Magma Dynamics at CIG

Laurent G. J. Montési

University of Maryland

Contributions from:

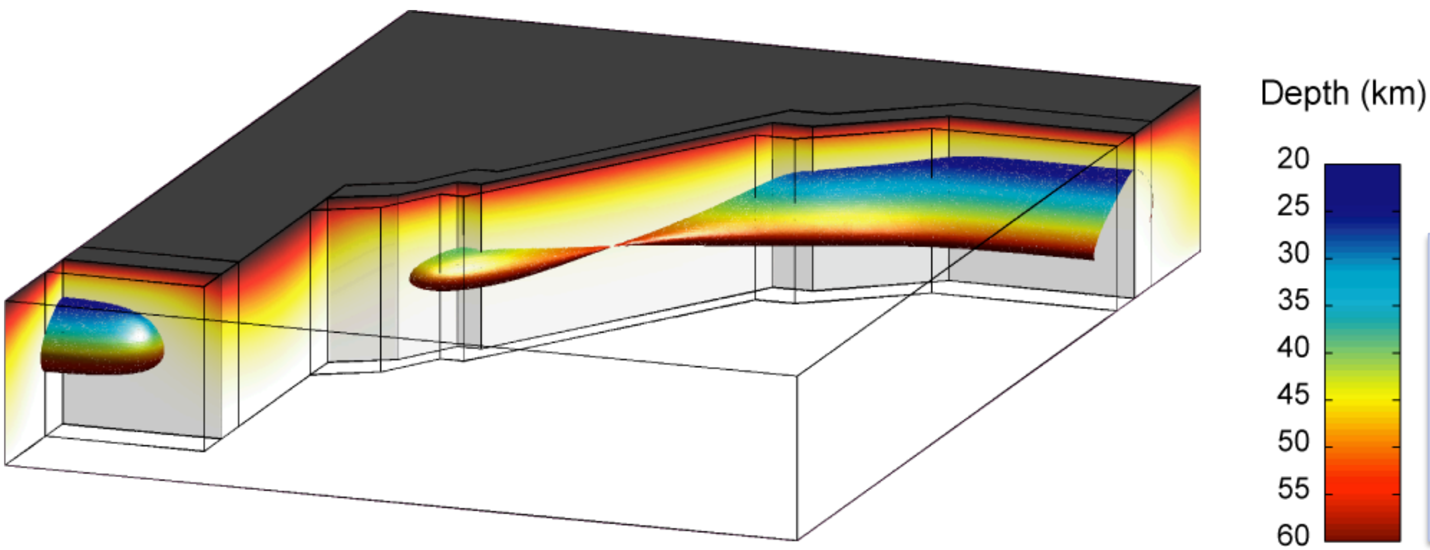
Marc Spiegelman (Columbia)

Steve Quenette, David Lee (VPAC)

Wolfgang Bangerth (Texas A&M)

Why Magma Dynamics?

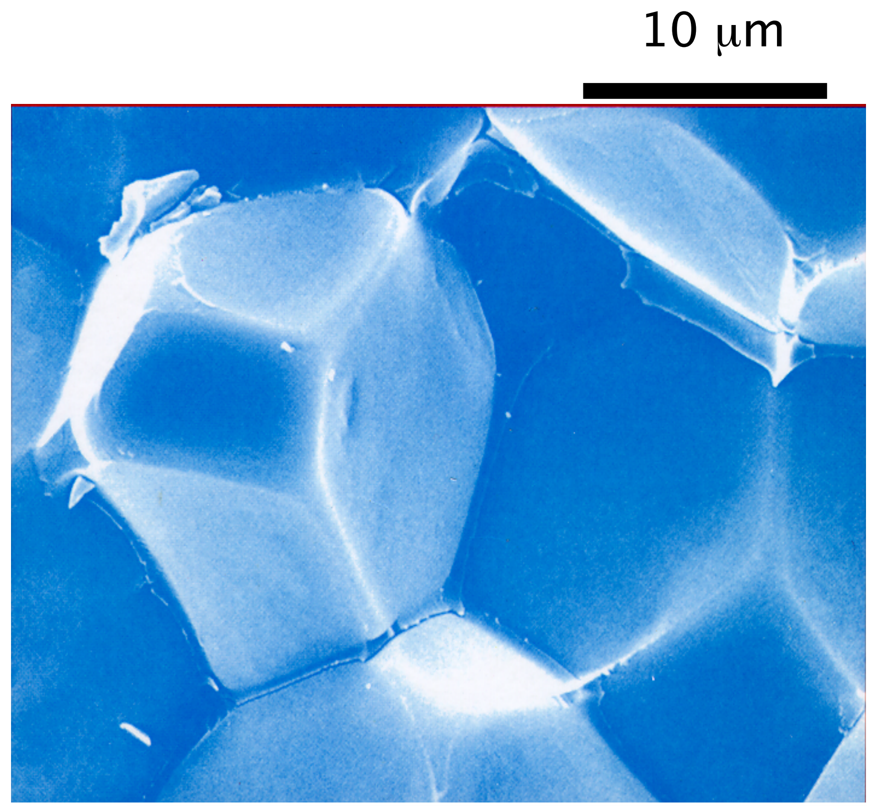
- Probe into the mantle
  - Flux influenced by geodynamics (temperature, flow, pressure)
  - Composition depends on source and transport process
  - Complements geological and geophysical investigation
- Influences mantle dynamics
  - Differentiation / partitioning of heat producing elements
  - Buoyancy of melt and residuum



