Reservoir Simulation
Magma Dynamics

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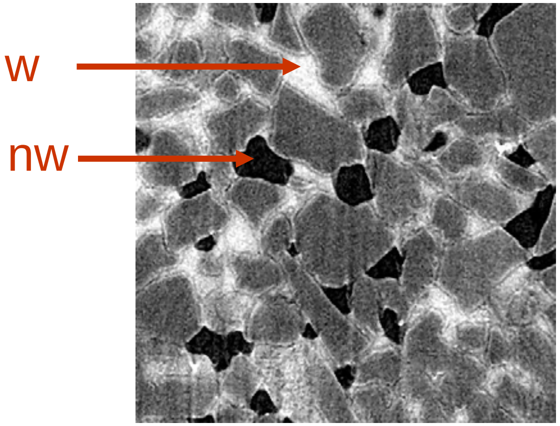
Outline

1. Introduction
   1. Reservoir Simulation (RS)
   2. RS vs. Magma Dynamics (MD)
2. Numerical Issues
   1. Permeability Heterogeneity
   2. Permeability Anisotropy
   3. Grid Adaptivity
   4. Multiscale Methods (MSM)
3. Summary

(High-Resolution/Order Schemes)

Idealized RS Equations I

matrix is rigid $\Rightarrow \phi = \phi(\vec{x})$, wetting & non-wetting phase



Benson et al. 06

saturations: $S_p = \varphi_p / \phi \quad S_w + S_{nw} = 1$

saturation eqn.: $\phi \frac{\partial S_w}{\partial t} + \nabla \cdot \vec{u}_w = \Gamma_w$

Darcy's law: $\vec{u}_p = -\lambda_p \left(\nabla p_p + \rho_p \vec{g} \right)$

mobility & rel. perm.: $\lambda_p = \frac{K(\vec{x})}{\mu_p} k_{r,p}(S_w) \qquad k_{r,p} = S_p^2$

capillary pressure: $P_c(S_w) = p_{nw} - p_w$

Idealized RS Equations II

assuming $P_c = 0$ and $\nabla \cdot \left(\vec{u}_w + \vec{u}_{nw}\right) = 0$ introducing

the fractional flow $f_w(S_w) = \dfrac{\vec{u}_w}{\vec{u}_T} = \dfrac{\lambda_w}{\lambda_T}\left[1 - \dfrac{k_{r,nw}}{\vec{u}_T} N_g \vec{n}_z\right]$

where $\lambda_T = \lambda_w + \lambda_n$ and $\vec{u}_T = \vec{u}_w + \vec{u}_{nw} = -\lambda_T \nabla p$

saturation equation: $\phi \dfrac{\partial S_w}{\partial t} + \vec{u}_T \cdot \nabla f_w = \Gamma_w$

pressure equation: $\nabla \cdot \left[\lambda_T(S)\nabla p\right] = \sum_p \left(\Gamma_p - N_g \vec{g} \nabla \cdot \lambda_p\right)$

Comparison RS vs. MD

	RS	MD
hyperbolic:	$\phi \frac{\partial S_w}{\partial t} + \vec{u}_T \cdot \nabla f_w(S_w) = \Gamma_w$	$\frac{\partial \phi}{\partial t} + \vec{v}_s \cdot \nabla \phi = \frac{P}{\xi} + \Gamma_w$
elliptic:	$\nabla \cdot [\lambda_T \nabla p] = R$	$\nabla \cdot (k_\phi \nabla P) - \frac{P}{\xi} = -\nabla k_\phi \cdot \vec{g}$

open problem: Dynamics of 2 fluids in compacting matrix

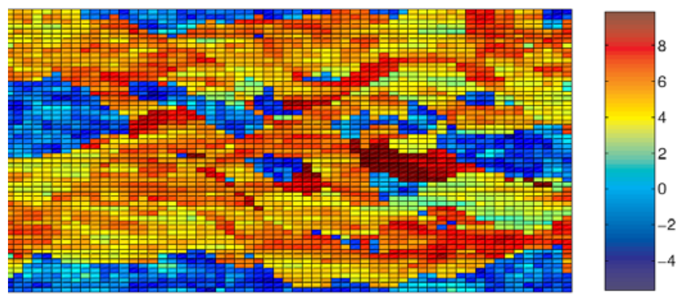
applicable to: 1) flux melting in subduction zones
2) hydrates in soft sediments
3) brine – hydrocarbon – salt dynamics

interesting maths problem: $\lim_{\xi \to 0} \Rightarrow$ quasi-linear hyperbolic system

