Transient Rheology: Lab Experiments, Materials Theory and the Challenges of Spatiotemporal Scaling

Reid F. Cooper

Department of Geological Sciences

Brown University

Acknowledgements: Donald Stone, Thawatchai Plookphol, Marshall Sundberg, Christine McCarthy

***Foundation*:**

E.W. Hart, Constitutive equations for the nonelastic deformation of metals, J. Eng. Mater. Technol. **98**, 193-202 (1976).

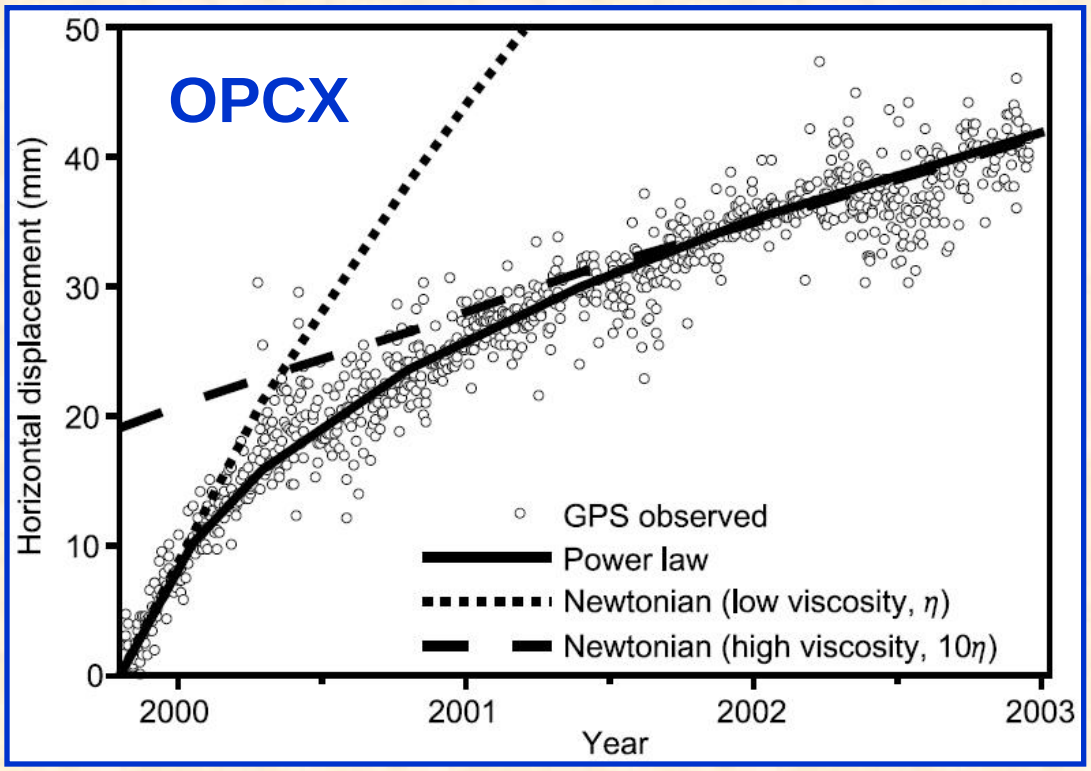
D.S. Stone, Scaling laws in dislocation creep, *Acta Metall. Mater.* **39**:599–608 (1991).

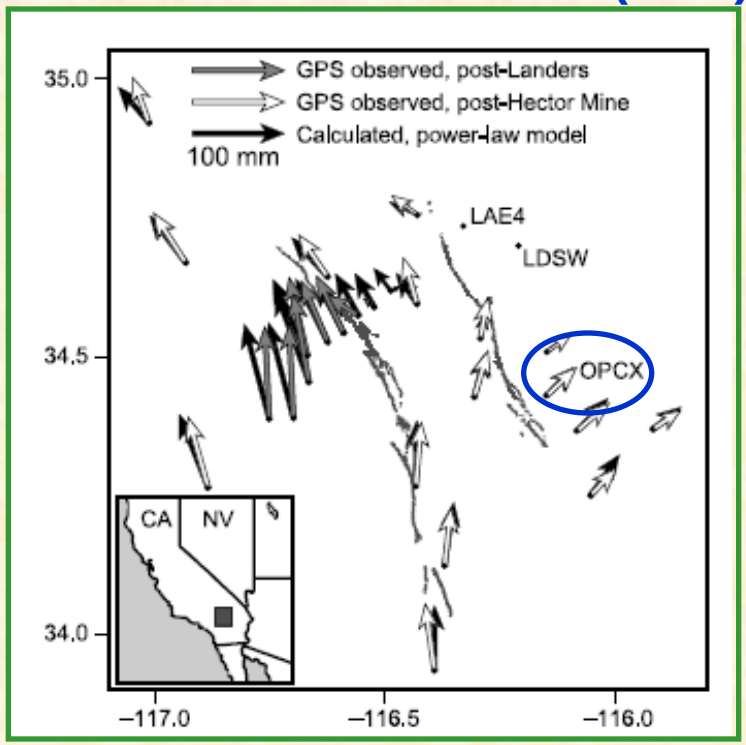
D.S. Stone, T. Plookphol & R.F. Cooper, Similarity and scaling in creep and load relaxation of single-crystal halite, J. Geophys. Res. **109**:B12201, doi:10.1029/2004JB003064 (2004)

CIG 2009 Workshop on Numerical Modeling of Crustal Deformation and Earthquake Faulting, Golden, CO, 24 June 2009

Freed & Bürgmann: Post-Seismic Transient Relaxation, 1992 Landers & 1999 Hector Mine Earthquakes

Nature (2004)





Aplite (Strong) Crust; Wet Olivine Upper Mantle
T_{40km}= 1225°C; n = 3.5

$$\dot{\varepsilon} = A\sigma^{n} \exp\left(\frac{-E_a}{RT}\right); \text{or}$$

Model approach assumed a *stepwise set of steady states* following the wet-olivine flow law:

$$\eta_{eff} = \frac{\sigma}{2\dot{\varepsilon}} = \frac{1}{2A}\sigma^{(1-n)} \exp\left(\frac{E_a}{RT}\right)$$

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Weertman "Average Dislocation Model" of Power-Law Creep

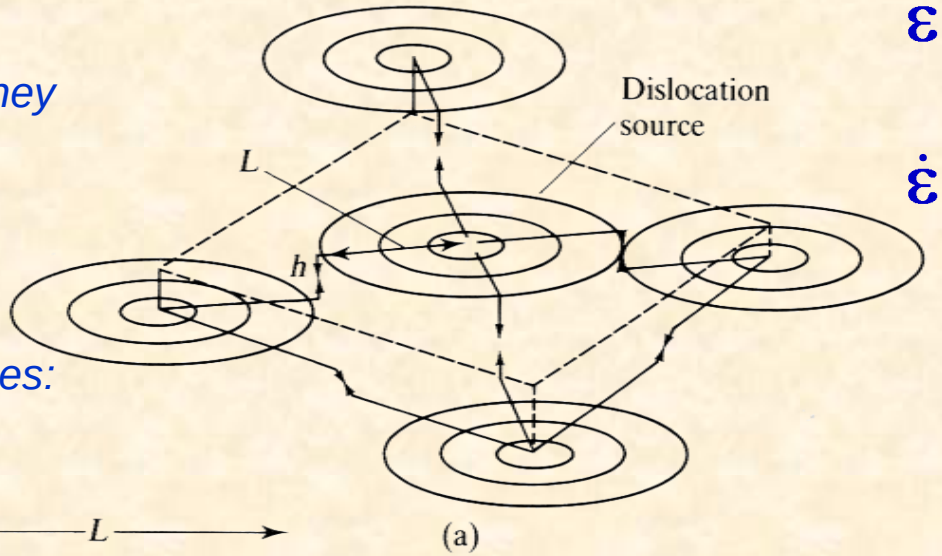
Re σ Sensitivity of M
--D.S. Stone

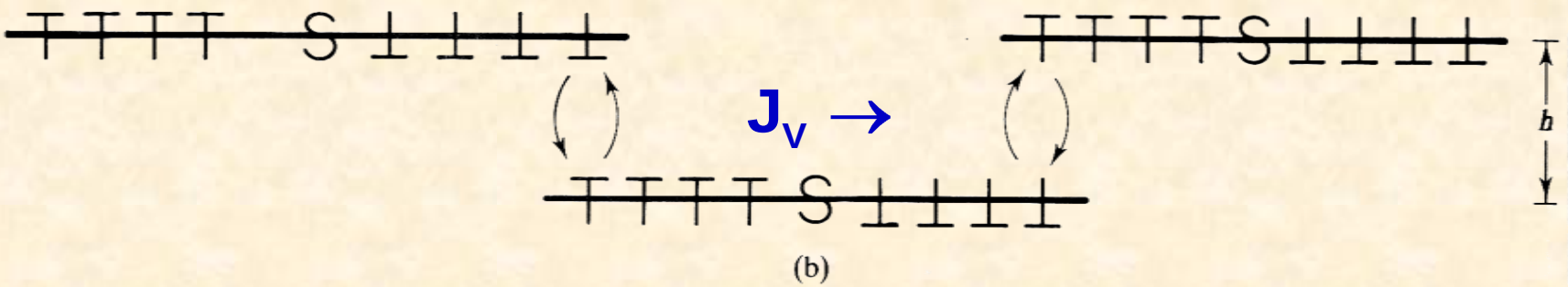
Same sensitivity have they but density entails volume; slip plane length.

Stress squeezes: distance between sources: inverse; volume per source: inverse cube.

$$\dot{\varepsilon} \propto \sigma^n$$

$$\dot{\varepsilon} \propto \exp\left(\frac{-E_a}{RT}\right)$$





$$\dot{\varepsilon} \propto \frac{D_L}{h^{3.5}M^{0.5}}\left(\frac{\sigma\Omega}{kT}\right)$$

for $h \propto \sigma^{-1}$ and $M \propto \sigma^{0}$, power law n = 4.5:
symmetry is broken

for $h \propto \sigma^{-1}$ and $M \propto \sigma^{3}$, power law n = 3:
self-similarity

Steady State: *nonequilibrium stationary state* (Prigogine)

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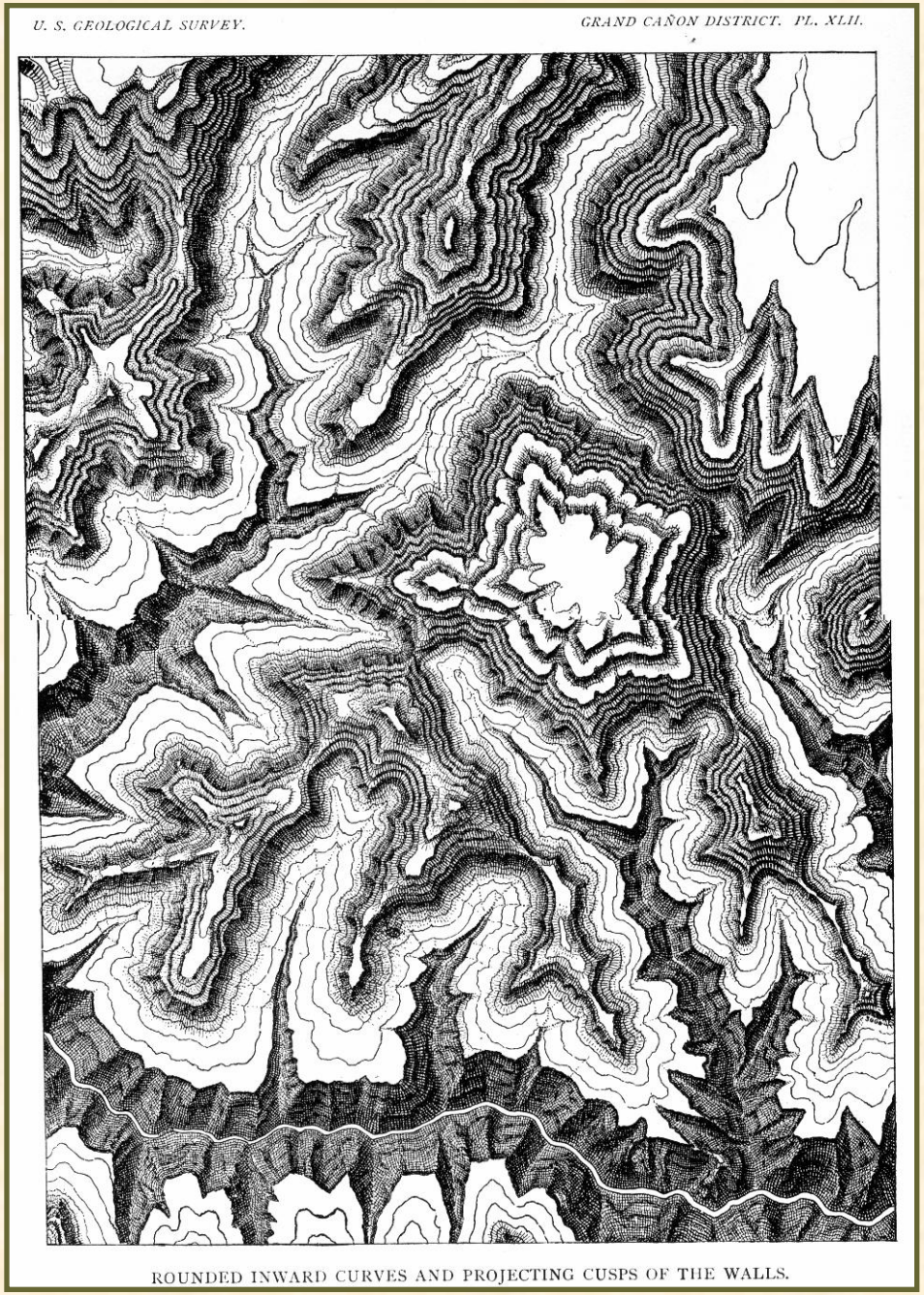
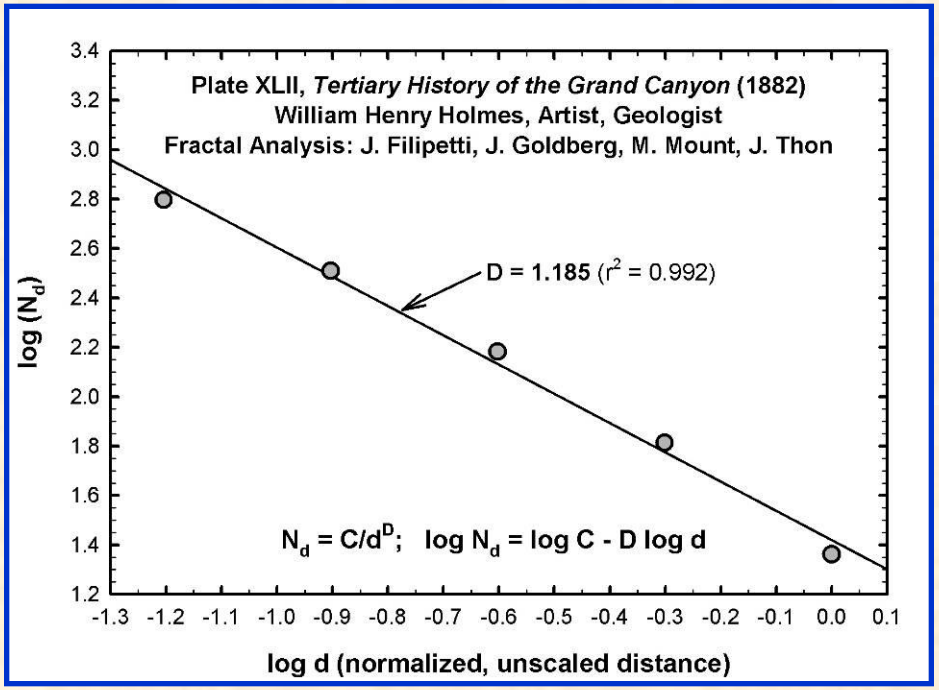


Spatial Scales of Energy Dissipation: *Self-Similarity*; *Self-Organized Criticality*

Fractal Analysis of Erosion Topography of the Grand Canyon

Geol 1960F—*Patterns: in Nature, in Society*
Spring 2009



Natural Landscapes: D = 1.2 ± 0.05

—C.E. Dutton, *Tertiary History of the Grand Cañon District*, USGS, Washington, 1882.

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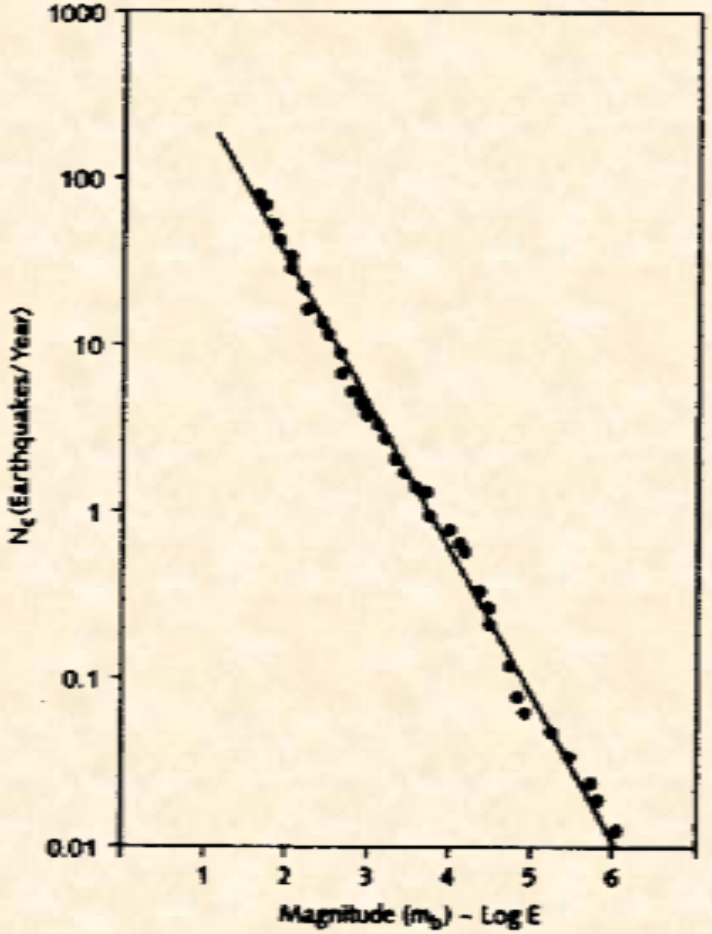
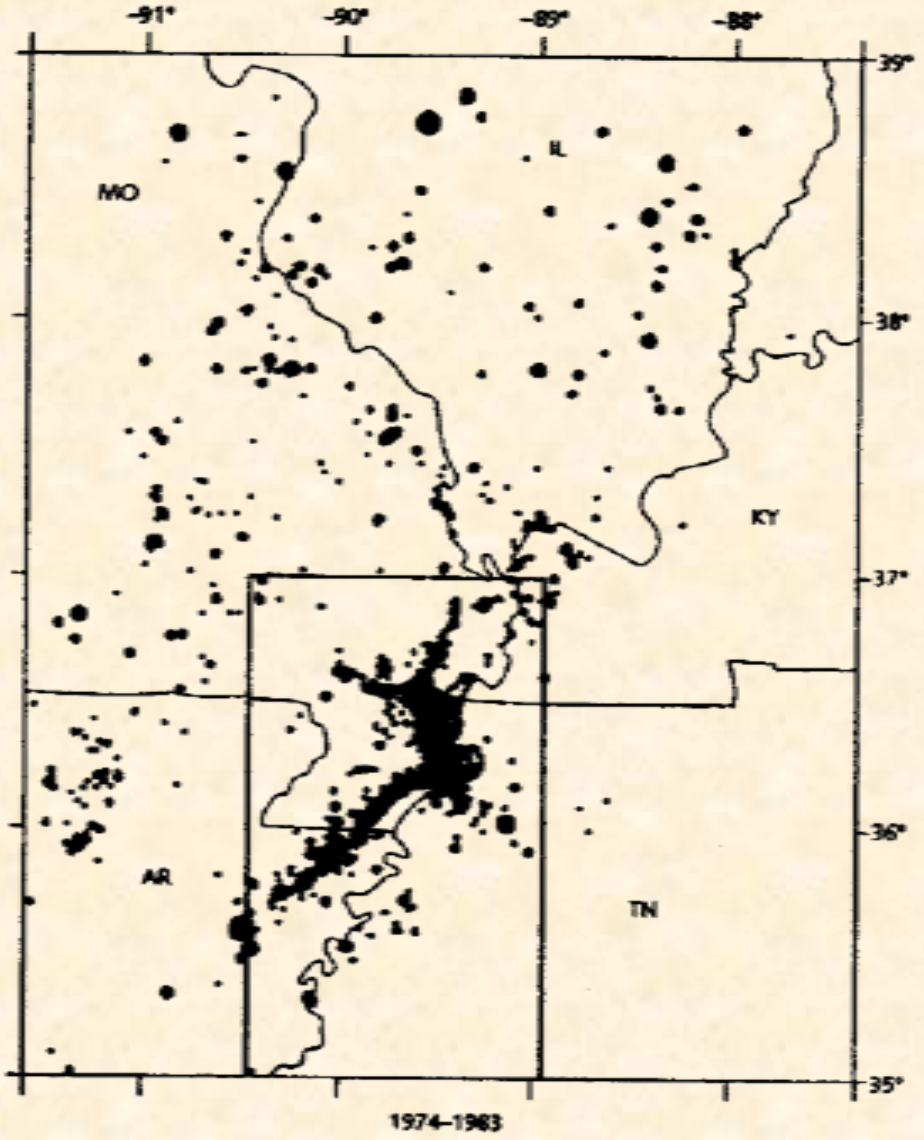
Self-Organized Criticality: Gutenberg-Richter Law

Log Frequency (a^{-1}) v. Log Magnitude





Johnston & Nava, JGR, 90, 6737 (1985)

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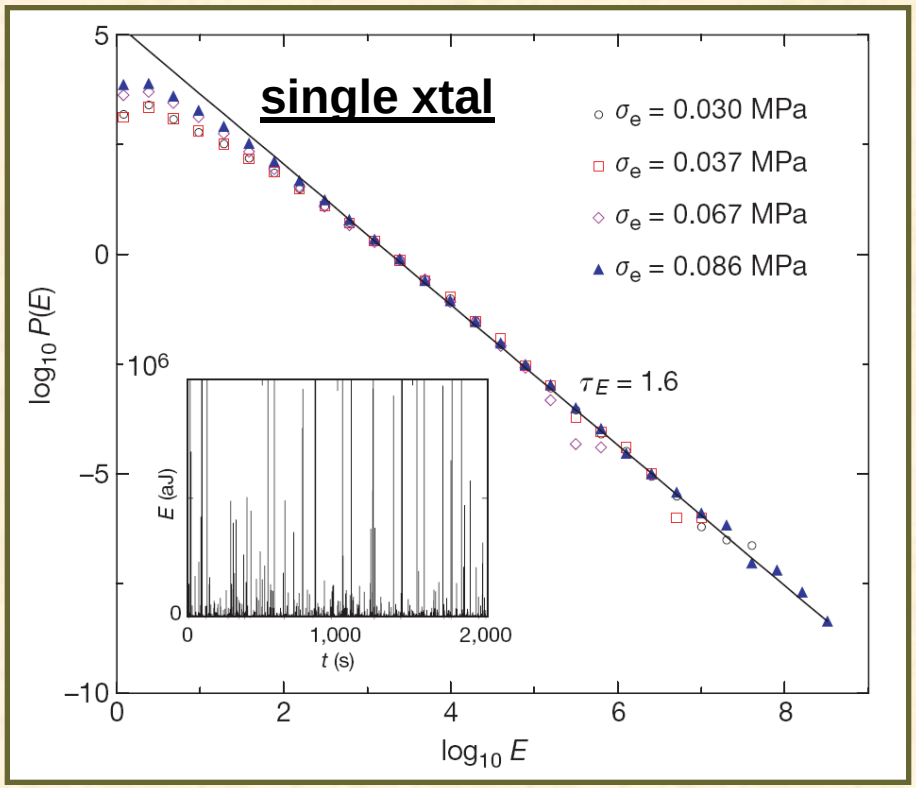
Criticality in the Dislocation Plasticity of Ice

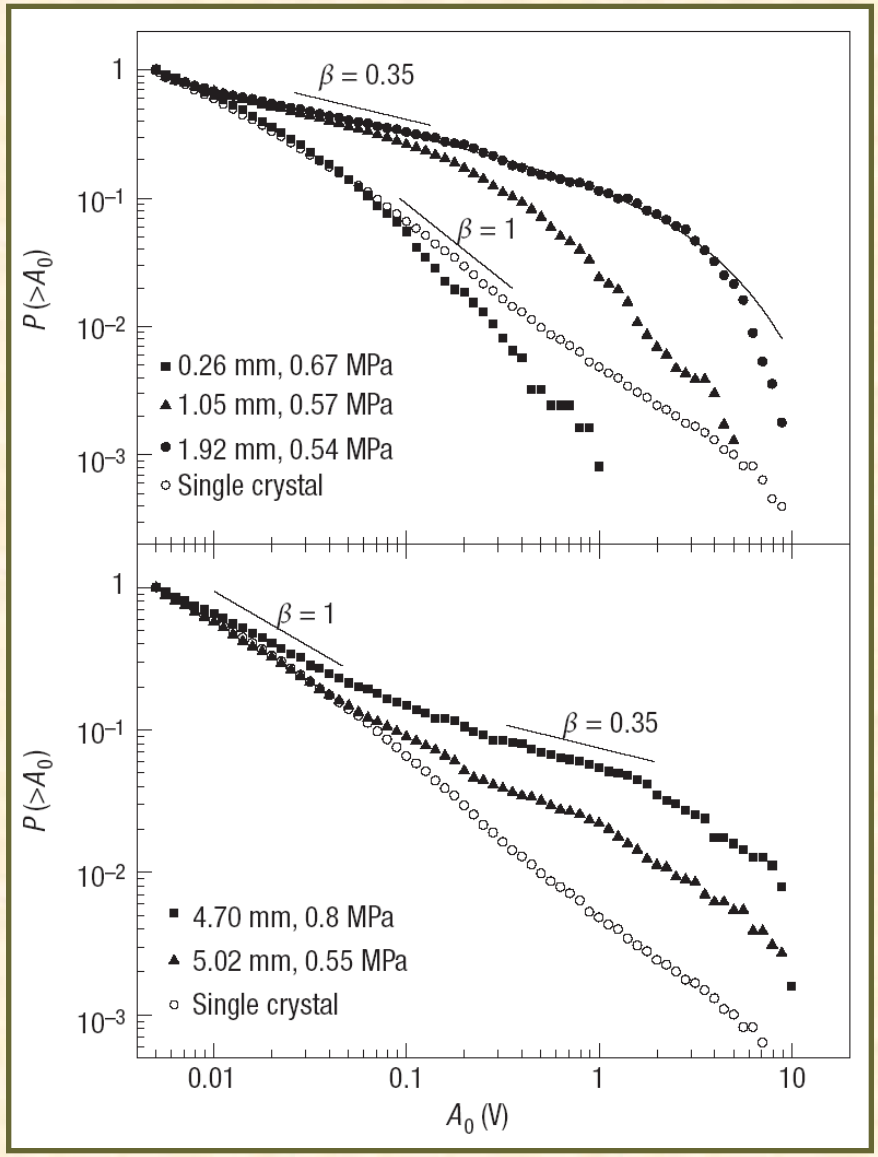


Miguel et al., Nature, 410, 667 (2001)

