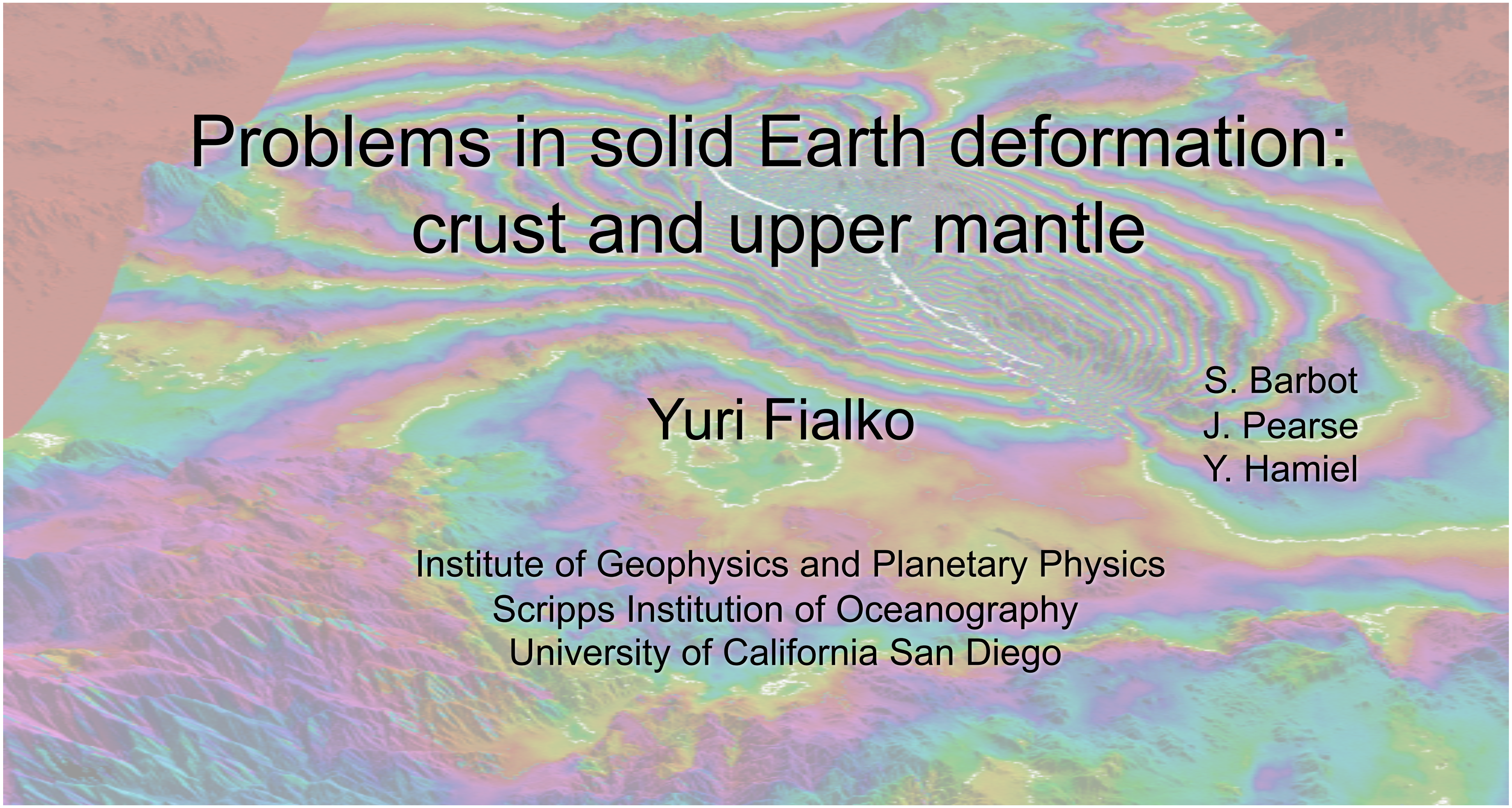

Problems in solid Earth deformation: crust and upper mantle

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