Adaptive Mesh Refinement (AMR)

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Outline

- Motivation
- Types of adaptivity
- Parallel partitioning
- Space filling curves
- Forest of octrees
- Octree-based software
- Adaptivity and mantle convection
- Adaptivity and CIG

Promise of adaptivity

Invest computational work selectively
where it promises the highest gain in accuracy.

pros

- improve $\frac{\text{accuracy}}{\text{runtime}}$
- implement goal-oriented refinement
- mitigate curse of dimensionality
- turn intractable problems into tractable ones

cons

- non-trivial neighborhood relations
- non-trivial mesh partition
- non-trivial node ownership
- surplus of complexity costs development time
- surplus of topological operations costs runtime

Indications for adaptivity

Use adaptivity when error, energy, activity, ...
is distributed non-uniformly in space.

less likely

- when multiscale behaviour permeates the domain
- when activity spreads through the whole system

more likely

- when spatial resolution increases
- when physical heterogeneity increases
- when quantities of interest are localized

examples

– Turbulence
? Wave propagation
+ Mantle convection

Growth in computing power points towards the more likely regime?

Large scale adaptivity

static AMR (i.e., up-front adaptation)

- Mesh and parallel partition are known before program start
- Mesh setup can be precomputed and hand-optimized
- Cannot adapt to moving phenomena
- Change in setup can be costly

dynamic AMR (i.e., mesh changes over time)

- Additionally requires coarsening capability
- Mesh adaptation is integral part of the code
- Mesh adaptation occurs frequently
- Parallel repartition occurs frequently

> Adaptivity algorithms need to be at least as scalable as the numerical algorithms

Large scale adaptivity

mesh partitioning

- Each element is assigned to a unique processor core
- Distributed storage – no processor stores the whole mesh
- Numerical information exchanged between neighbor elements
- Connectivity information between elements known or stored

connectivity information – local

- Find neighbor elements
- Find owner processor of a neighbor
- Find degrees of freedom (DOF)

connectivity information – global

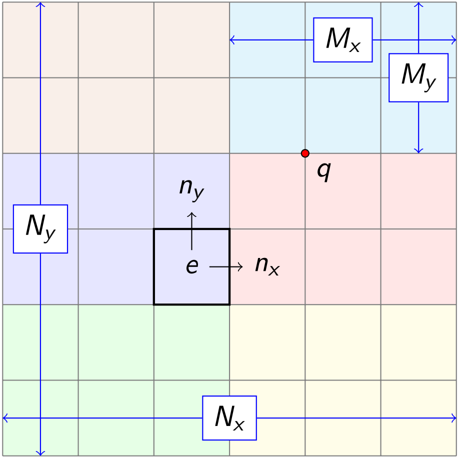
- Parallel partitioning – global redistribution of elements

Encode connectivity information with minimal storage

Types of adaptivity differ in choice of encoding

Types of adaptivity

local information (uniform mesh)



find neighbors

$$\#n_x = \#e + 1$$
$$\#n_y = \#e + N_x$$

find owner

$$P(e) = \left\lfloor \frac{e.x + N_x \lfloor \frac{e.y}{M_y} \rfloor}{M_x} \right\rfloor$$

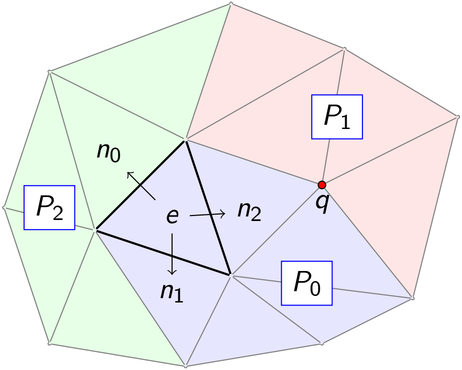
find DOF

$$\#q = q.x + (N_x + 1)q.y$$

global shared information: 4 integers N_x, N_y, M_x, M_y

Types of adaptivity

local information (unstructured mesh)