Creep Experiments: –10°C

E, A_0 ≡ acoustic energy bursts: dislocation glide

$P(E, A_0)$ ≡ frequency of occurrence



Richeton et al., Nature Mater., 4, 465 (2005)

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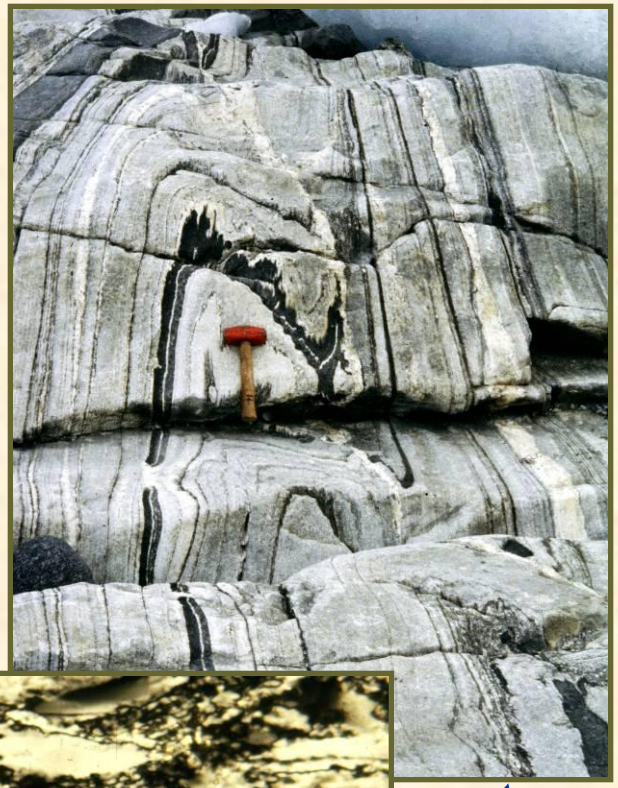
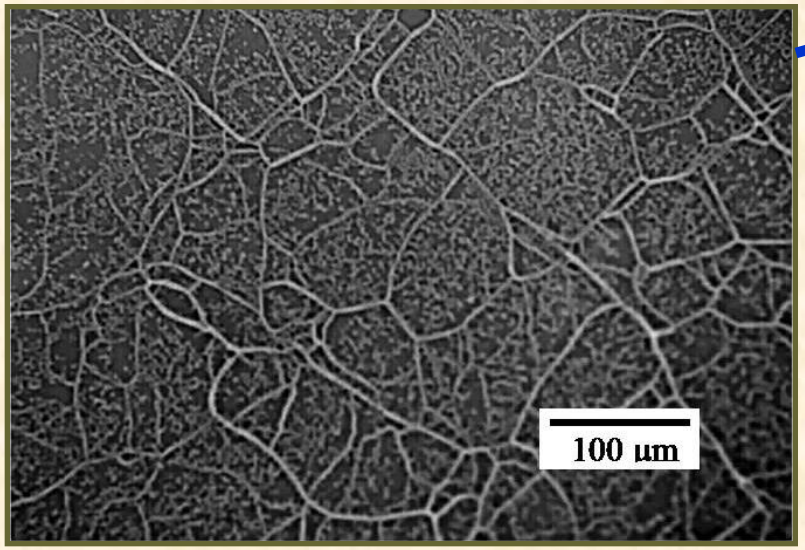
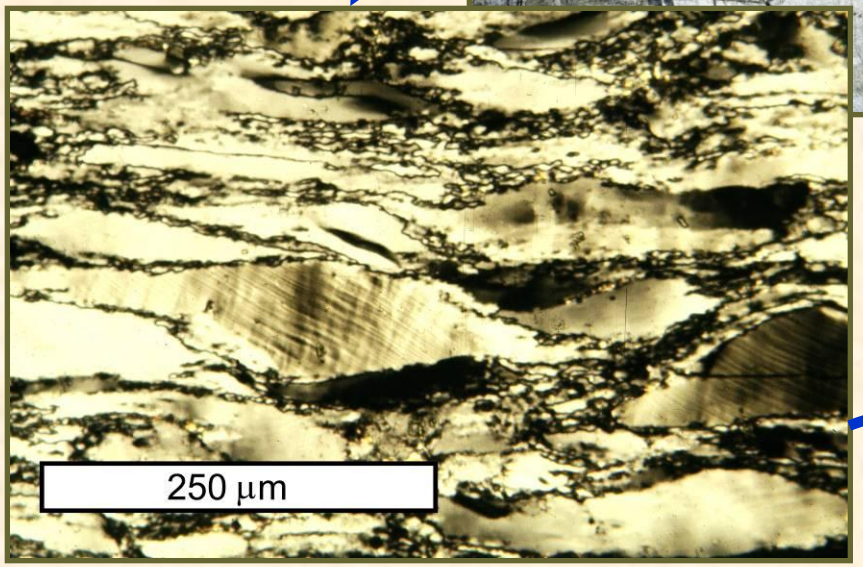
An Energy-Dissipation Perspective…

The dynamic geological setting consists of rock being actively deformed—rock experiencing a thermodynamic state that includes a relentless deviatoric stress in the range 1–100 MPa. The accumulation of plastic strain promotes texture—"self-assembly"—on a variety of scales, from nanometer to kilometer:

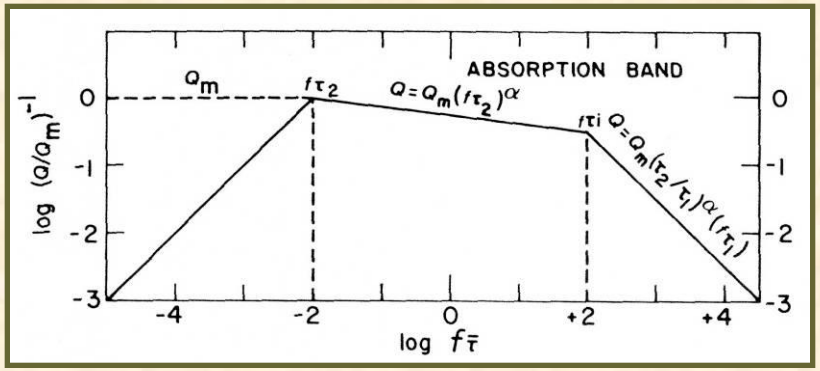




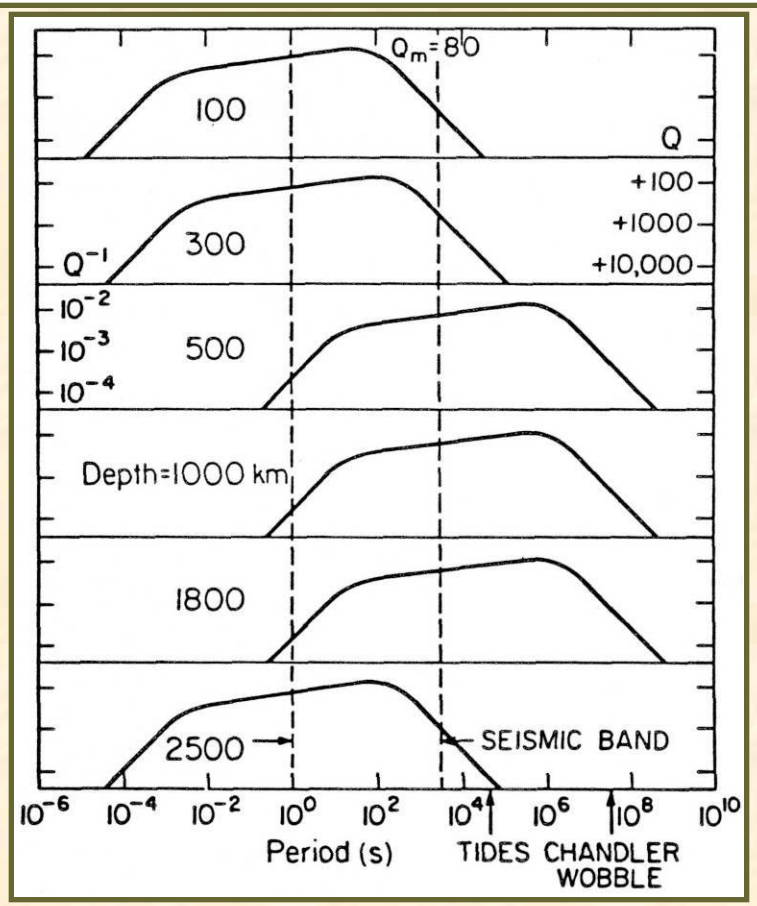


How does one place this phenomenology in the context of self-organized critical behavior—particularly in the recovery after large energy cascades?

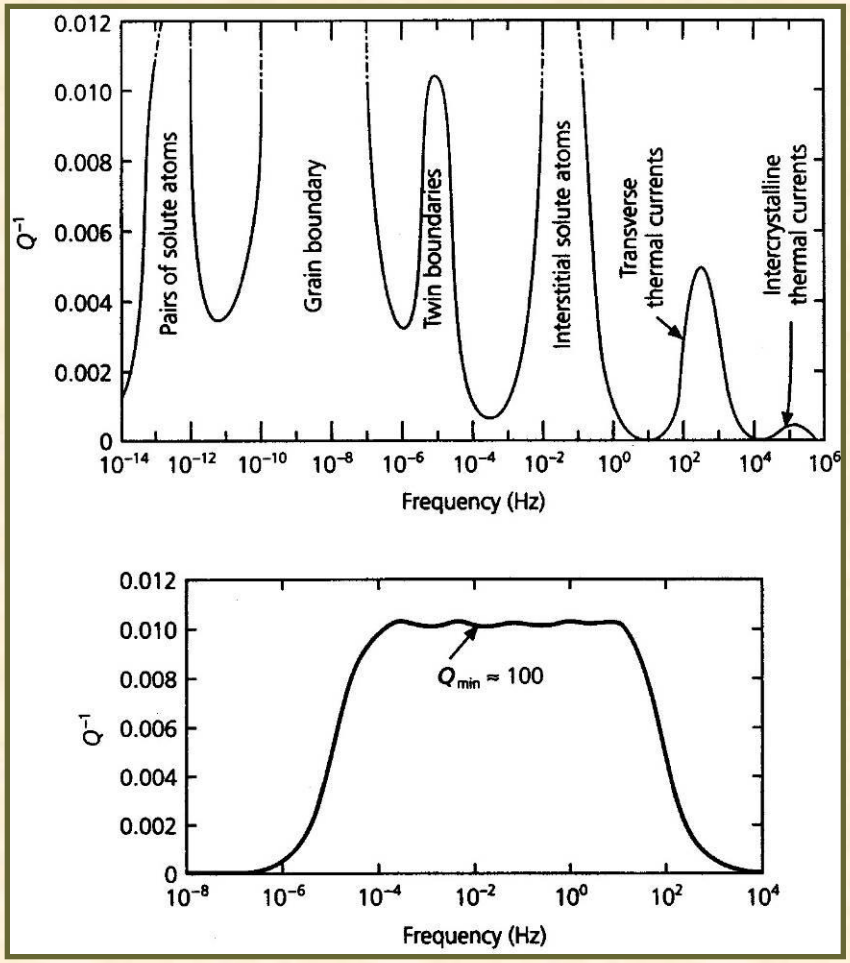
Viscoelasticity & Attenuation: The Absorption Band…*How?*



Conventional perspective: superposition of standard *(exponential-decay)* solid models; a mix of mechanisms and/or microstructural scales…



Anderson and Given (1982)



Liu et al. (1976)

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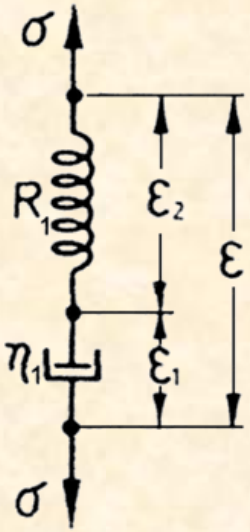
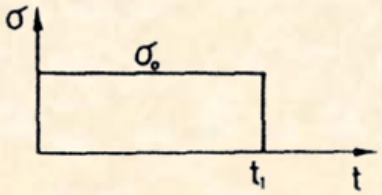
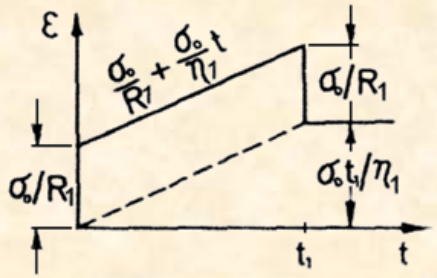
Linear Viscoelasticity: Mechanics Perspective (1)

Maxwell Solid



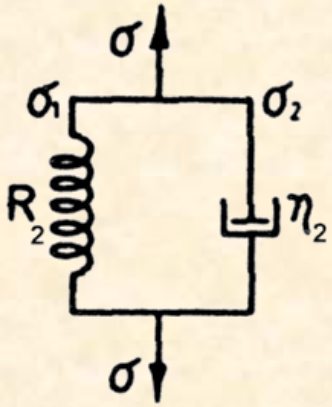
$$\dot{\varepsilon} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 = (\dot{\sigma}/R_1) + (\sigma/\eta_1)$$
$$\varepsilon(t) = (\sigma_0/R_1) + (\sigma_0/\eta_1)t$$



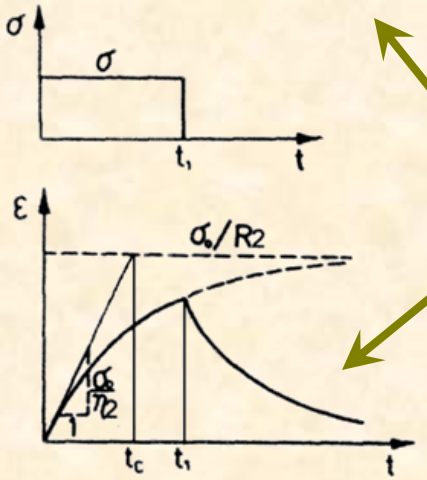


→ *no anelasticity*

Voigt/Kelvin Solid



$$\sigma(t) = \eta_2\dot{\varepsilon}(t) + R_2\varepsilon(t)$$
$$\varepsilon(t) = (\sigma_0/R_2)(1 - \exp[-R_2 t/\eta_2])$$



→ *no plasticity*

- linear elastic spring: R_i
- linearly viscous dashpot: η_i

exponential decay

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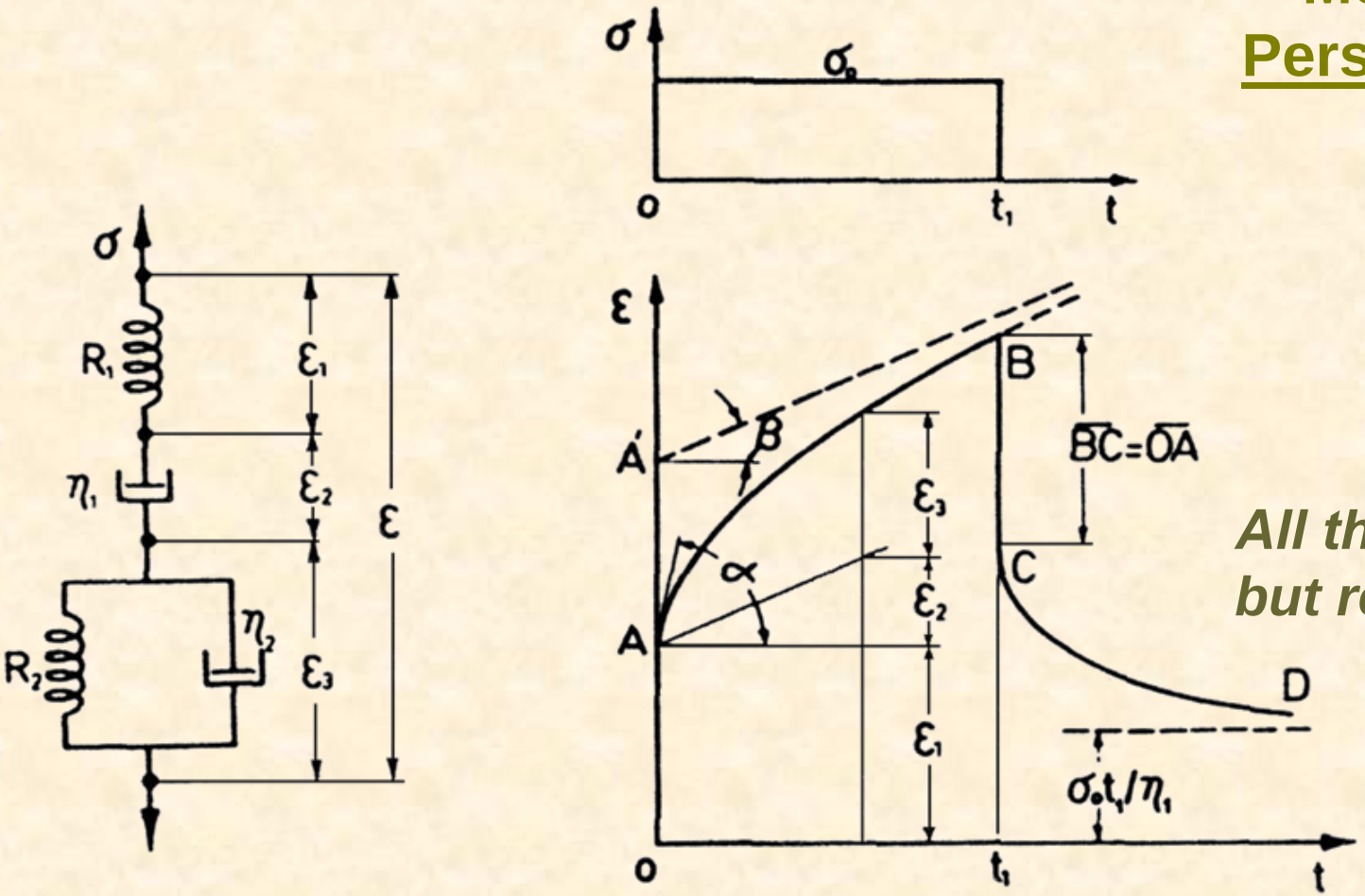
Burgers Solid: Boltzmann superposition of Maxwell and Voigt/Kelvin models

Linear Viscoelasticity: Mechanics Perspective (2)



All the elements … but real materials?

$$\dot{\varepsilon}(t) = (\sigma_0/\eta_2)(\exp[-R_2 t/\eta_2]) + (\sigma_0/\eta_1)$$

$$\varepsilon(t) = (\sigma_0/R_1) + (\sigma_0/R_2)(1-\exp[-R_2 t/\eta_2]) + (\sigma_0/\eta_1)t$$

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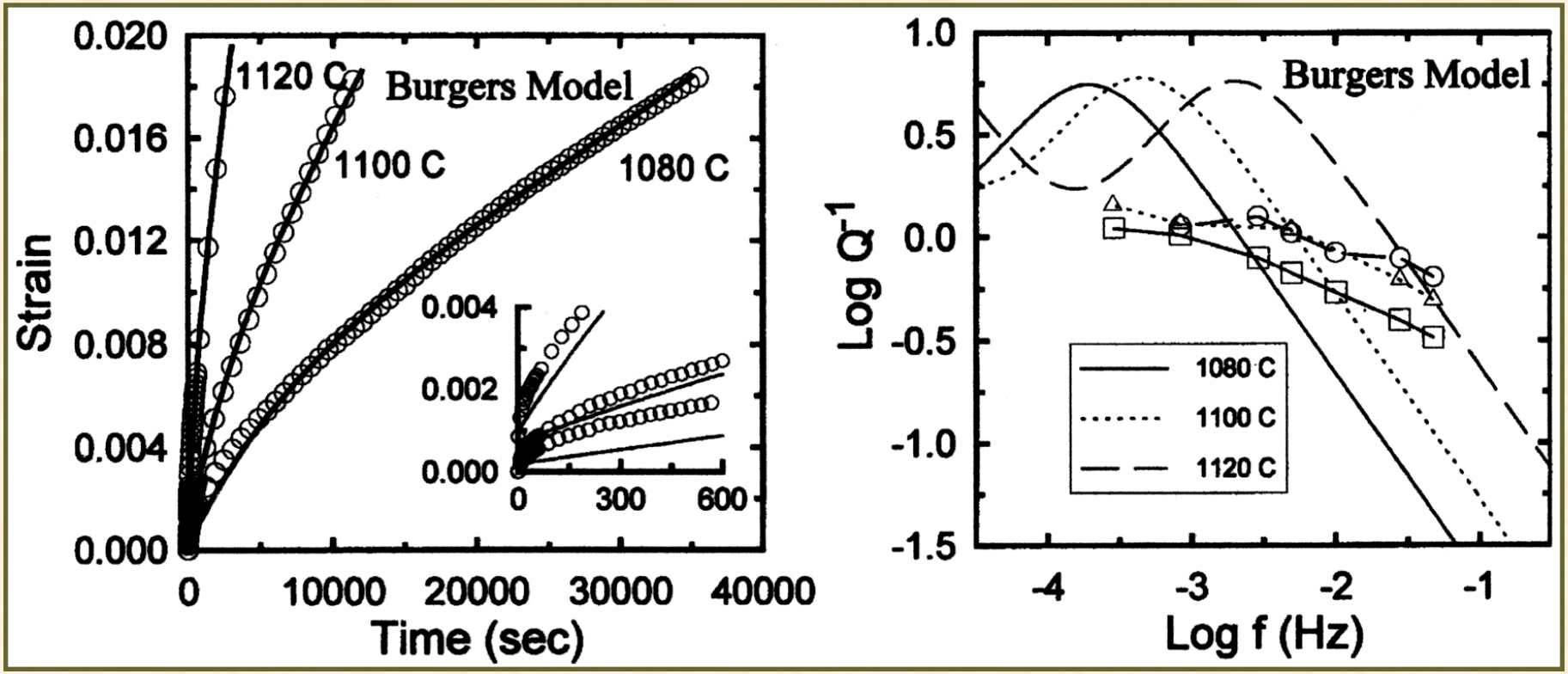
Linear Viscoelasticity: Burgers Solid Analysis



orthoenstatite glass-ceramics/4-point flexure: Gribb and Cooper (1995)

Burgers Solid: $\varepsilon(t) = (\sigma_0 / R_1) + (\sigma_0 / R_2)(1 - \exp[-R_2 t / \eta_2]) + (\sigma_0 / \eta_1)t$

$$\dot{\varepsilon}(t) = (\sigma_0 / \eta_2)(\exp[-R_2 t / \eta_2]) + (\sigma_0 / \eta_1)$$

Laplace transform: $$Q^{-1} = \frac{p_1 q_2 [2\pi f]^2 + \eta_1 (1 - p_2 [2\pi f]^2)}{p_1 \eta_2 [2\pi f] - q_2 [2\pi f](1 - p_2 [2\pi f]^2)}$$

$$p_1, p_2, q_2 (\eta_1, \eta_2, R_1, R_2)$$

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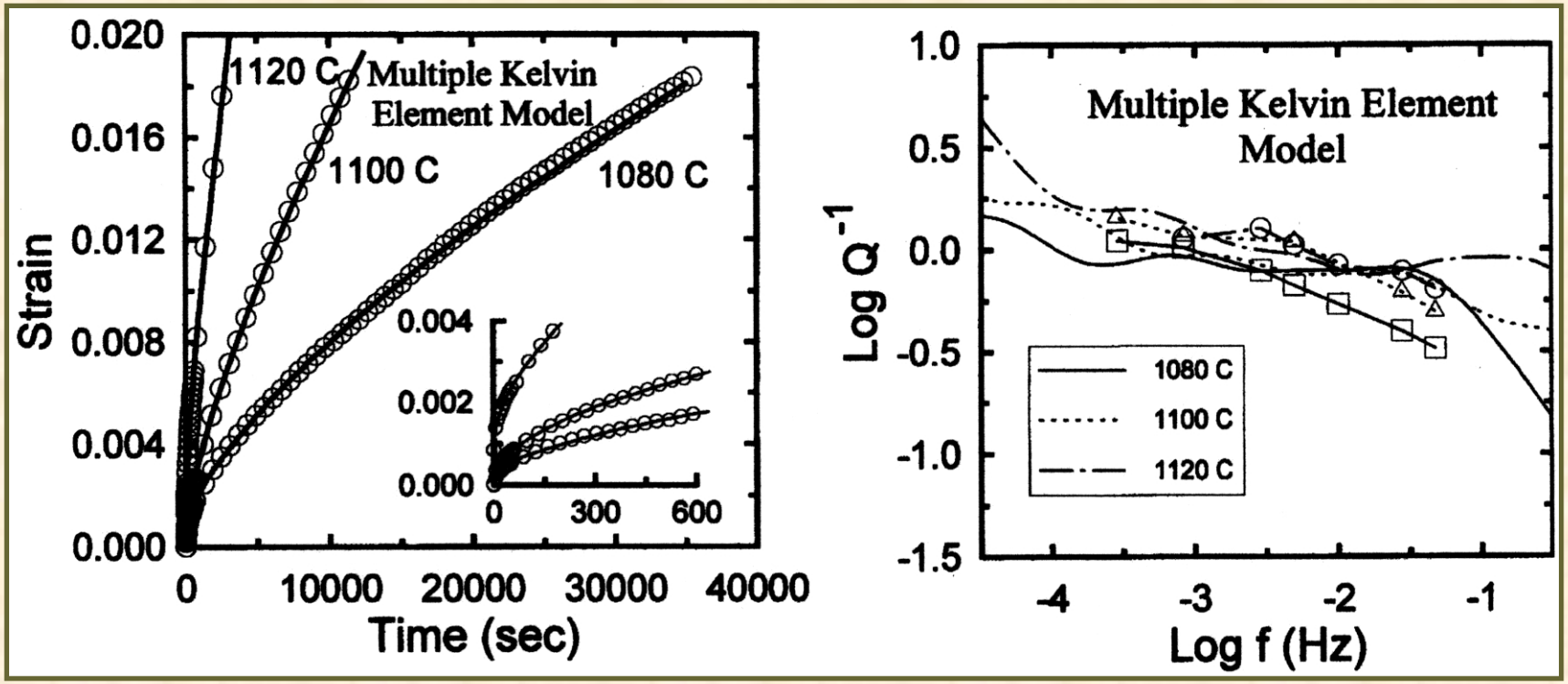
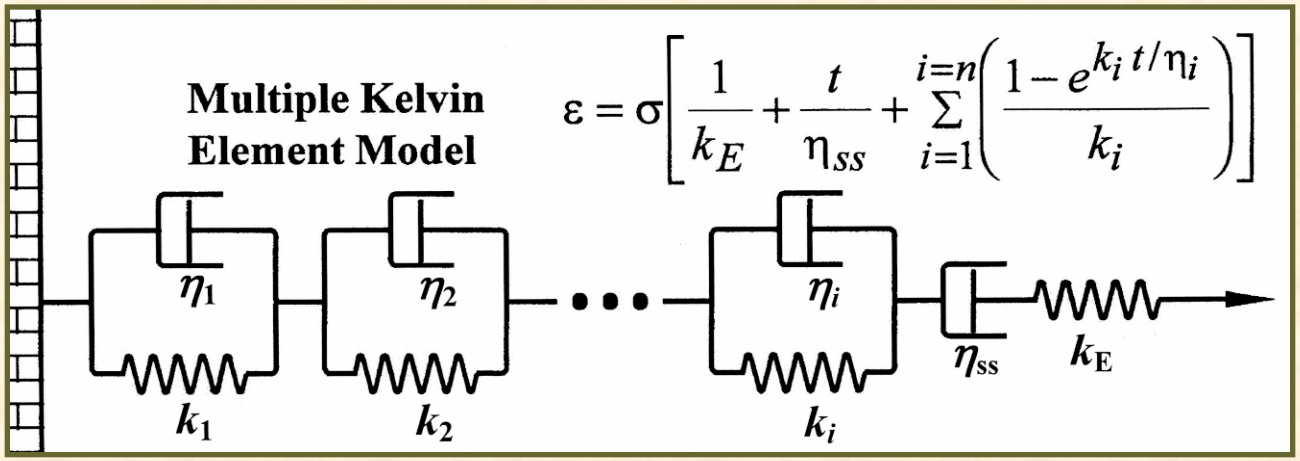
Hmm…Hit It Harder: Boltzmann Superposition



orthoenstatite glass-ceramics/4-point flexure: *Gribb and Cooper* (1995)



