3-D geometry of fault surfaces

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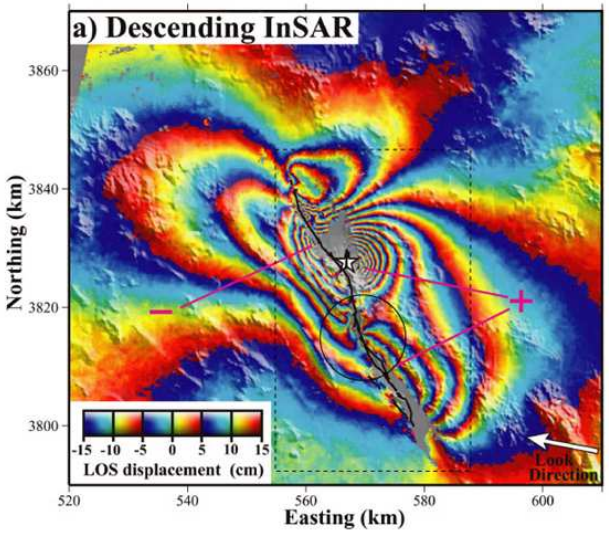
- Ovunc Mutlu
- Amir Sagy & Emily Brodsky, UCSC
- Stanford Rock Fracture Project
- Stanford Dept. Geol. & Env. Sciences

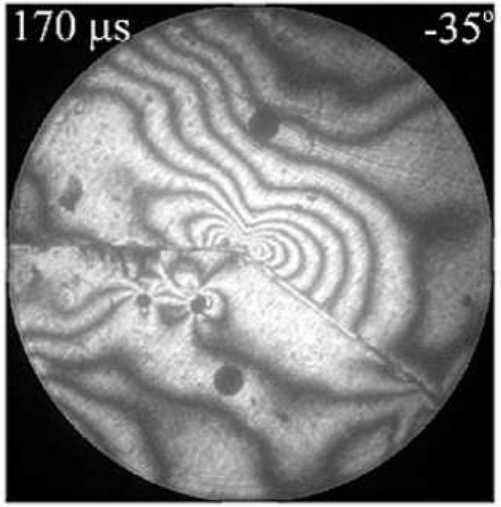
Overview

Motivation

- Effects of non-planarity of fault surfaces significantly affect fault mechanics, e.g. slip nucleation & cessation, off-fault deformation
- Knowledge of non-planar geometry is limited



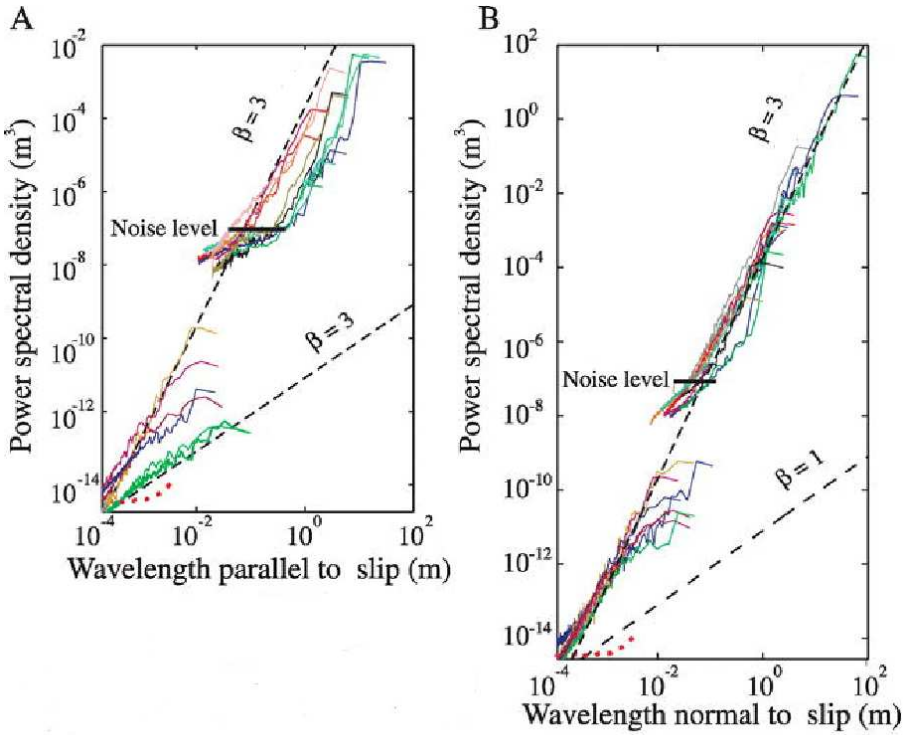
Johnson et al., 2002



Rousseau and Rosakis, 2002

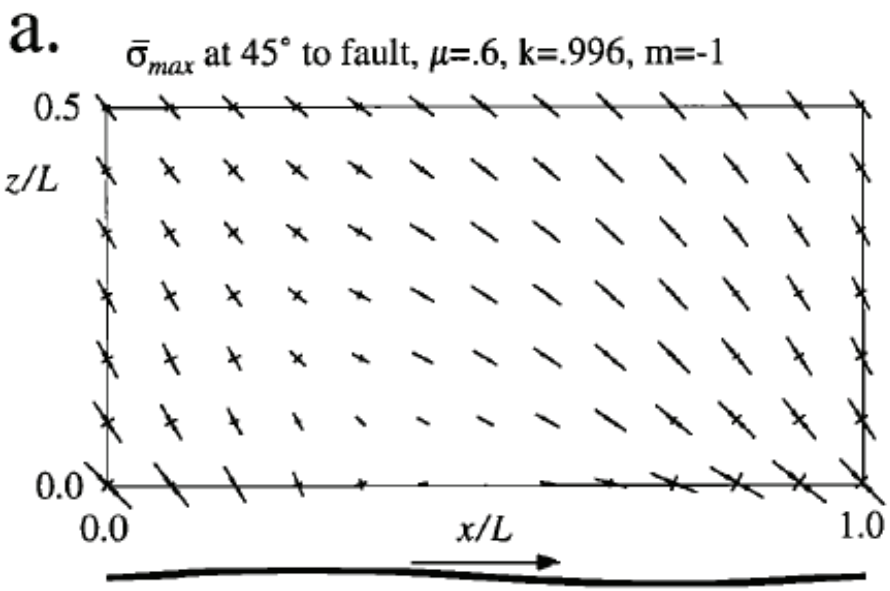
Motivation

- Geometric description: roughness measures
  - Past findings: Scale independence of roughness (Power & Tullis, 1987)
  - Sagy et al. 2006 argue that roughness measures are scale dependent
  - Inherent to roughness measurements: spatial incoherence



Sagy et al., 2007

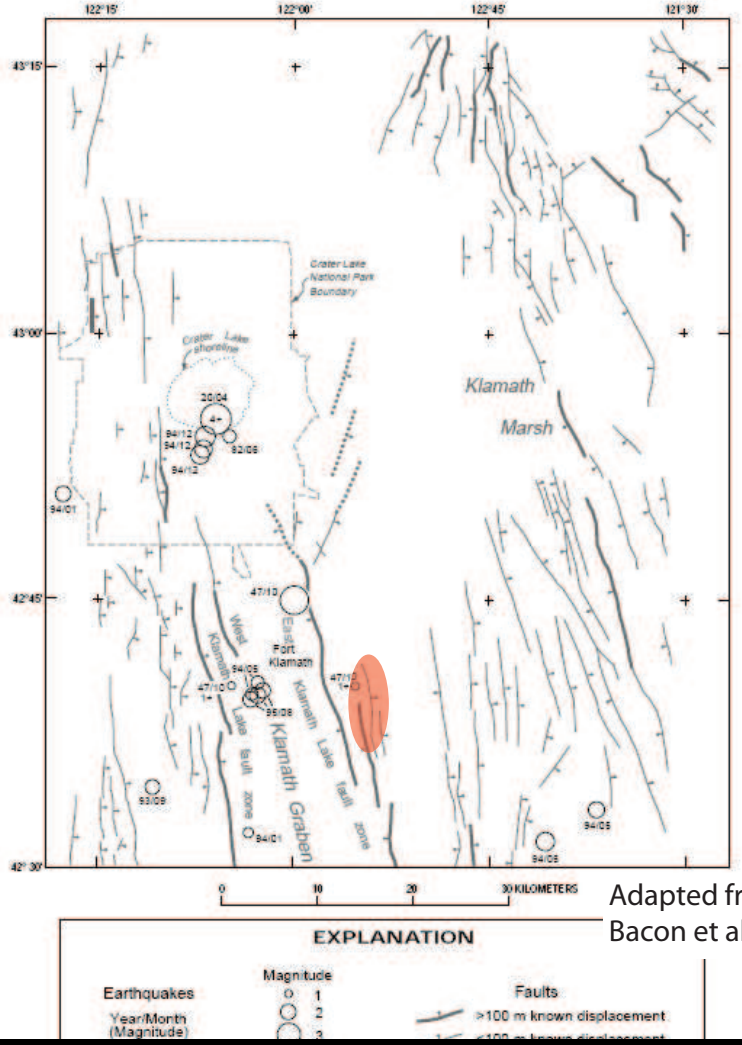
- Modeling efforts generally assume planar geometry
- Wavy fault models simplify mechanics (Chester & Chester, 2000)



Goals

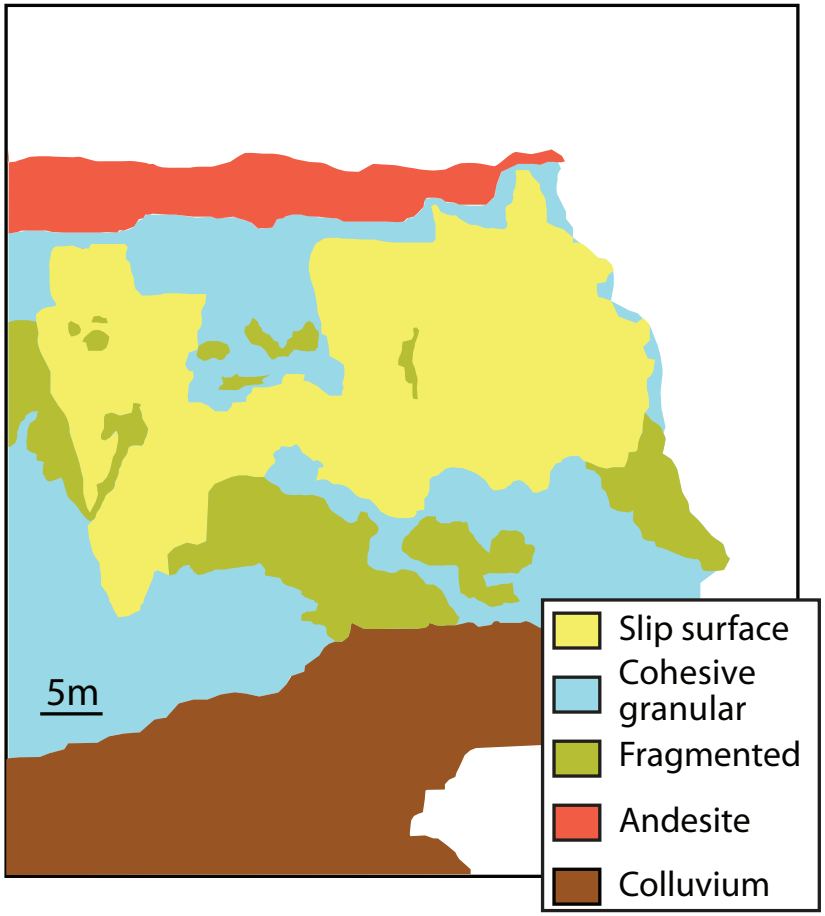
- Alternate geometric description of fault surfaces
- Investigate effects on sliding mechanics