find neighbors

store $e.n[3]$

find owner

store $e.P$

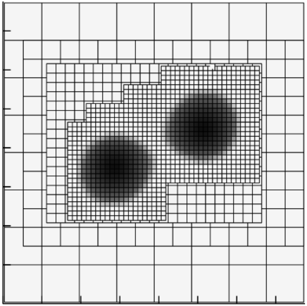
find DOF

store $\#q$

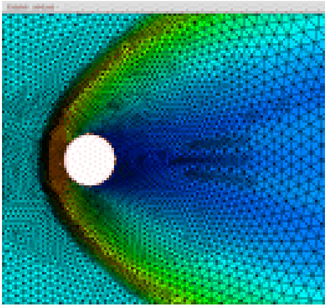
Loss of structure $\longrightarrow$ loss of information
Required information must be stored

Types of adaptivity

block-structured AMR [1,2]



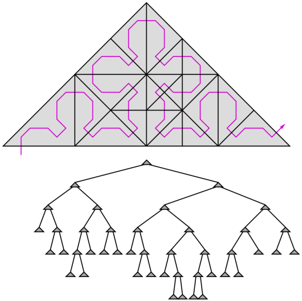
unstructured AMR [3]



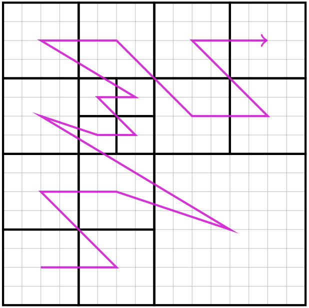
[1] E. Evans, S. Iyer, E. Schnetter et.al. 2005
[2] www.cactuscode.org
[3] www.openchannelfoundation.org/projects/Pyramid

Types of adaptivity

hierarchical triangles [1]



hierarchical hexahedra



[1] M. Bader, S. Schraufstetter, C.A. Vigh, J. Behrens 2008

Parallel partitioning

partitioning – global exchange of information

- Operation: redistribute elements among processors
- Objective: miminize overall run time

useful criteria?

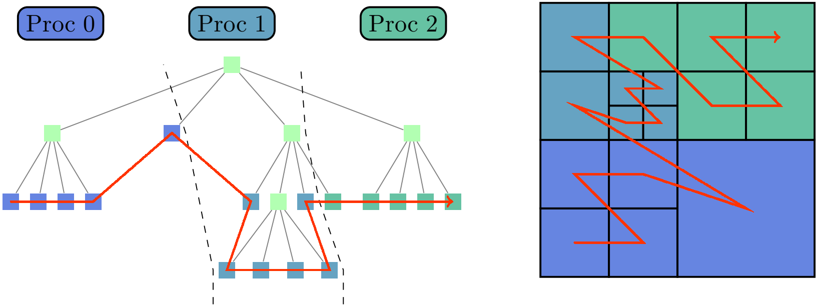
- Balance element counts between processors
- Balance node counts between processors
- Minimize number of neighbor processors
- Minimize number of elements on processor boundaries

tools available?

- Unstructured AMR: represent mesh connectivity as graph; use graph-partitioning heuristics (NP-hard), e.g. `Zoltan`
- Hierarchic AMR: space filling curves (SFC)

Parallel partitioning

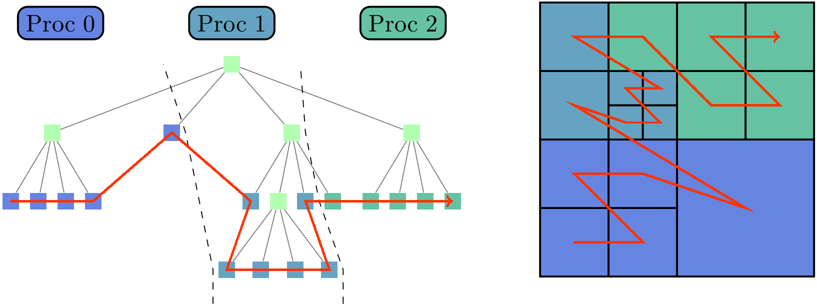
octrees and space filling curves (SFC)



- 1:1 relation between octree and SFC → efficient encoding
- Map a 1D curve into 2D or 3D space → total ordering
- Recursive self-similar structure → scale-free
- Tree leaf traversal → cache-friendly