Computer arbitration following *Thigpen et al.* (1983); result: three independent exponential-decay mechanisms. *Physics?!*

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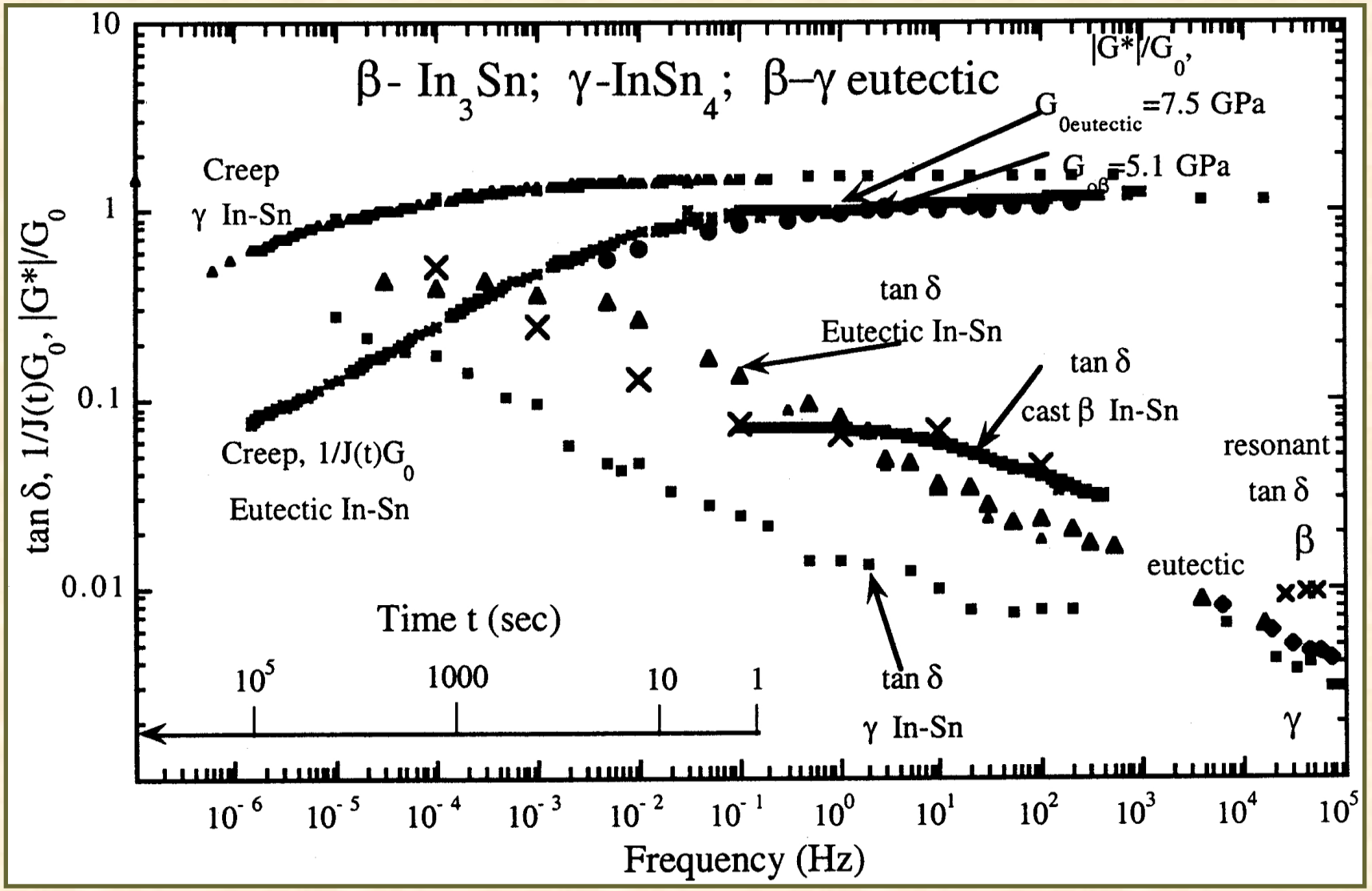
But How Can You Justify 10^{10} in Length Scale?!

Q_G^{-1} v. f; $Q_G^{-1} \equiv \tan\delta$



***McMillan et al.* (2003)**

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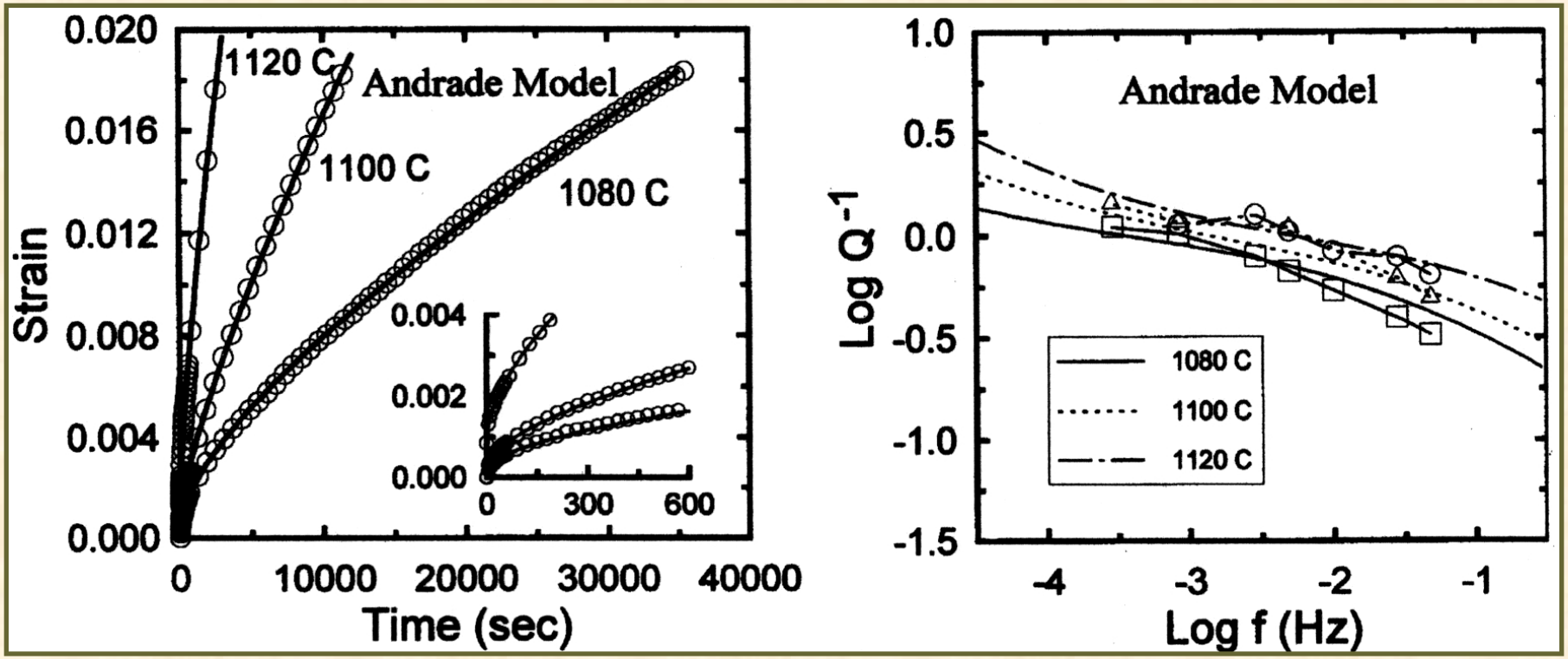
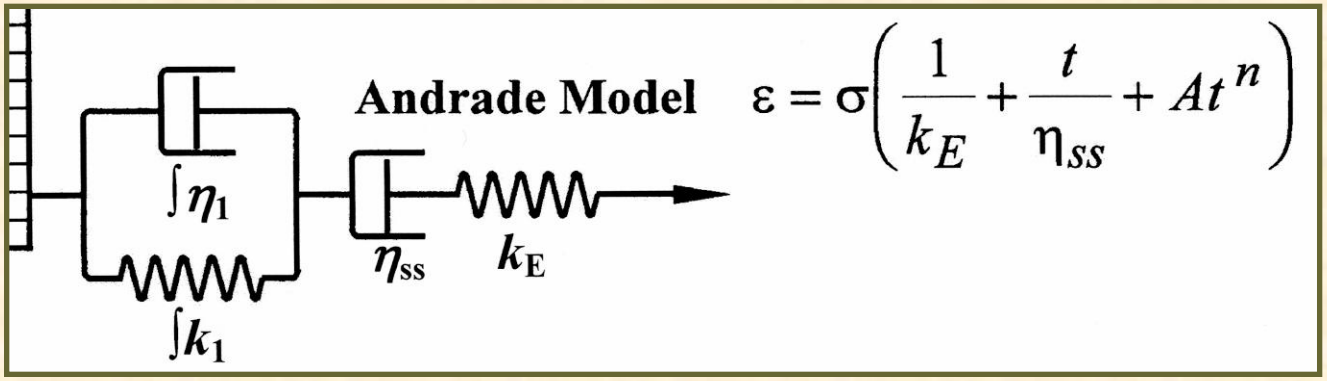
Go to the Limit: Andrade Model



orthoenstatite glass-ceramics/4-point flexure: *Gribb and Cooper* (1995)



Continuous distribution of compliances: nice fit (no surprise!) *but where's the physics?*

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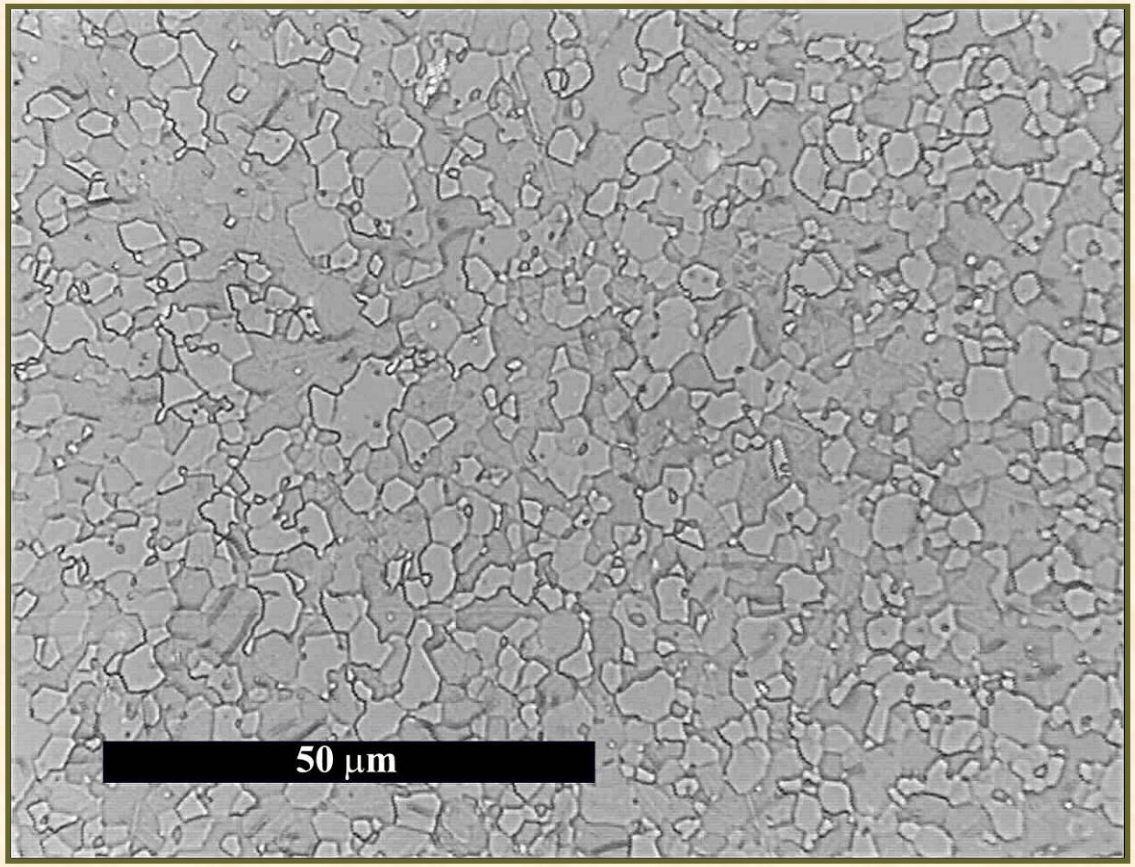
Isolating the Physics: Uniform *d*; No Dislocations‡



Balsam Gap (NC) Dunite: Fo_{92}
(this specimen: $\bar{d} = 4.8 \pm 0.4$ μm)

- $\rho \approx 0.98+ \rho_{th}$; chemically stable
- $4 \leq \bar{d}$ (μm) ≤ 17 in five distributions
- uniform: no grain growth during testing

- fluid-energy pulverized to ~1-μm particles
- cold-pressed (100 MPa)
- vacuum dried/reacted (0.1 torr; 1000°C, 1 h)
- vacuum sintered (0.1 torr but at Ni:NiO; 1350–1380°C, 1 h; ± *T* spike to 1465°C, ¼ h)

‡$\sigma_{xy} \approx 2Gbl\lambda$ where $\lambda = d$: need olivine $d \leq 20$ μm for $\sigma_{xy} = 4$ MPa

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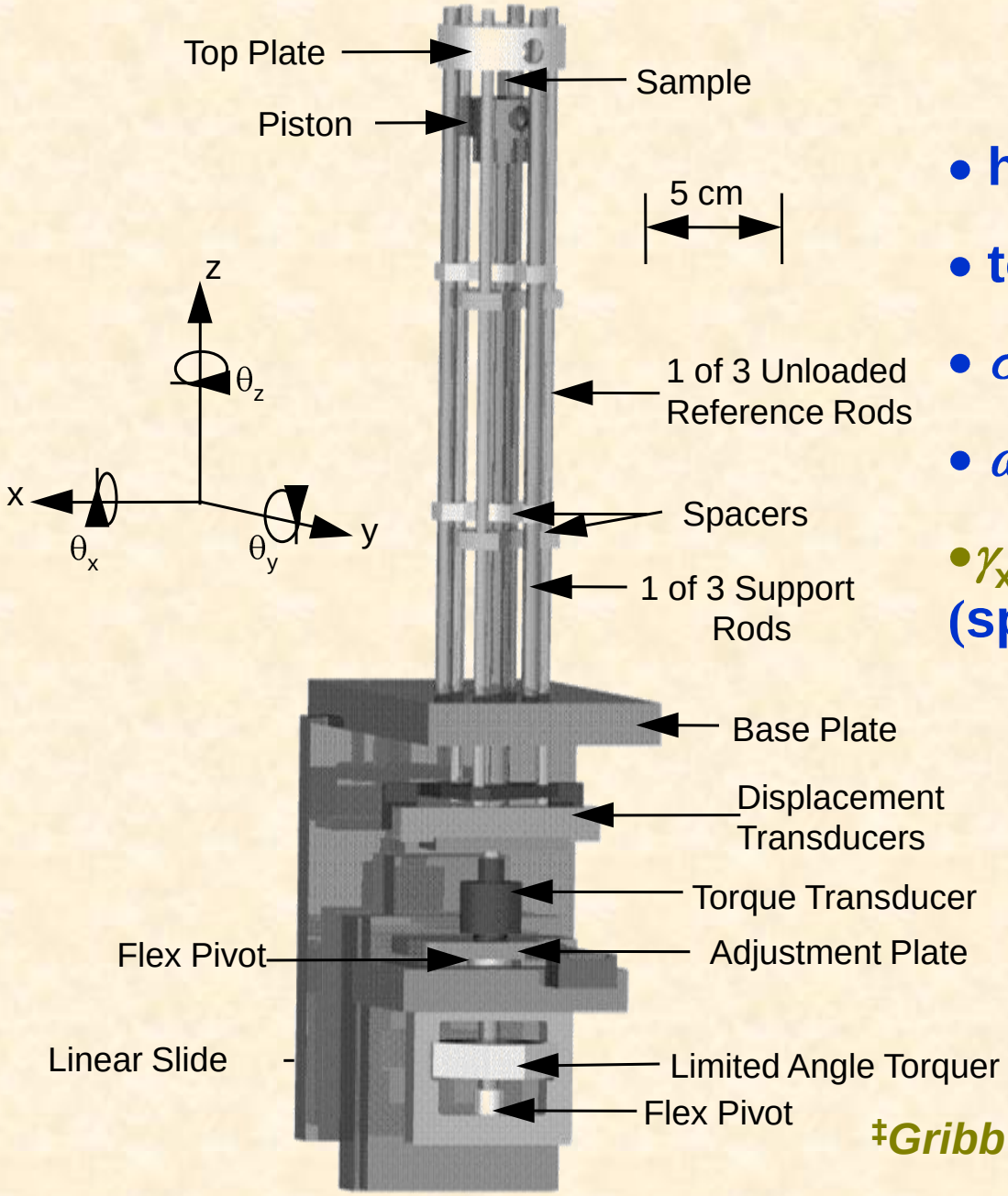

Apparatus‡

- high-T components: TZM
- torque res.: 2×10^{-5} N·m
- σ_{xy} res.: ~2 kPa
- ω res.: 5×10^{-7} rad
- γ_{xy} res.: $\sim5\times10^{-8}$

(specimen size: 3×3×8 mm)

‡*Gribb & Cooper, RSI* (1998)

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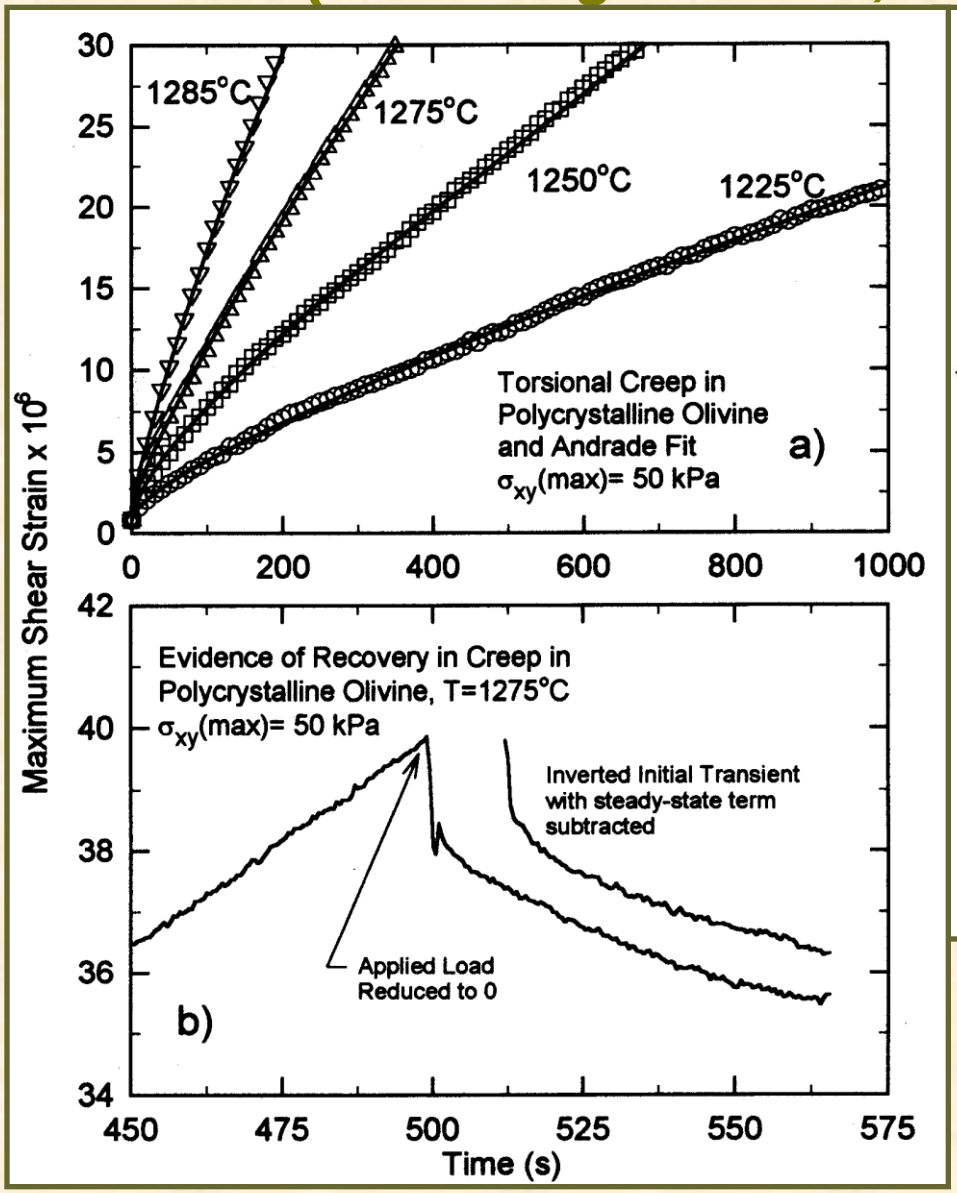
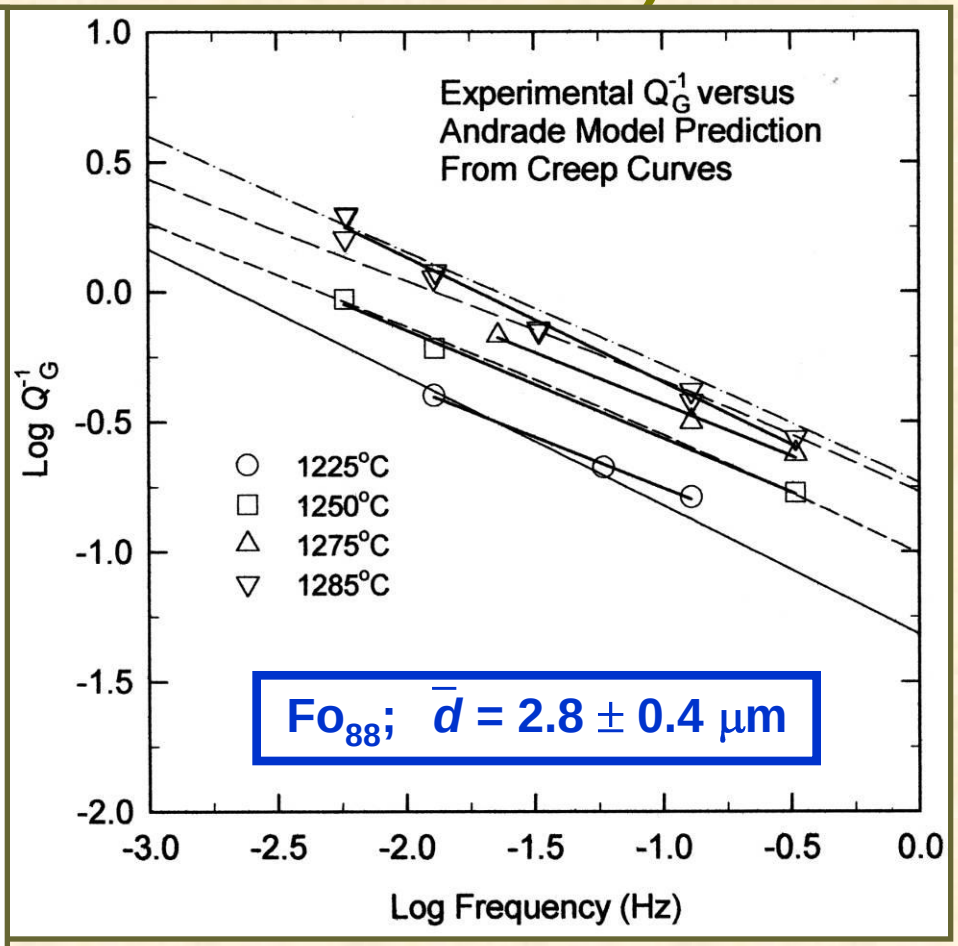
Attenuation Response of Polycrystalline Olivine (*uniform grain size; no lattice dislocations*)





Andrade Fit: ***single-mechanism, non-exponential-decay relaxation***; **consistent with** $\varepsilon_a \propto t^{1/2}$.