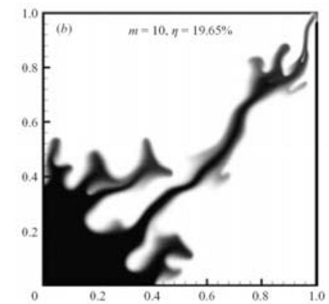
Heterogeneity in RS

pressure equation $\nabla \cdot \left(\lambda_T(x) \nabla p \right) = R$

Jasper National Park



bottom layer SPE 10 (Aarnes et al. 05)



Riaz & Meiburg 04

lithological heterogeneity: preexisisting

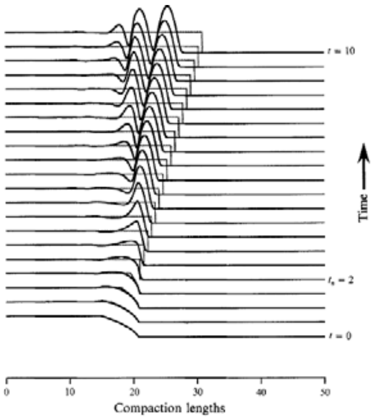
$\mathrm{Var}(\lambda_T) = 10^8$

dynamic heterogeneity: mobility (viscosity)

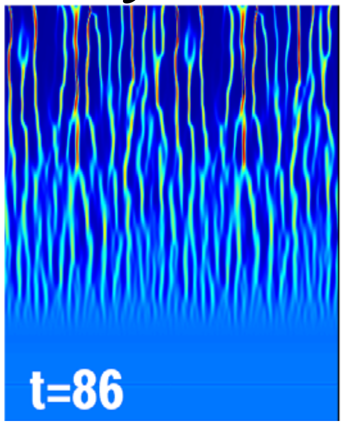
$\mathrm{Var}(\lambda_T) = 10^2$ viscous fingering

Heterogeneity in MD

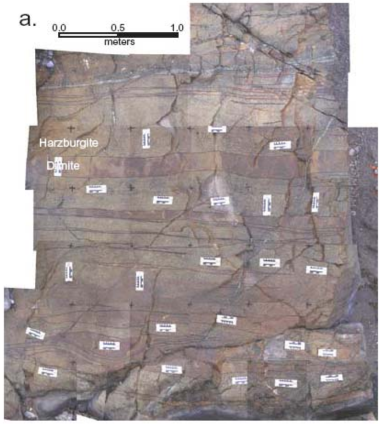
Helmholtz eqn: $\nabla \cdot \left(k_\phi \nabla P\right) - \frac{P}{\xi} = -\nabla k_\phi \cdot \vec{g}$



Spiegelman 93



Spiegelman et al. 01



Braun & Kelemen 02

lithological heterogeneity: preexisting ?, generated during melt extraction

dynamic heterogeneity: (de)compaction ⇒ solitary waves

reactive infiltration instability ⇒ channels

Heterogeneity in RS & MD

Reservoir Simulation

preexising het. dominant:

- ⇒ pervasive
- ⇒ all length scales
- ⇒ random component
- ⇒ long correlation length

essentially stationary

discontinuous

$Var(\lambda_T) = 10^8 \ (10^{12})$

Magma Dynamics

focus on dynamic het.:

- ⇒ localized to solution (solitons & channels)
- ⇒ dynamic length scales

transient

continuous & discont.

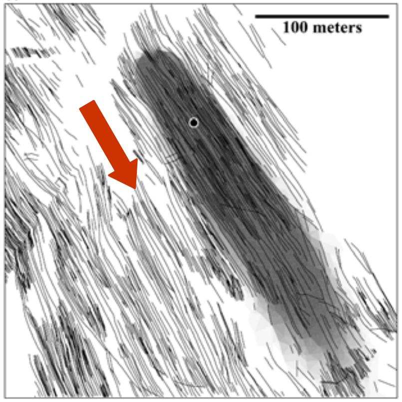
$Var(k_\phi) = 10^4$

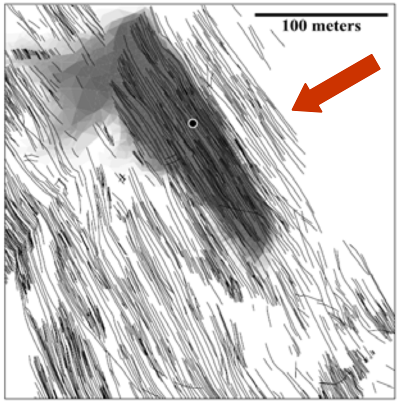
Is preexisting lithological heterogeneity important in MD?