Large scale adaptivity

octrees and space filling curves (SFC)

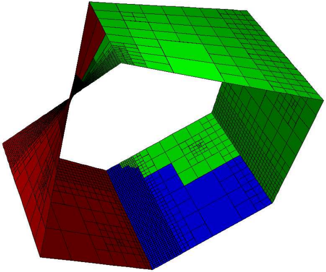


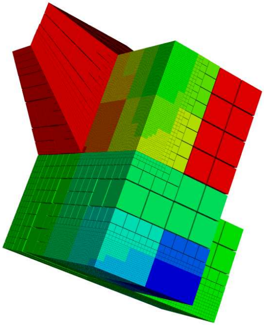
local information

- Find parent or children $\longrightarrow$ vertical tree step $\mathcal{O}(1)$
- Find on-processor neighbor $\longrightarrow$ tree search $\mathcal{O}(\log \frac{n}{p})$
- Find owner of off-processor neighbor $\longrightarrow$ binary search $\mathcal{O}(\log p)$

Forest of octrees

a conforming macro-mesh of adaptive octrees





- Connectivity between octrees is interpreted purely topological
- Any # of octrees (=≠ 4) can connect through common edge
- Any # of octrees (=≠ 8) can connect through common corner
- 2:1 balance condition across faces, edges and corners
  is honored within and between octrees (optional)

Octree-based parallel adaptive software

reinventing the wheel? (can be great fun! takes time though.)

- deal.II (W. Bangerth, R. Hartmann, G. Kanschat; general purpose)
- libMesh (G. Carey, D. Gaston, B. Kirk, J. Peterson, R. Stogner)
- AFEAPI (A. Patra et.al.)
- octor (T. Tu; closed source, pointer-based, ripple propagation)
- Dendro (R. Sampath; linear octree, insulation layers)
- `p4est` (C. Burstedde, L. C. Wilcox; forest of linear octrees)

Many of the headaches of parallel adaptivity
are happily encapsulated in a software library

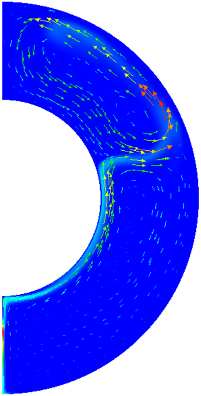
deal.II

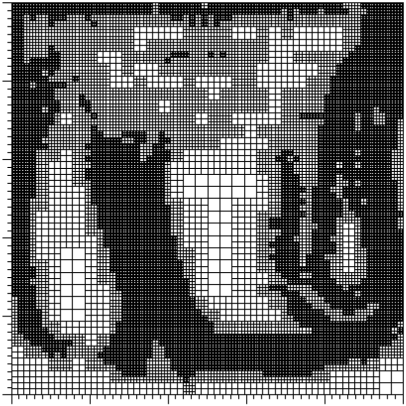
all-in-one finite element package

- Forest of octrees mesh topology
- Wealth of finite element spaces
- Problem assembly and linear algebra
- Direct and iterative numerical solvers
- Wealth of visualization formats
- Wealth of documentation
- Wealth of tutorials (including geodynamics!)
- Moderate parallelism ($\approx$ 100 processor cores)
- Directly available for download (`www.dealii.org`)

Get up and running quickly

fluid dynamics examples [1]





[1] W. Bangerth 2008

p4est

lightweight parallel adaptive mesh library (not a FE code!)

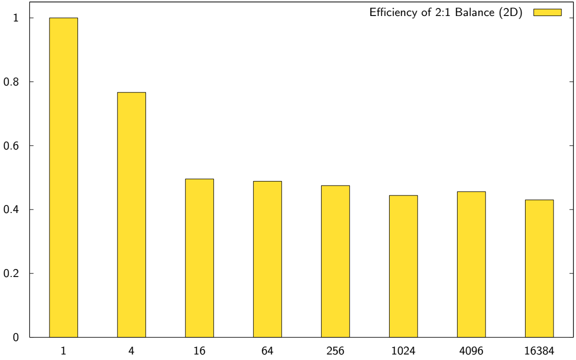
- Forest of octrees mesh topology
- Designed for uncompromised parallel scalability
- Almost arbitrary connectivity and periodicity of the domain
- Small memory footprint

local storage	24 bytes per element
global storage	32 bytes for each processor

- Ongoing work on integration into deal.II as mesh backend
- Ongoing work on generic FE mesh interface