***Gribb & Cooper, JGR* (1998)**

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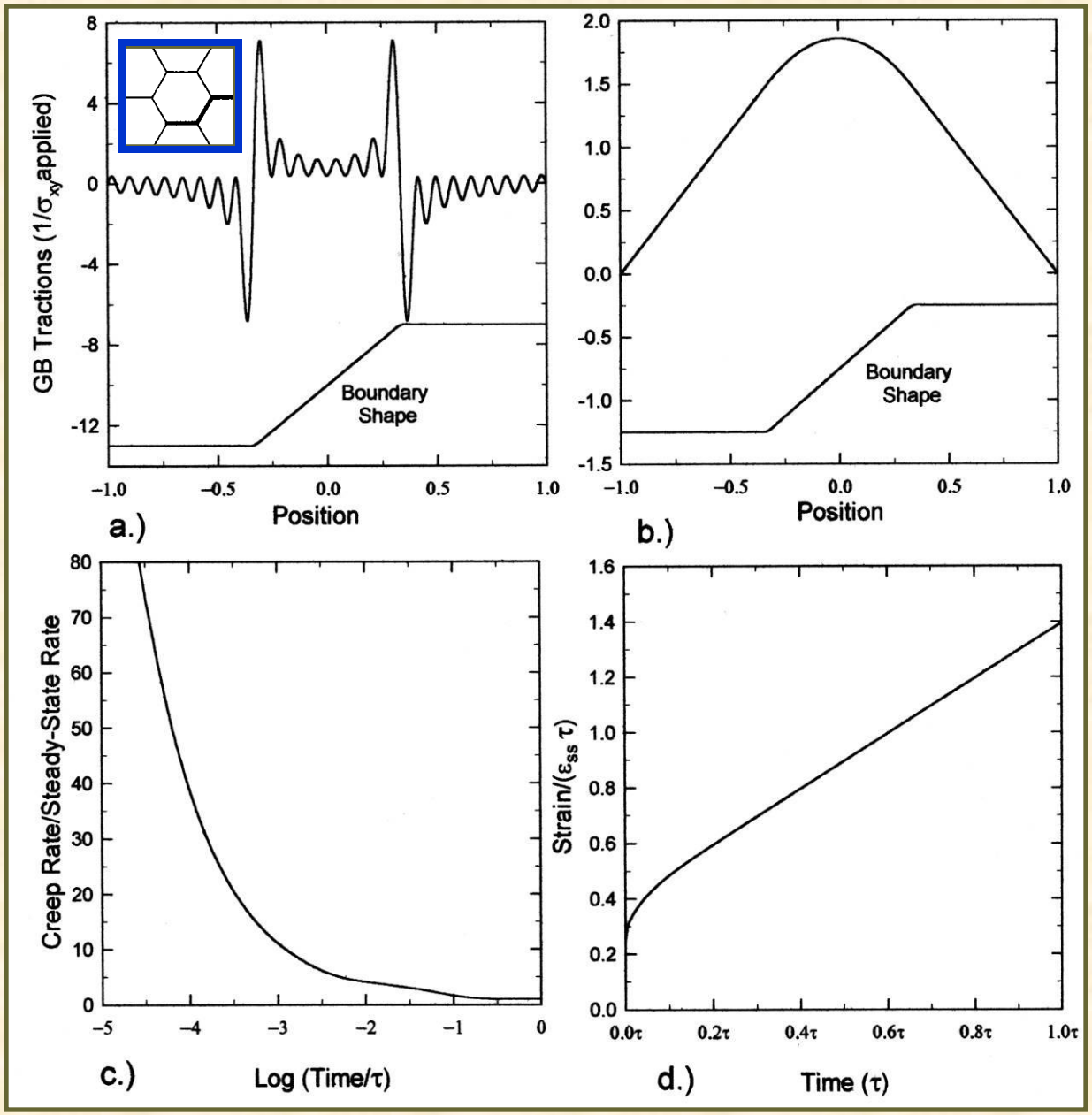
Physical Interpretation of Andrade Behavior:
Intrinsic transient in diffusional creep (after *Raj*, 1975)



- **Fully Newtonian material; Diffusional Creep (threshold?).**
- **Grain boundaries shear-inviscid—gives rise to (a)**
- **Apparent broad distribution of compliances, *but no distribution in a microstructural variable.* *Andrade behavior intrinsic to the creep mechanism: chemical diffusion in a diminishing potential*.**

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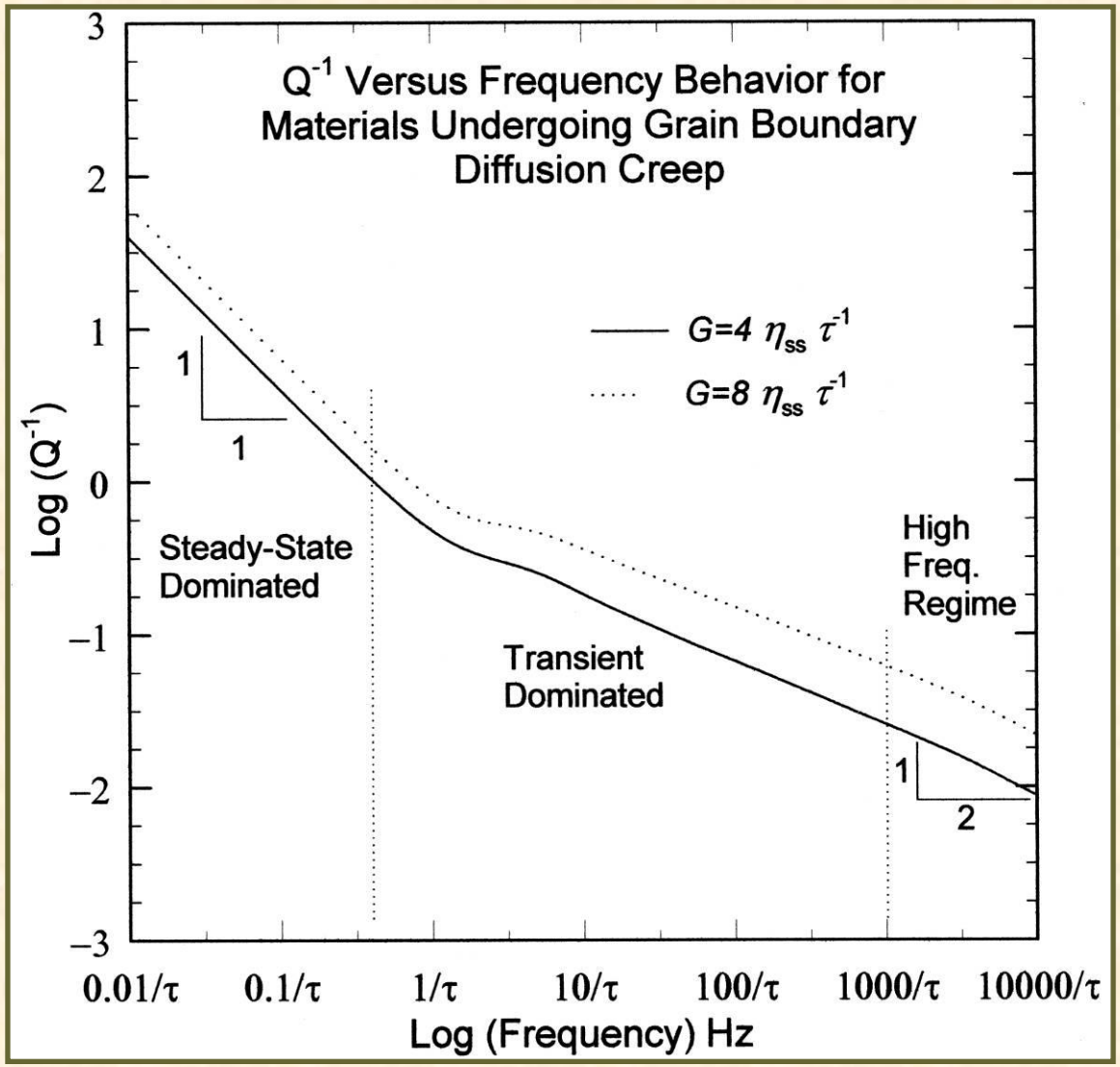
Universal Q^{-1} Spectrum from Universal Creep Curve



Transient—chemical diffusion in the context of a diminishing potential:

$$\tau = \frac{3\sqrt{3}\ (1-\nu^2)\ d^3 kT}{2\pi^3 E D_b \delta \Omega}$$

Steady-State—*Coble* (1963) creep:

$$\dot{\gamma}_{ss} \cong \frac{132\sigma_{xy}\Omega\delta D_b}{kTd^3} = \frac{\sigma_{xy}}{\eta_{ss}}$$

$$\tau \cong \frac{10\sigma_{xy}}{E\dot{\gamma}_{ss}} = \frac{10\eta_{ss}}{E} \cong \frac{4\eta_{ss}}{G}$$

→ ***Entirety of dynamic behavior can be described by two variables,*** η_{ss} ***and*** **E.**

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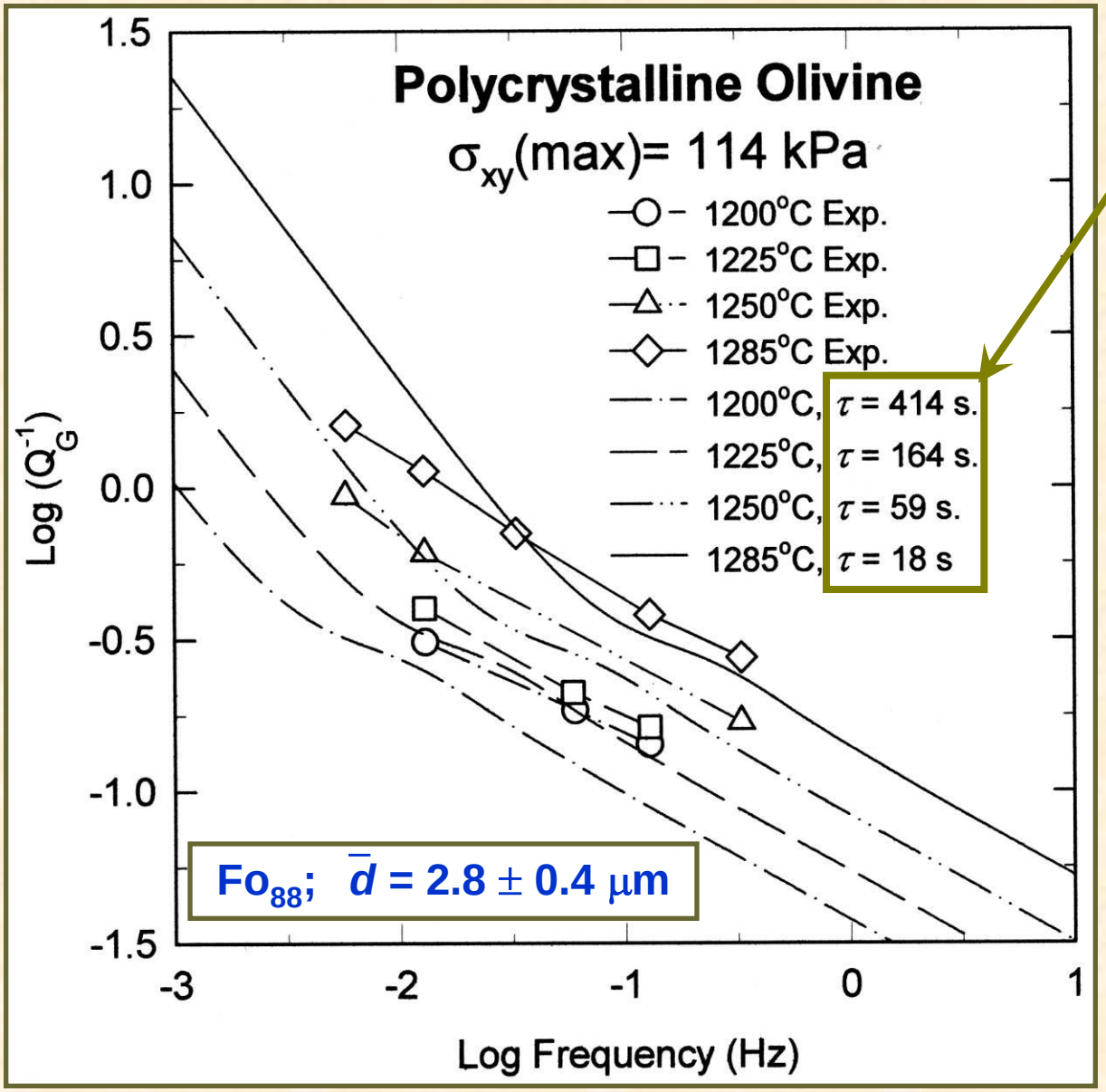
Prediction of Attenuation Response



$$\tau \cong \frac{10\sigma_{xy}}{E\dot{\gamma}_{ss}} = \frac{10\eta_{ss}}{E} \cong \frac{4\eta_{ss}}{G}$$

- E from *Simmons and Wang* (1971)
- η_{ss} from unidirectional creep response

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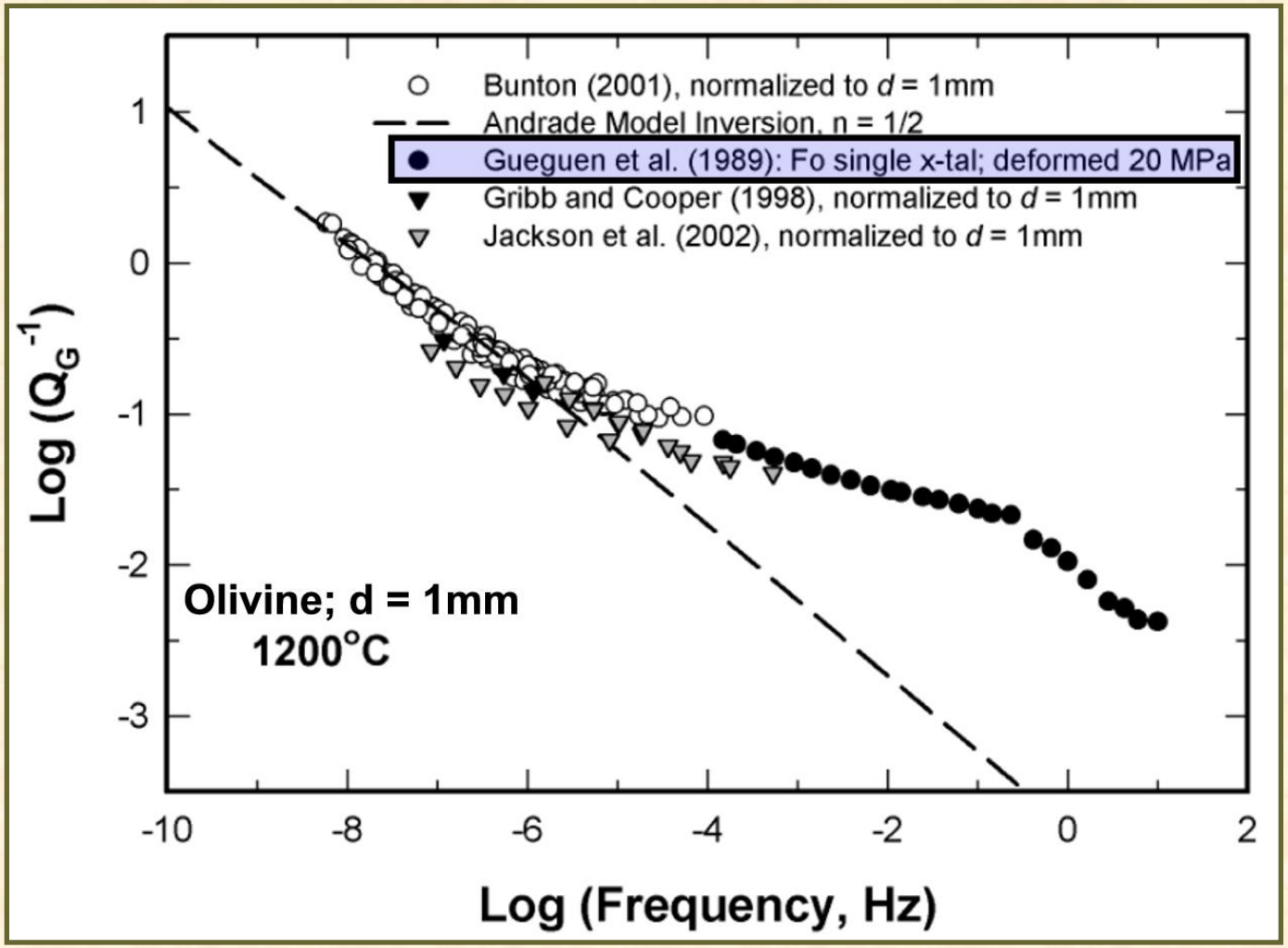
Applicability to Material Deforming by Dislocation Creep?



Polycrystalline data deviate from the Andrade-based model when local grain boundary processes start to dissipate mechanical energy.

Data "map" into deformed-single-crystal response, suggestive of a primary role of subgrain boundaries in effecting absorption.

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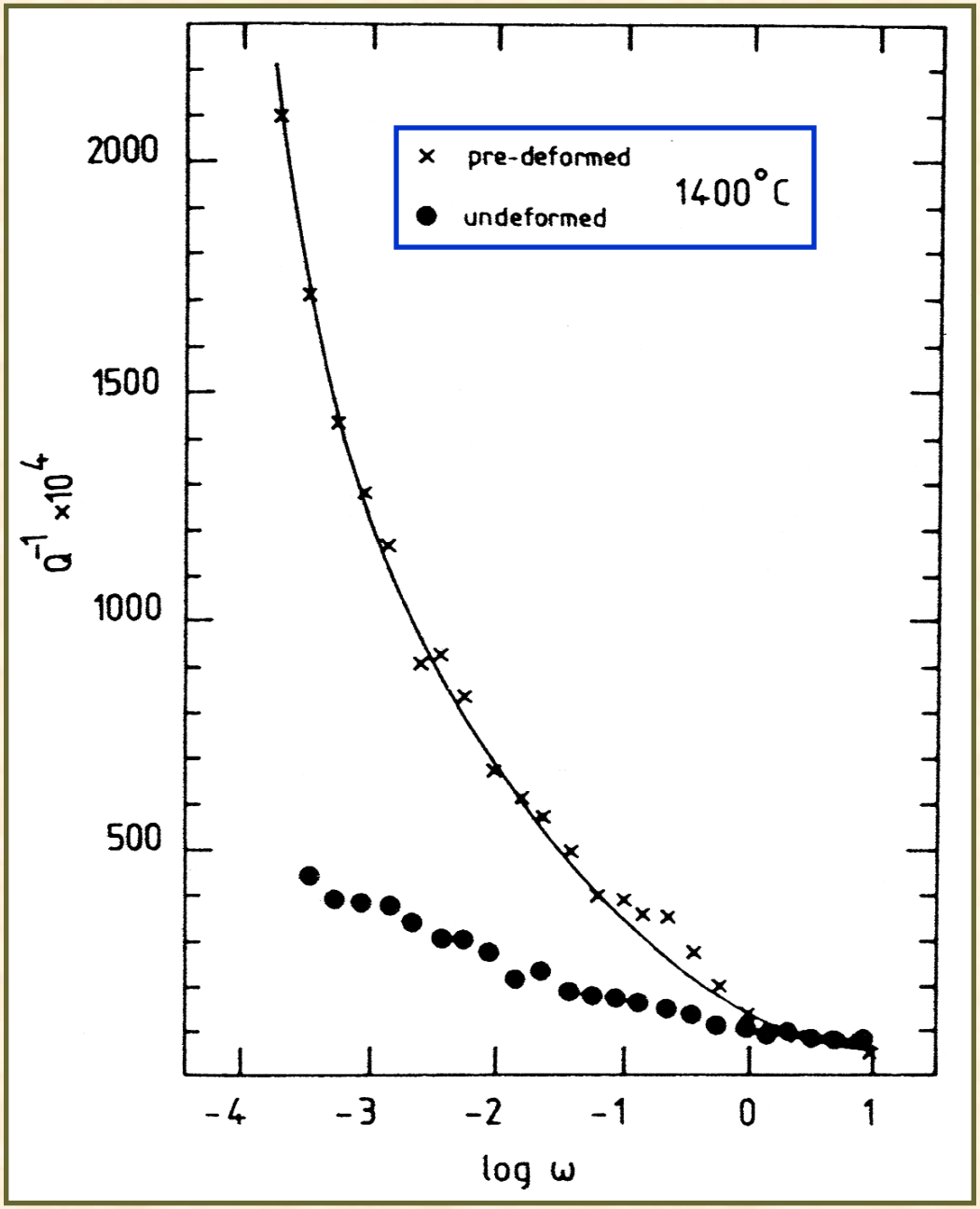
Single Crystal Attenuation : The Role of "Hardness"



Synthetic Forsterite

"Pre-deformation:"

1600°C; 20 MPa; $[111]_C$

Steady-state creep to 1% strain

Anticipated mean subgrain size: ~20 μm

Attenuation:

Sub-resonant torsion

Gueguen et al., *PEPI* (1989)

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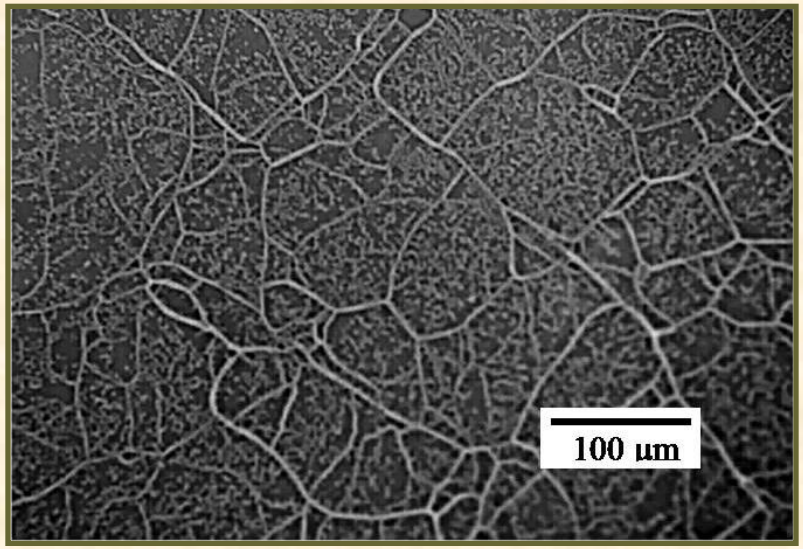
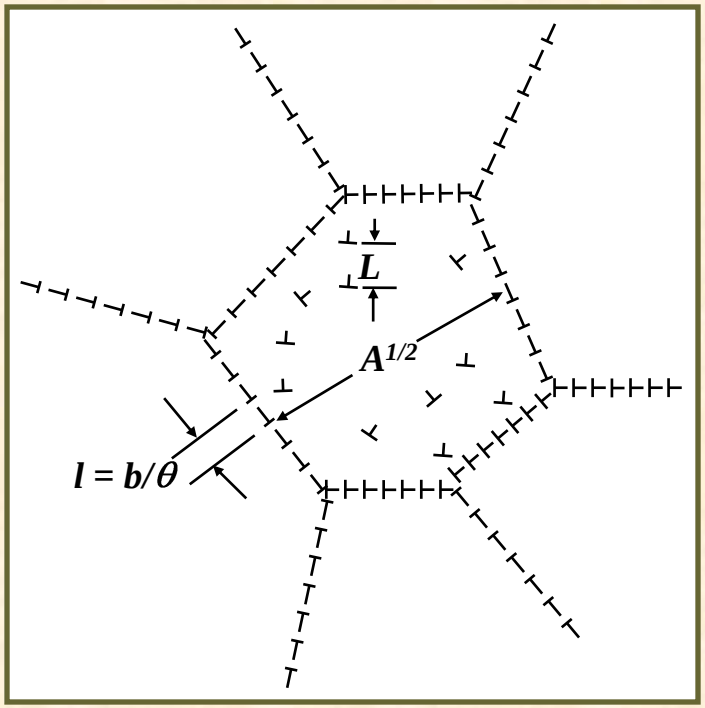
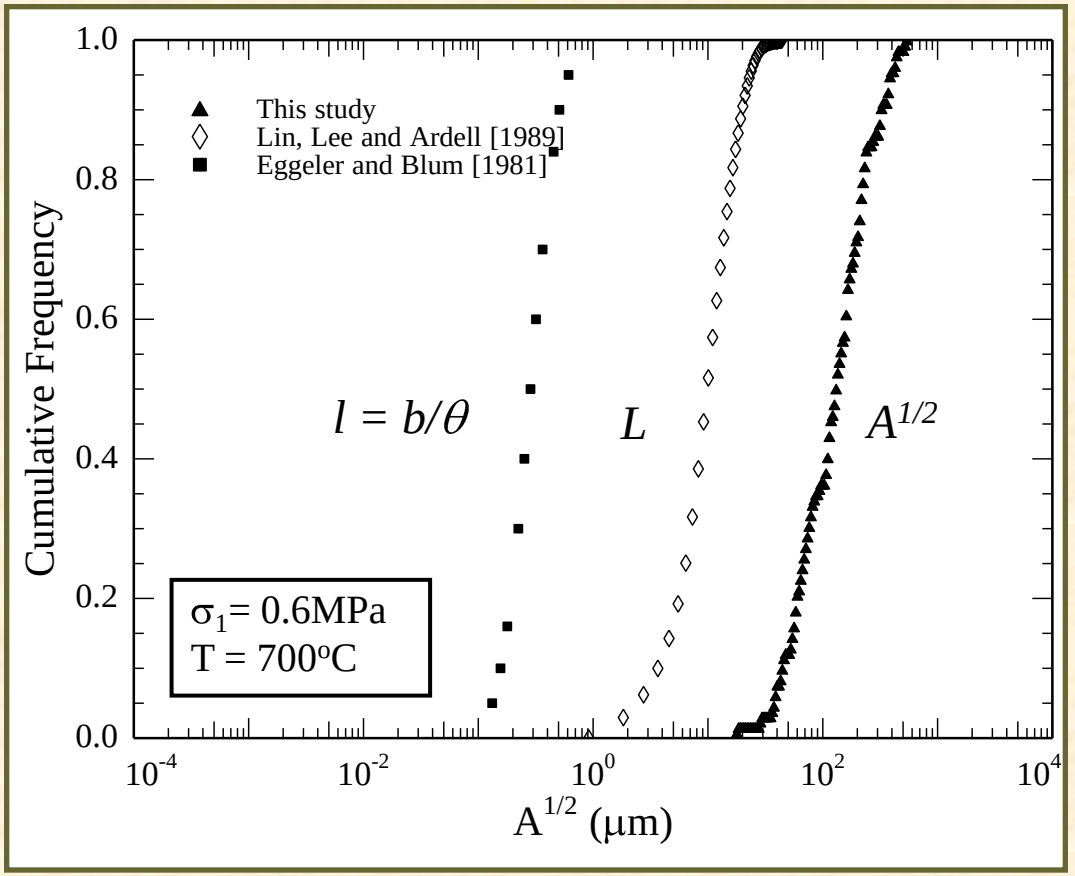
Microstructural Self-Similarity







Self-similarity implies, too, microstructural aspects at a larger scale, e.g., the statistical angular relationship amongst groups of subgrains.

l ≡ SGB dislocation spacing; L ≡ dislocation link length;
$A^{½}$ ≡ D ≡ subgrain size

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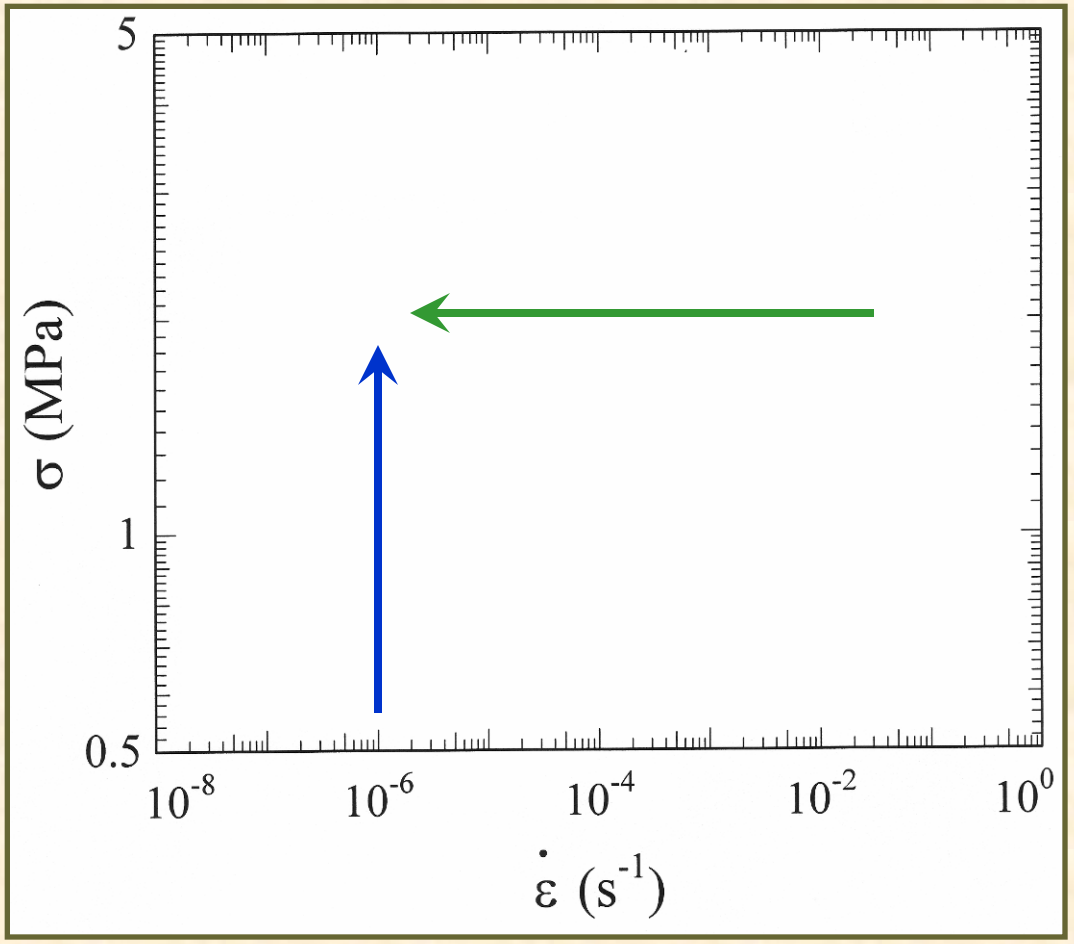
A State-Variable Approach to Plasticity?