Permeability Anisotropy in RS

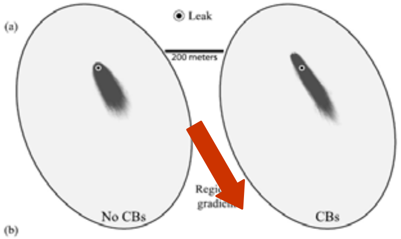
direct simulation:

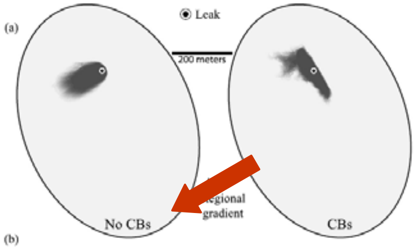
Sternloff et al. (2006)





reservoir scale:

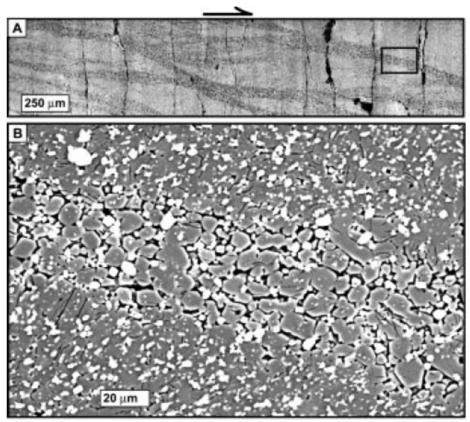




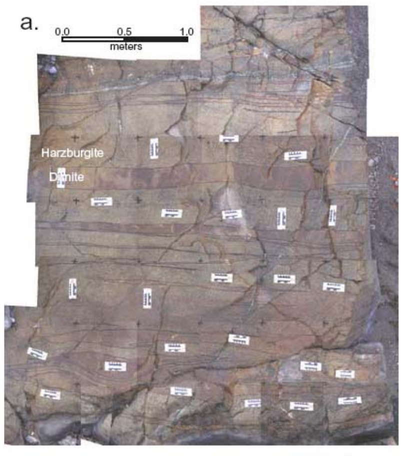
$\Rightarrow$ large natural anisotropies (1:1000)

$$K = \begin{bmatrix} k_{xx} & k_{xy} \\ k_{xy} & k_{yy} \end{bmatrix}$$

Origins of Anisotropy in MD



Holtzman et al. 03



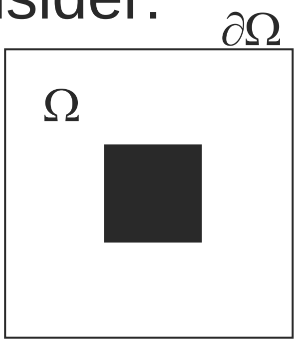
Braun & Kelemen 02

What is the magnitude of this anisotropy?

Anisotropy dynamically related to deformation of matrix!

Monotonicity & Anisotropy

consider:



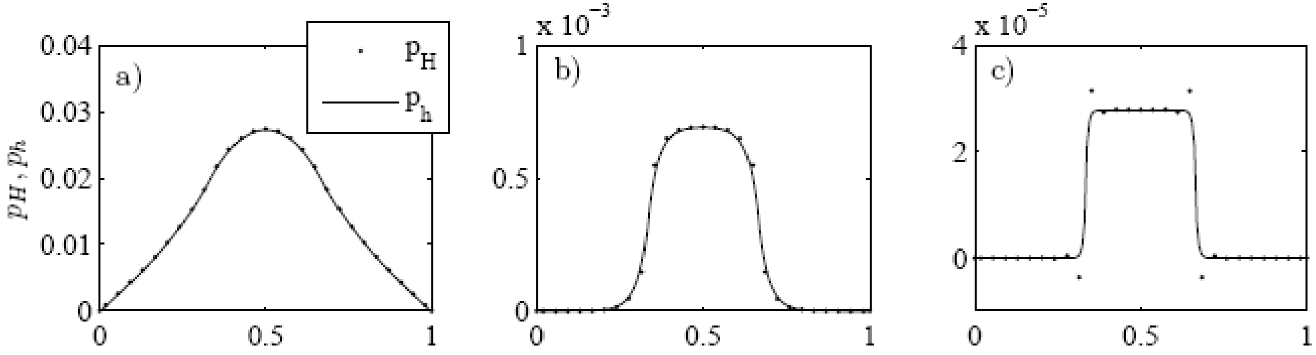
$$-\nabla\cdot\left(\lambda_T \nabla p\right) = R(x) \quad x\in\Omega$$