Melt distribution model underneath ultraslow ridges Montési, Hebert, Behn, Barry, 2009

A multiphysics problem

- Two-phase flow
- McKenzie 1984 formulation
  - Flow of melt controlled by Darcy's law
  - Compaction of matrix controlled by bulk viscosity
  - Stokes flow
- Additional complexity
  - Surface tension, grain boundary processes, viscoelasticity, failure, thermodynamics
- Use magma dynamics as testbed for multiphysics software



Granitic melt in quartz (Laporte et al. 1997)

Magma equations

- Solve for ϕ, V, $\mathcal{P}$, P^*

Mass Conservation	$$\frac{\partial \phi}{\partial t} + V \cdot \nabla \phi = (1-\phi)\frac{\mathcal{P}}{\xi} + \frac{\Gamma}{\rho_s}$$
Compressibility	$$\nabla \cdot V = \mathcal{P}/\xi$$
Darcy/Helmholtz	$$-\nabla \cdot \frac{K}{\mu}\nabla \mathcal{P} + \frac{\mathcal{P}}{\xi} = \nabla \cdot \frac{K}{\mu}\left[\nabla P^* + \Delta\rho g\right] + \Gamma \frac{\Delta \rho}{\rho_f \rho_s}$$
Stokes	$$\nabla P^* = \nabla \cdot \eta\left(\nabla V + \nabla V^T\right) - \phi \Delta \rho g$$
Permeability	$$K = K_0 \phi^n$$

- Compute other variables

$$\phi v = \phi V - \frac{K}{\mu}\left[\nabla\left(P^* + \mathcal{P}\right) + \Delta \rho g\right]$$