Paths

***Creep:* constant stress**
(decelerating transient)

Constant strain rate
(work hardening)

Constant Microstructure?



Hart (1976): Each point in (σ, $\dot{\varepsilon}$) space (T constant) corresponds to a singular internal structure ("hardness") of the material—e.g., a specific distribution of lattice defects. Can microstructure be described as a state variable? Can we discover *path independence*?

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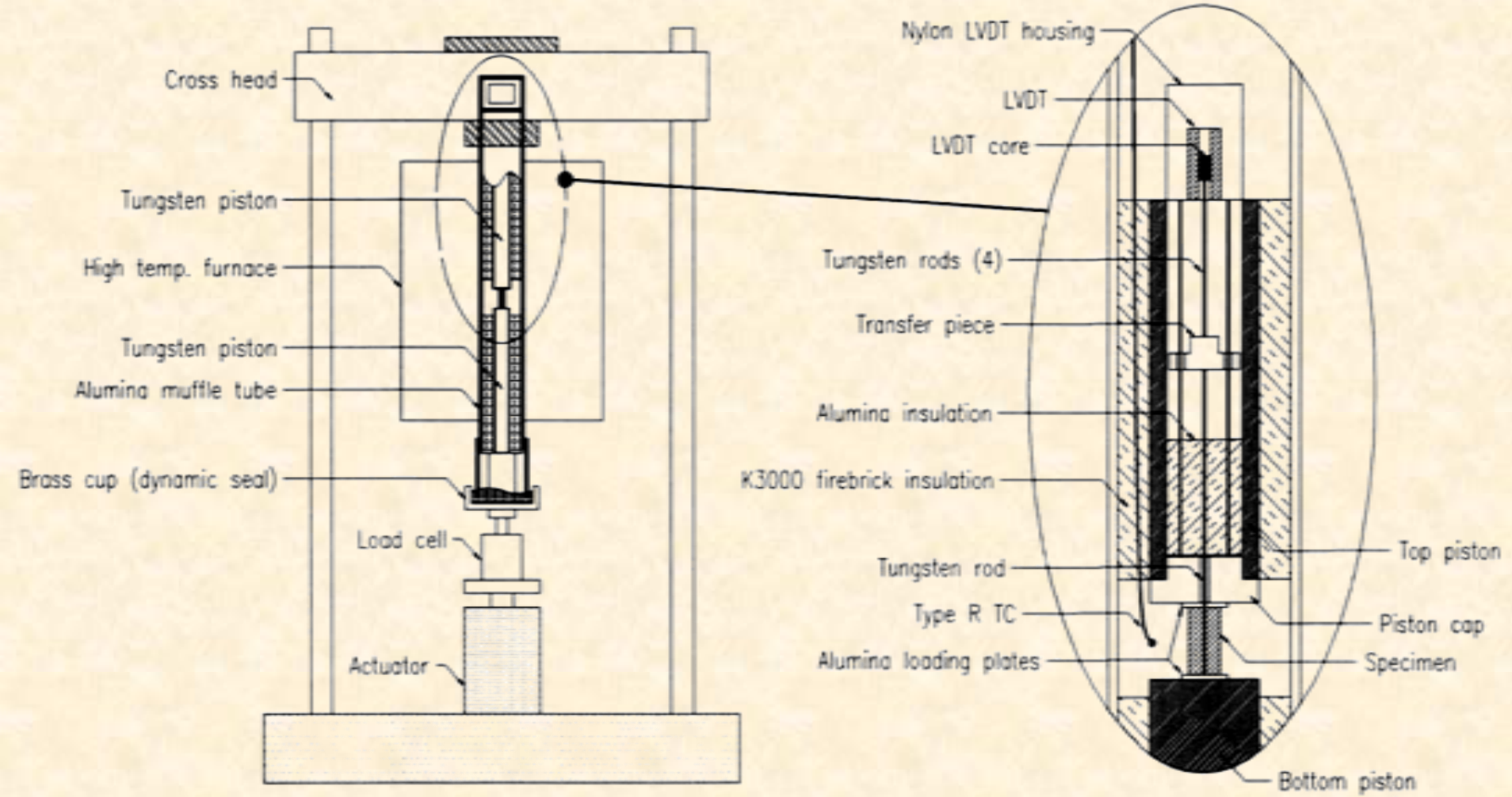
Single-Crystal Deformation: Experimental Apparatus



- **Servomechanical actuator**
- **Environmental chamber w/frictionless Si-oil dynamic seal**
- **Gravity-fed displacement extensometer**
- **Force 0.5 N; strain 2×10^{-6}; temperature 1 C**

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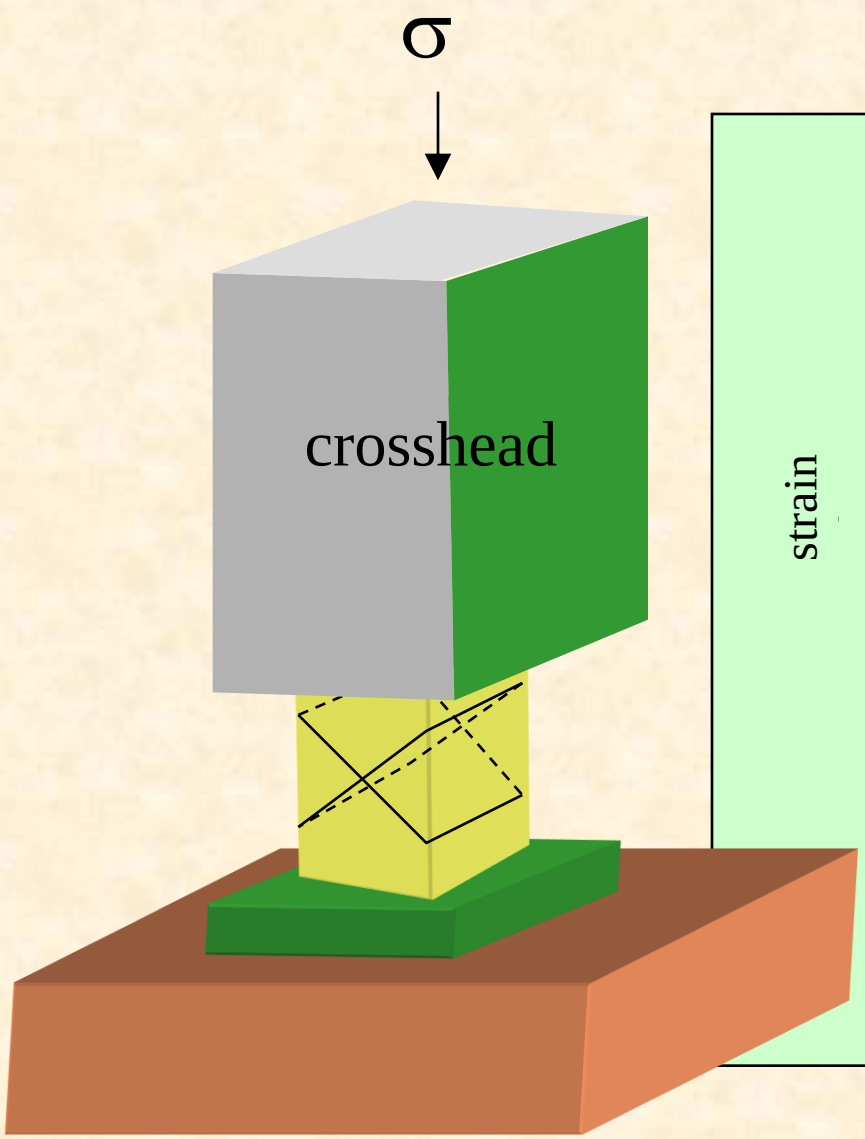
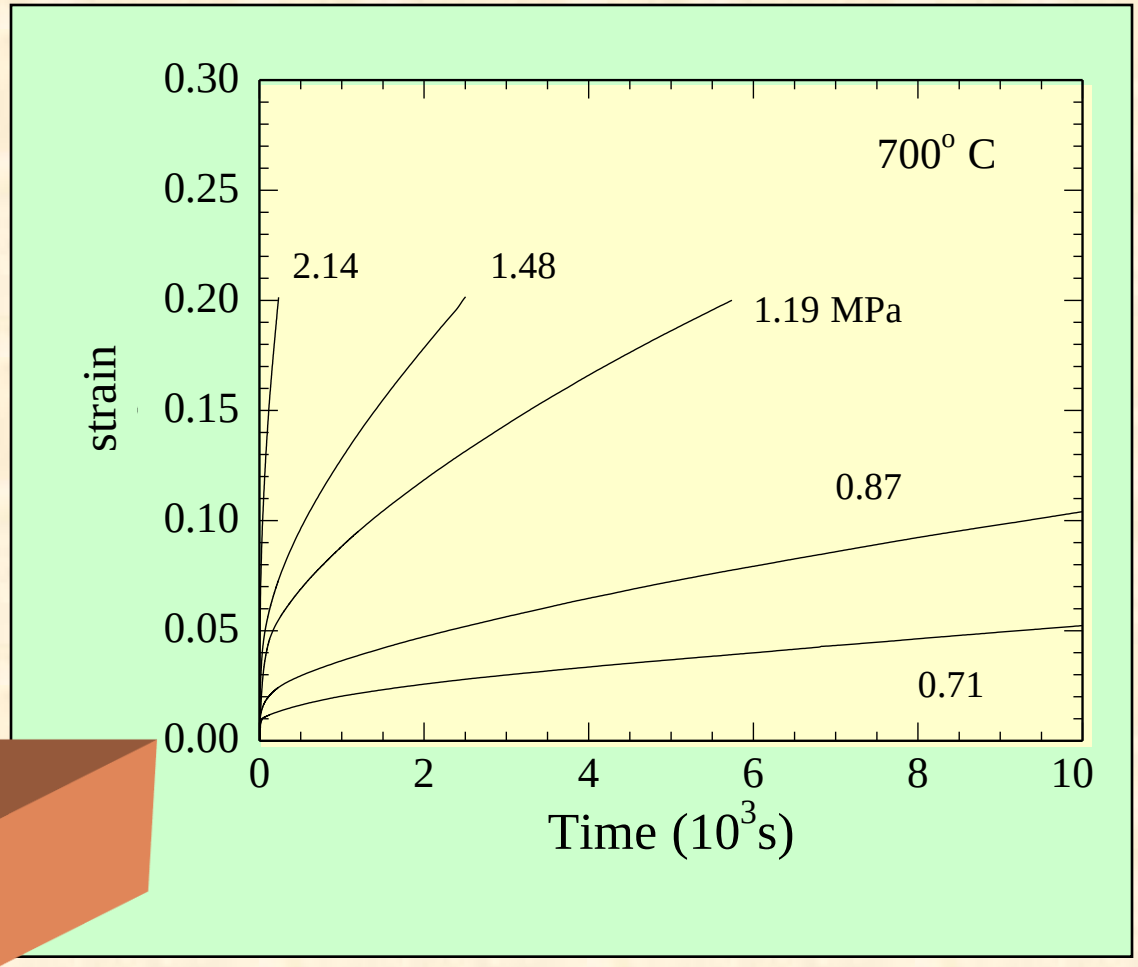
Creep at constant stress: Single-crystal halite





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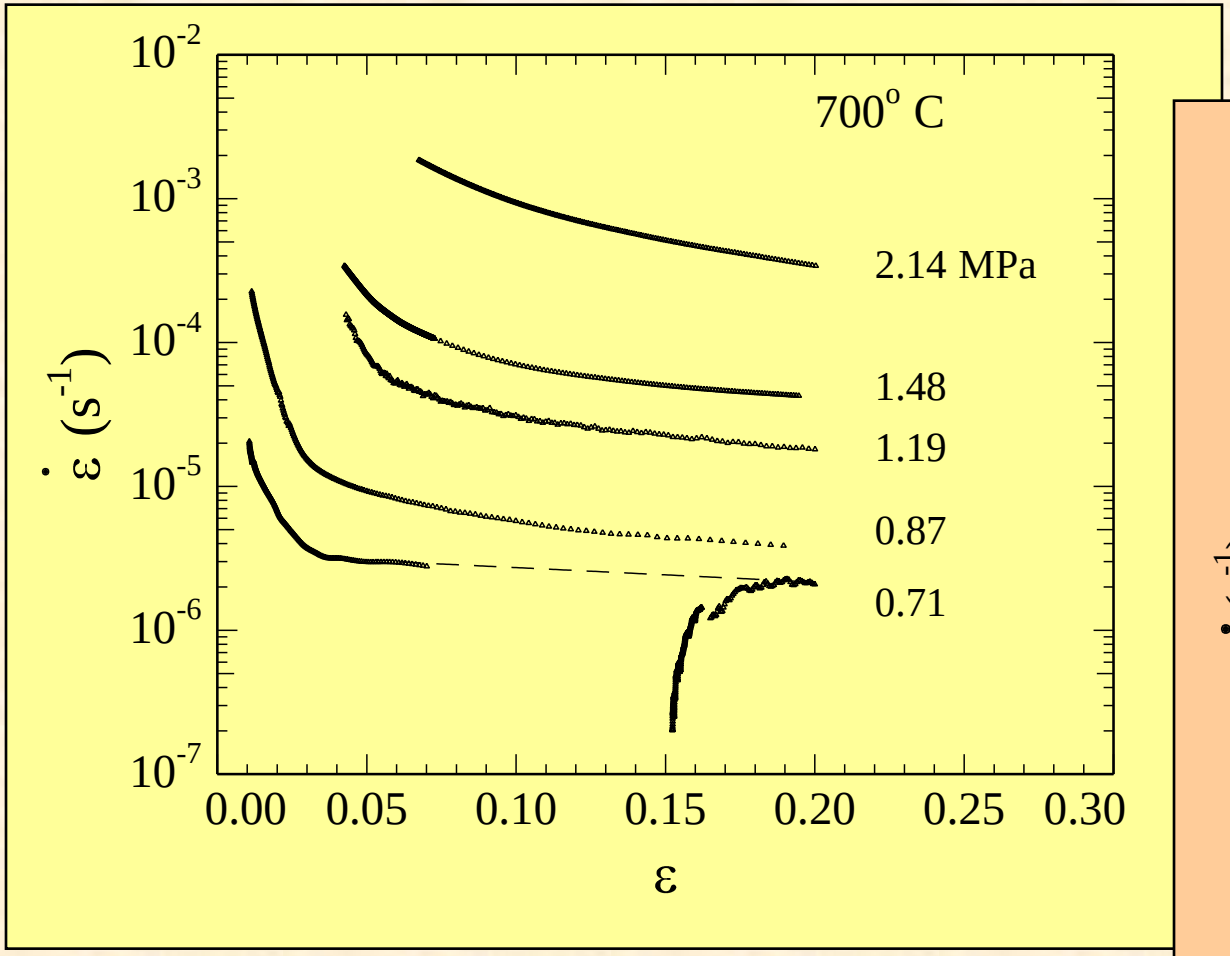
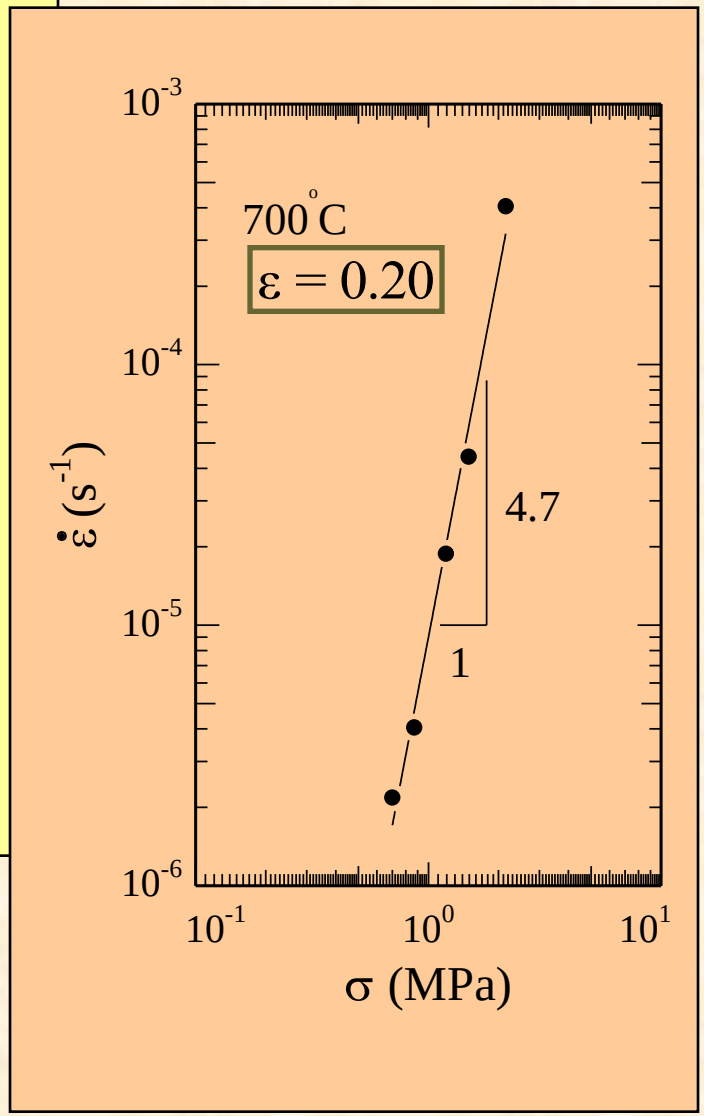
Creep Experiments: Halite Single Crystals





→ **Thermodynamic steady state (*nonequilibrium stationary state*) not achieved, even after 20% strain.**

→ $d \log \dot{\varepsilon} / d\varepsilon = f(\sigma)$

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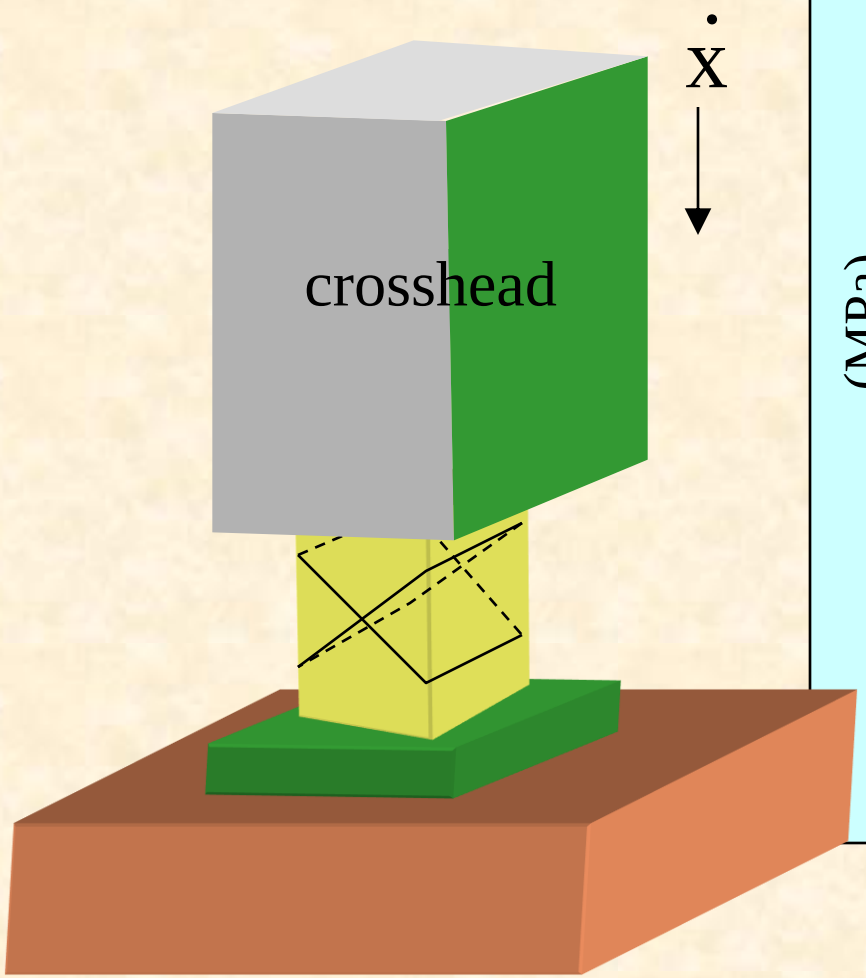
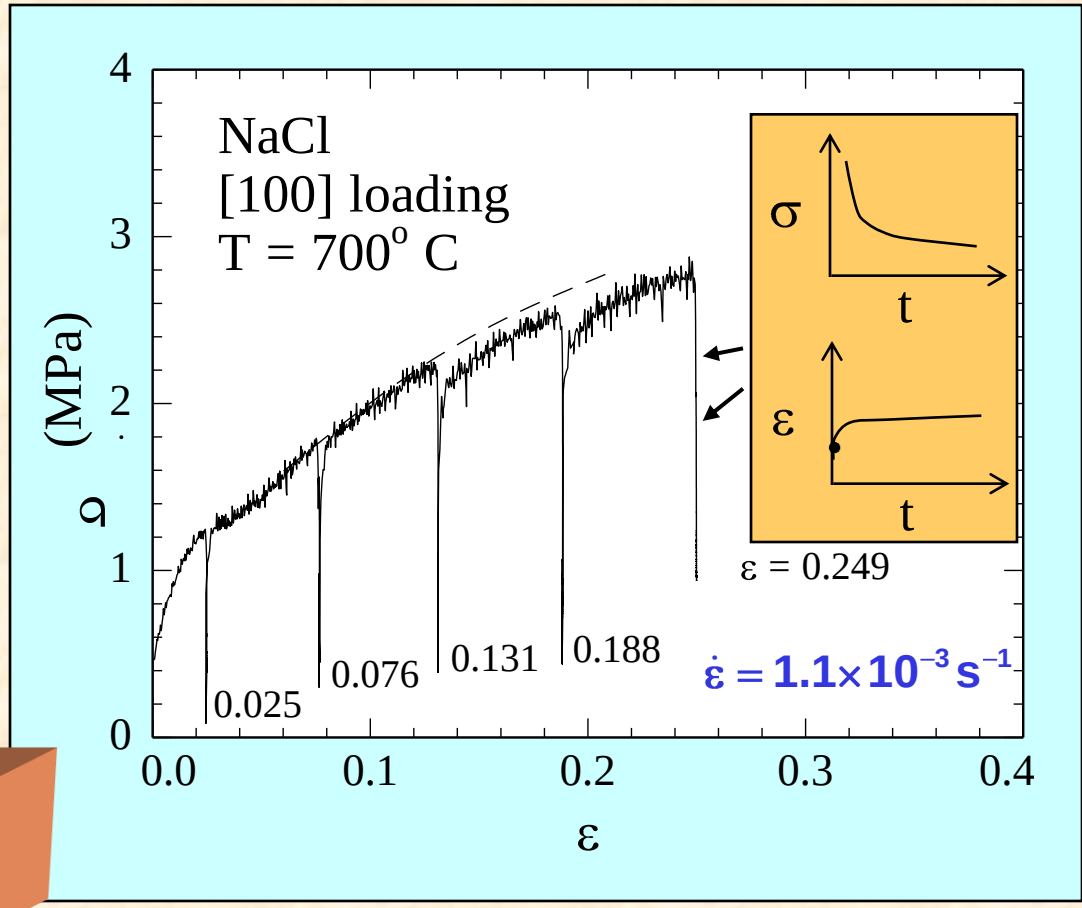
Stress relaxation: Single-crystal halite

stress as a function of strain rate at (nearly) constant strain





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Mechanical equation of state with one internal variable (Hart)

$$\dot{\varepsilon} = \dot{\varepsilon}(\sigma, \sigma^*, T)$$

flow behavior at constant structure

Measured in a load relaxation test

$$\sigma^* = \text{"internal structure"}$$

$$\Gamma = \frac{d \ln \sigma^*}{d\varepsilon} \quad \text{"absolute hardening parameter"}$$

evolution of structure during deformation

Behavior under any arbitrary loading path

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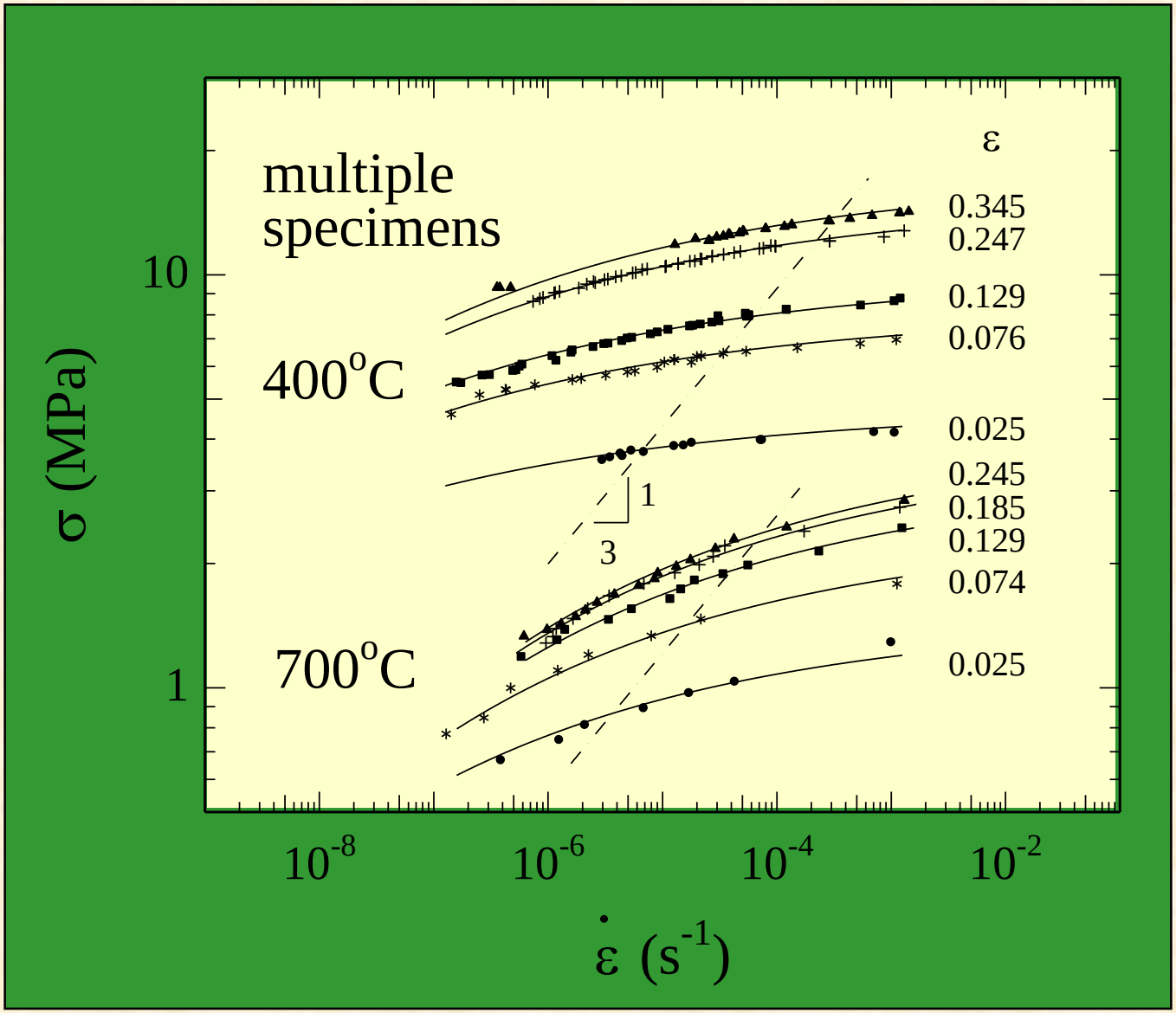
Stress Relaxation Data: Halite Single Crystals

