Investigating Surface-wave Propagation Using Spectral Element Method

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Current Researches

1 Near-Field Surface-wave Sensitivity Kernels

In global seismic tomography, three-dimensional (3-D) surface-wave sensitivity kernels have been used to improve the resolution of lateral heterogeneities in the upper mantle. To date, Born sensitivity kernels formulated in the framework of surface-wave mode summation are based upon a far-field approximation which may not be valid in regions close to the receiver (or source), especially for long period surface waves. In this work, we go beyond the far-field approximation and compute the exact 3-D surface-wave sensitivity kernels based upon calculations of the exact Legendre function of fractional orders.

1.1 Exact Surface-wave Green tensor

In the frequency domain, the exact surface-wave Green tensor can be expressed as (Dahlen and Tromp,1998, Section 11.3):

$$\mathbf{G}_{\mathrm{rs}}(\omega)_{EXA} = \frac{1}{2}\sum_n \mathbf{D}_n\,\mathbf{D}_n'\,Q^{(1)}_{k_n-\frac{1}{2}}(\cos\Delta)\,e^{i(s-1)\pi/2} \tag{1}$$

Where $Q^{(1)}_{k_n-\frac{1}{2}}(\cos\Delta)$ is the traveling-wave Legendre function which is approximated by $Q^{(1)}_{k_n-\frac{1}{2}}(\cos\Delta)_{FFA}$ in the far-field Green tensor $\mathbf{G}_{\mathrm{rs}}(\omega)_{FFA}$. The difference between exact and approximated traveling-wave Legendre function $E_f(\Delta)$ is shown in Fig.1 (a). We define the critical distance Δ^* as the maximum distance where FFA breaks down(ie. $E_f(\Delta) \geqslant 10\%$).

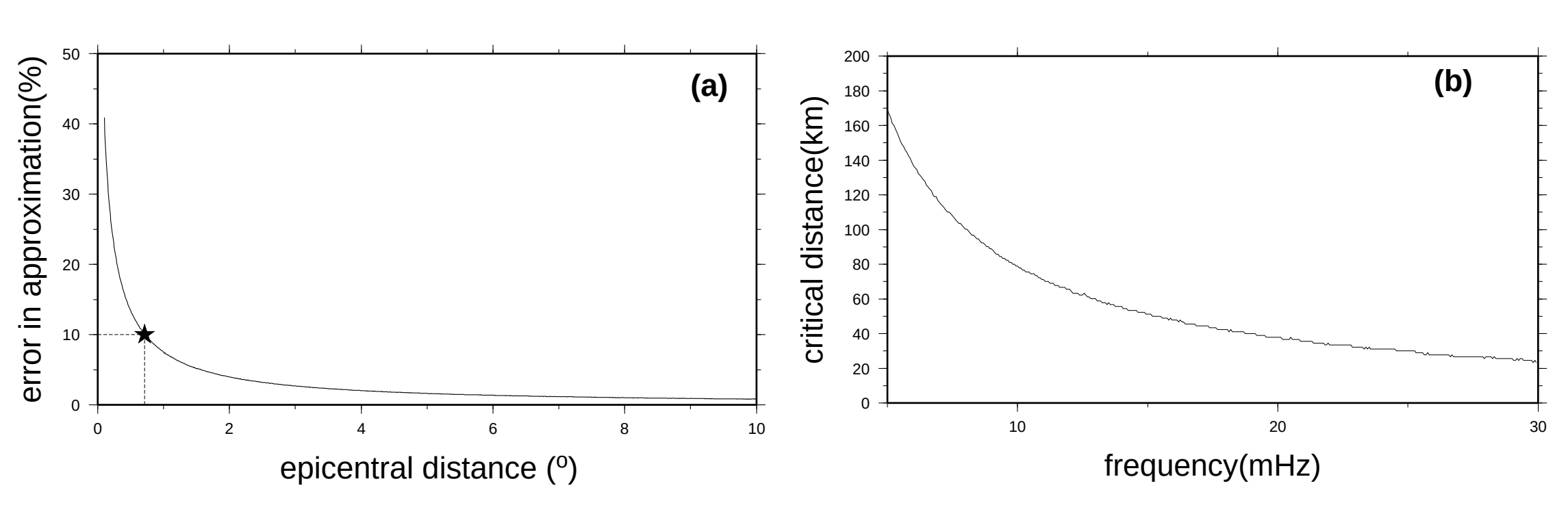

Figure 1: (a) Error in approximation $E_f(\Delta)$ for a 10 mHz Love wave; (b) Critical distance $\Delta^*(f)$ as a function of frequency at 5 mHz to 30 mHz