Hesse et al. 08

$$p(x) = 0 \quad x \in \partial\Omega$$

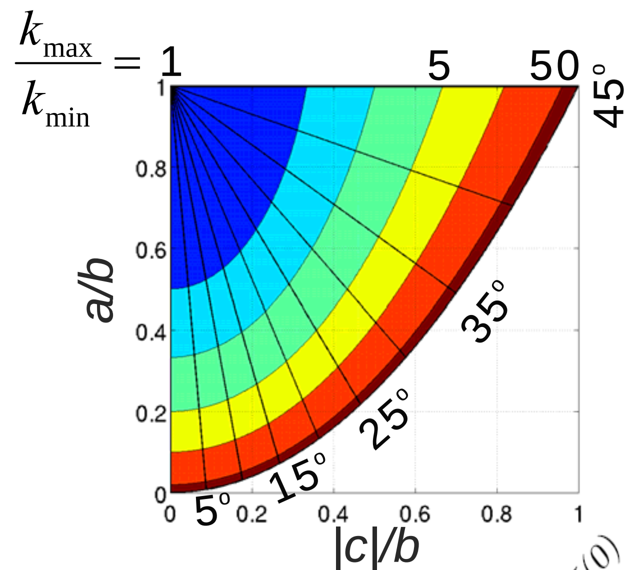
$$\lambda_T = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \qquad R(\vec{x}) = \begin{cases} 1 & x\in \blacksquare \\ 0 & x \in \square \end{cases}$$

Hopf's lemma: If $R(x)\geq 0$ in Ω, then p has no minimum in Ω.



b/a = 1 *b/a = 100* *b/a = 2500*

Discretization for Anisotropy

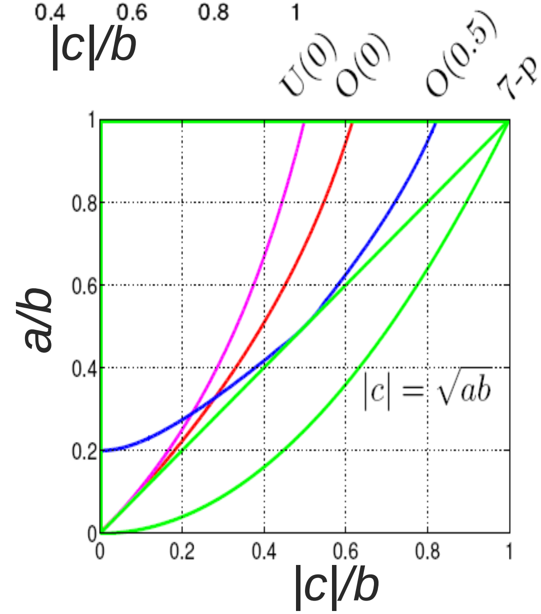


Nordbotten et al. 07

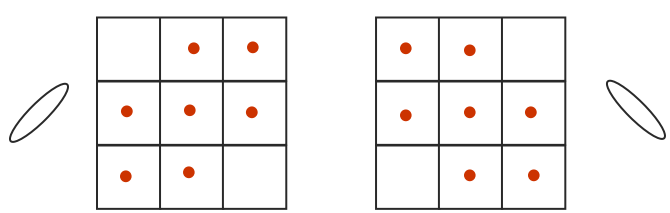
Discrete analog of Hopf's lemma conditions monotonicity of stencil

- quadrilateral grid
- conservative
- 9-point stencils
- 1^{st} order or higher

⇒ monotonicity regions



stencil with optimal monotonicity (7 pt)



What does this mean for MD?

Hopf's lemma $\Rightarrow$ identify numerical artifacts in Poisson eqn

Similar properties for Helmholtz eqn? $\Rightarrow$ more difficult

Sign change ($\phi < 0$) due to numerical artifacts $\Rightarrow$ blow up

Helmholtz eqn. more sensitive than Poisson eqn.?

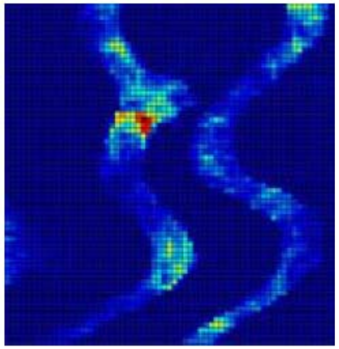
$\Rightarrow$ Caution is necessary with anisotropy!