Geologic Setting



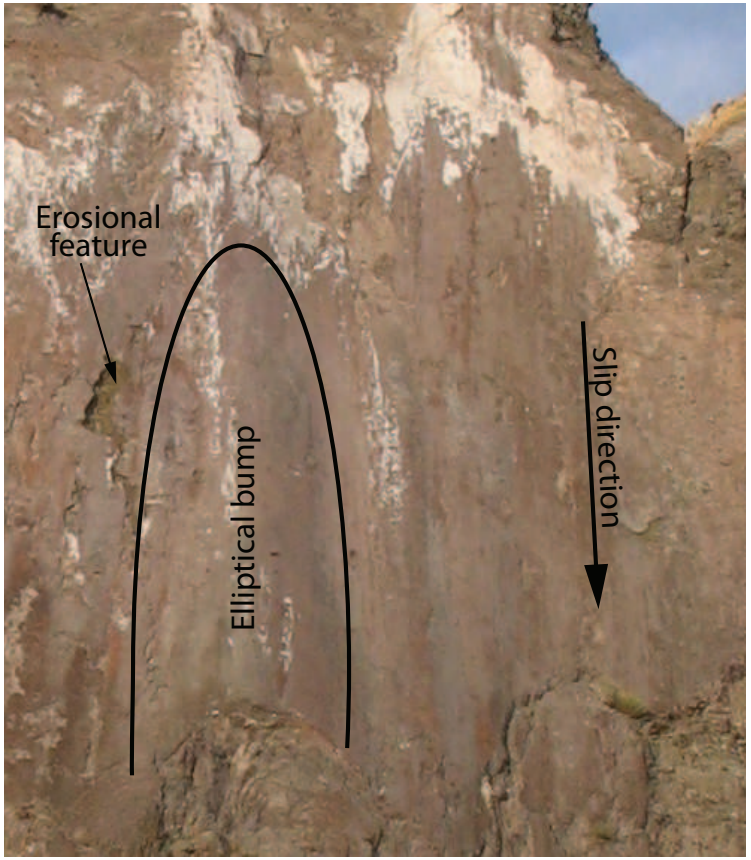
Adapted from :
Bacon et al., 1997

Fault zone constituents

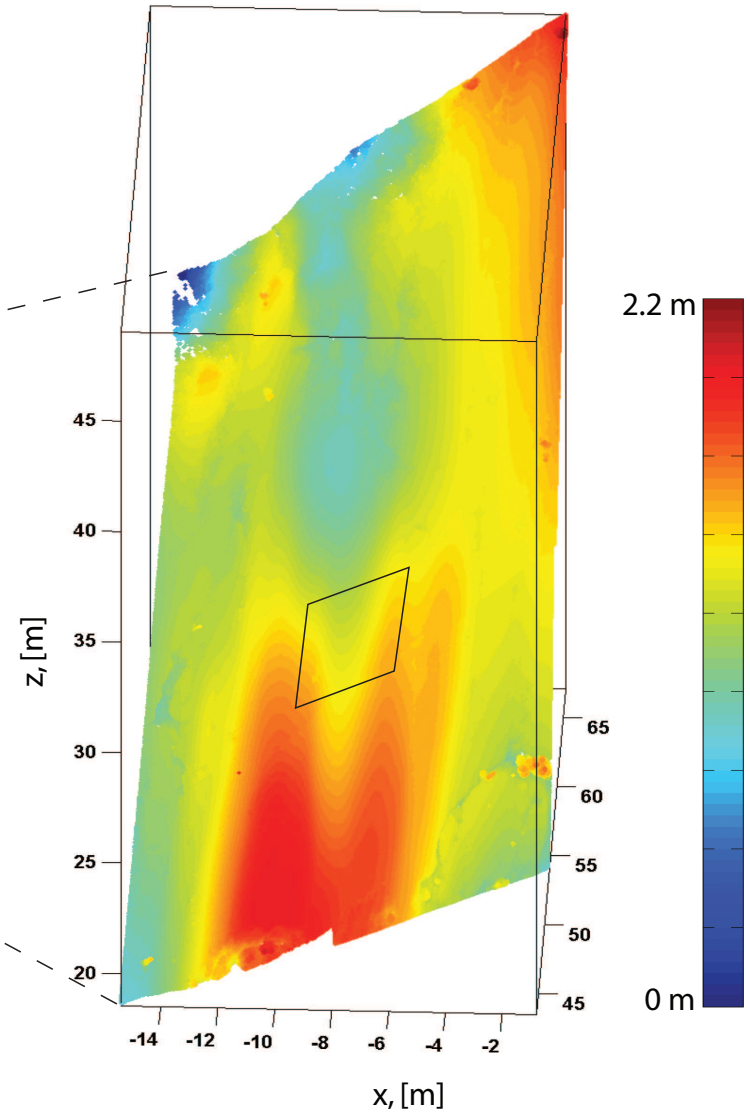


Large scale topography

- Erosional features contained within surface
- Elliptical bumps/troughs



Data: from A. Sagy & E.E. Brodsky

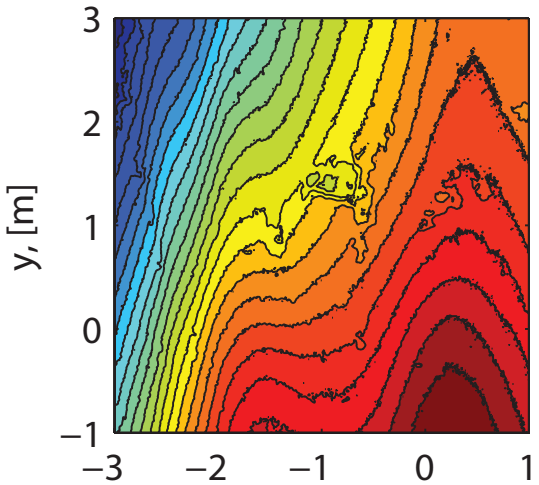


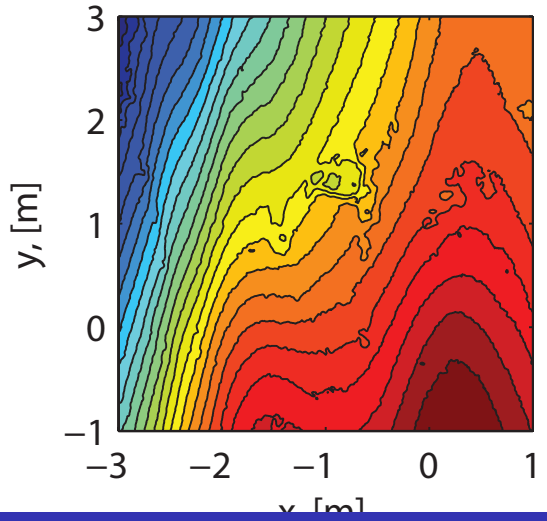
Data Collection

- Ground-based LiDAR
- Point clouds merged
- ∼11 million data points
- 1cm resolution
- 3mm precision

Data

- Interpolate onto regular grid
- Remove noise
- Remove different frequencies: larger wavelength topography
- Moving average filter (Hamming window)





Differential Geometry

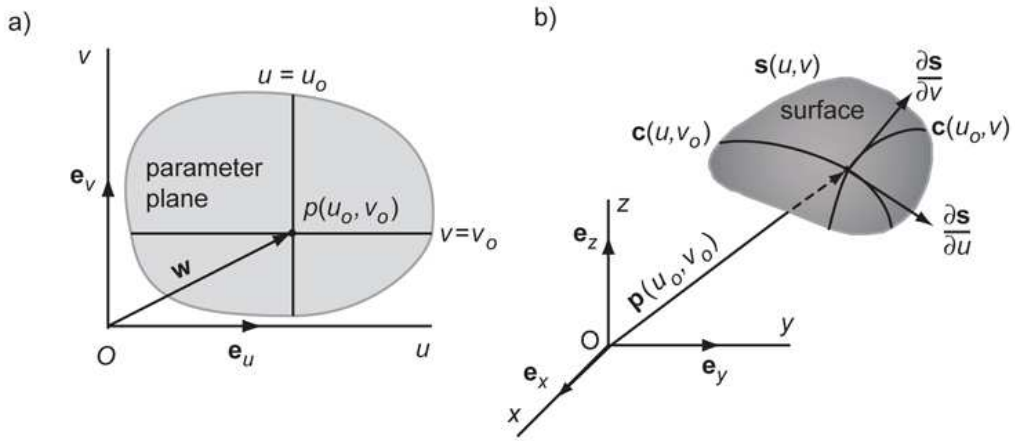
- Quantify geometric properties completely and uniquely
- Two fundamental forms:

First fundamental form:

$$\begin{aligned} \mathrm{I} &= d\mathbf{c} \cdot d\mathbf{c} \\ d\mathbf{c} &= \frac{\partial \mathbf{s}}{\partial u} du + \frac{\partial \mathbf{s}}{\partial v} dv \end{aligned}$$

Second fundamental form:

$$\begin{aligned} \mathrm{II} &= -d\mathbf{N} \cdot d\mathbf{c} \\ d\mathbf{N} &= \frac{\partial \mathbf{N}}{\partial u} du + \frac{\partial \mathbf{N}}{\partial v} dv \quad \text{and} \\ \mathbf{N} &= \left[\frac{\partial \mathbf{s}}{\partial u} \times \frac{\partial \mathbf{s}}{\partial v} \right] / \left| \left[\frac{\partial \mathbf{s}}{\partial u} \times \frac{\partial \mathbf{s}}{\partial v} \right] \right| \end{aligned}$$