2D scalings

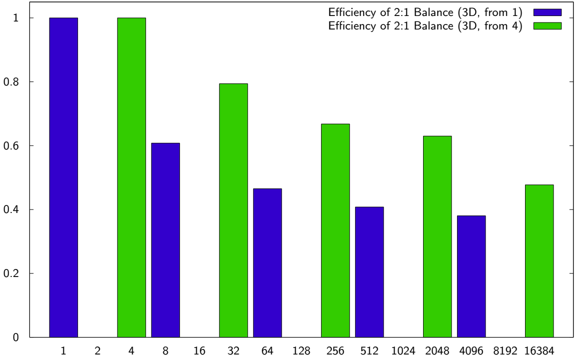
Star-shaped domain, 6 trees, parallel efficiency of 2:1 balance



Largest run: 6 trees, 32,768 cores, 74 billion elements

3D results

Spherical shell domain, 24 trees, parallel efficiency of 2:1 balance



Largest run: 24 trees, 62,464 cores, 256 billion elements

Adaptivity and mantle convection

the Rhea code [1,2]

- Global adaptive mantle convection simulation
- Continuous trilinear elements for both velocity and pressure
- AMG preconditioned MINRES iterations for Stokes
- SUPG predictor-corrector time integration
- Spherical shell resolved with 24 octrees by `p4est`
- Scaled up to 16,384 cores on TACC/Ranger

[1] C. Burstedde, O. Ghattas, M. Gurnis, E. Tan,
G. Stadler, T. Tu, L. C. Wilcox, Z. Zhong 2008.
Finalist paper for the '08 Gordon Bell Prize
[2] ongoing work

Adaptivity and mantle convection

the Rhea code – multigrid scalings [1]

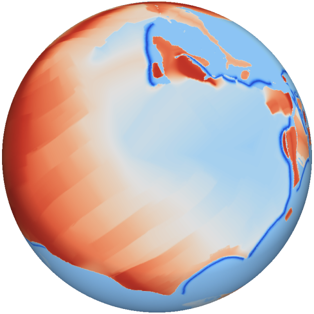
#cores	#dofs	MINRES #iter	AMG setup	AMG V-cycle
1	170K	66	1.45	18.06
8	1.1M	76	1.60	22.91
32	4.6M	88	2.22	33.20
128	17.9M	81	3.41	30.22
2,048	294M	63	15.12	70.53
16,384	2.35B	71	26.91	84.96

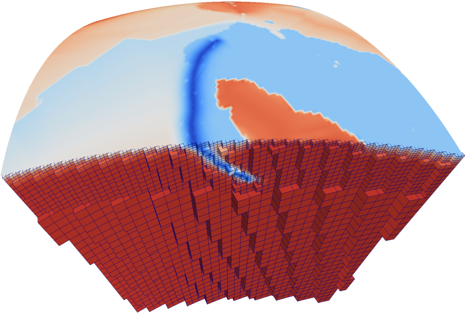
- Sum of setup and V-cycle times increase by a factor of 5.5
- All other FE computations scale roughly linearly

[1] With ML solver from Trilinos

Adaptivity and mantle convection

the Rhea code – present day slab dynamics [1]





[1] ongoing work with L. Alisic, O. Ghattas, M. Gurnis, G. Stadler, L. C. Wilcox

Adaptivity and CIG

high-level code

- deal.II works up to small computer clusters
- deal.II works for uniform and adaptive meshes
- Worth considering when starting a new high-level code

technology transfer

- `p4est` creates new scalable technology
- This technology propagates, e.g. into deal.II
- Whoever uses these codes will benefit without extra effort

special purpose codes

- Codes that bring their own FE logic could use `p4est`
- Mesh management would need to be separated
- Active collaboration with code development on both sides