1.2 3-D Sensitivity Kernels for Inter-station differential Measurements

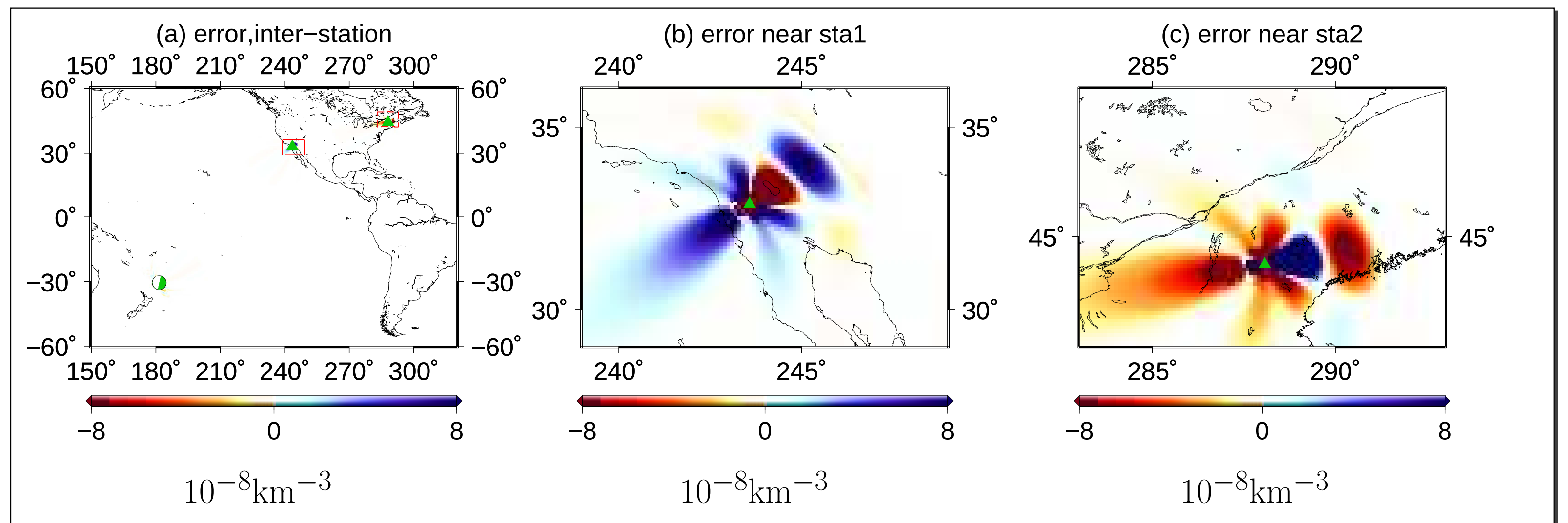

Figure 2: Comparison of exact and far-field approximated sensitivity kernels for inter-station differential measurements. All kernels are plotted at at the depth of 108 km for a 10mHz Love wave.

$$\delta\phi(\omega)_{st1-st2} = \delta\phi(\omega)_{st1} - \delta\phi(\omega)_{st2} = \iiint_{\oplus} K^m_\phi(\mathbf{x},\omega)_{st1-st2}\,\delta m(\mathbf{x})\,d^3\mathbf{x} \tag{2}$$

where $K^m_\phi(\mathbf{x},\omega)_{st1-st2} = K^m_\phi(\mathbf{x},\omega)_{st1} - K^m_\phi(\mathbf{x},\omega)_{st2}$ is the phase delay kernel for inter-station differential measurement. In Fig.2 we compare phase kernels for inter-station differential measurements with and without far-field approximation and conclude that the differences between the exact and far-field approximated kernels are significant only in regions close to the two stations, while they are not significant in regions close the source. In the future we will compare near-field kernels with kernels calculated by the adjoint method (*Tromp, et.al. 2005*).

2 The effects of 3-D Q structure on surface-wave phase delays

The Earth's anelasticity structure (Q) is important for understanding the thermal and compositional state of the mantle because anelasticity has strong sensitivity of to temperature and weak sensitivity to compositional variations. In present global anelasticity (Q) tomography practices, the effects of 3-D anelasticity effects on seismic travel time (phase delay) – 3-D anelastic dispersion- have been ignored. In this study, we quantify the effects of 3-D anelasticity on surface wave phase delays by simulating wave propagation in 3-D anelastic earth models using SEM.