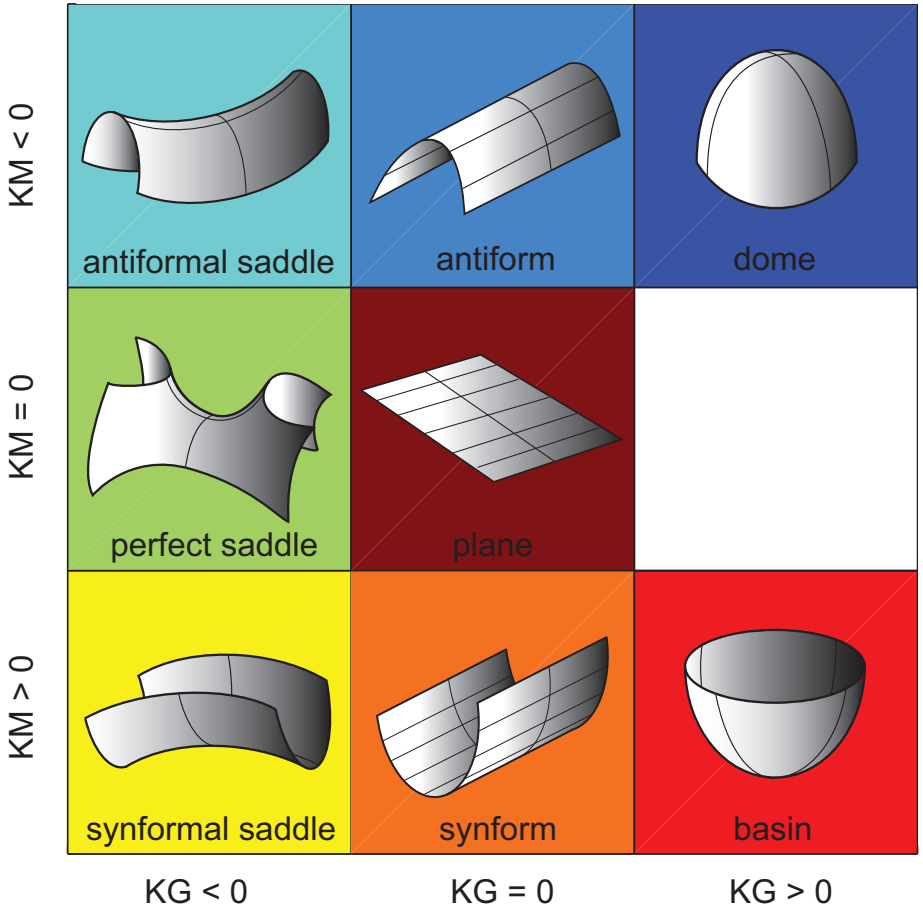
Pollard et al., 2004

Differential Geometry

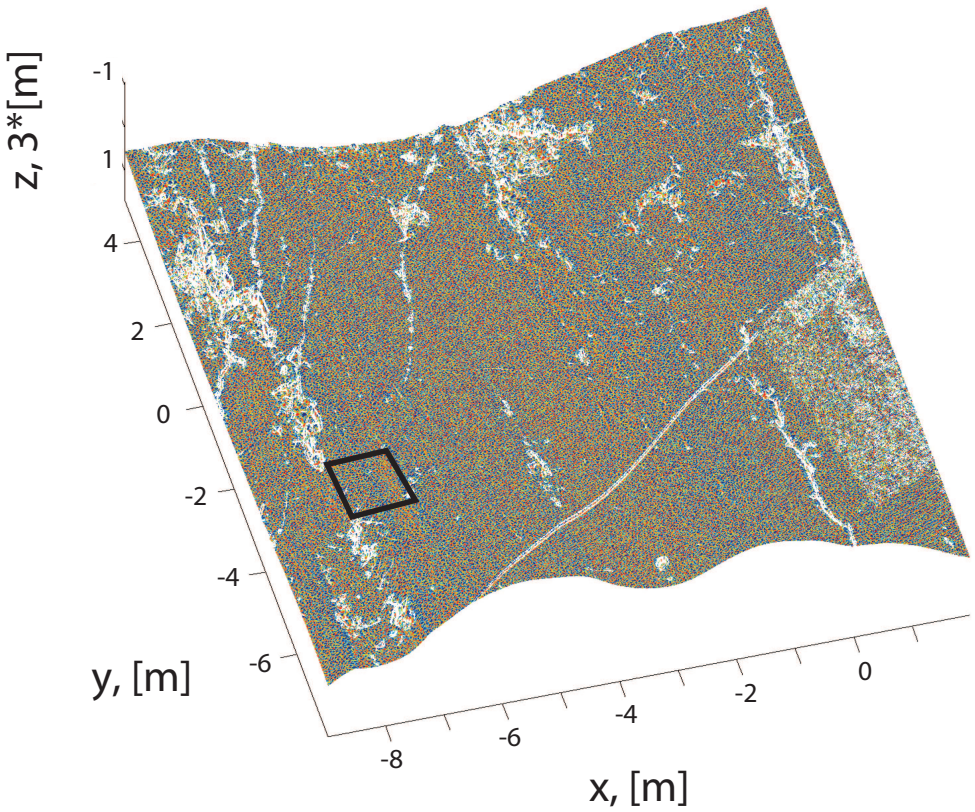
- Shape operator:

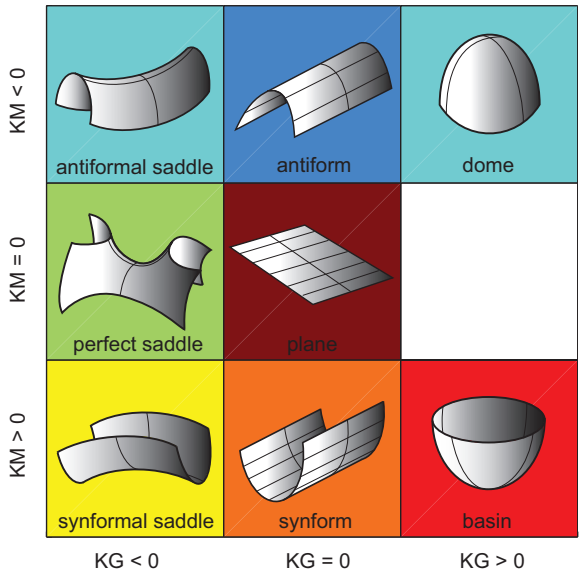
$$\mathbf{L} = \mathbf{I}^{-1}\mathbf{II}$$

- Principle normal curvatures, κ_1 & κ_2
- Useful curvature measures:
  Gauß, $\kappa_G = \kappa_1\kappa_2$,
  mean normal,
  $\kappa_M = \frac{1}{2}(\kappa_1 + \kappa_2)$



Results: low-pass r=0.02m





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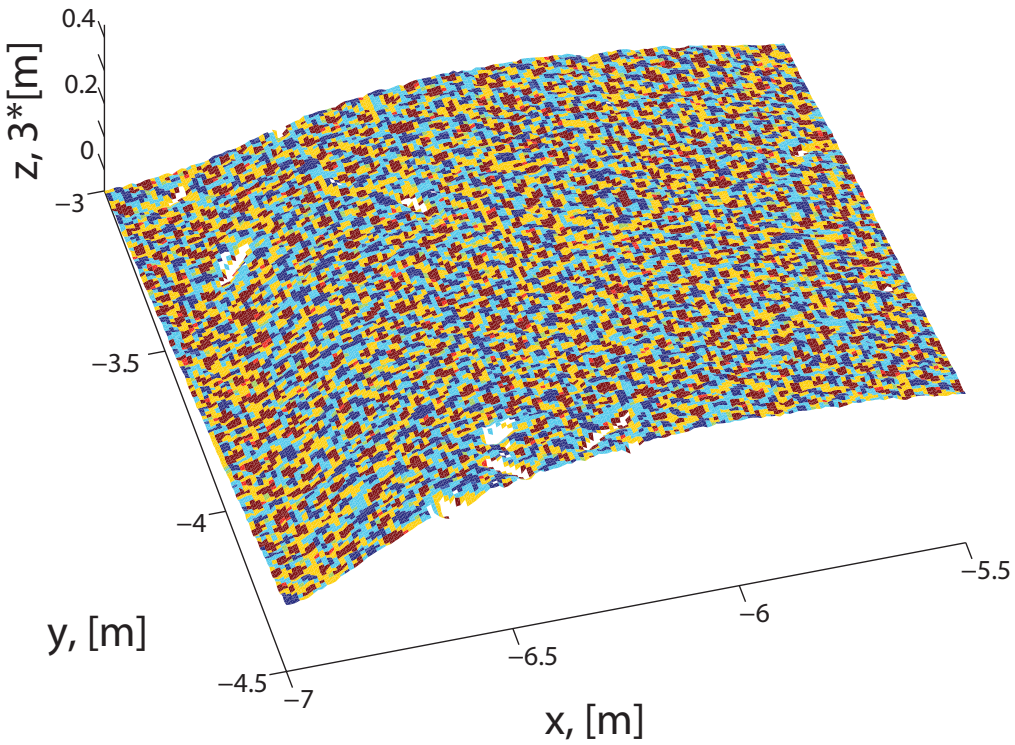
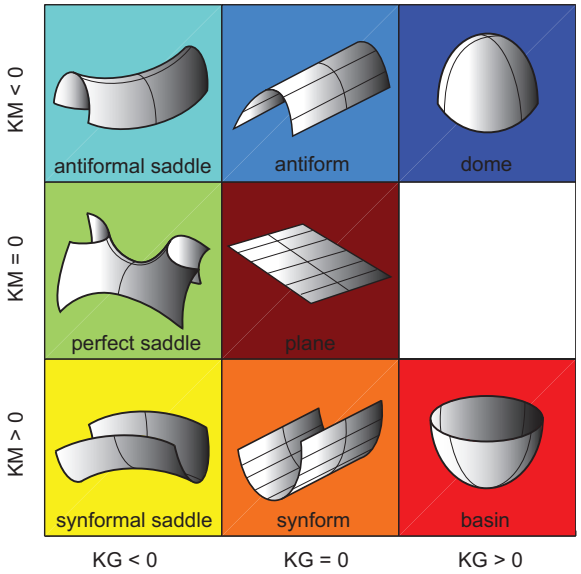
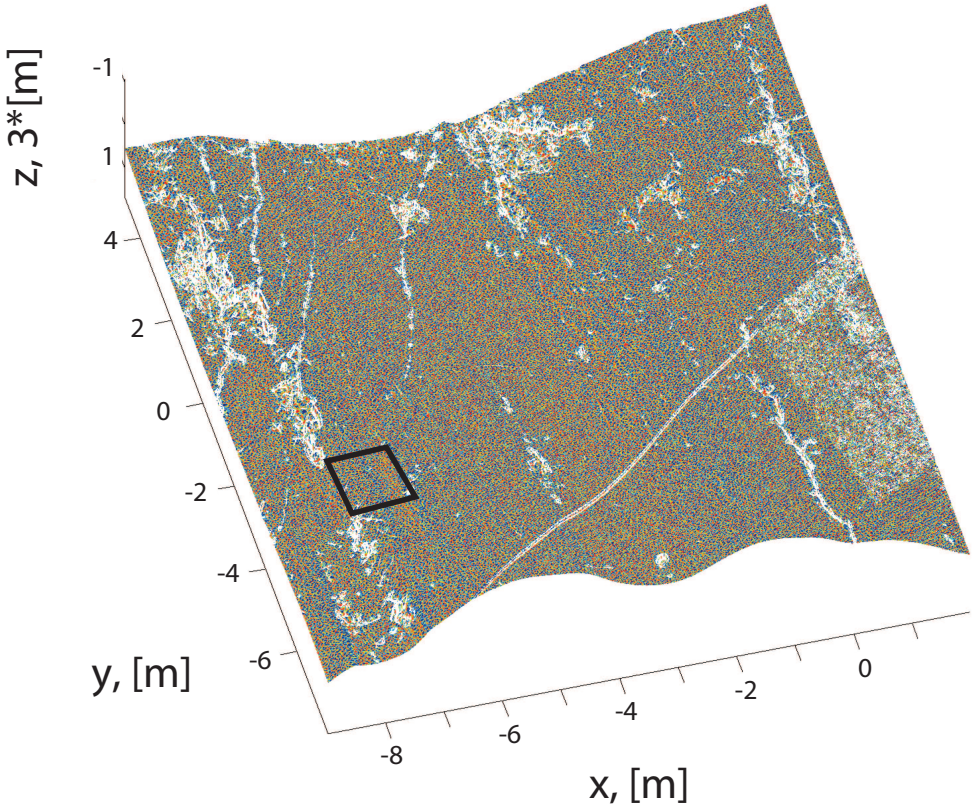
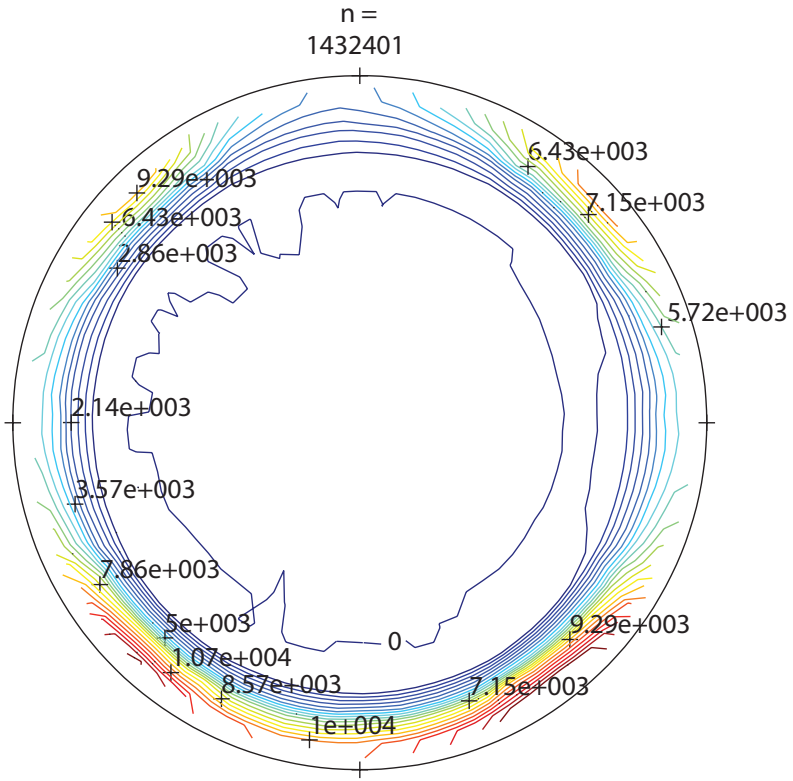