$\dot{\varepsilon} = \dot{\varepsilon}(\sigma, \sigma^*, T)$ flow behavior at constant structure



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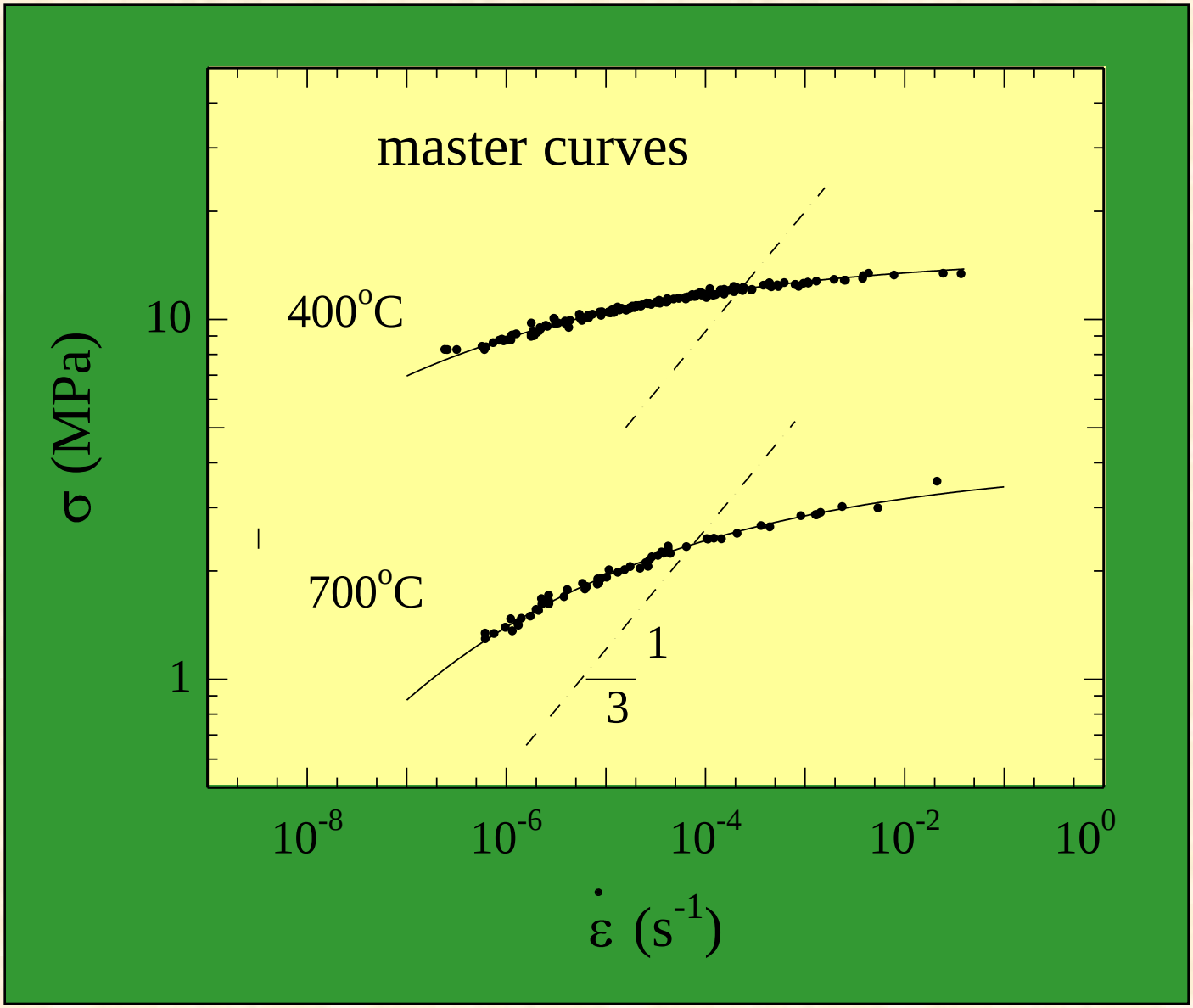
Master Curves: Halite Single Crystals



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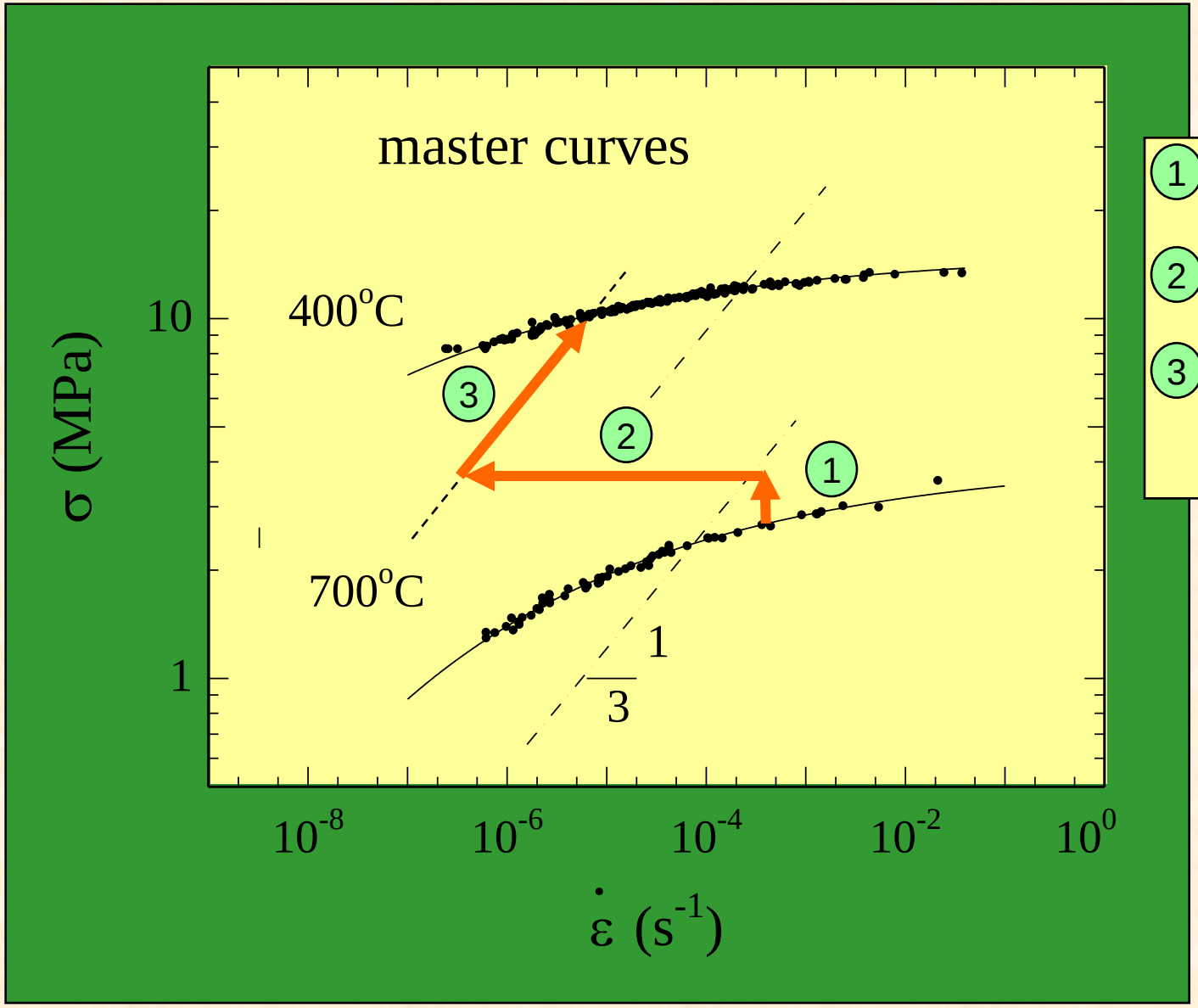
Adjust for differences in temperature



1. Effect of temperature on shear modulus
2. Activation energy (137 kJ/mol)
3. Adjust for differences in hardness level

Step (2) would include, too, effects of chemical potentials on point defect populations.

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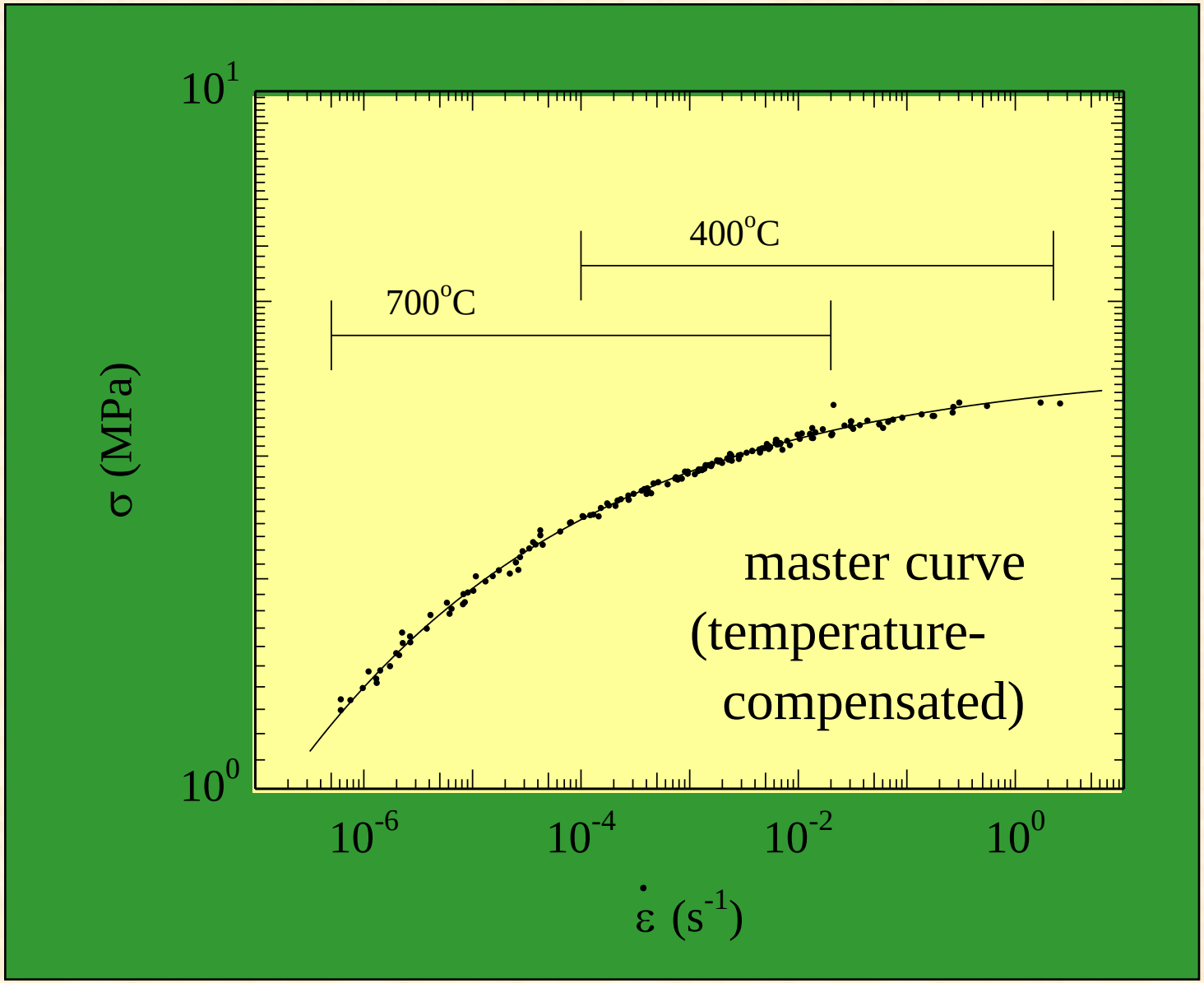
Temperature-compensated master curve: Halite



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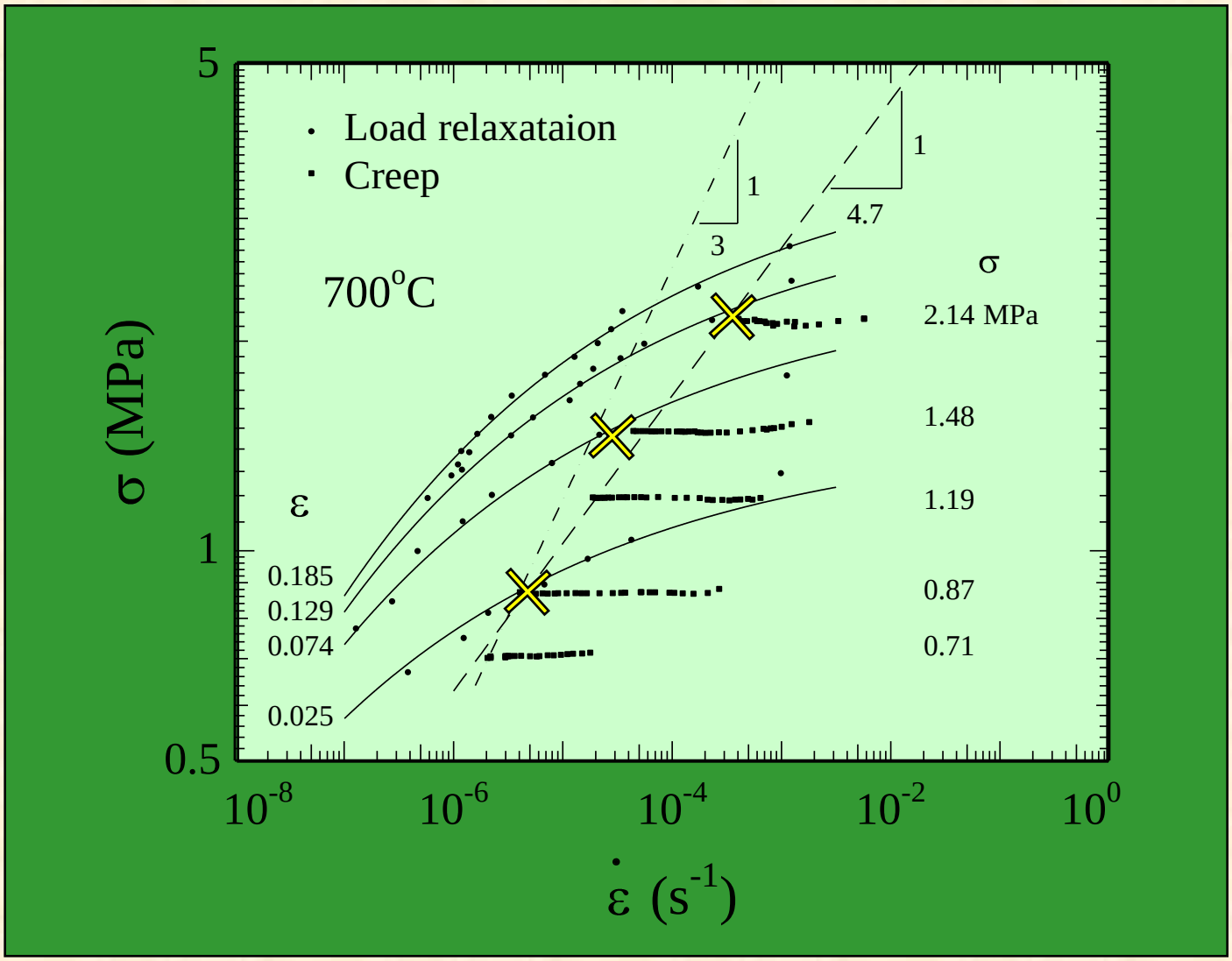
Creep v. Load Relaxation



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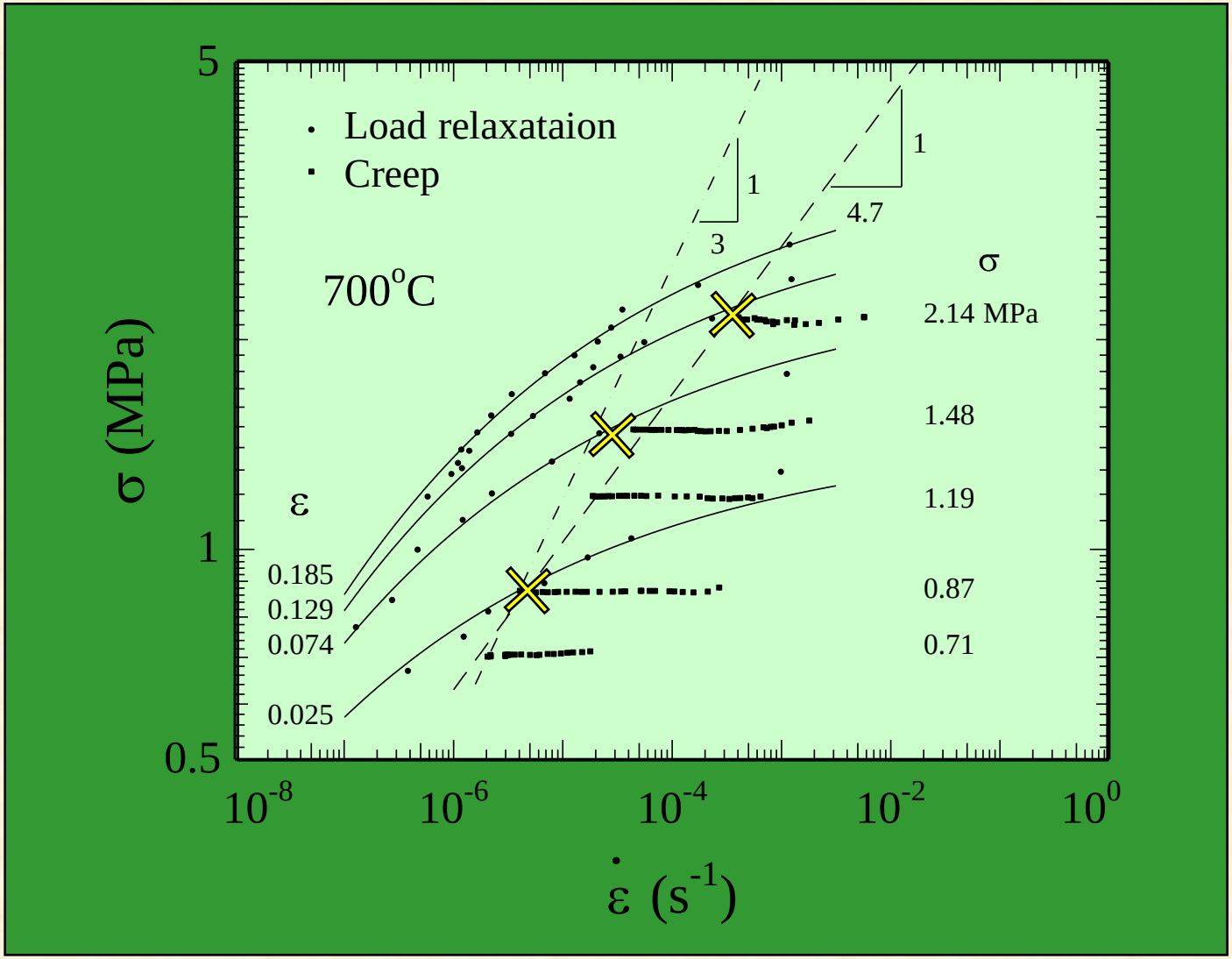
Creep v. Load Relaxation



"Hart's model? Is that what geophysicists are thinking about? We tried Hart's model 20 years ago: it doesn't do work hardening."

—Prof. Rod Clifton, Brown Mechanics

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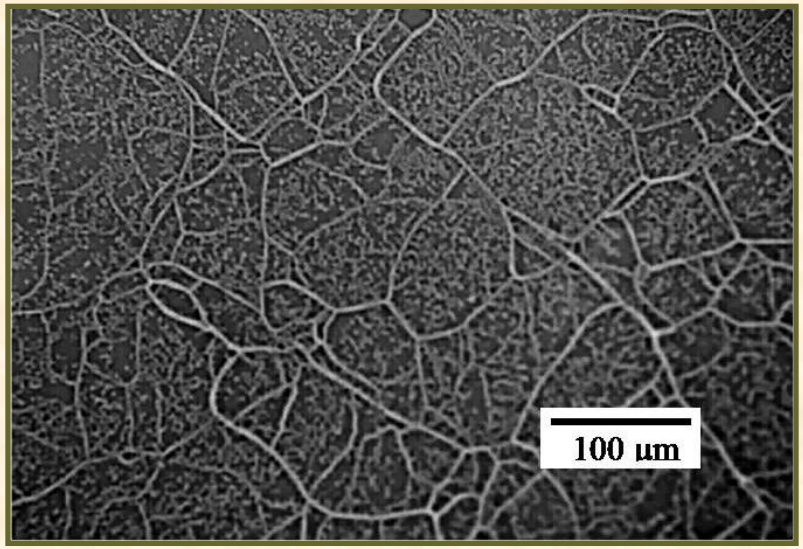
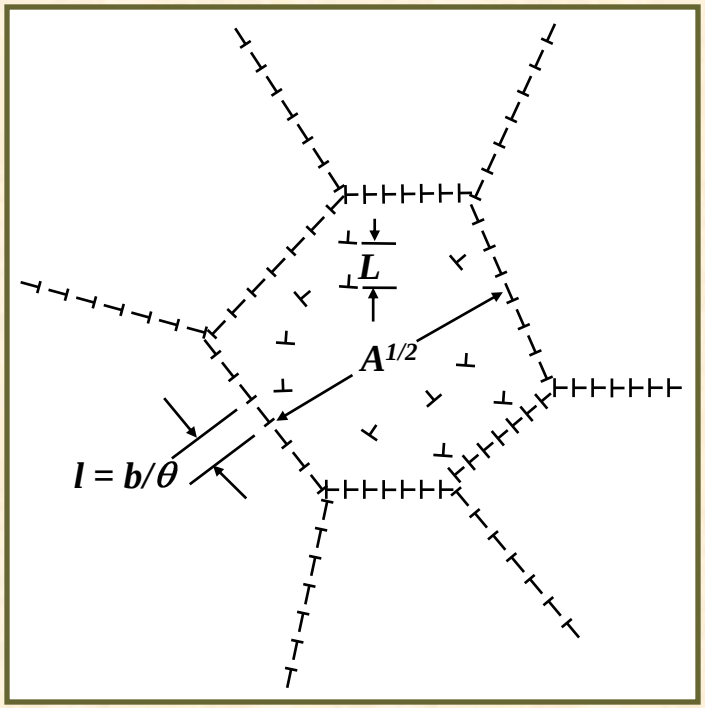
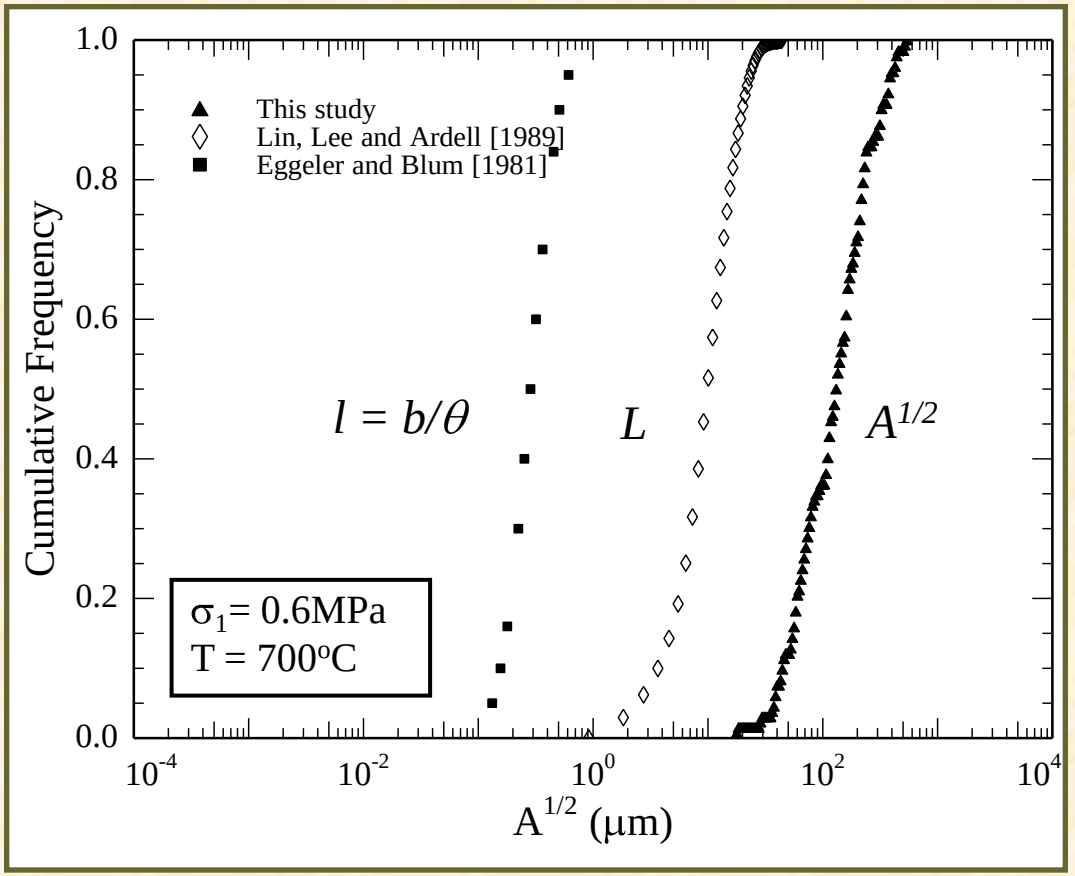
Microstructural Self-Similarity