$$P = \rho_s^0 g z + \mathcal{P} + P^*$$

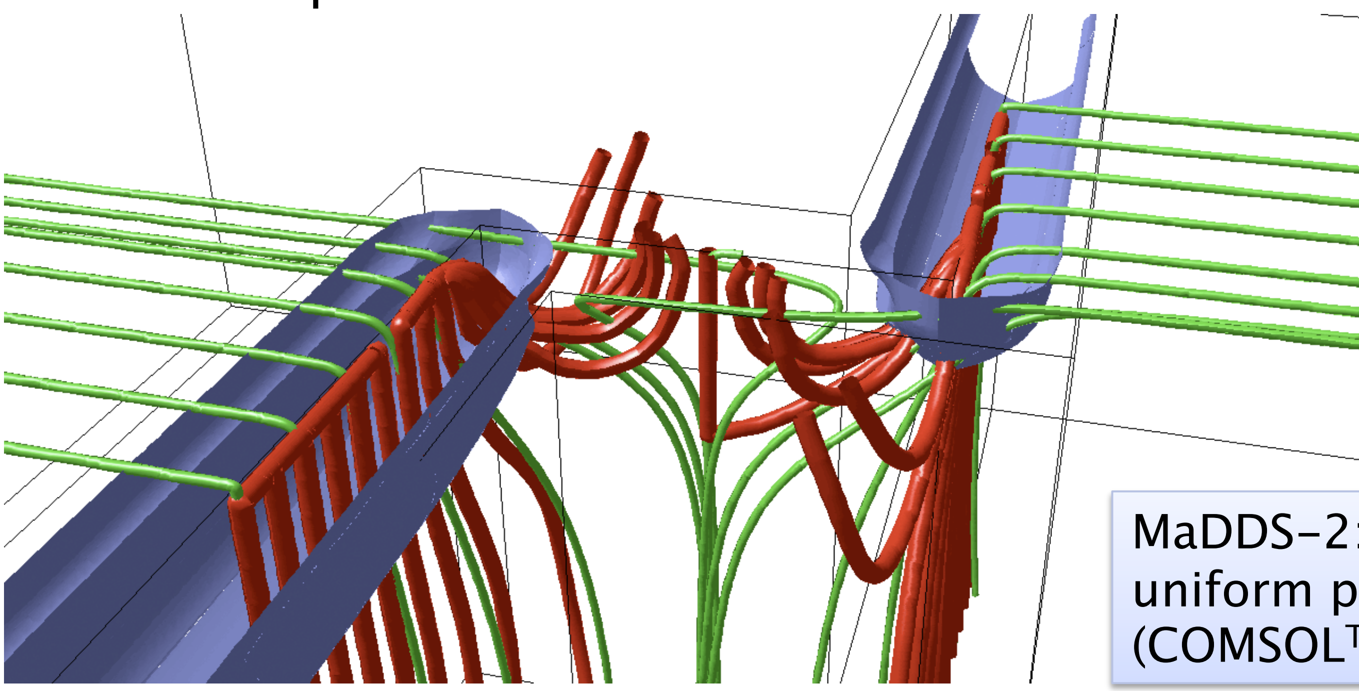
Spiegelman, Katz, and Simpson, 2007

The MaDDs project

- **Ma**gma **D**ynamics **D**emonstration **S**uite
- 5 demonstration problems, described in online document at CIG
    - 1: 2D ridge / 3D segmented ridge (Stokes only)
    - 2: Constant porosity ridge (Stokes + Darcy)
    - 3: Shear bands
    - 4: Solitary wave (Darcy flow + compaction)
    - 5: Coupled ridge, forced melting
- Test several software libraries / platforms
- No “Gold Standard” code!

MaDDS platforms / software

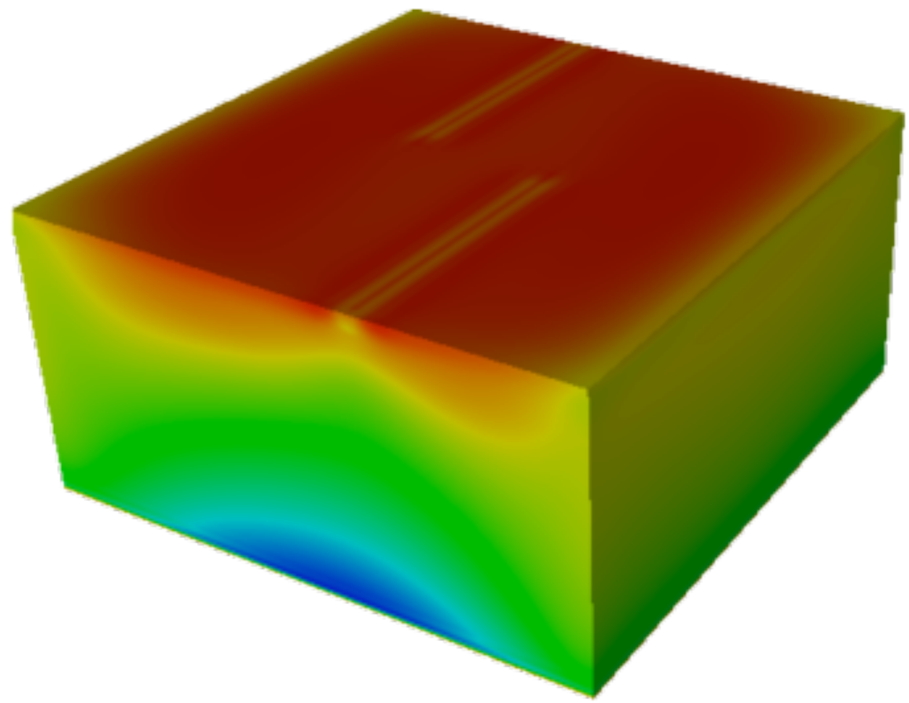
- stgMaDDS project
  - David Lee (VPAC)
  - Based on StGermain
- Deal.II
  - Wolgang Bangerth (TAMU)
  - Adaptive mesh refinement
- MultiMaDDS:
  - Laurent Montési (Maryland)
  - COMSOL multiphysics™
- FEniCS/PETSc hybrid
  - Marc Spiegelman (Columbia)
  - Automatic code generation



MaDDS-2: Stokes flow and uniform porosity melt migration (COMSOL™, Montési.)

