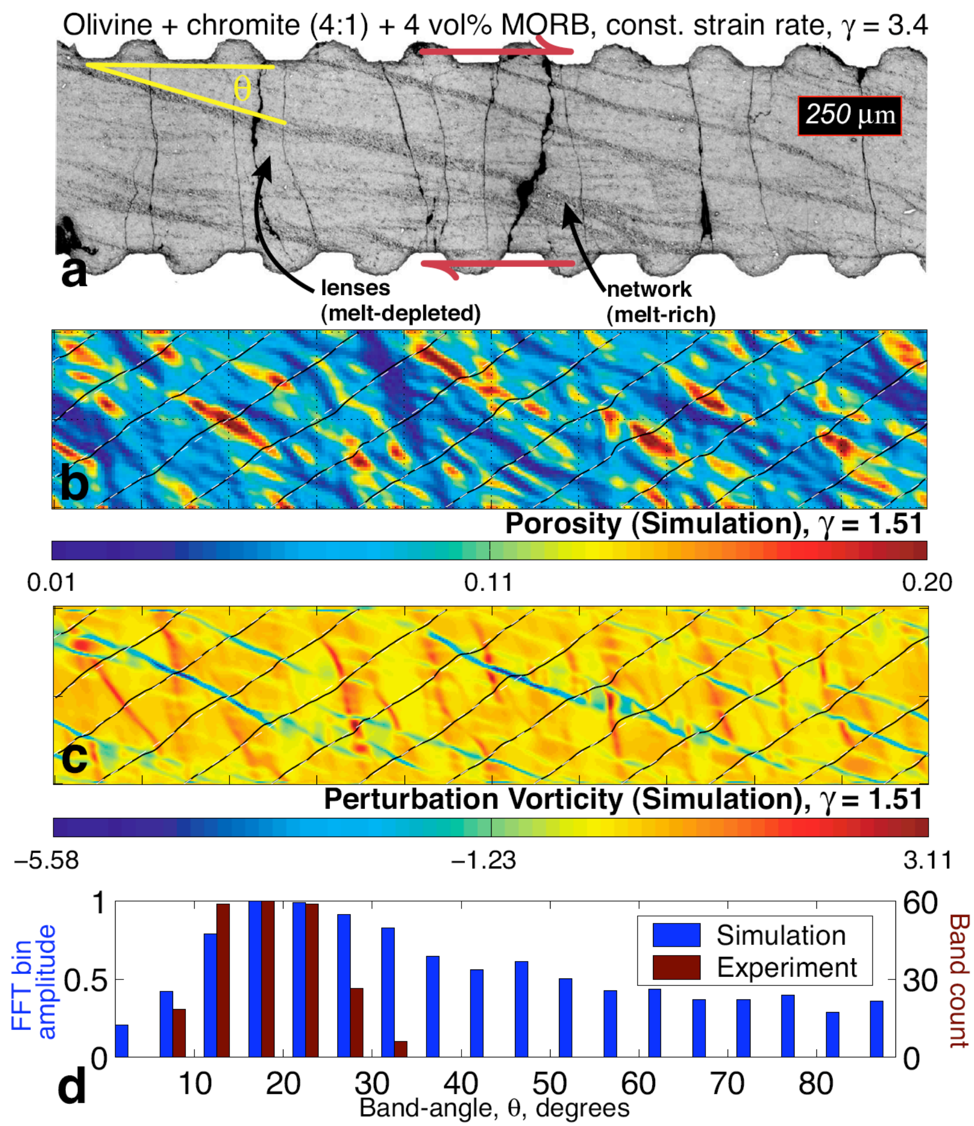
Non-linear Numerical solutions
(Katz & Spiegelman, PETSc)

Strong viscosity feedback (α=–30, n=1, white noise IC)