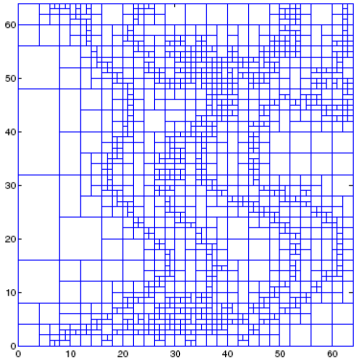
Use a stencil that is monotone for Poisson, hope for best!

Open research problem for a numerical analyst!

Adaptivity in Heterogenous Media

Gerritsen & Lambers 08



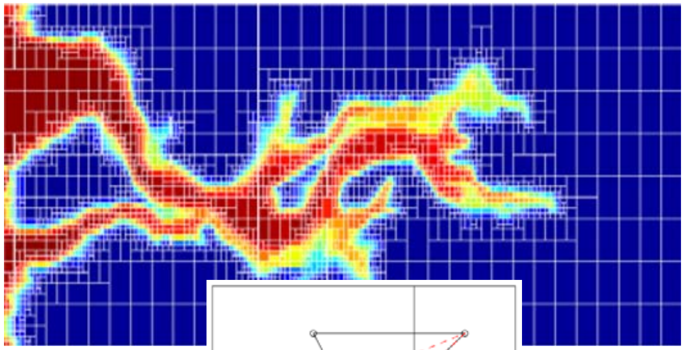


Cartesian Adaptive Refinement:

- data structures
  simple & efficient
- anisotropic refinement

1) static refinement (difficult)
   channels, wells, faults

2) dynamic refinement
   saturation fronts



Problems:

hanging nodes ⇒ unstructured method DG
effective parameters ⇒ multiscale methods

Adaptivity RS vs. MD

Reservoir Simulation	Magma Dynamics
heterogeneity:	
at all length scales strong random component	localized to solution solitons & channels
coarse operator:	
complicated $\Rightarrow$ multiscale	simpler
adaptivity criteria:	
complicated (edge detection)	simpler, based on solution
discretization:	
FV $\Rightarrow$ hanging nodes	unstructured DG, Spectr. Elem,

Adaptive gridding simpler and more promising for MD than RS.

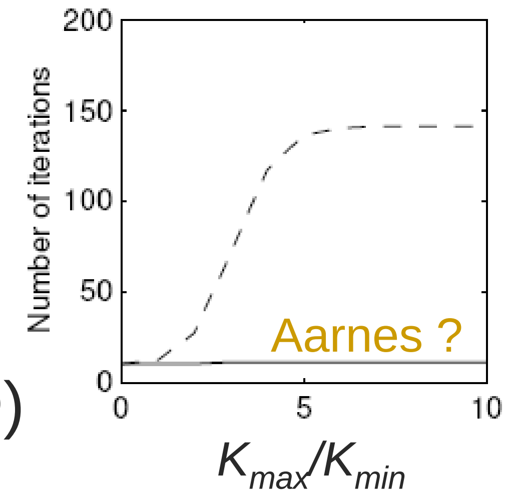
Multiscale Methods (MsM)

Poisson equation: $\nabla \cdot \left[\lambda_T(x)\nabla p\right] = R$

Solve eqn on coarse grid, and reconstruct fine solution.

Problems:

1) no scale separation
   $\Rightarrow$ sample entire field, O(N)
2) field is transient
   $\Rightarrow$ iterative solvers (MG, NODD)



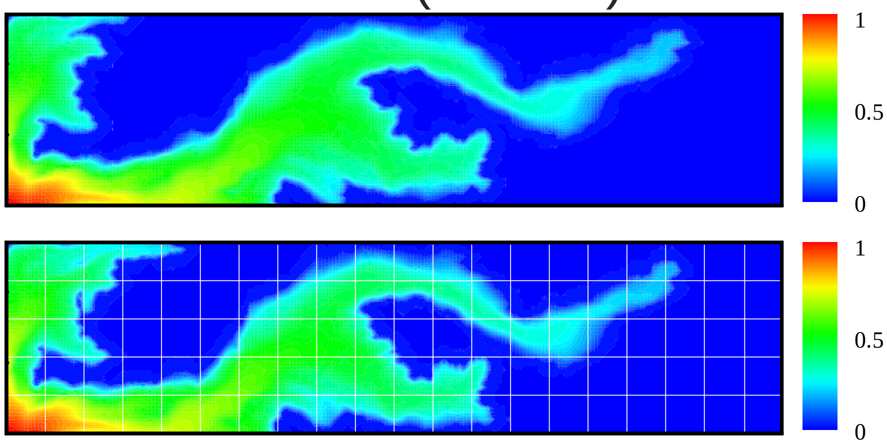
MG and DD: simple coarse operators and iterate

MS: complex coarse operator and don't iterate

Multiscale Methods (MsM)

Is error is acceptable?