Status of stgMaDDS

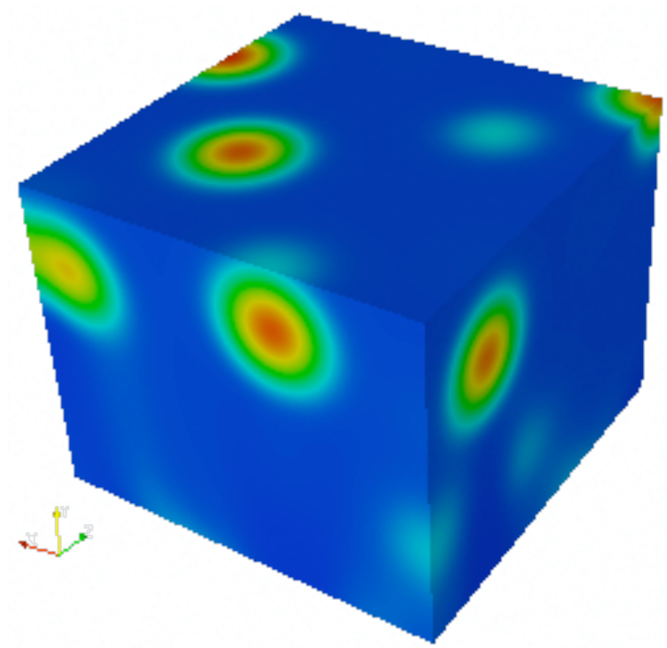
- Most problems done
  - Stokes benchmark
  - Constant porosity ridge
  - Solitary wave
  - Ridge with melting
- Alpha version released through CIG's mercurial repository and StGermain project



Compaction pressure for MaDDS-5: Fully coupled 3D segmented isoviscous ridge with melting.
(Quenette and Lee one-pager)

Evaluation of stgMaDDS

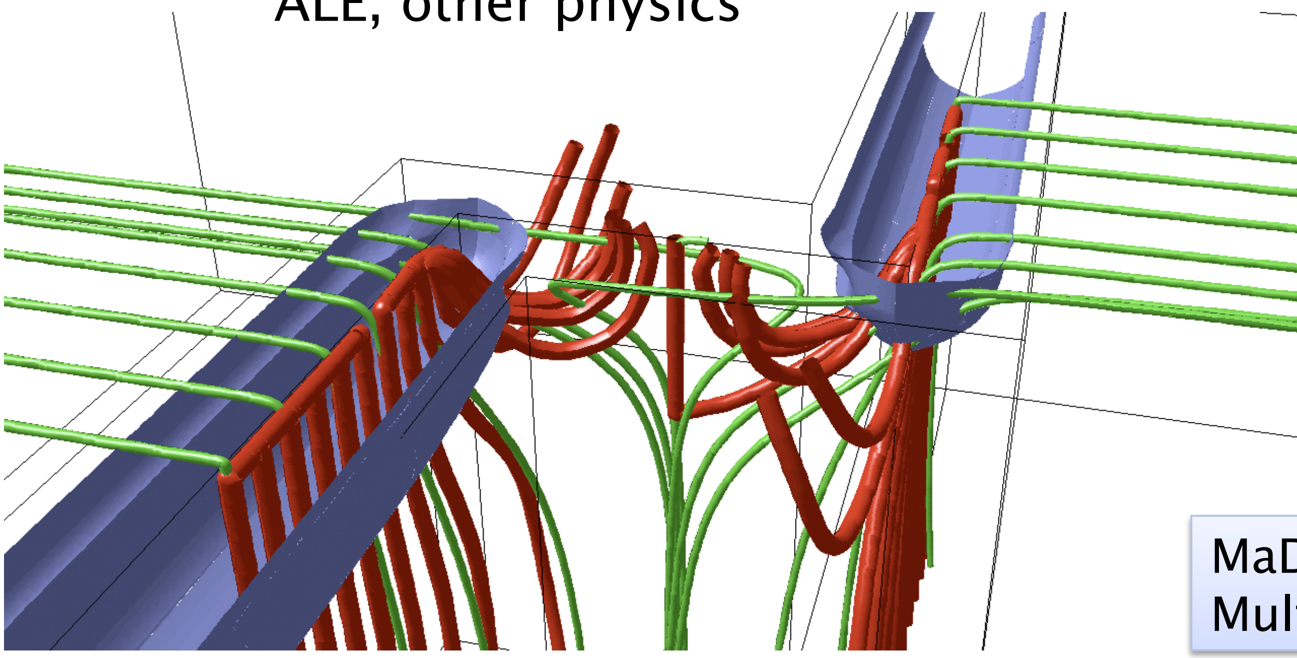
- It works, and you can get it!
  - Publically available software, parallel
  - In principle, compatibility with other StGermain-based applications (Underworld, SNAC, GALE)
- It wasn't easy!
  - Framework bring complexity
  - Solver flexibility
  - Performance/scalability?
- User base?