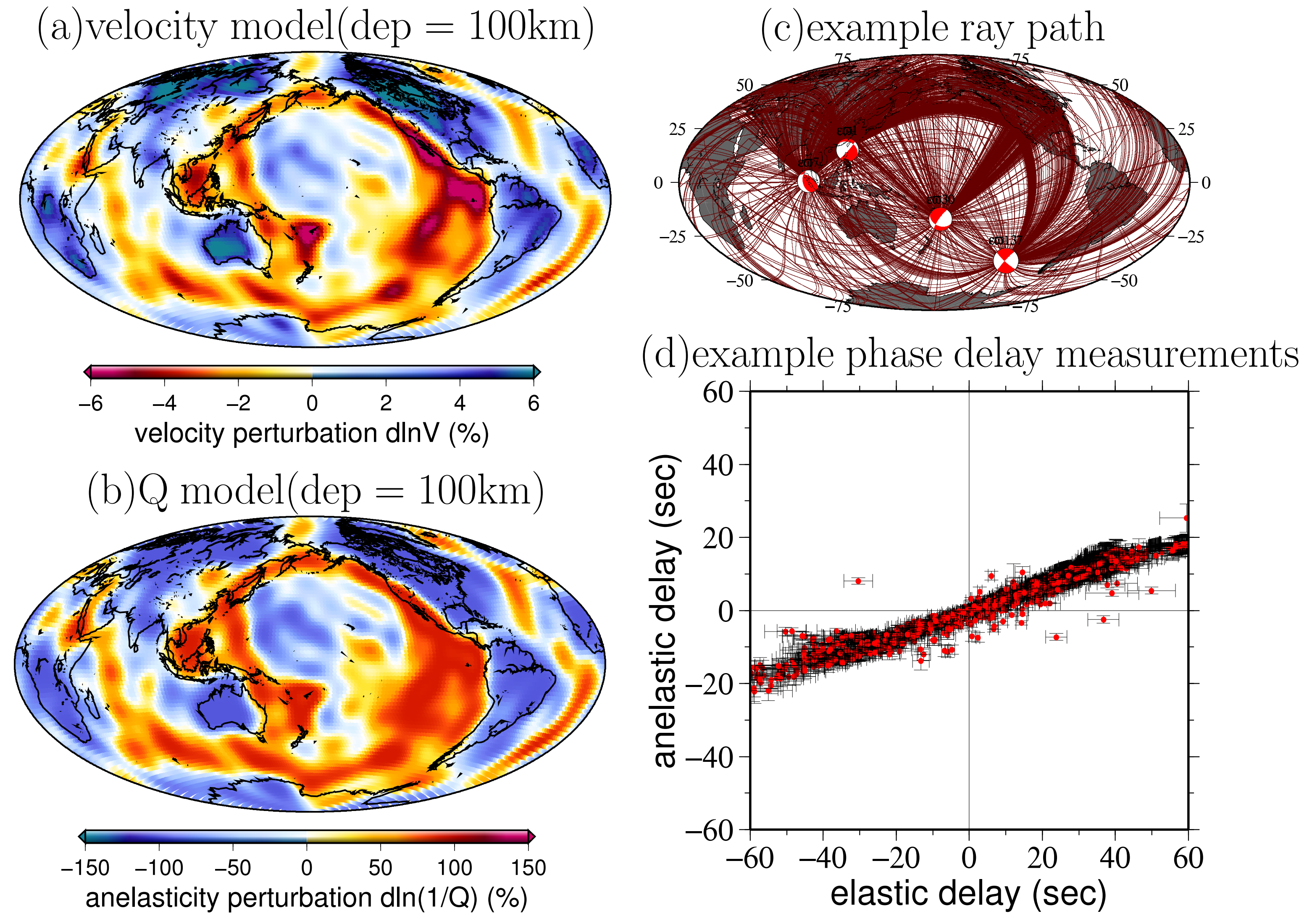

Figure 3: Examples of phase delays for 100s Rayleigh waves caused by 3-D velocity structure and 3-D Q structure from global SEM wave propagation simulations in realistic earth models.

2.1 Background Models

The dependence of anelasticity upon temperature can be expressed as the function of rheology parameters (Karato and Spetzler, 1990; Jackson, 2000):

$$Q(\omega,T) \quad = \quad A\,\omega^\alpha\,\exp(\alpha\,\frac{E^* + PV^*}{R\,T}). \tag{3}$$

E^* and V^* are activation energy and activation volume respectively, P is pressure, R is the gas constant and α is frequency dependence constant (ω^α) which varies from 0.1 to 0.4 (Shapiro & Ritzwoller; Karato).