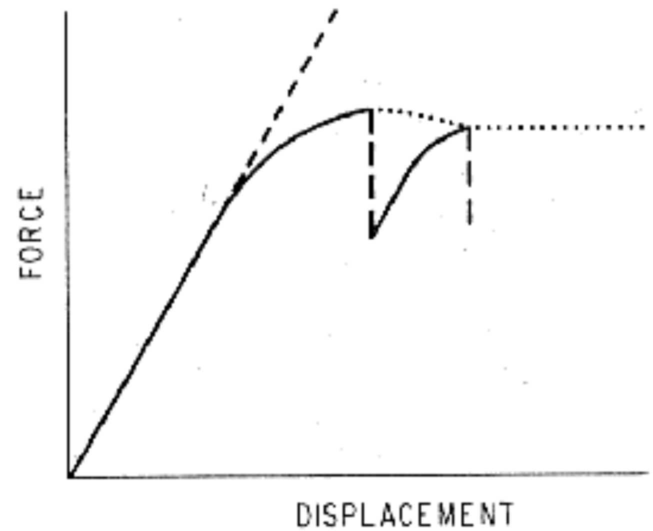
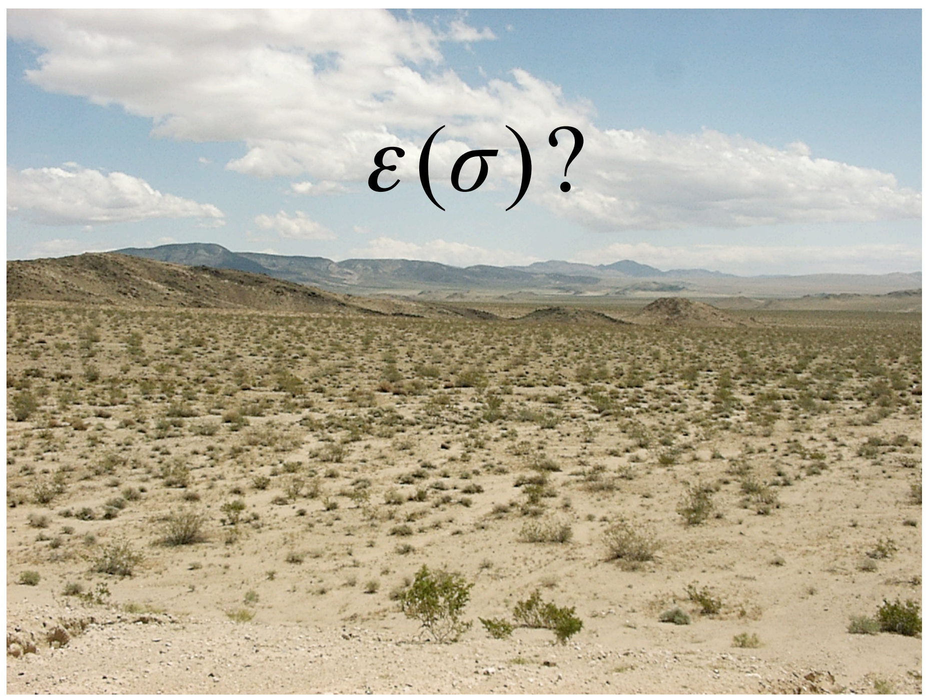
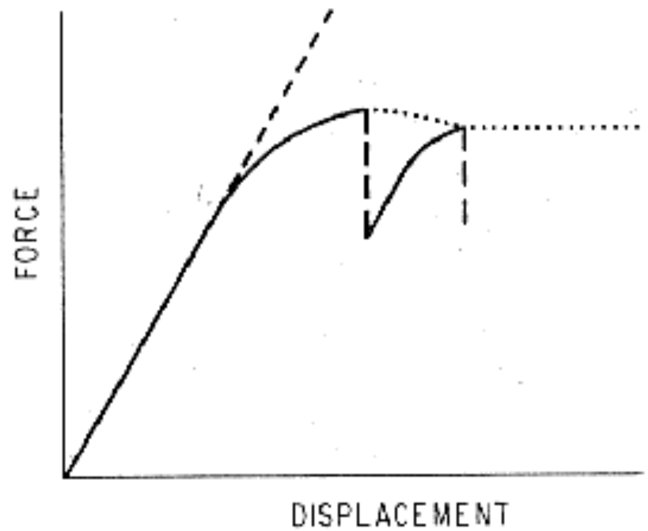
CIG, Sep. 16, 2008

Overview:

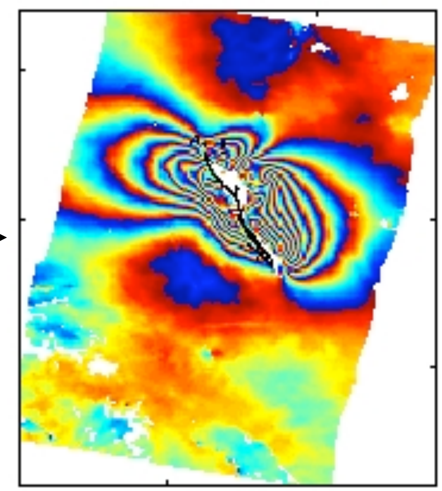
- Data-driven models
- Quest for “realistic” constitutive relationships for lithosphere, including upper (brittle) crust, lower crust, and upper mantle
- Data *require* material heterogeneity, non-linear rheologies, localization
  - models need to be sufficiently flexible to resolve multiple spatial and temporal scales
  - sufficient flexibility/efficiency in generating many realizations for inverse modeling
- Are FEM an ultimate answer?

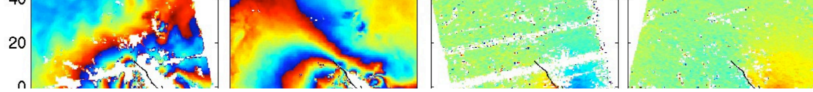

FORCE
DISPLACEMENT


$\varepsilon(\sigma)$?