MaDDS-4: 3D solitary wave (Quenette and Lee one-pager)

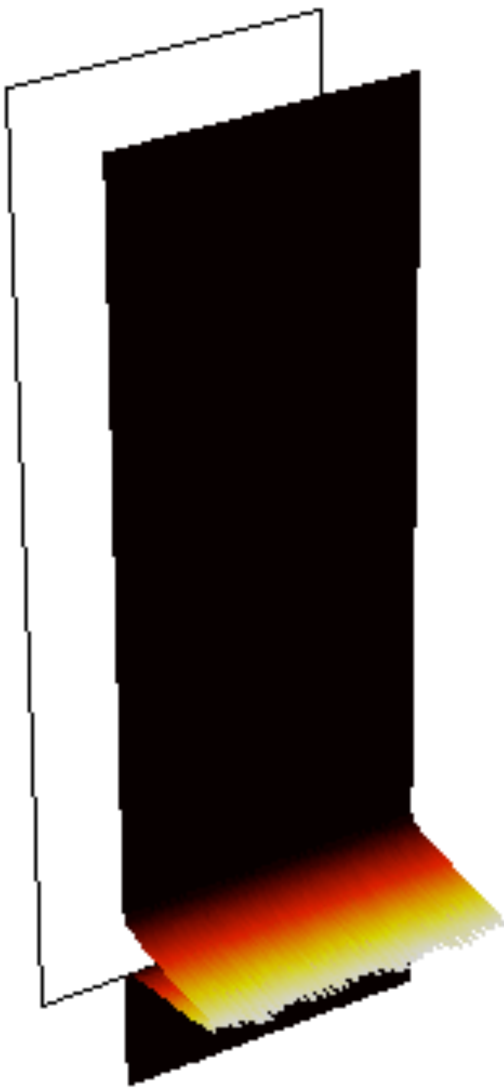
Commercial multiphysics software

- COMSOL Multiphysics™
  - 3D Stokes Flow, Darcy flow, and Helmholtz equation all implemented
  - Possibility of adaptive meshing (steady-state), ALE, other physics
- Evaluation
  - Great for small, rapid applications.
  - Easy to learn, even with little numerical knowledge
  - Can't tinker with solver



MaDDS-2 with COMSOL Multiphysics™ (Montési)