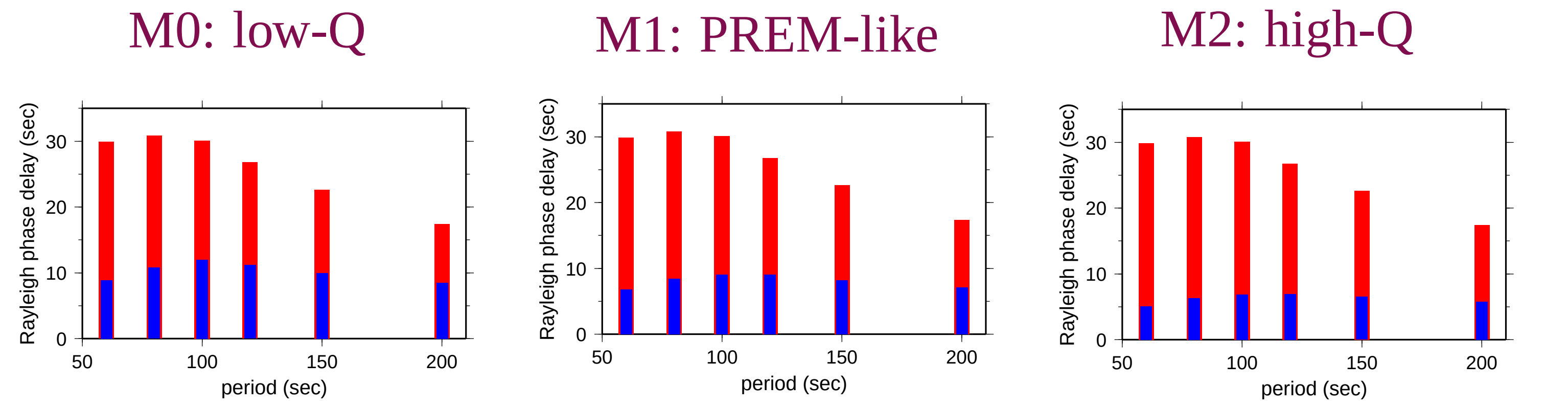

Figure 4: Measurements for M0 (lower background Q), M1 (moderate background Q) and M2 (higher background Q) respectively. Red bar present average elastic delays and blue bar means average anelastic delays. The contribution of 3-Q effects on surface wave phase delay varied from 30% for M0 to 20% for M3.

3 Conclusions

1. For inter-station differential measurements the differences between the exact and far-field approximated kernels are significant only in regions close to the two stations, while they are not significant in regions close the source.
2. 3-D anelastic effects on surface-wave phase delays are significant. Their contribution on phase delays (1) depends upon frequency of the wave and more prominent for lower frequencies; (2) depends upon background Q model, and vary from 30% for lower-than-PREM background Q to 20% for higher-than-PREM background Q.

Future works

It is known crust is the most heterogeneous part of the earth, and we shall take advantage of the spectral element method to investigate wave propagation in the crustal at a global scale. The current SEM software does not accurately account for the effects of the first-order discontinuity due to its meshing techniques. We will modify the meshing method and implement model based meshing strategy to handle the 2-D variation of crustal thickness. We expect this theoretical investigations to provide important guidelines for mapping global crustal structure using seismic surface waves.

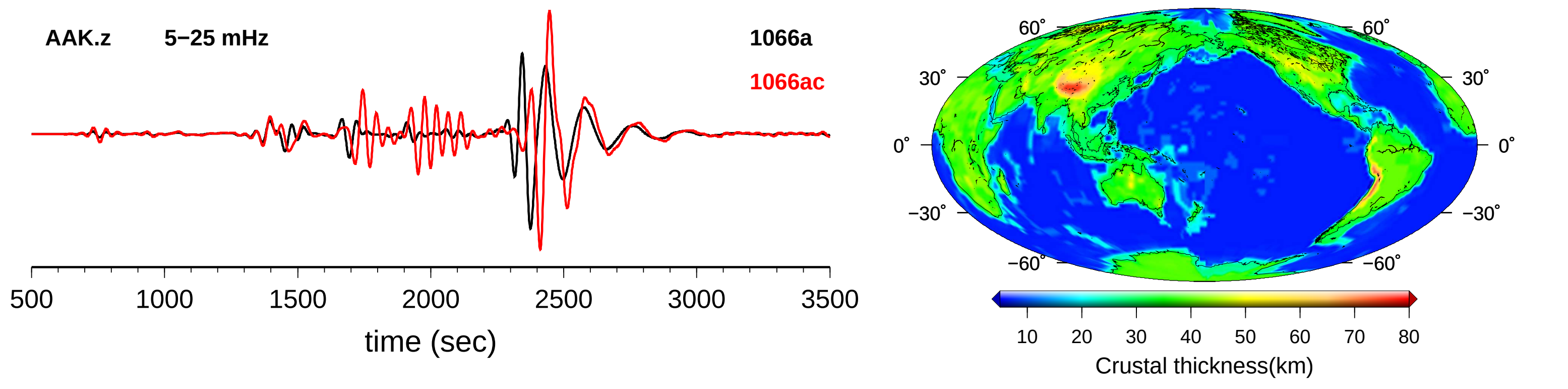

Figure 5: Example of the effects of variation in crustal thickness on the seismic wave propagation and crust 2.0 model.