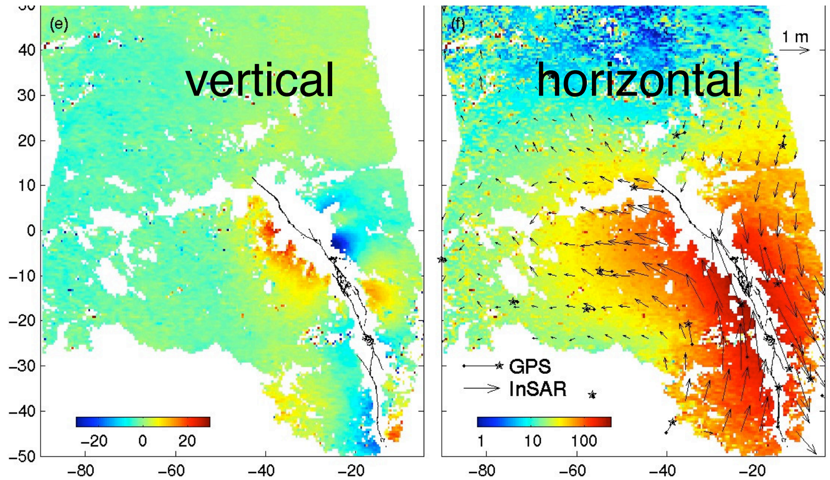


$$\varepsilon(t) = F(\sigma, E, t, D, \ldots)$$

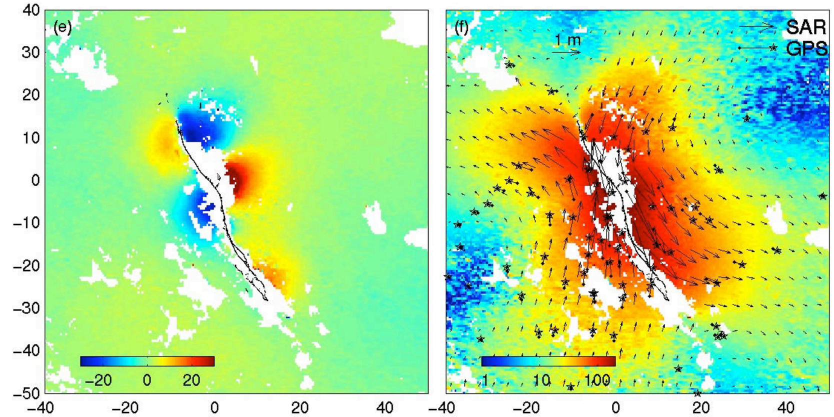