MultiMaDDS status

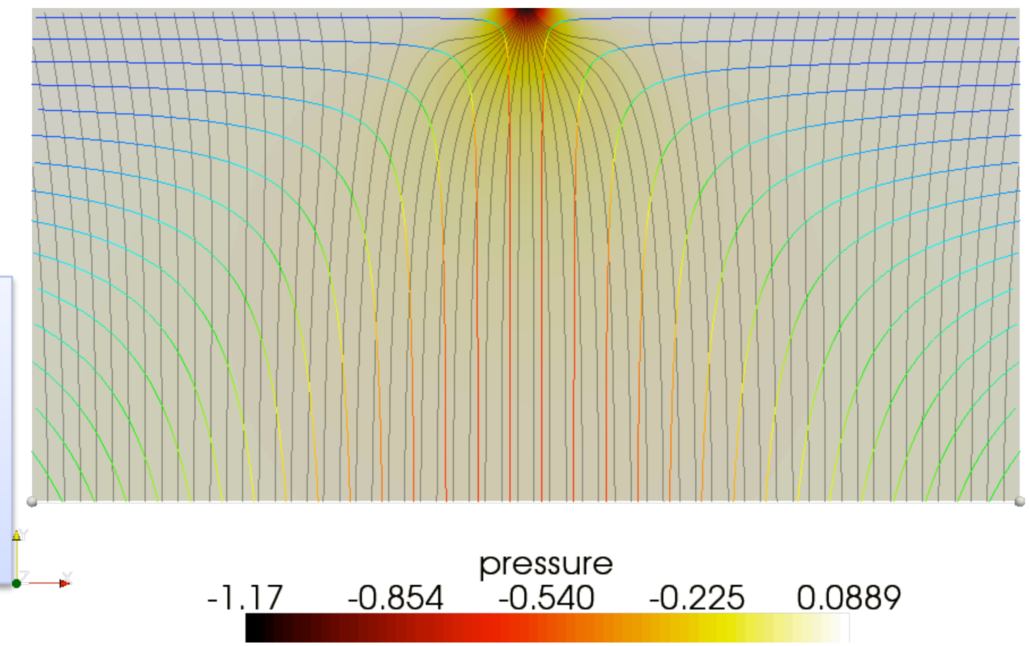


- MaDDS-1: Stokes solver
  - Higher order element
  - Unstructured mesh (ALE and AMR possible)
  - Stress and temperature – dependent rheology (with Mark Behn)
- MaDDS-2: Stokes coupled with Darcy equation
- MaDDS-4: Solitary waves: Custom PDE and Helmholtz equation
- 2D and 3D