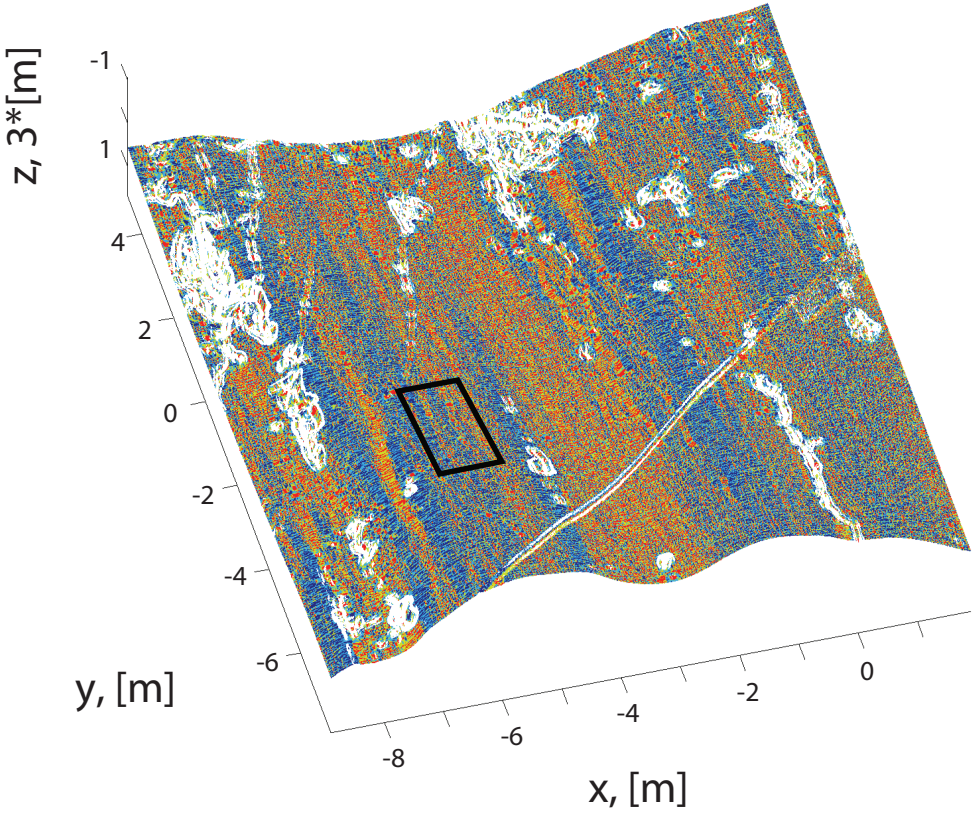


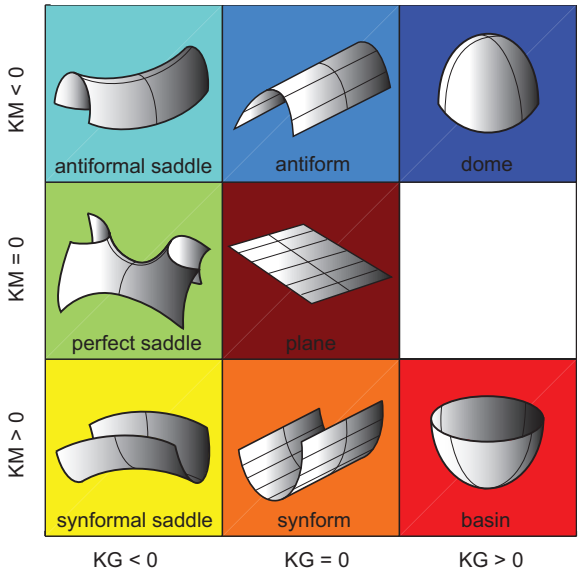
Results: low-pass r=0.02m



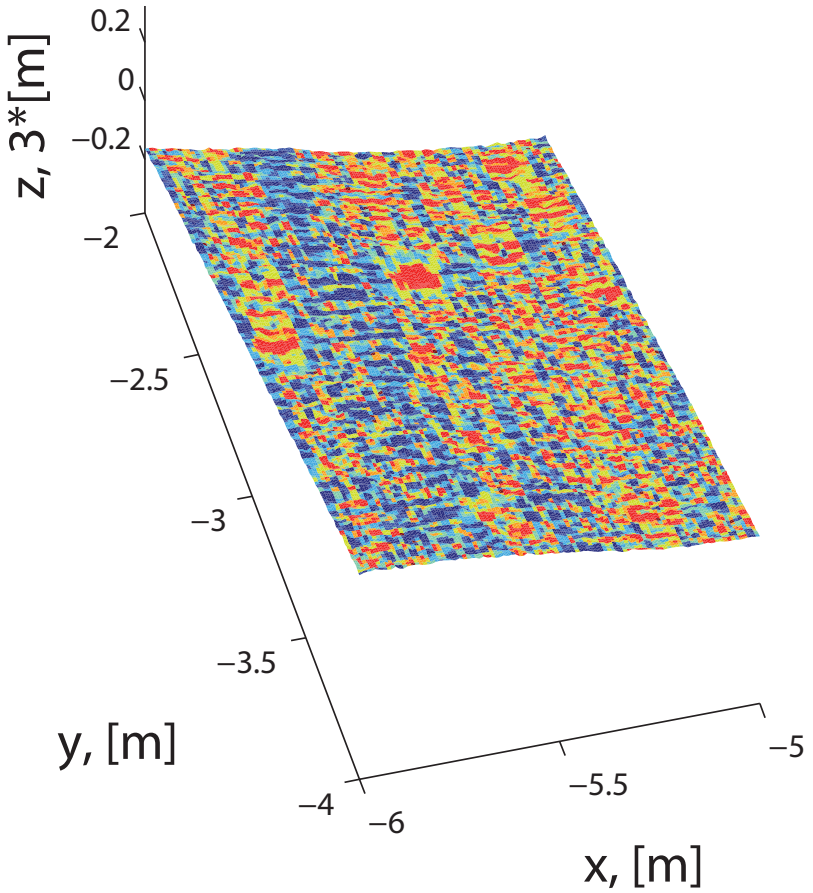


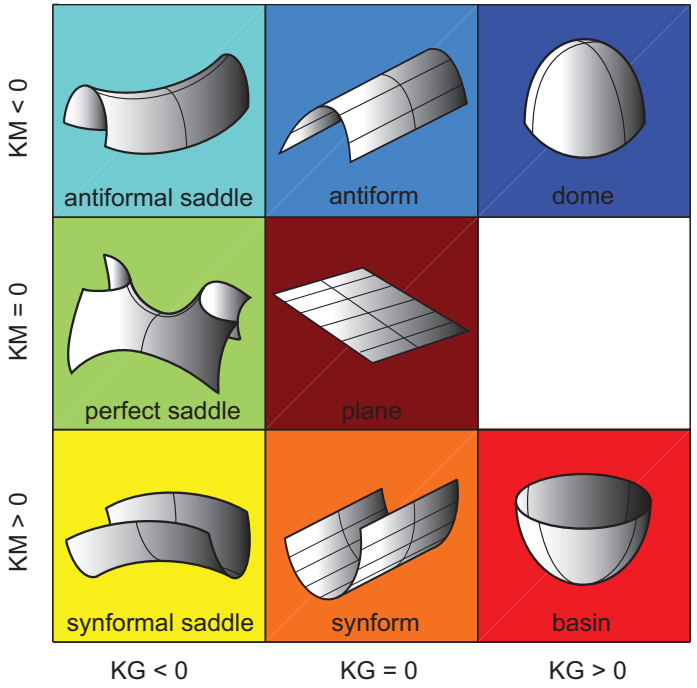
Results: low-pass r=0.1m



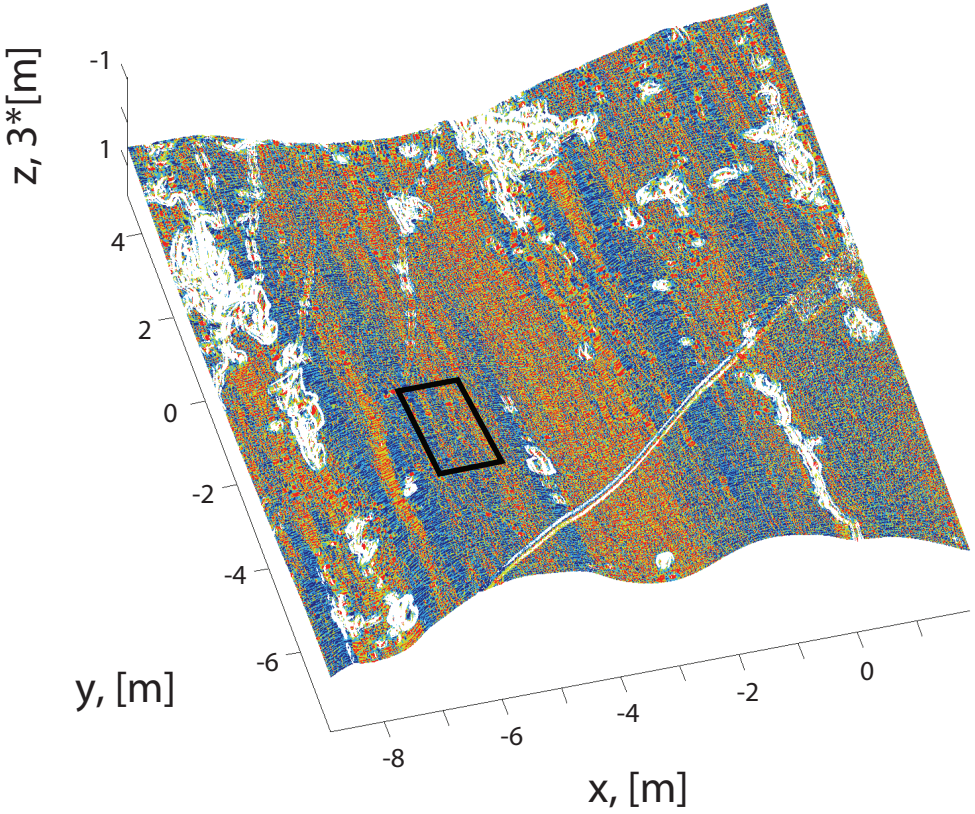


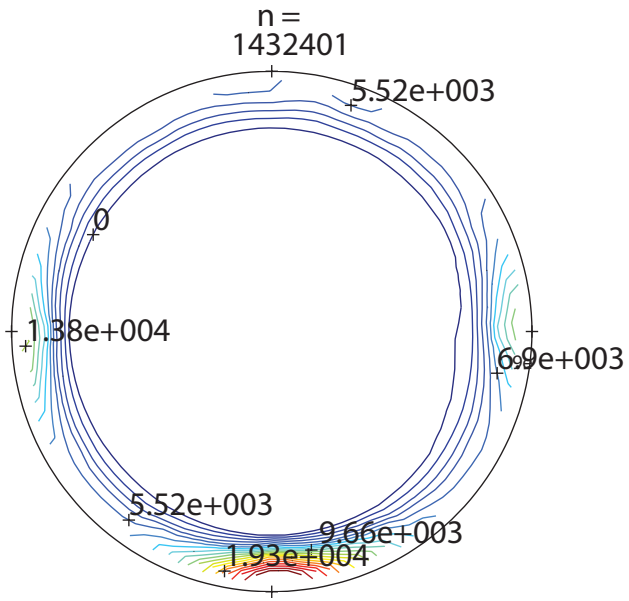
Results: low-pass r=0.1m



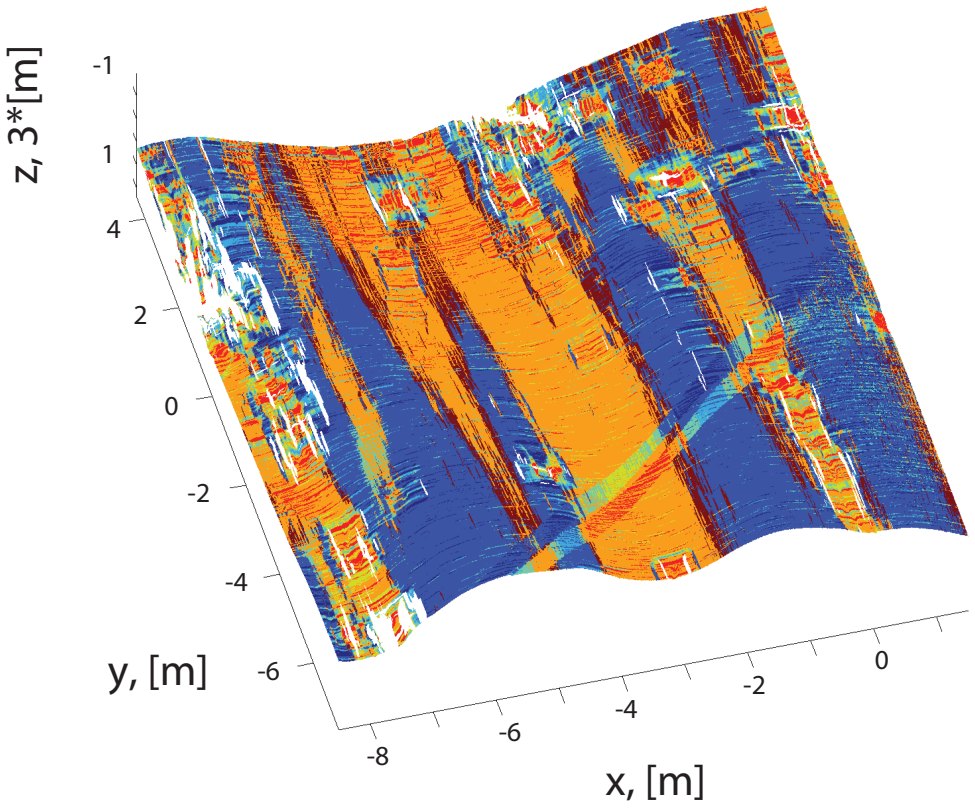


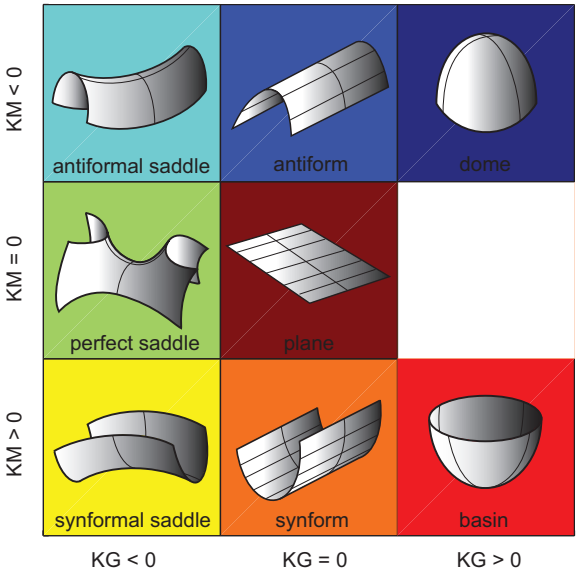
Results: low-pass r=0.1m