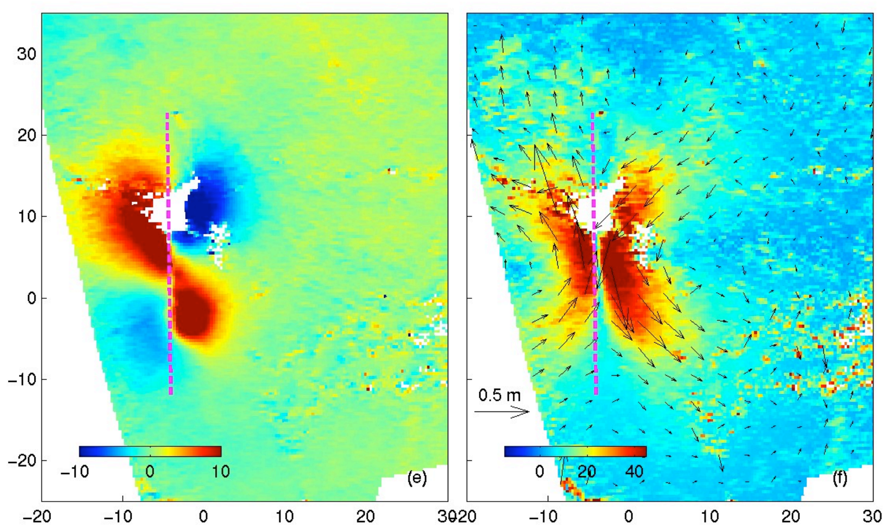
M7.3 Landers, 1992



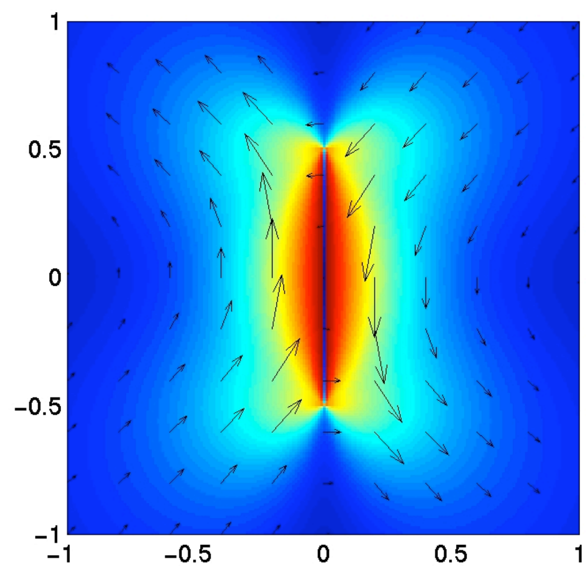
M7.1 Hector Mine, 1999

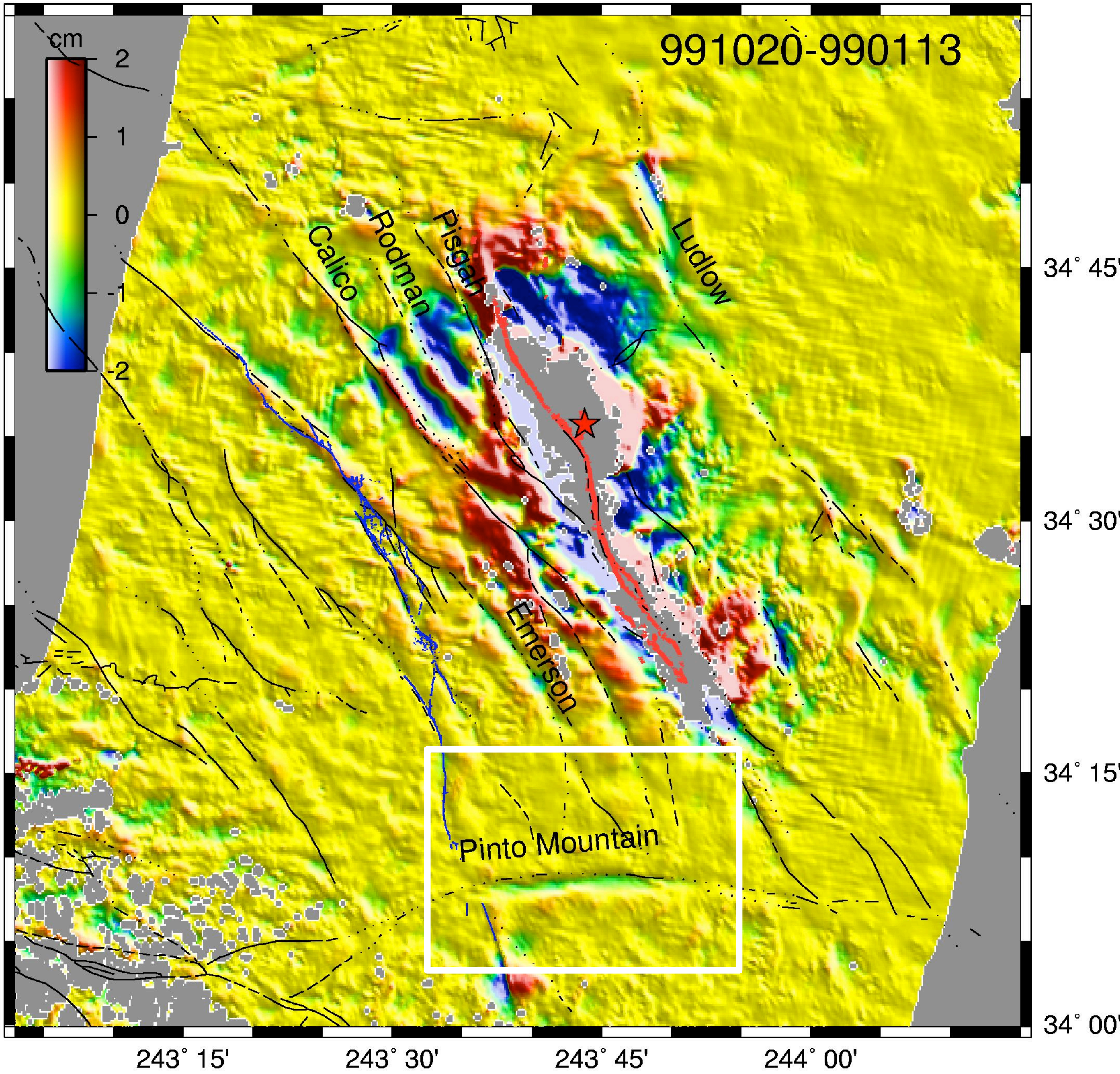