Stokes flow and uniform porosity melt migration with COMSOL Multiphysics™

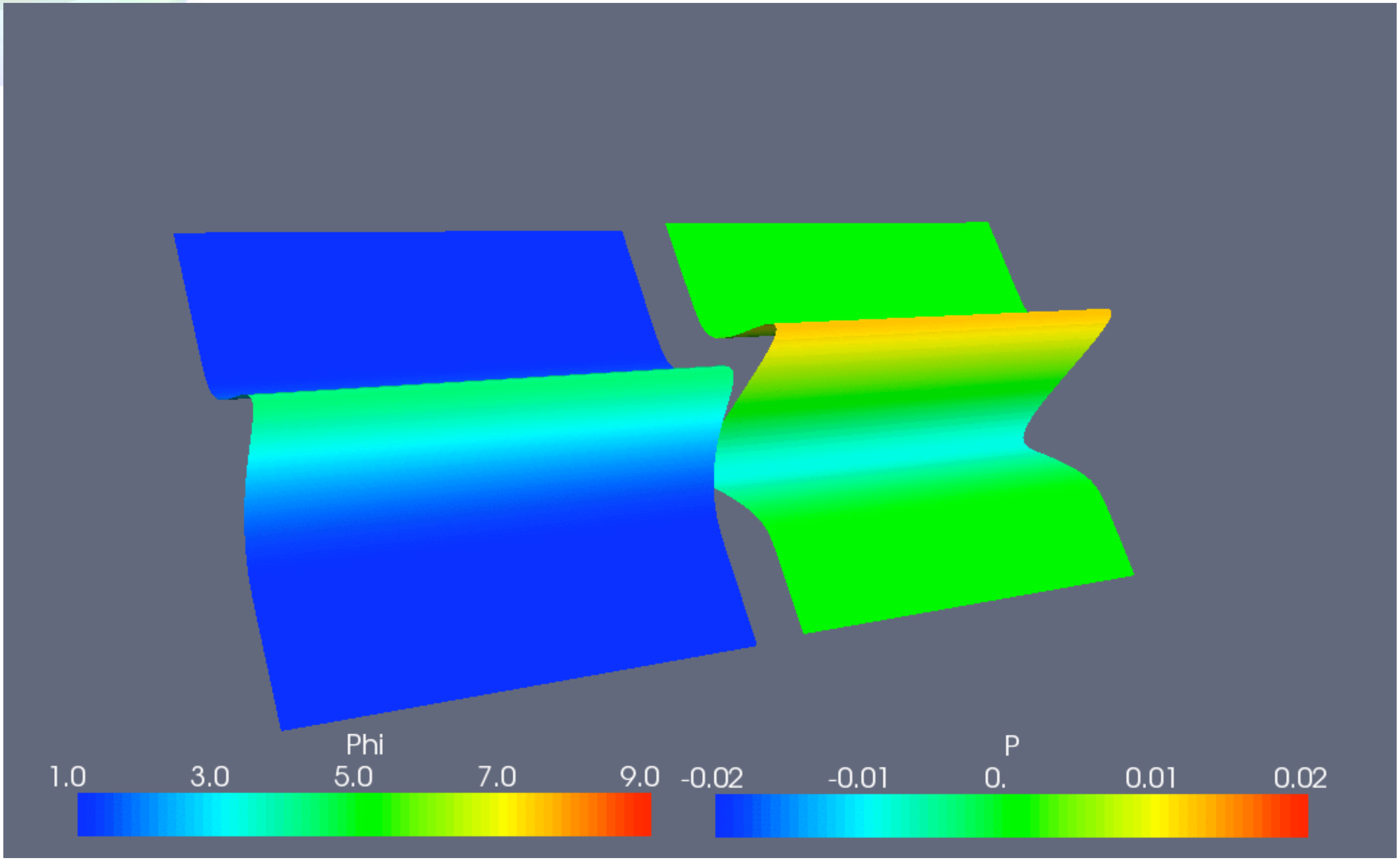
FEniCS/PETSc hybrid

- Developed by Marc Spiegelman
- Automated code generation (FFC/Dolfin) and PDE-based block-preconditioners (PETSc)
- MaDDS 1, 2, 4 done; Preliminary MaDDS-5



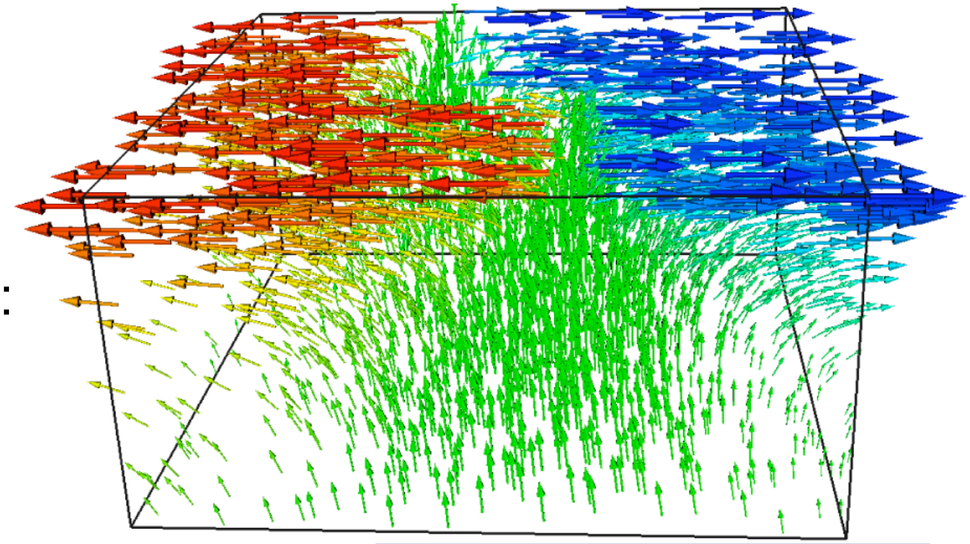
MaDDS-2 with FEniCS/PETSc
_ Stokes flow with corner flow boundary conditions
_ Melt trajectory for constant porosity

MaDDS-4: Solitary wave (FEniCS)