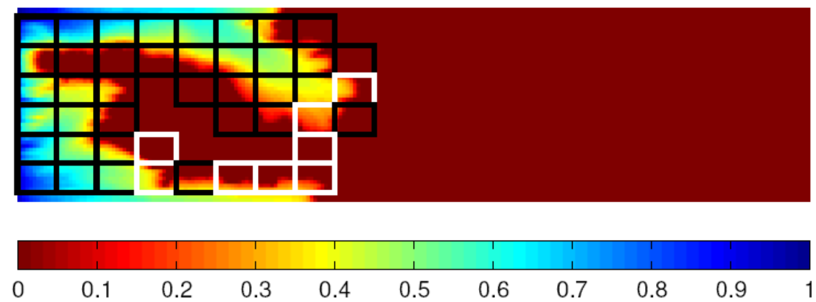
reference (220x55)



MSFV (20x5)

Hadjibeygi et al. 08

Is the cost competitive?



Result at $t = \tau_0$	
e_p	7.23e-5
e_s	3.55e-5
basis(%)	3.76
velocity(%)	4.20
transport(%)	15.55

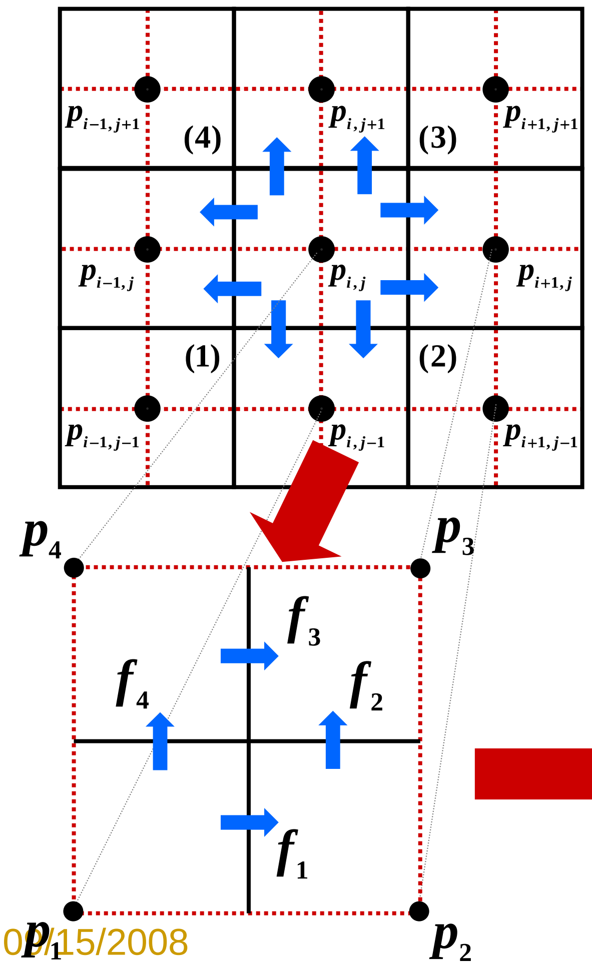
Zhou et al. (in prep.)

Yes, most of the time.

“Yes”, through adaptivity.