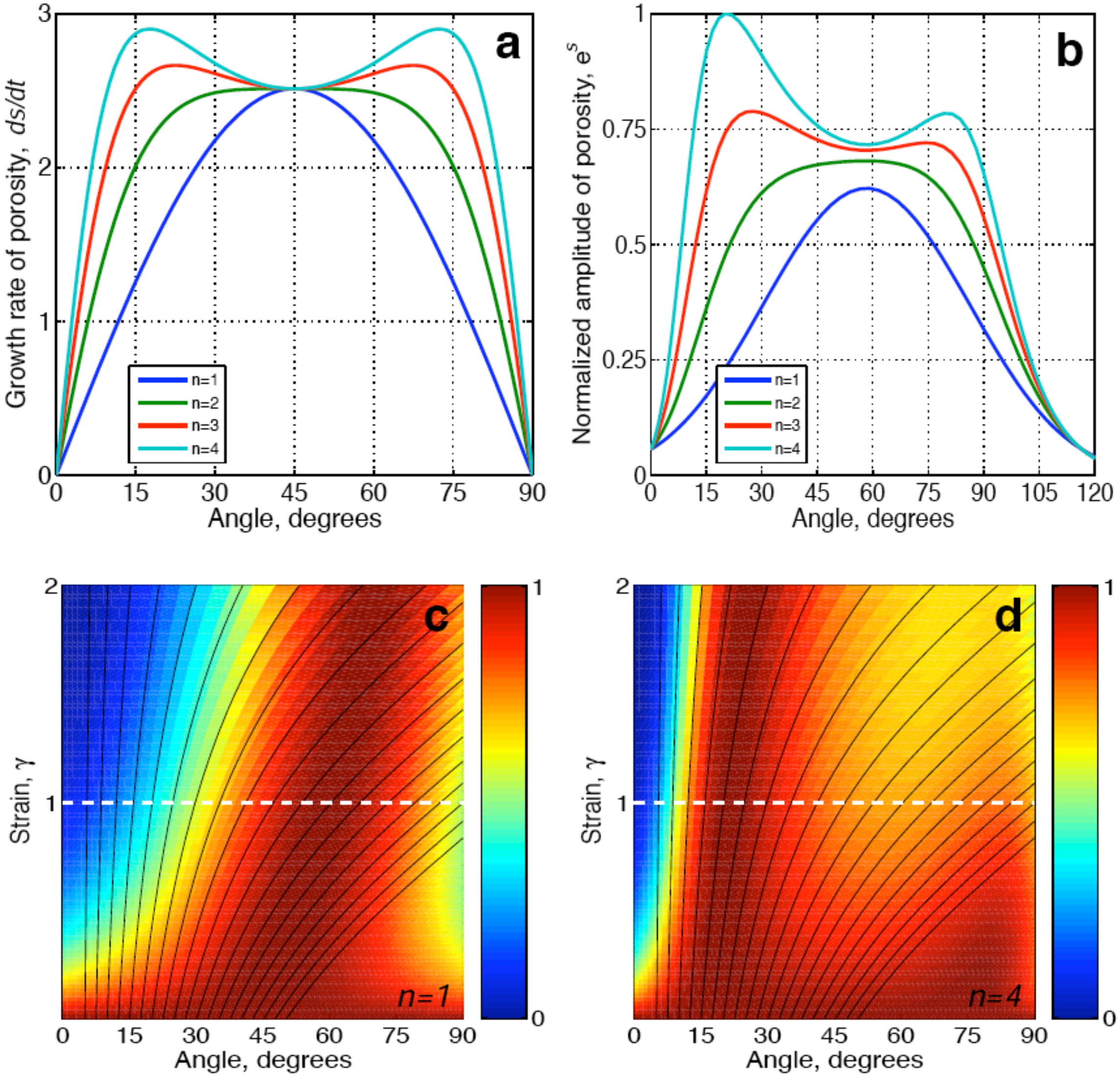
Shear Band formation

(n=4,α=30°,ξ=.1)