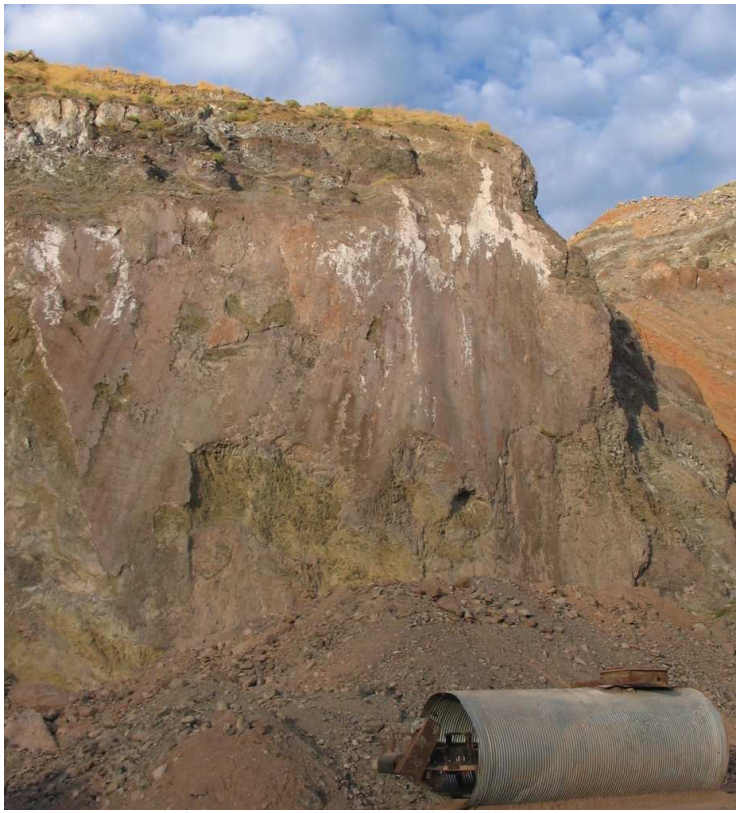
Results: low-pass r=0.5m





Summary of intermediate results

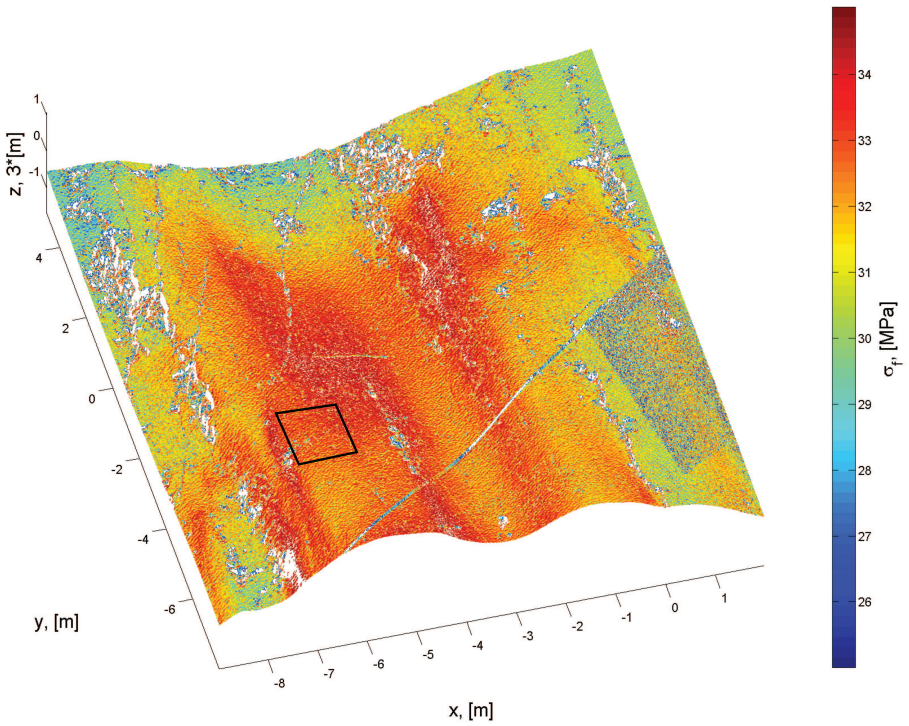
- Longer wavelengths: 'ideal' geometry
- Medium / short wavelengths: slip-parallel undulations
- Elliptical bumps: are also slip-parallel undulations, but with larger a/λ
- D.G. quantification highlights these differences
- Scale dependent!
- What are the important (length) scales?