Finite Volume Formulation

Jenny et al. 03, Giniting 04, Hesse et al. 08



□ control volume

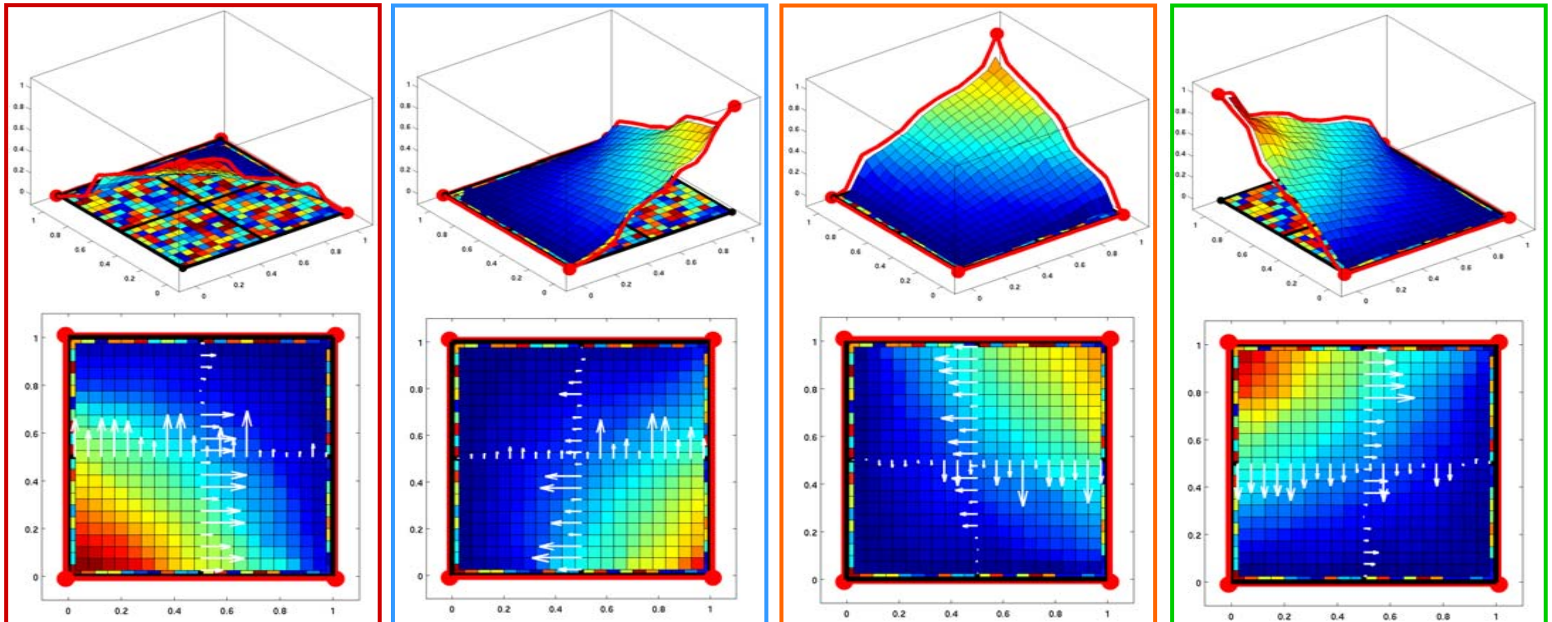
⬚ interaction region

$$L_h p_h = \begin{bmatrix} m_{-1,1} & m_{0,1} & m_{1,1} \\ m_{-1,0} & m_{0,0} & m_{1,0} \\ m_{-1,-1} & m_{0,-1} & m_{1,-1} \end{bmatrix}_h p_h$$

$$m_{1,0} = -t_{1,2}^{(3)} - t_{4,2}^{(3)} + t_{4,3}^{(2)} - t_{3,3}^{(2)}$$

$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} \end{pmatrix} \cdot \begin{pmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix}$$

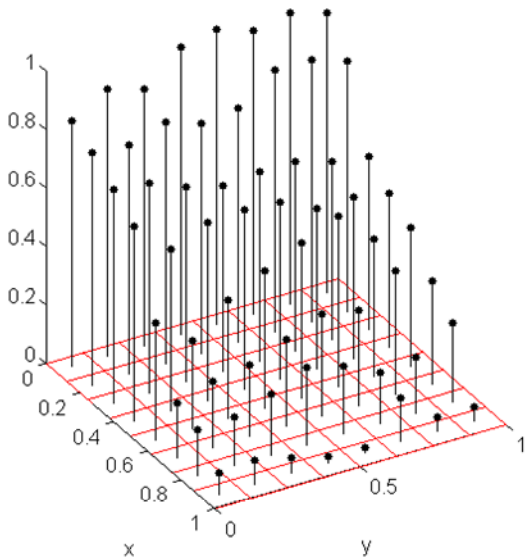
Multiscale Methods RS



$$\begin{pmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{pmatrix} = \begin{pmatrix} t_{1,1} & t_{1,2} & t_{1,3} & t_{1,4} \\ t_{2,1} & t_{2,2} & t_{2,3} & t_{2,4} \\ t_{3,1} & t_{3,2} & t_{3,3} & t_{3,4} \\ t_{4,1} & t_{4,2} & t_{4,3} & t_{4,4} \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