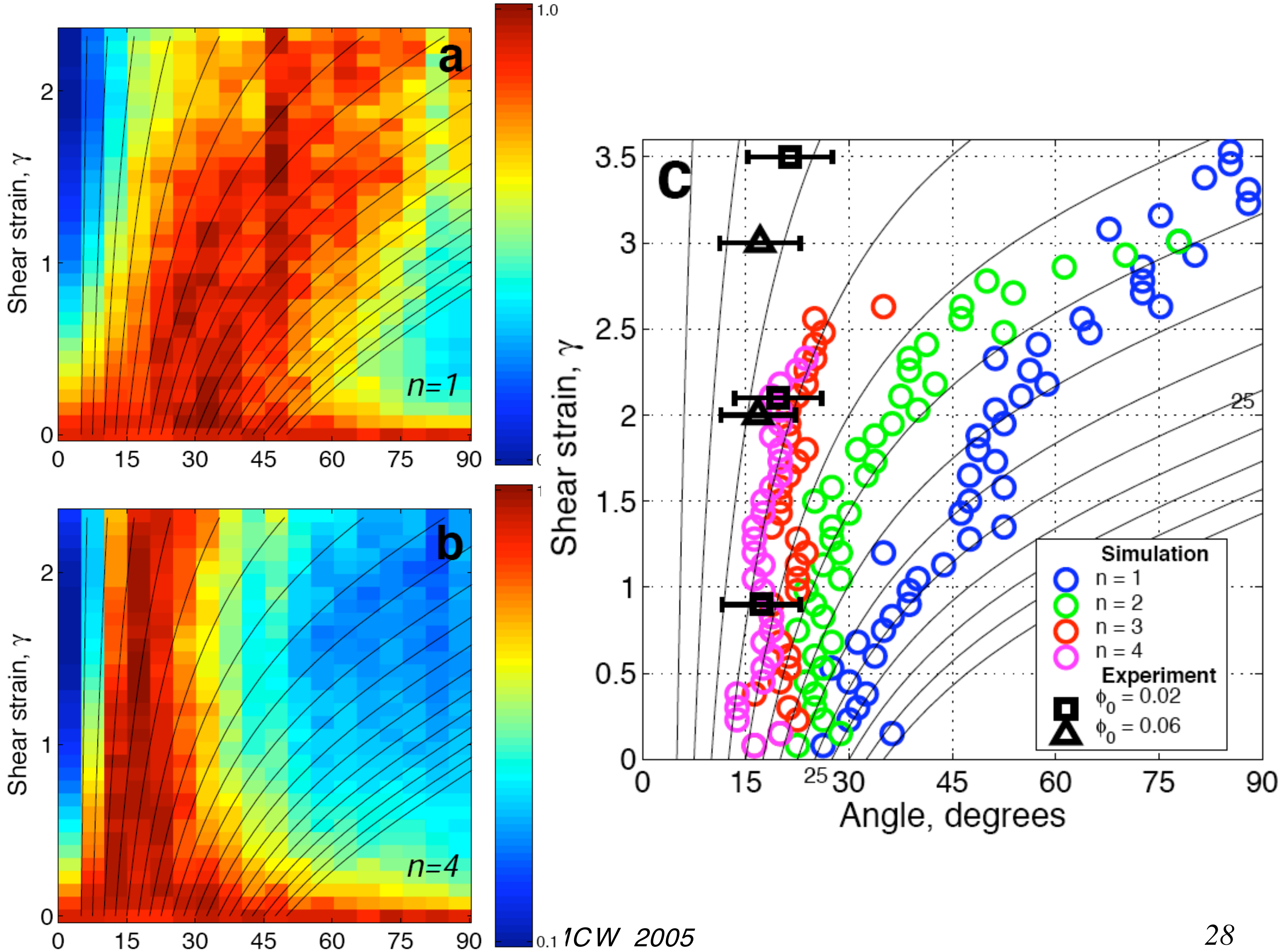
Shear Band formation

(Linear Analysis)



Shear Band formation

(non-Linear Analysis, comparison to experiments)



Conclusion/ Future Directions

- Inclusion of magma dynamics can have significant impact on rheology/chemical transport
- Fluid/solid system are likely to localize
  - Develop/resolve features $<\delta$
  - Observable consequences in chemistry/seismic anisotropy
- Require accurate multi-scale, multi-physics codes (embedded meshes, accurate PV stokes solvers)
- CIG may finally let everyone play…