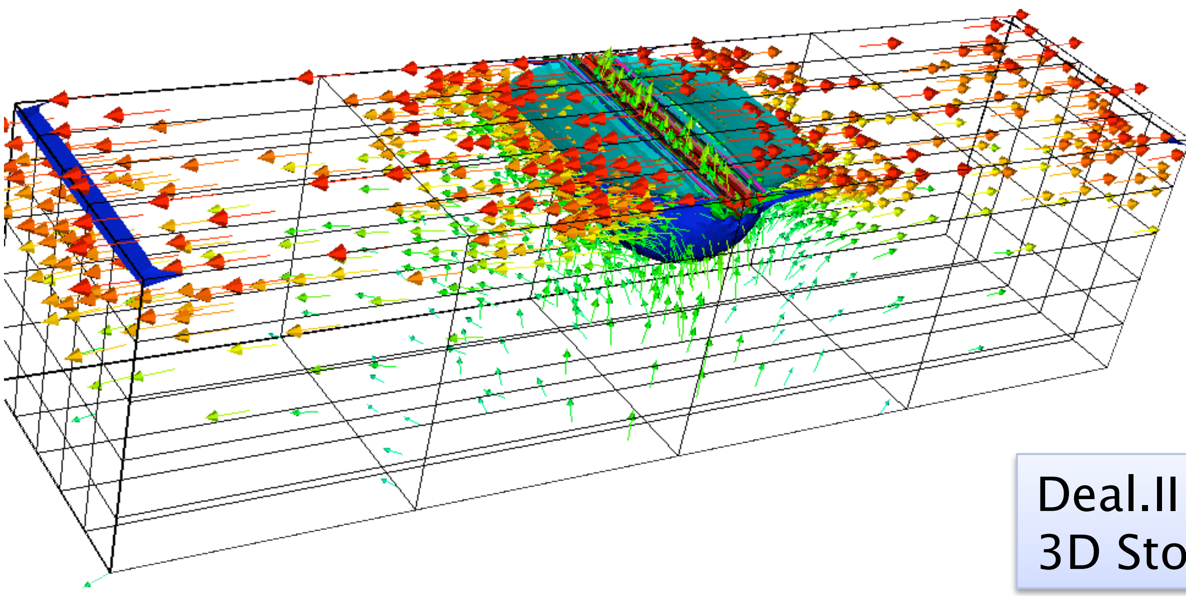


Adaptive refinement

- Deal.II (Bangerth)
  - Existing Darcy flow tutorials
  - Stokes flow tutorial complete: Step 22
  - Compaction equation under development



Stokes flow for segmented ridge with adaptive refinement (Burstedde et al.)



Deal.II tutorial step 22
3D Stokes flow underneath a ridge

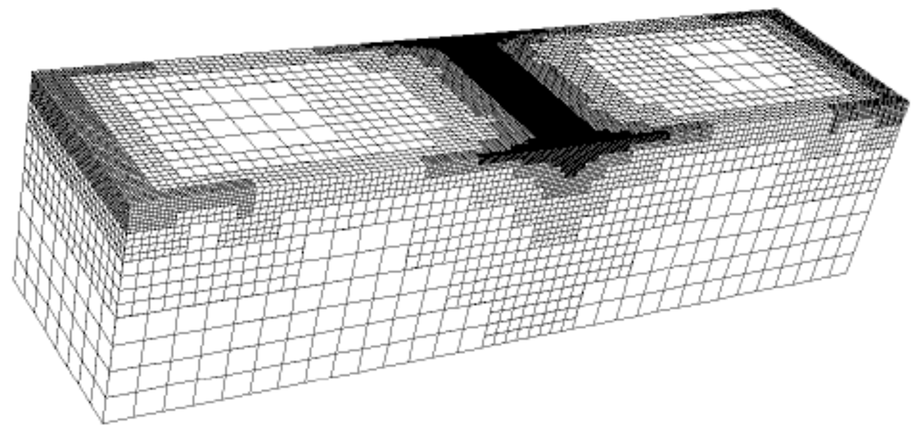
Summary / Evaluation

- Multiphysics is a challenge!
  - No ideal solution

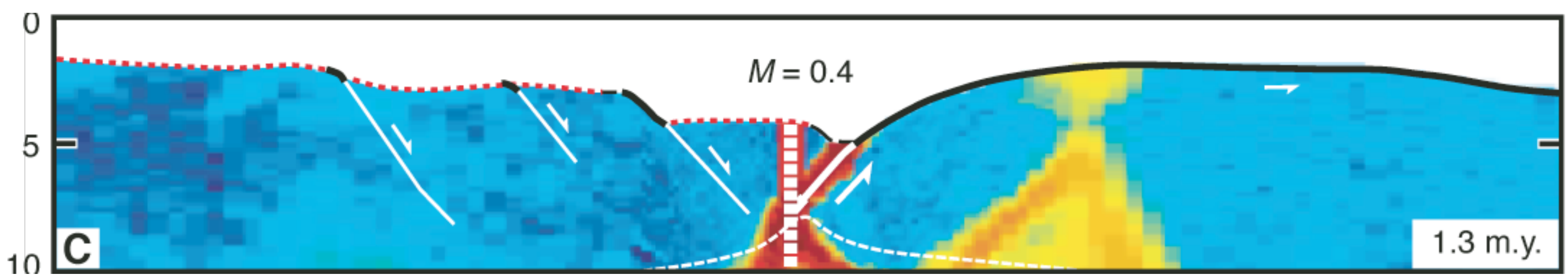
- StGermain
  - Flexible, but complex
- COMSOL Multiphysics™
  - Easy to use but impossible to modify
  - Limited to “small” problems
- FEniCS/PETSc
  - Newly adopted: Stay tuned!
- Deal.II
  - Flexible library
  - Excellent tutorials

Possible future goals

- Multiphysics coupling
  - Mantle convection
  - Tectonics
    - Rheology/failure
  - Computational thermodynamics
- Adaptive refinement



Deal.ii step 22 adaptively refined mesh (Bangerth, 2008)



Mid-ocean ridge tectonics influences by dike intrusion
Tucholke, Behn, Buck, 2008