Self-similarity implies, too, microstructural aspects at a larger scale, e.g., the statistical angular relationship amongst groups of subgrains.

l ≡ SGB dislocation spacing; L ≡ dislocation link length;
$A^{½}$ ≡ subgrain size

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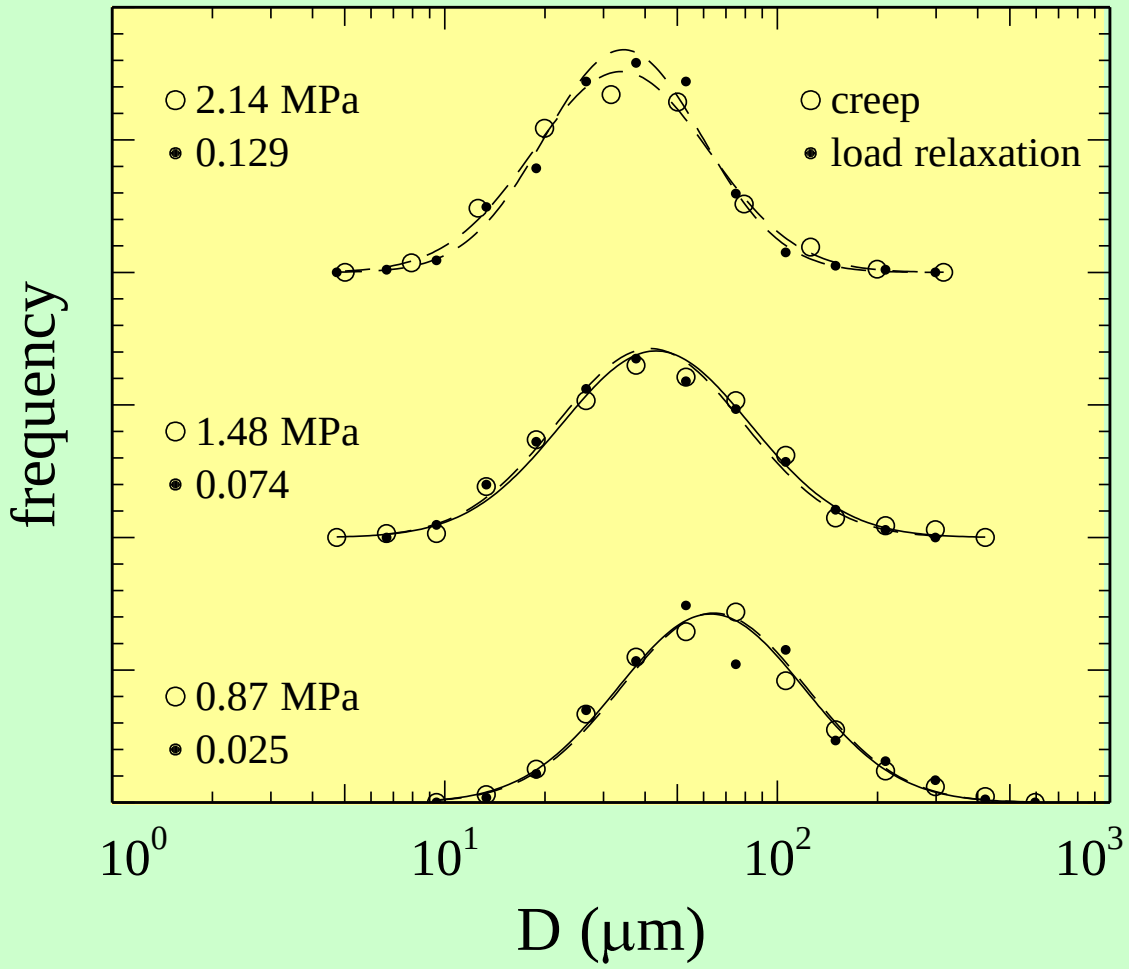
Comparison between subgrain size distributions from load relaxation and creep: *Identical!*



Subgrain size distributions measured following creep & load relaxation

$$\sigma^* \cong \frac{50Gb}{D_{mean}}$$

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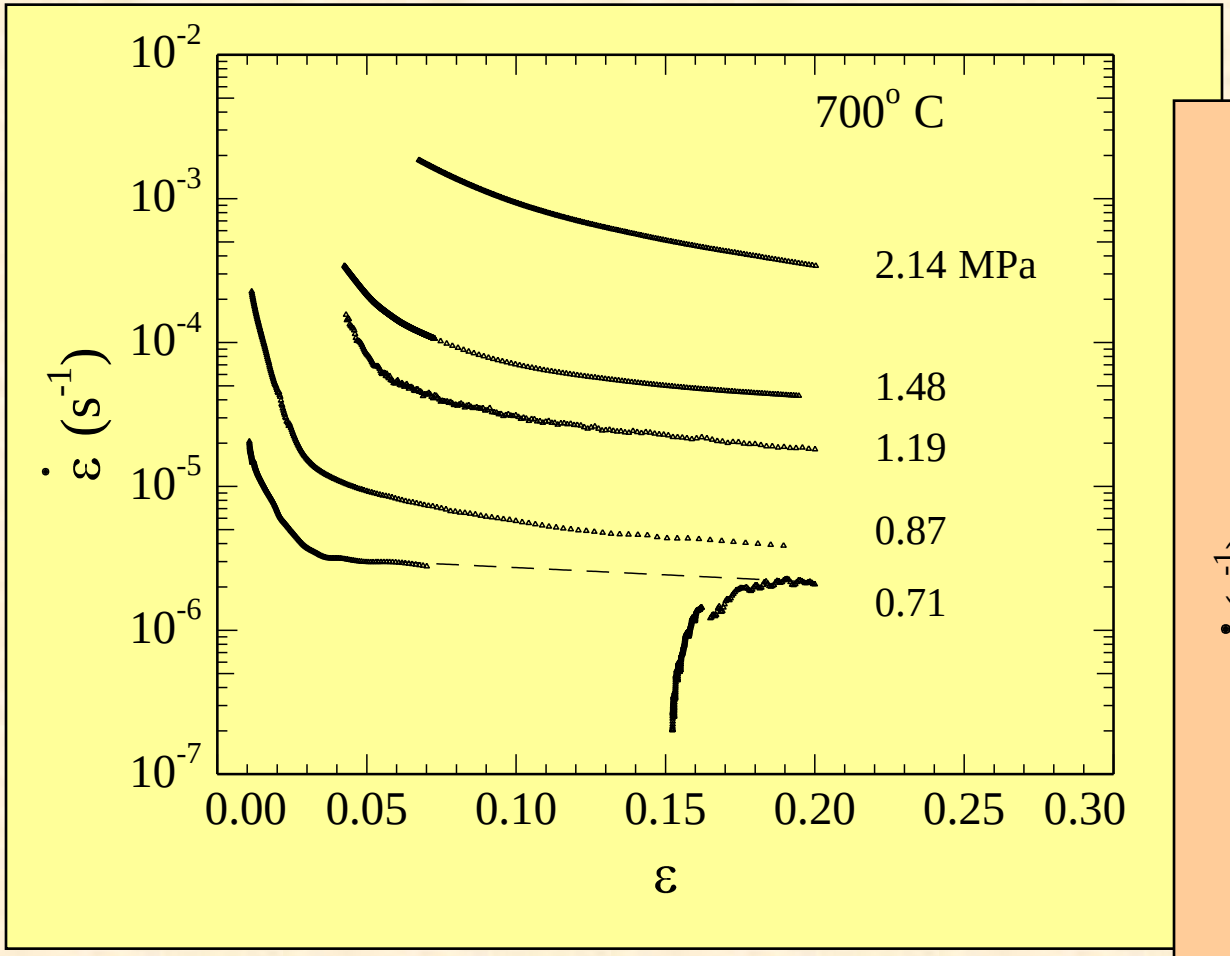
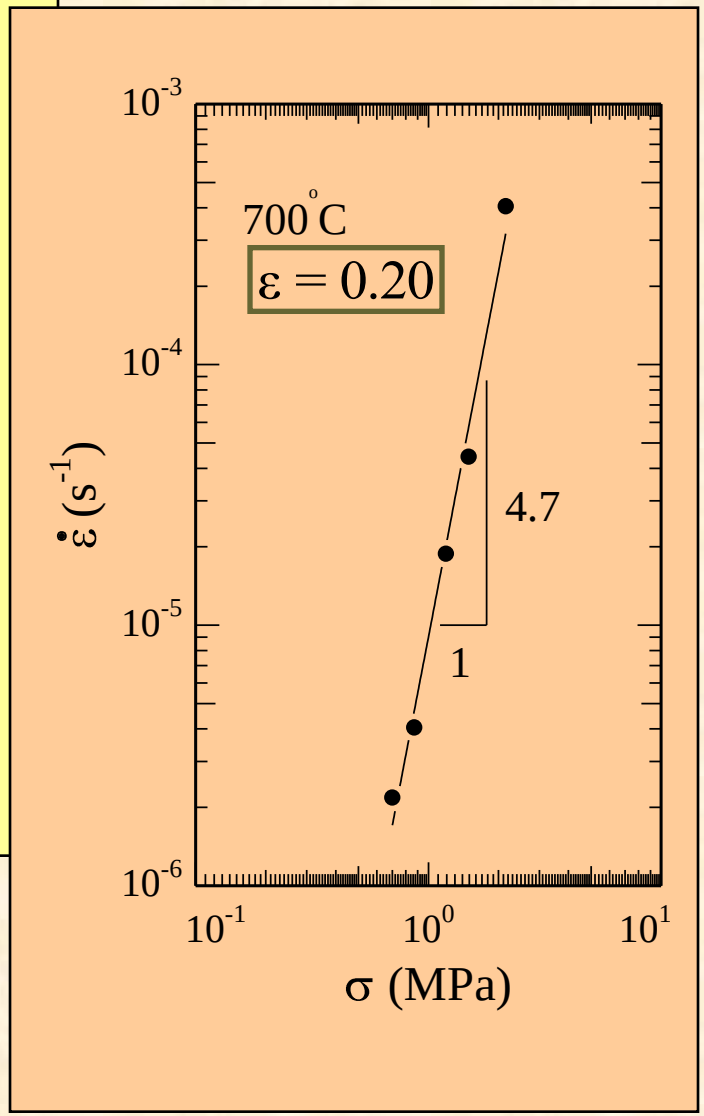
Creep Experiments: Halite Single Crystals





→ **Thermodynamic steady state (*nonequilibrium stationary state*) not achieved, even after 20% strain.**

→ $d \log \dot{\varepsilon} / d\varepsilon = f(\sigma)$

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Thermodynamic Landscape and Prigogine's Bifurcations

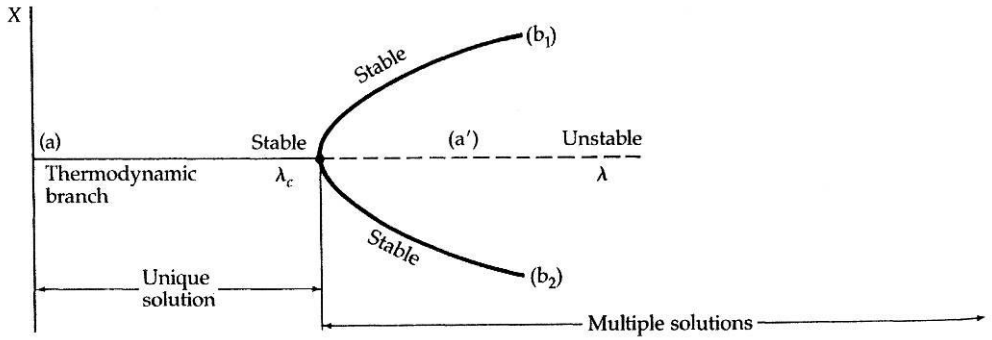


Figure 30 Bifurcation diagram showing how a state variable X is affected when the control parameter λ varies. A unique solution *(a)*, the thermodynamic branch, loses its stability at λ_c. At this value of the control parameter new branches of solutions (b_1, b_2), which are stable in the example shown, are generated.

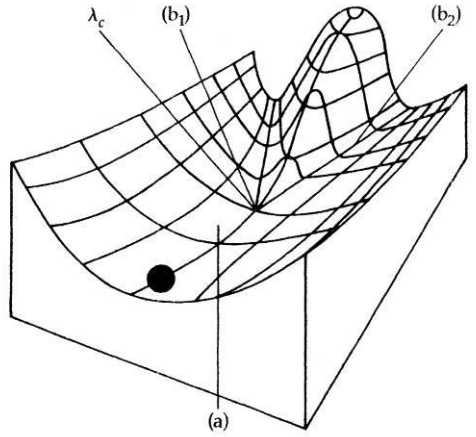
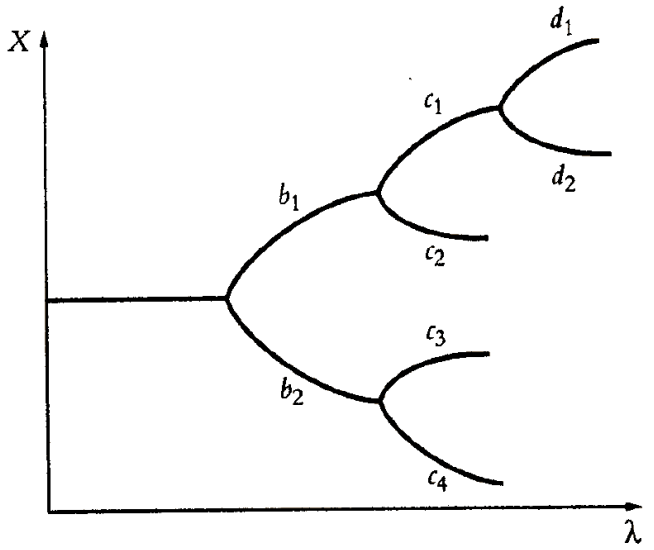


Figure 31 Mechanical illustration of the phenomenon of bifurcation.

Successive Bifurcations with Increasing Distance from Equilibrium



The old future's gone…
—John Gorka

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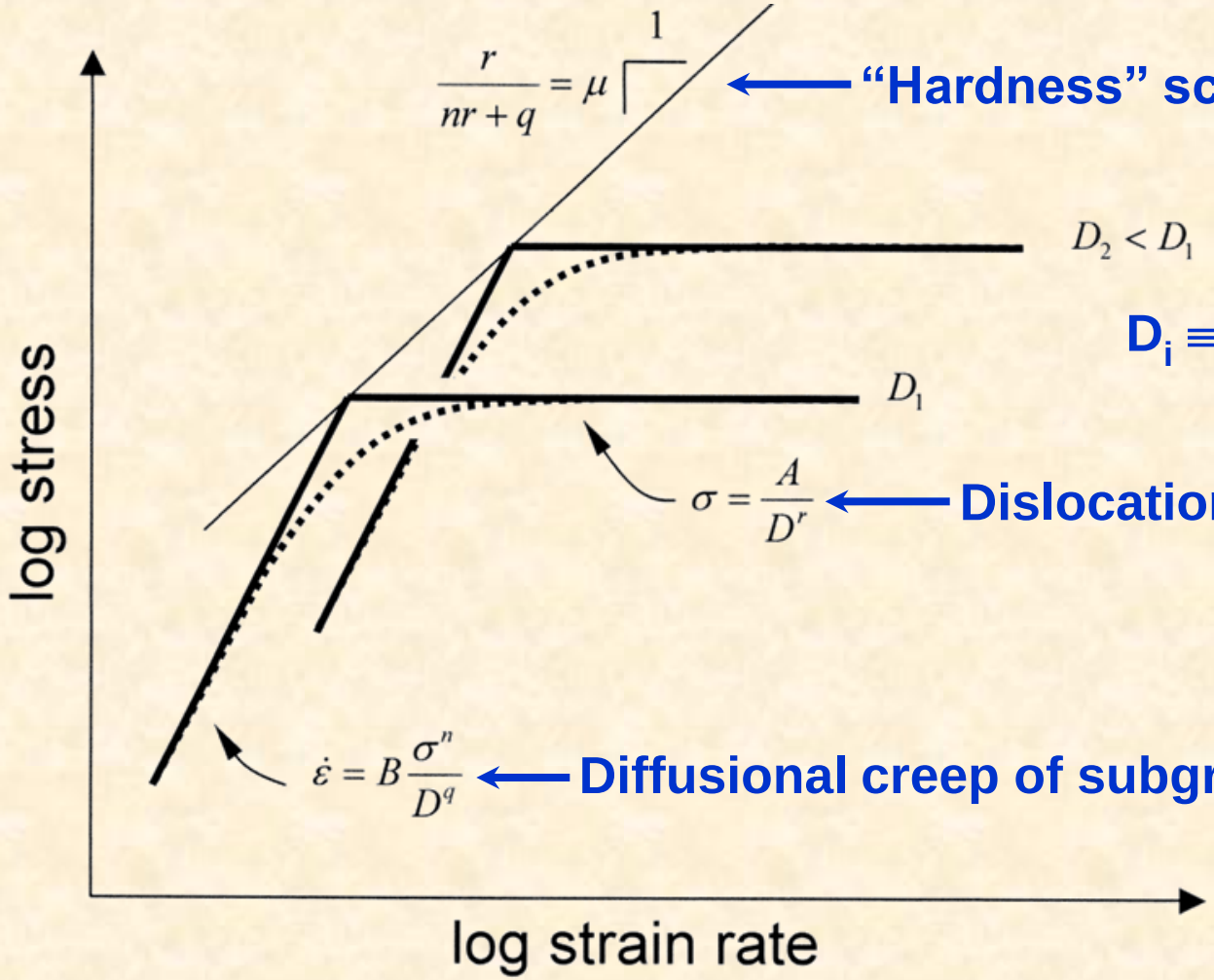
The Physics: Serial Kinetic Processes of Subgrain Diffusional Creep and Dislocation Glide (Stone, 1991)



← "Hardness" scaling slope

D_i ≡ subgrain size—shown here as monodispersed

← Dislocation glide: ***non-linear***; r ~ 1

← Diffusional creep of subgrains: ***linear***; n = 1; q = 2

Distribution of subgrain sizes allows broadening of hardness curve

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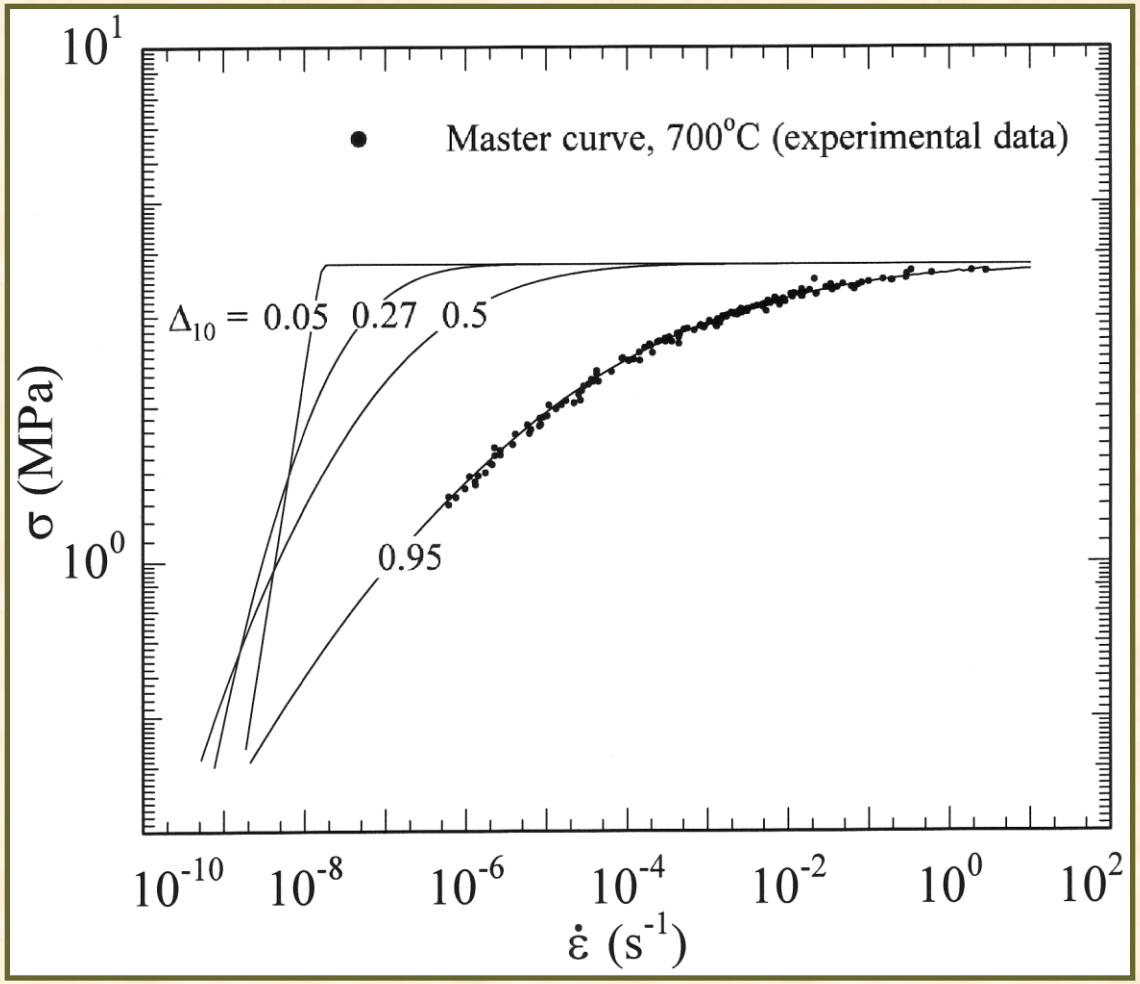
Matching Data to the Model



Δ_{10} ≡ standard deviation of lognormal distribution of subgrain size

Measured: Δ_{10} = 0.27

Reasonable maximum: Δ_{10} = 0.50

Best fit: Δ_{10} = 0.95

The more compliant nature of the data compared to the model suggests that microstructural features at a scale larger than the subgrain size—and yet related by similarity—are an important contributor to the effective viscosity of the specimens.

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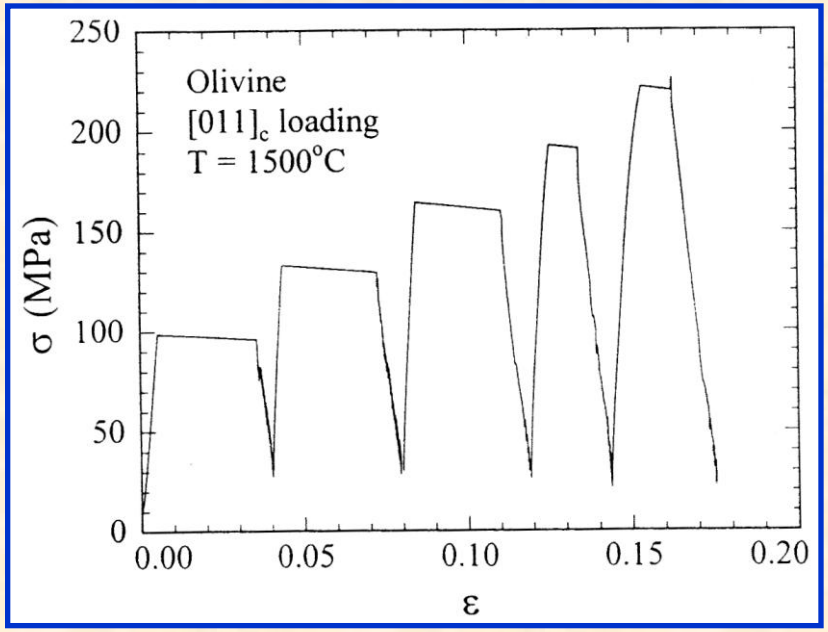
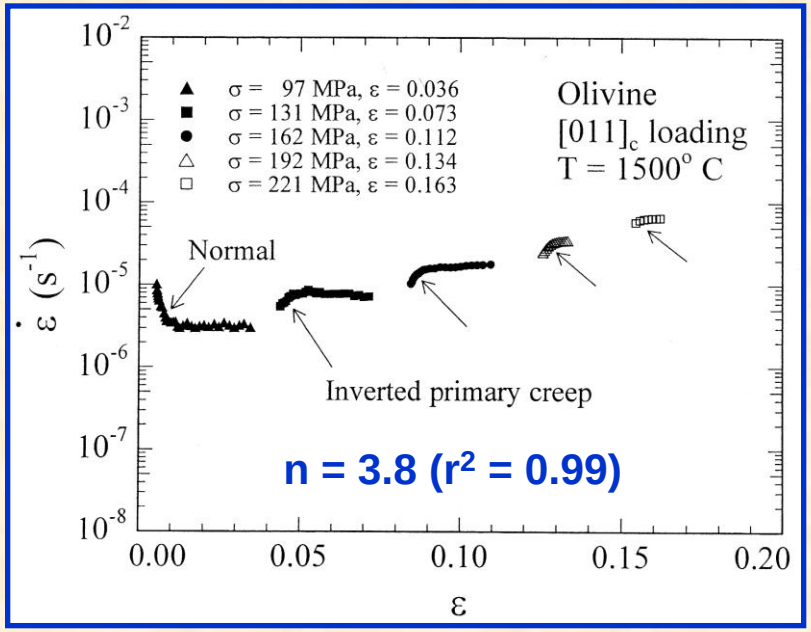
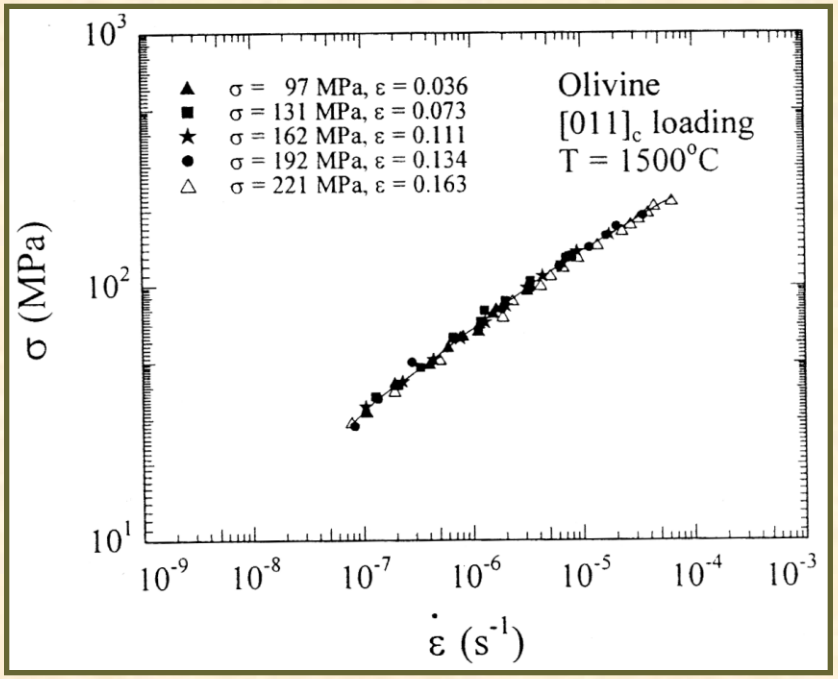
San Carlos Olivine: *Alloy Class Behavior*







All experiments at QIF–0.5

The data—both the transient creep & relaxation responses—are consistent with solid-solution strengthening (surprise!). Oxygen fugacity may thus be first-order in experiments attempting to effect & study changes in σ*.