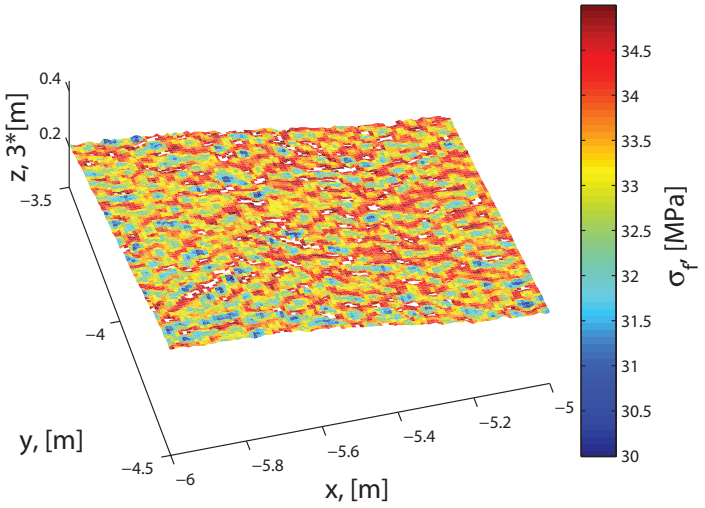


Implications for fault mechanics

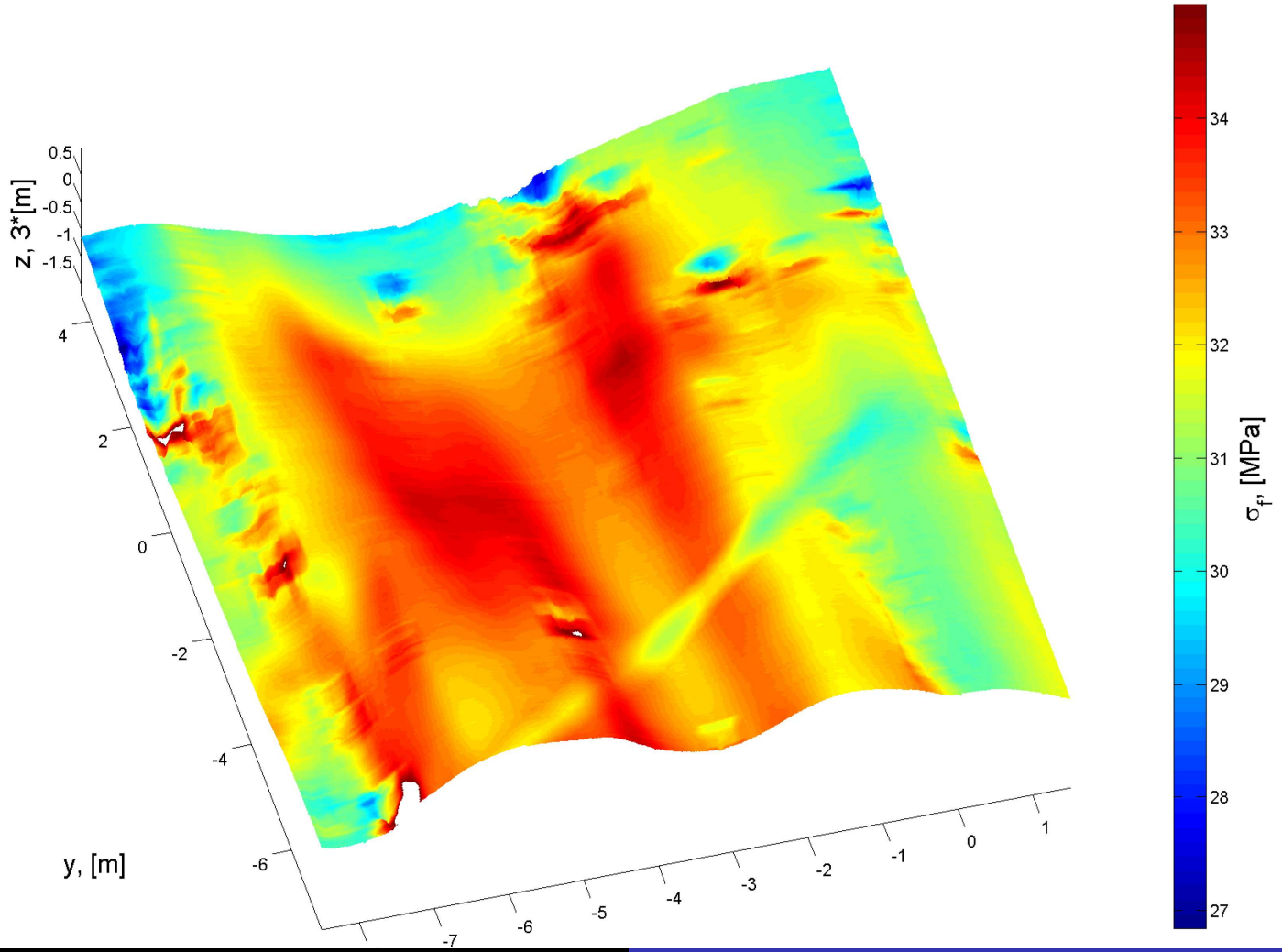
- Resolve tractions on filtered fault surface
- Solve frictionless 3D heuristic fault models
- Solve the (static) frictional sliding problem (2D)

Resolved Coulomb tractions (low-pass r=0.02m)