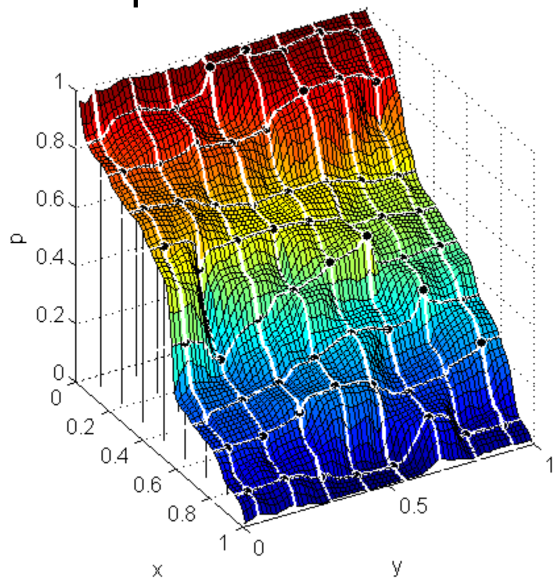
3 Pressure Fields

Coarse Pressure:



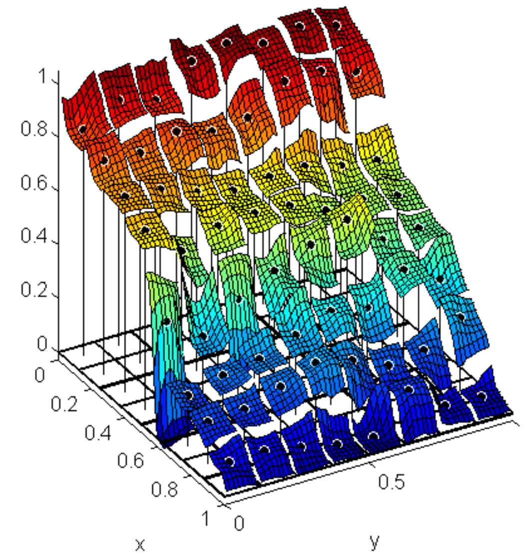
monotonicity not guaranteed

Interpolated Pressure:



fine *p* continuous
fine flux discontinuous

Conservative Pressure:



fine *p* discontinuous
fine flux continuous

continuous flux is essential ⇒ accept discontinuous pressure

Iterative MsM

Need error control for MS methods!

global information improve bc's basis functions

ALG (Chen 03) ALG-MsM (Durlofsky et al. 07)

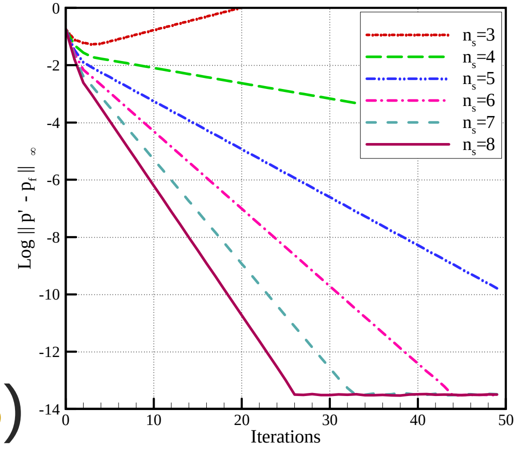
⇒ iterations, but not a solver

MsM extended to full solver:

Iterative MsM (Hadjibeygi et al. 08)

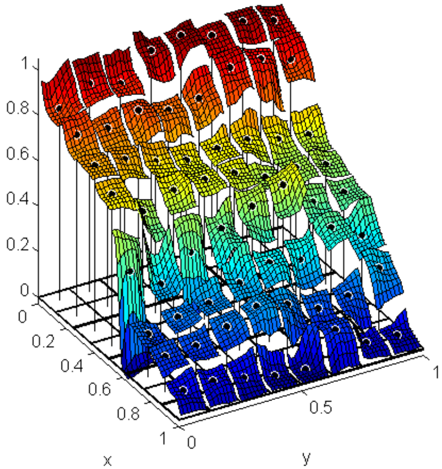
Solvers with Ms ideas:

NODD-MsM (Nordbotten & Bjorstad 08)



MS for MD

$$\nabla \cdot \left(k_\phi \nabla P\right) - \frac{P}{\xi} = -\nabla k_\phi \cdot \vec{g}$$



continuous flux field ∇P

discontinuous pressure P

$\Rightarrow$ instability

large perturbation of coefficients by dynamics

$\Rightarrow$ need to update often, expensive

Multiscale simpler and more promising for RS than MD.

Summary

1. Governing equations
   ⇒ Poisson vs. Helmholtz
   ⇒ Combine RS and MD to look at 3-pase flow

2. Nature of heterogeneity is fundamentally different
   ⇒ adaptive gridding in MD
   ⇒ multiscale methods in RS
   ⇒ higher order methods more impact in MD

3. Anisotropy MD can benefit from work done in RS
   ⇒ dynamic anisotropy in MD challenging & novel
   ⇒ discretizations for anisotropic Helmholtz eqns

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