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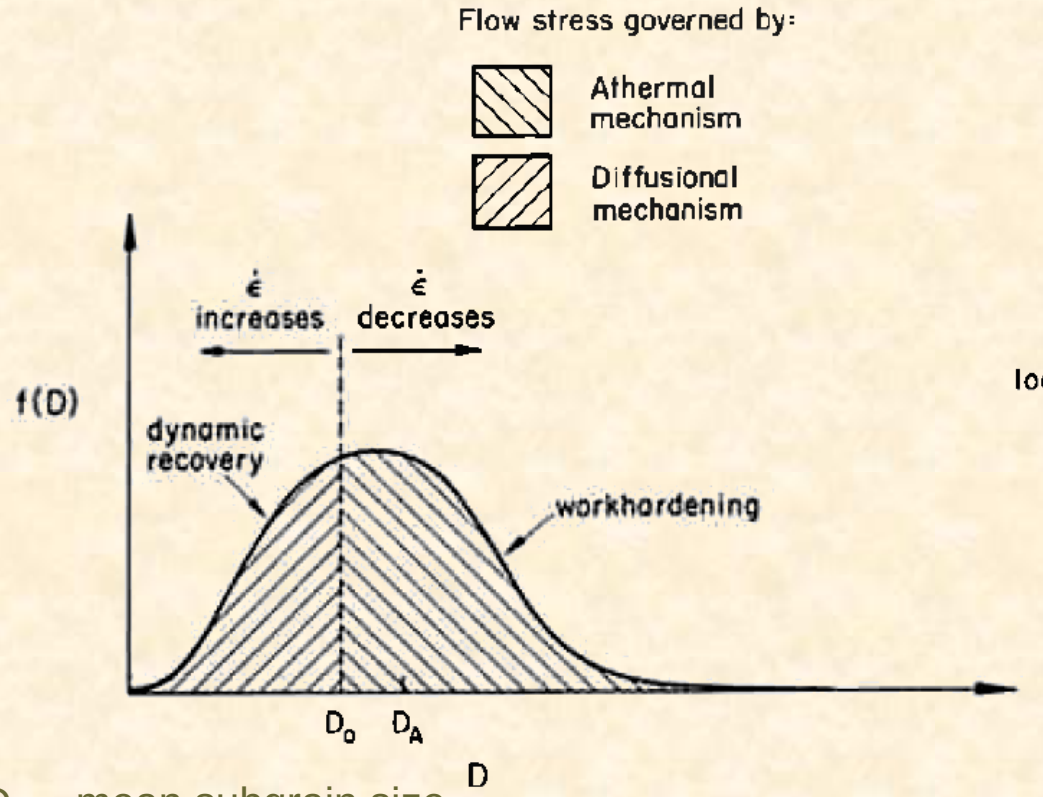
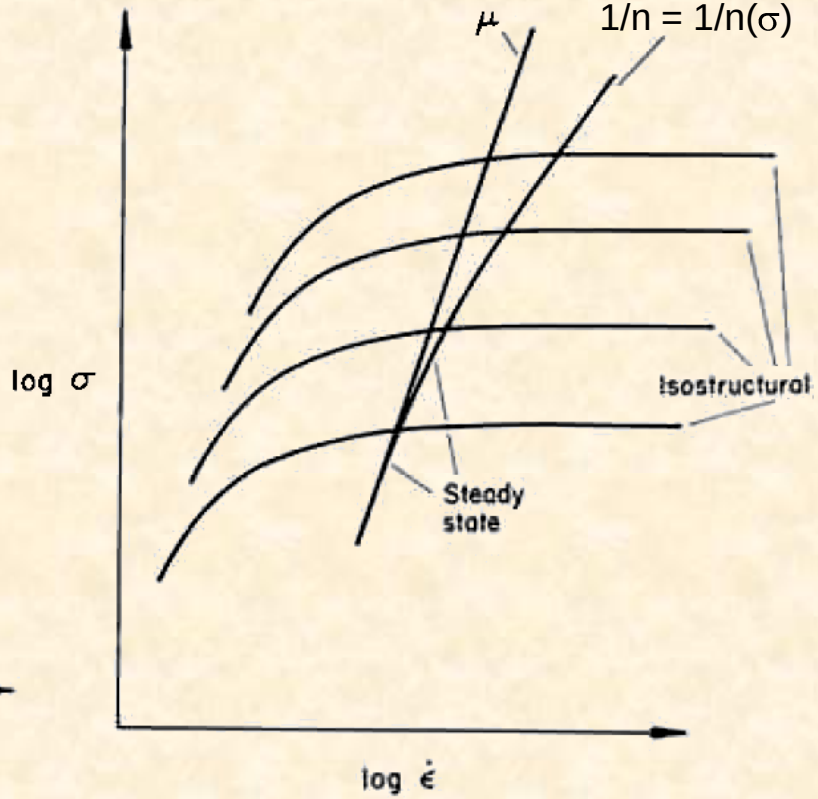
Prospects for Exploring/Understanding the Absolute Hardening Parameter, $\Gamma \equiv d\ln\sigma^*/d\varepsilon$





$D_A \equiv$ mean subgrain size

$D_o \equiv$ subgrain size dividing those subgrains dominated by diffusional from those dominated by dislocation dissipation.

Thermodynamic Steady State $\equiv (D_A/D_o) = \text{const.}$

In work hardening, the ability of the material to store dislocations decreases as the density of dislocations increases (e.g., for olivine, by cross-slip). Additional strain is accumulated beyond the dynamic that sets D_A relative to D_o. The effect makes 1/n deviate from μ. *Characterization of Γ depends on the careful study of the deviation.*

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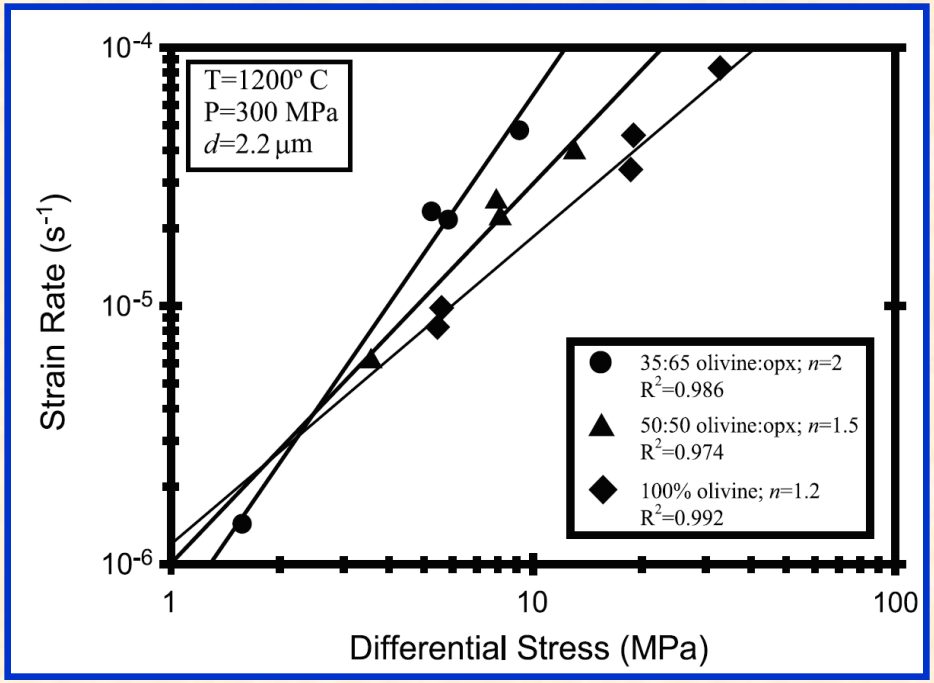
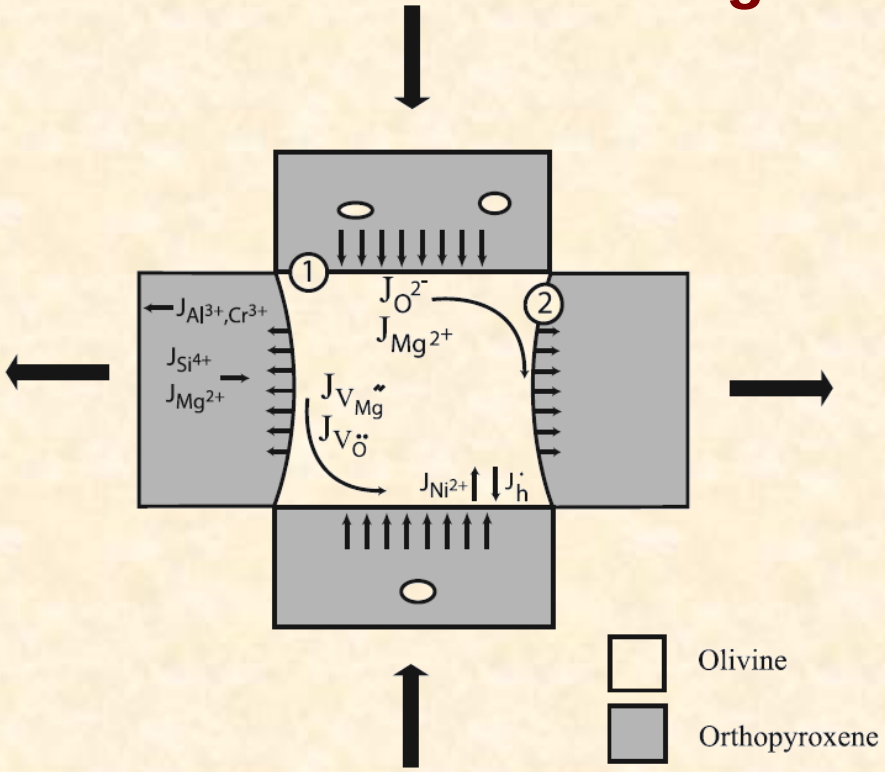
Grain-Size-Sensitive (Diffusion) Creep of Harzburgite

—Marshall Sundberg





Experiments indicate that, in the grain-size-sensitive regime, the viscosity of harzburgite is lower than that of dunite. Too, the grain-size sensitivity of viscosity diminishes, suggestive of interfacial reactions being rate-limiting…

Reaction along interfaces ①

$$V''_{Mg} + V_O^{\cdot\cdot} + Mg_2SiO_4 = Mg^{\times}_{Mg} + O^{\times}_{O} + MgSiO_3$$

Reaction along interfaces ②

$$Mg^{\times}_{Mg} + O^{\times}_{O} + MgSiO_3 = V''_{Mg} + V_O^{\cdot\cdot} + Mg_2SiO_4$$

***Pseudobinary system*: diffusional creep does NOT require transport of slowest-moving species…**

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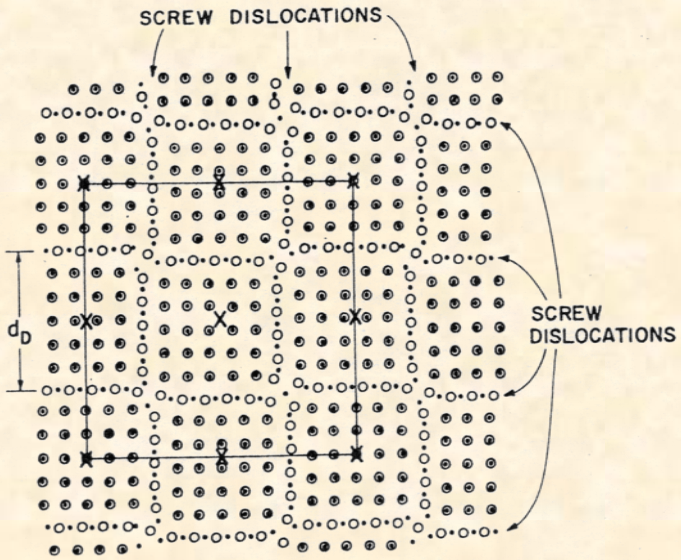
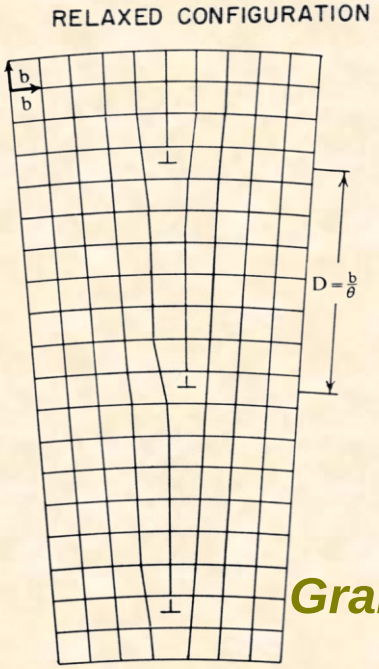
The Structure of Grain Boundaries: 2-D Crystals



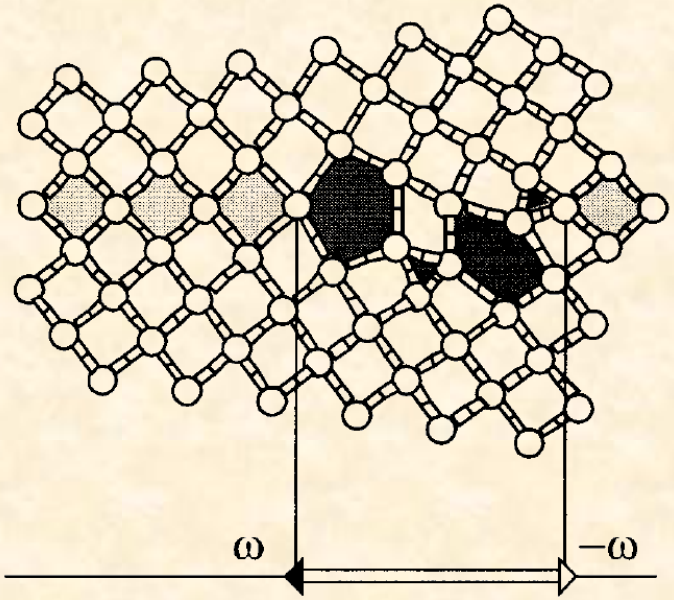


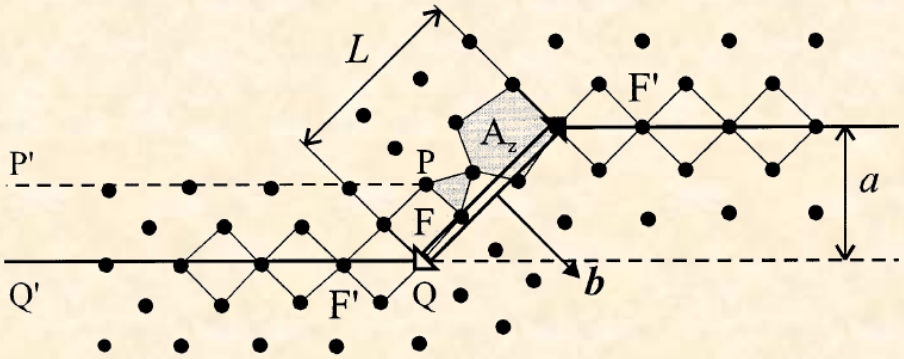
Low-angle boundaries: dislocation model:

$$\theta \cong \frac{b}{d}$$

High-angle boundaries: disclination model:

dislocation-based units with bending moments





Grain boundaries in rocks are not made of amorphous goo!

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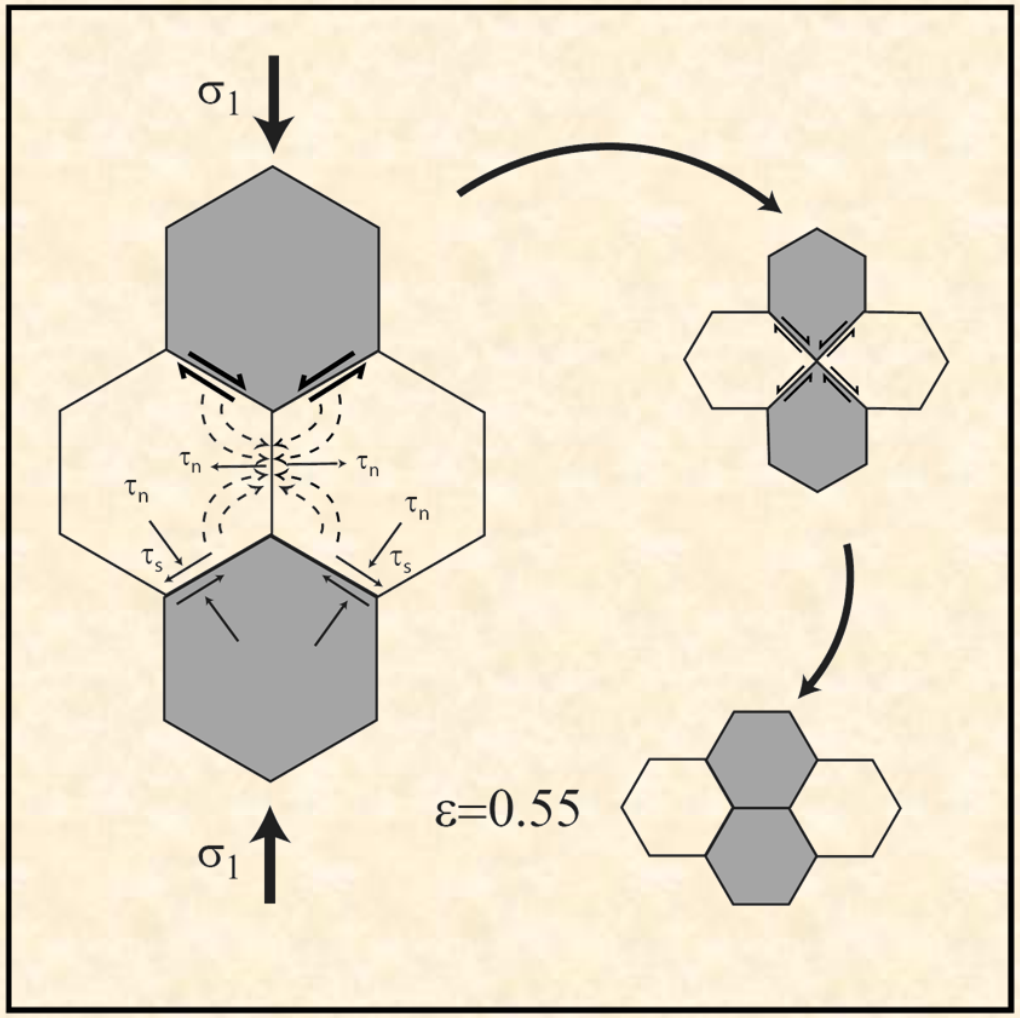
Two Ramifications:



GBS & diffusional creep: Ashby & Verall (1973)

Non-equilibrium thermodynamics requires a system pushed from equilibrium to respond kinetically along the path having the *highest energy dissipation rate*.

Thus:

1. In the grain-size sensitive flow regime, nature will select to optimize the spatial distribution of heterophase boundaries in this (ol-opx) system, that is, the phases will remain well mixed despite increasing strain; *&*

2. If heterophase boundaries have effective viscosities that are boundary-structure-sensitive…

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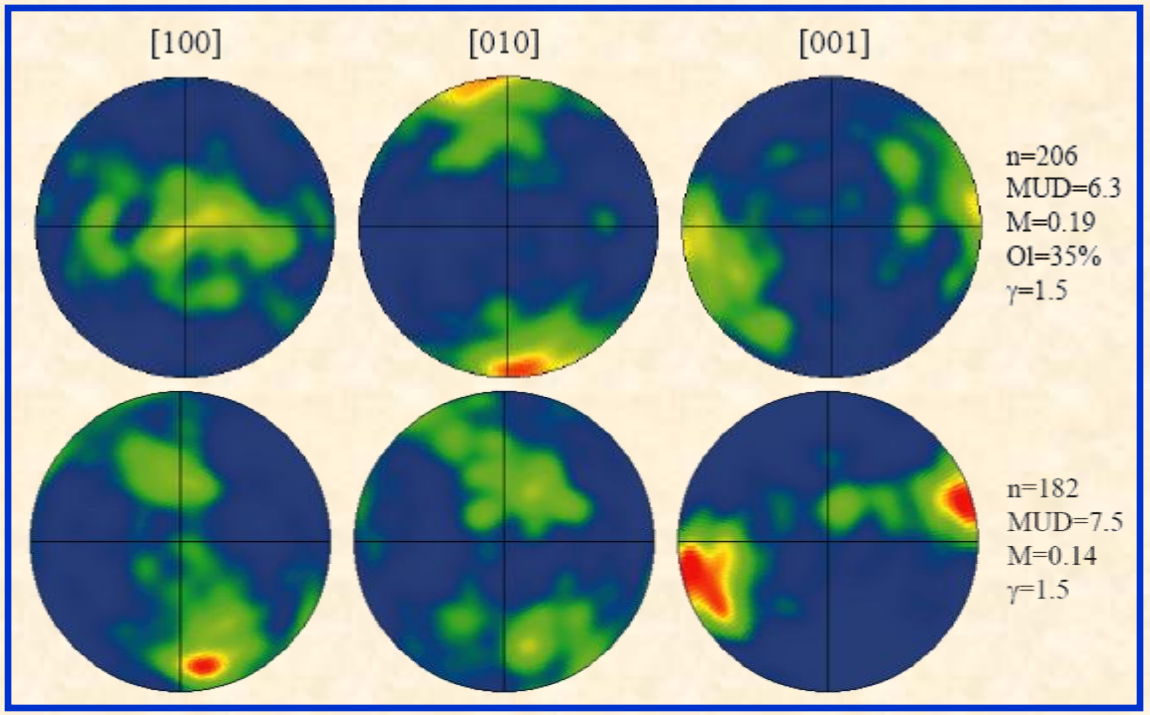
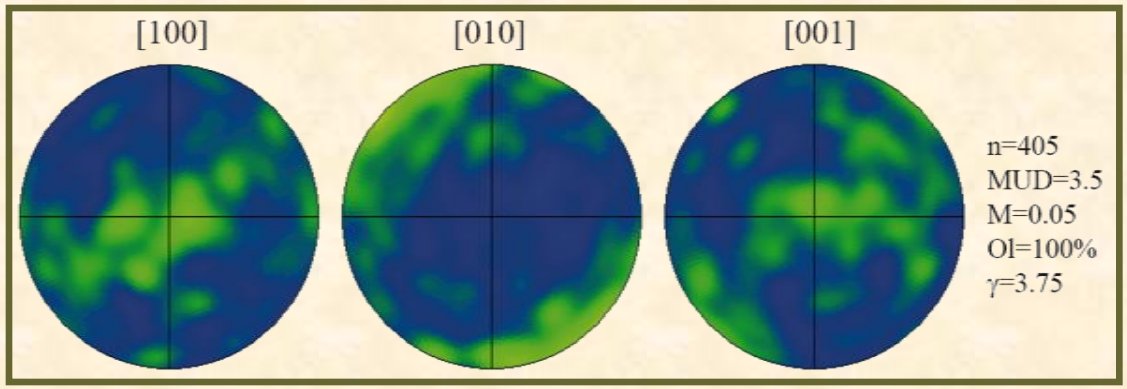
Strong, "B-Type" Olivine CPO at Low Stress:

nary a dislocation or a drop of water in sight…

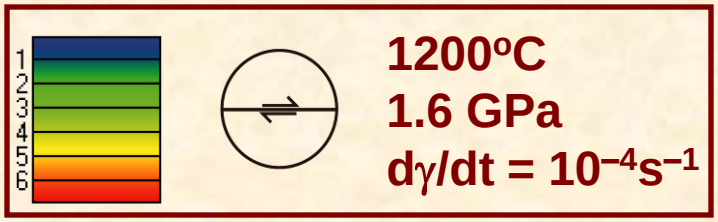
A strong, "B-type" CPO forms under thermo-dynamic conditions distinctly different than suggested by *Jung & Karato* (2001). *Chemical/structural constraints related to heterophase boundary sliding dictate the behavior.* These constraints hold for dislocation rheologies as well as diffusional ones.

Harzburgite: 35/65 (wt) ol/opx; d~5μm; γ~1.5





Dunite: d~8μm; γ~3.8



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Summary/Conclusions

- **Spatiotemporal scaling remains one of the significant challenges in rock physics. It's study is of first-order importance in understanding transient creep**
- **Plasticity, too, has a state variable associated with microstructure (i.e., beyond just grain size): intracrystalline and polycrystalline structures are σ- & ε-dependent—with effects on spatial distribution of phases and, thus, on strain localization.**
- **One can imagine an operative, integrative constitutive law based on the Hart model, most likely augmented by an additional state variable dealing with symmetry-breaking work-hardening—one goal of future rock-physics research at Brown.**