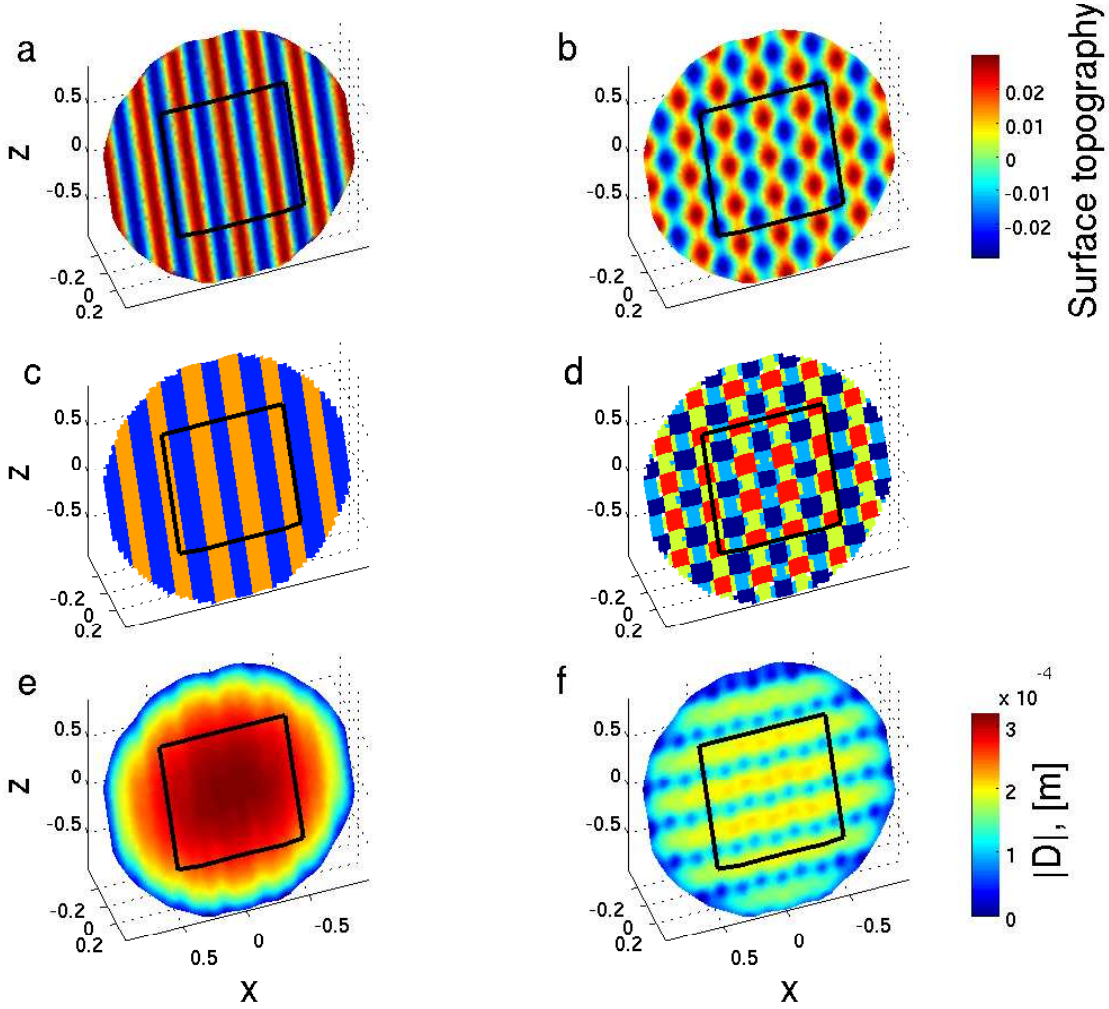




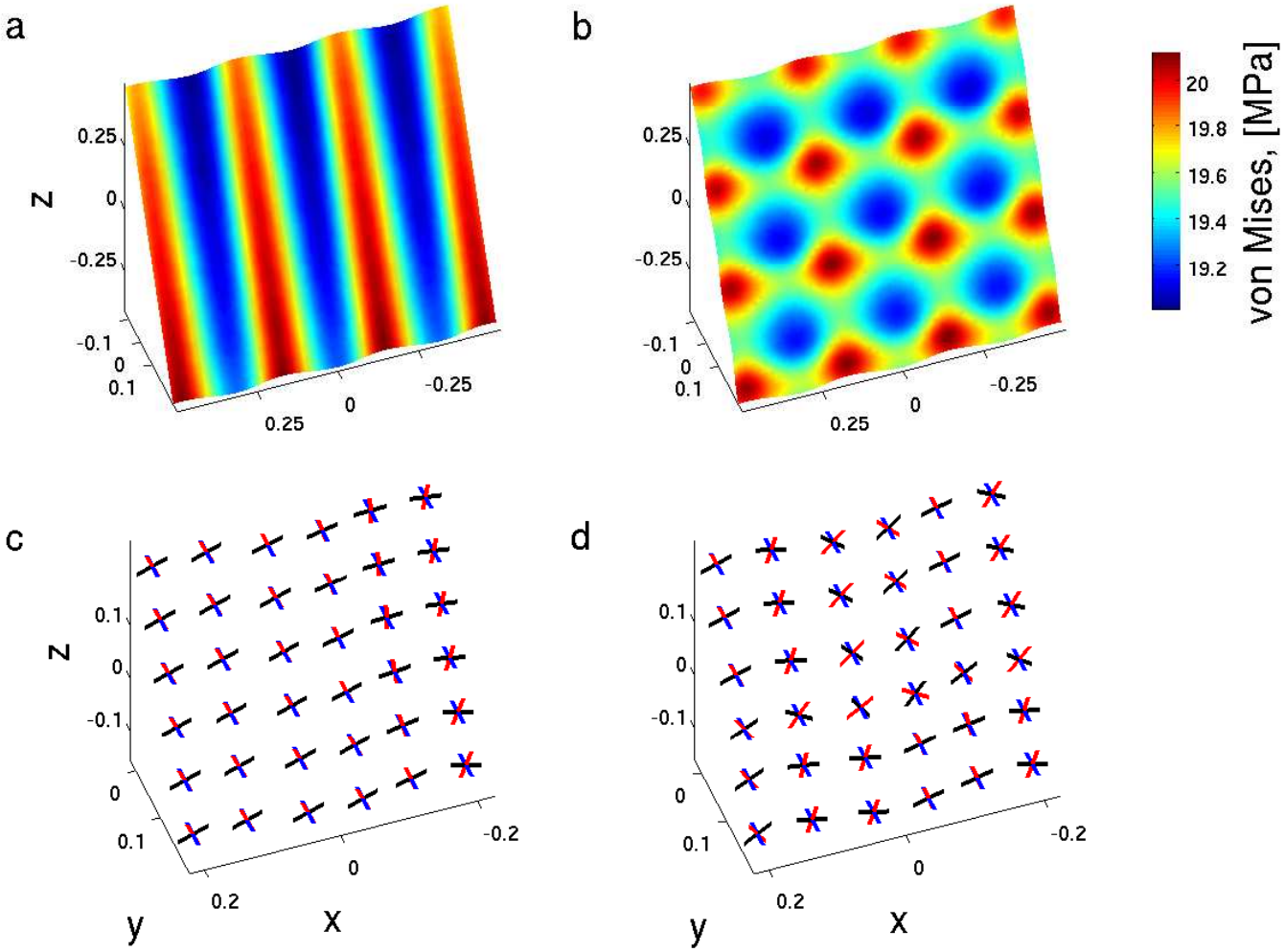
Resolved Coulomb tractions (low-pass r=0.5m)



Heuristic fault models

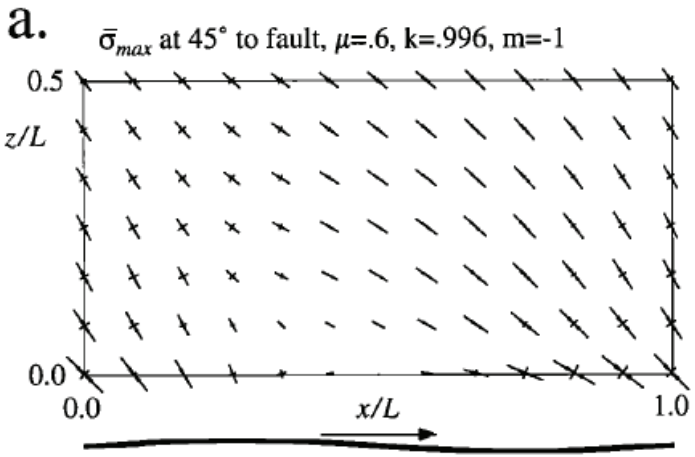


von-Mises stress & principal stress orientations



Modeling 2D frictional faults using boundary element methods

- Modeling efforts generally oversimplify the geometry
- Stick with non-planar geometry and treat boundary conditions somewhat differently

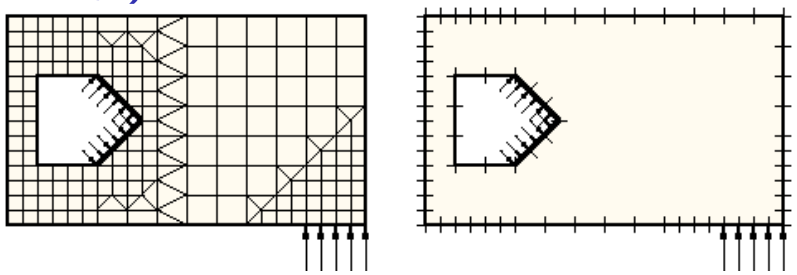


Chester and Chester, 2000

How to model finite faults (statically)?

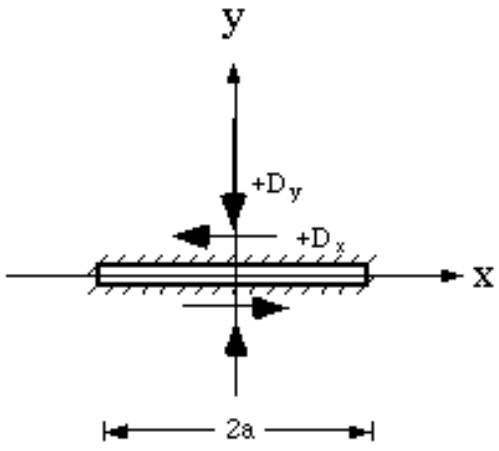
- Governing equations:

$$\sigma_{ij,j} = F_i \quad \in \Omega$$



- Discretization
- Faults: displacement discontinuities

$$D_i = U_i(x, 0^-) - U_i(x, 0^+)$$



- Boundary conditions:

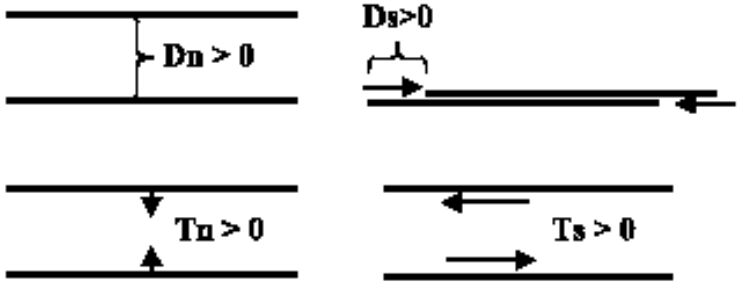
$$\begin{Bmatrix} T_s \\ T_n \end{Bmatrix} = \begin{bmatrix} A_{ss} & A_{sn} \\ A_{ns} & A_{nn} \end{bmatrix} \begin{Bmatrix} D_s \\ D_n \end{Bmatrix}$$

Crouch and Starfield, 1983

Friction Implementation in BEM: Complementarity

- Normal displ. & traction

$$T_n \geq 0$$
$$D_n \geq 0$$
$$T_n D_n = 0$$



- Shear slip & traction

$$\text{if} \quad |T_s| \geq 0, \mu T_n - T_s \geq 0,$$
$$\text{then} \quad D_s = 0$$
$$\text{if} \quad |T_s| - \mu T_n = 0,$$
$$\text{then} \quad D_s \neq 0$$

Pang et al., 1996; Mutlu and Pollard, 2008

Resulting numerical problem: Linear programming

- Rearranged algebraic expression

$$\begin{Bmatrix} D_n \\ U_s^+ \\ T_s^- \end{Bmatrix} = [M] \begin{Bmatrix} T_n \\ T_s^+ \\ U_s^- \end{Bmatrix} + q$$

- Numerical problem to solve:

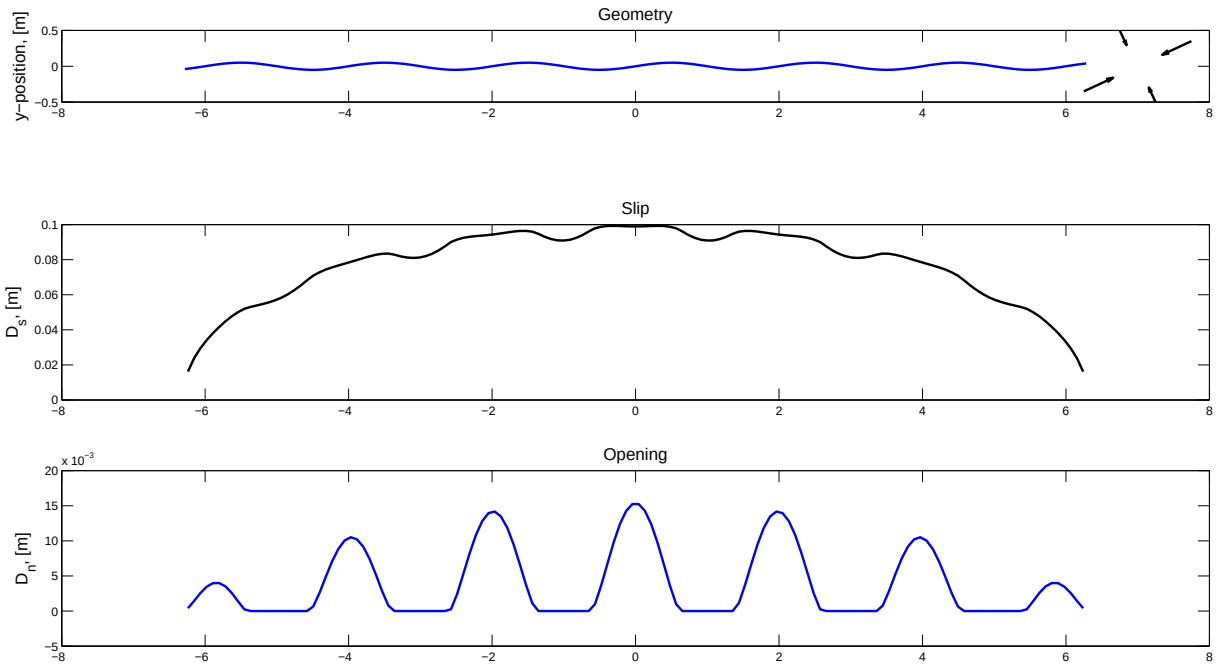
$$f(x) = Mx + q$$

subject to

$$x \geq 0, \quad f(x) = 0, \quad xf(x) = 0$$

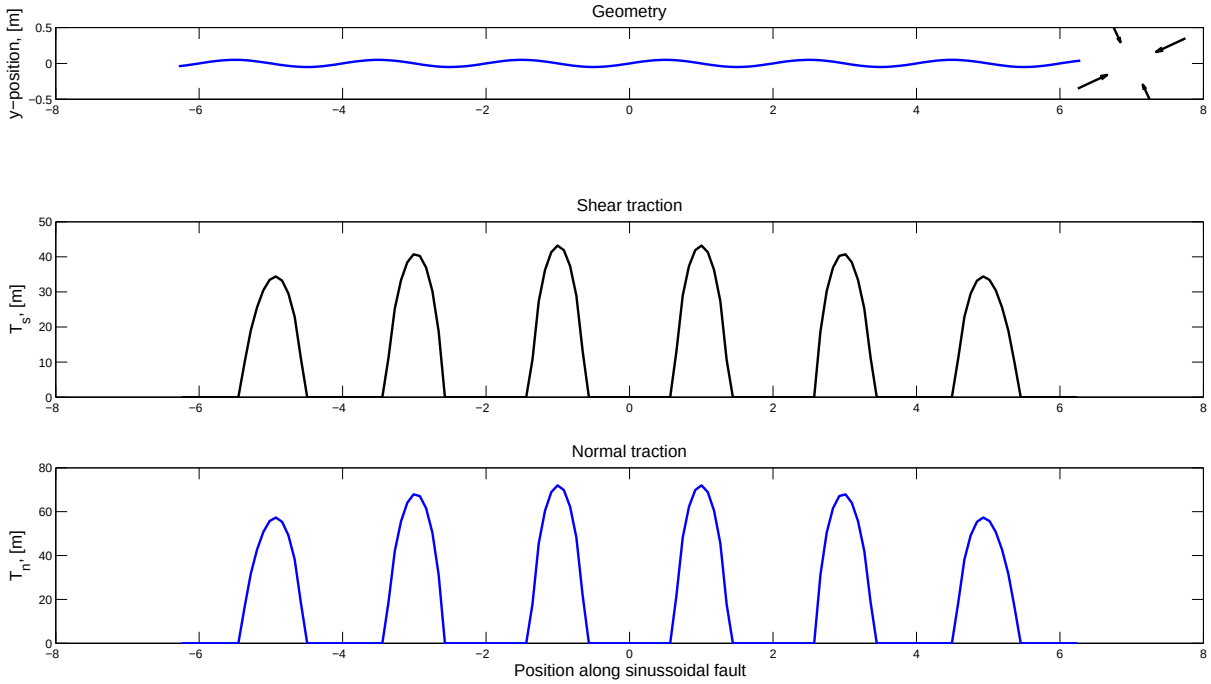
Boundary Element Model

- Finite length fault with wavy geometry
- Linearly elastic, isotropic, homogeneous
- Frictional contact $\mu = .6$, no cohesion
- $\nu = .25$, E = 5GPa

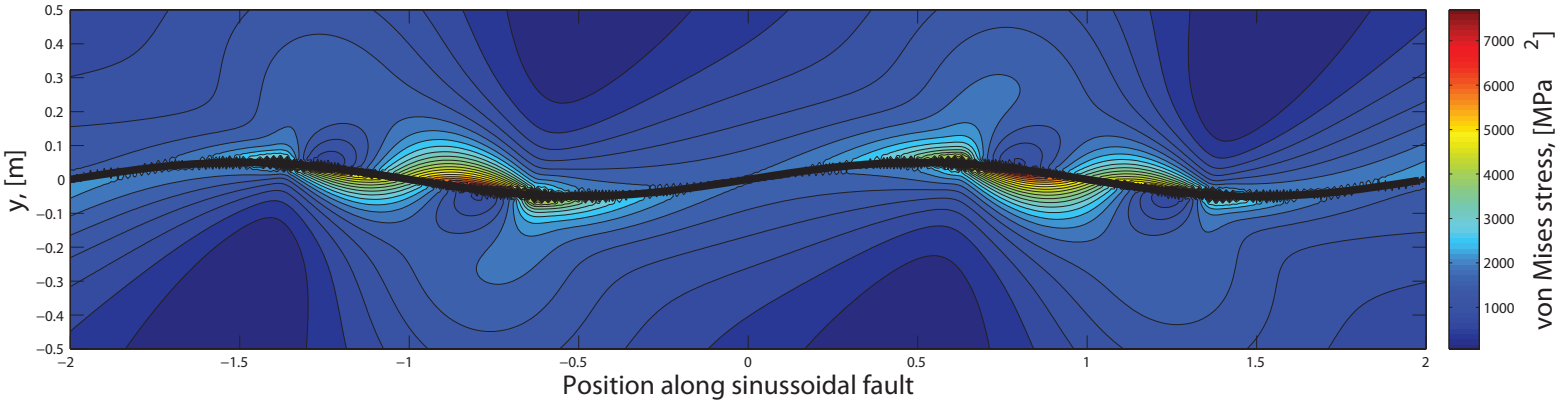


Boundary Element Model

- Finite length fault with wavy geometry
- Shear & normal tractions

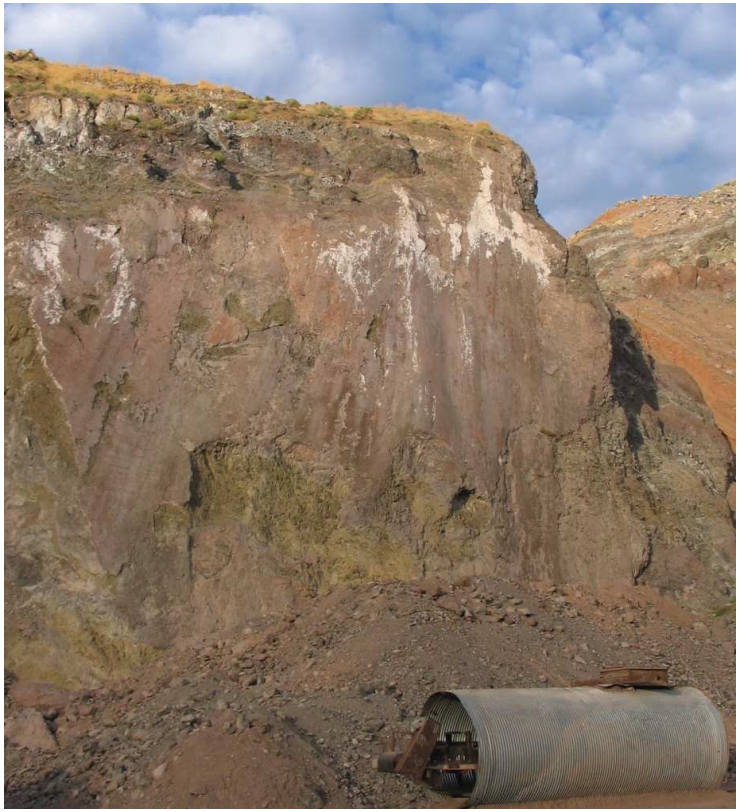


Off-fault deformation, onset of plastic yielding



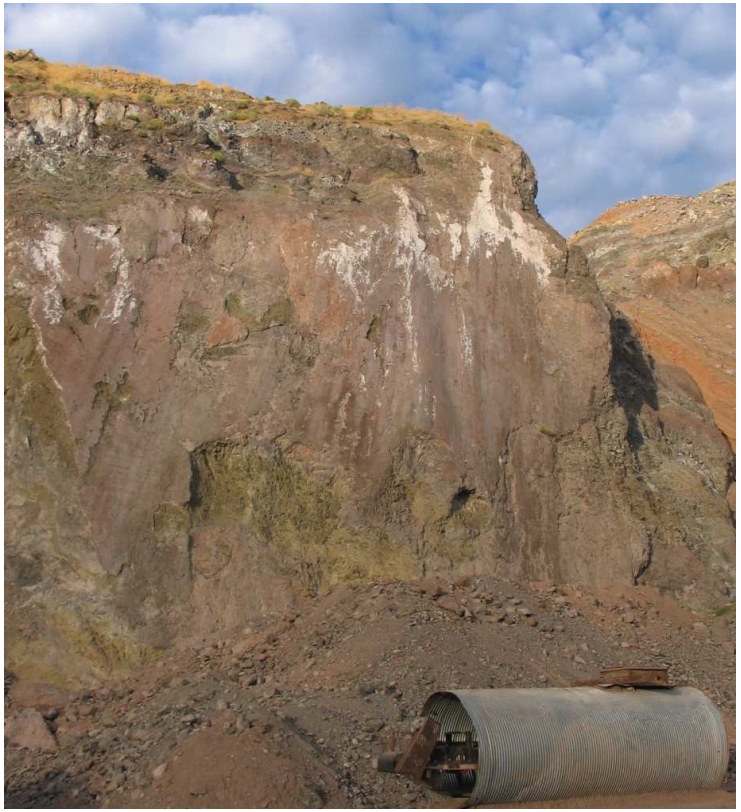
Conclusions

- Heterogeneous distribution of tractions
- Heterogeneous off-fault deformation
- Dilation happens under many loading conditions
- Non-constant friction law implementation underway (2D)
- Implementation in 3D pending (iterative solver for friction works)



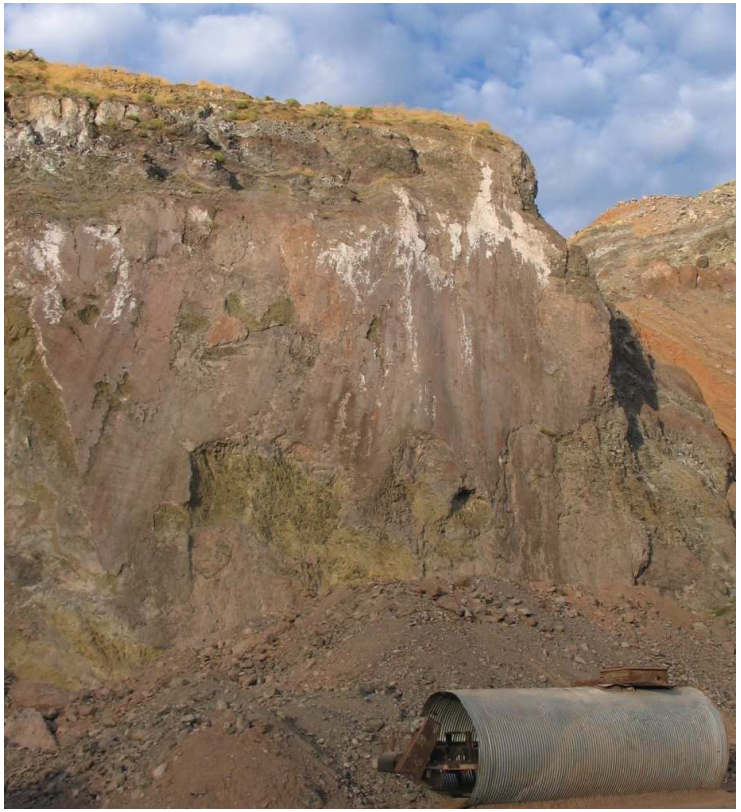
Conclusions

- D.G. quantification: spatial coherence, basic shapes
- Basic shapes affect mechanics of faulting
- Resolved tractions vary on the order of MPa
- Improved (static) modeling provides interesting results
- Quasi-static solutions with non-constant friction are likely to provide more insight into location of slip initiation etc.



Questions:

- Locked vs. creeping regions: geometric differences (resolvable)?
- Fluid flow along faults after slip
- Does off-fault deformation yield wavy surfaces?



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