M6.6 Bam (Iran), 2003

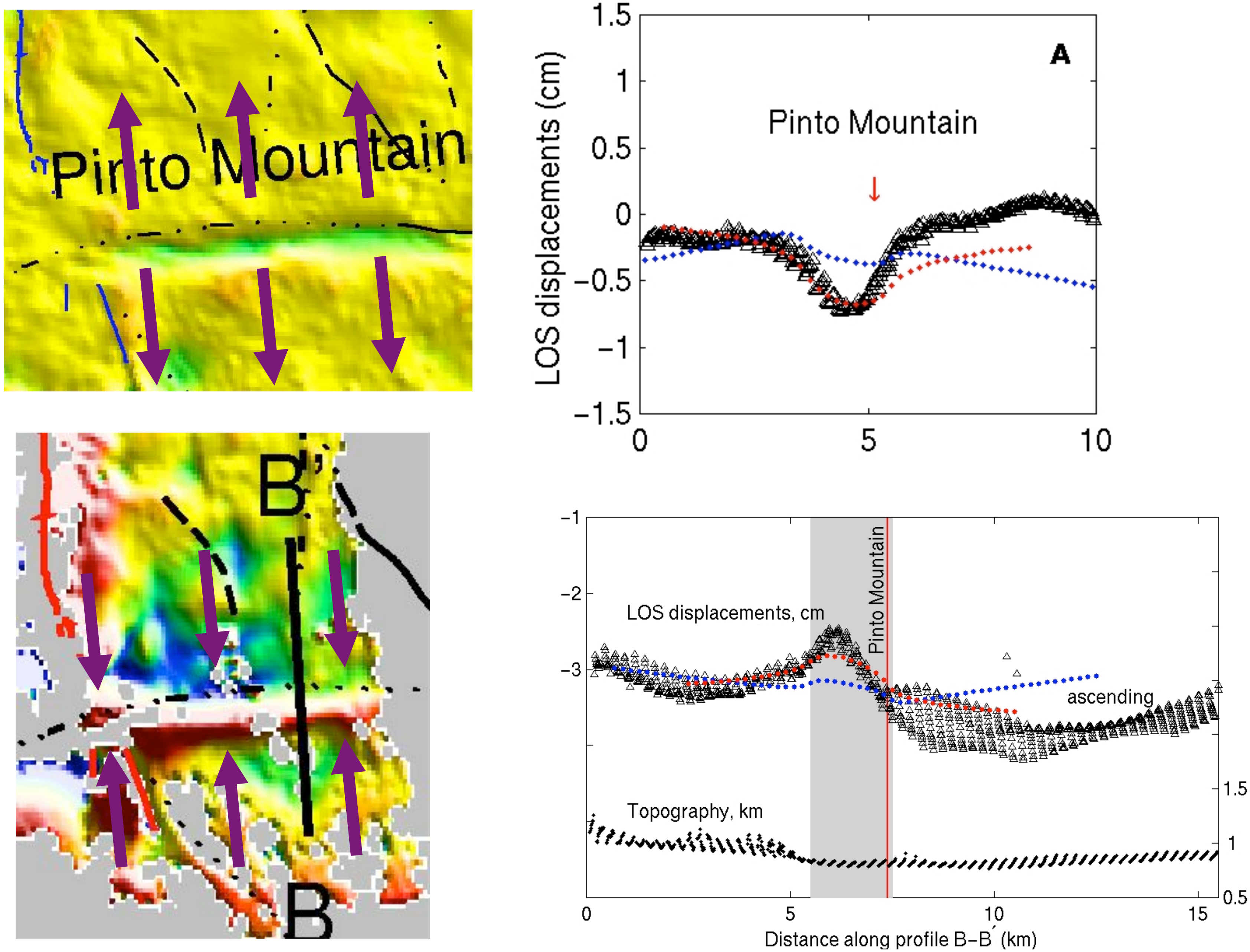


Dislocation



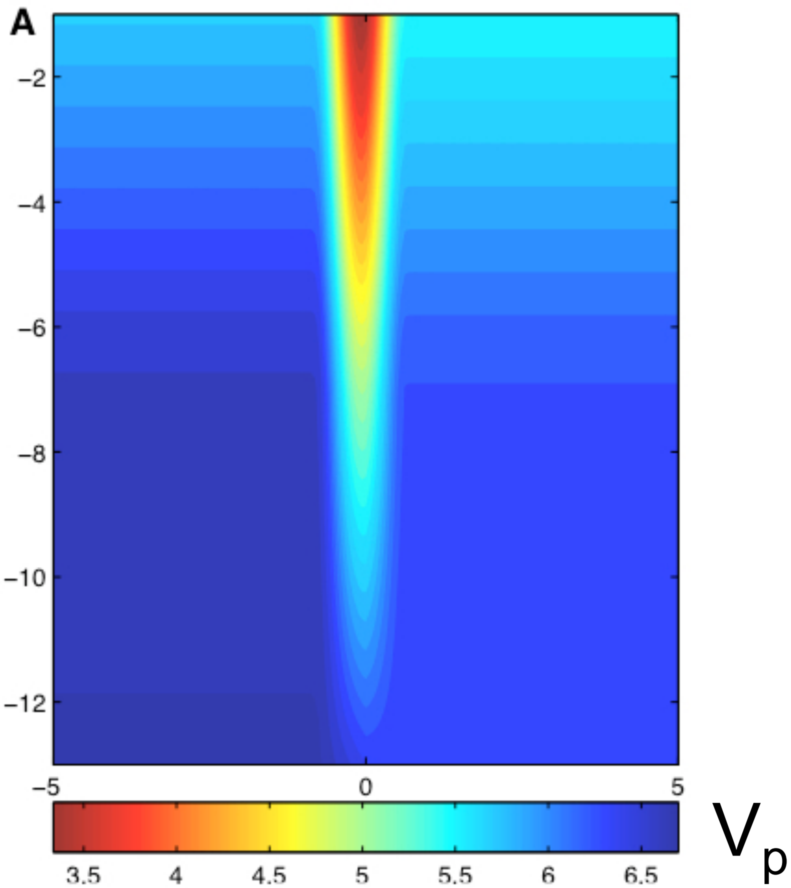


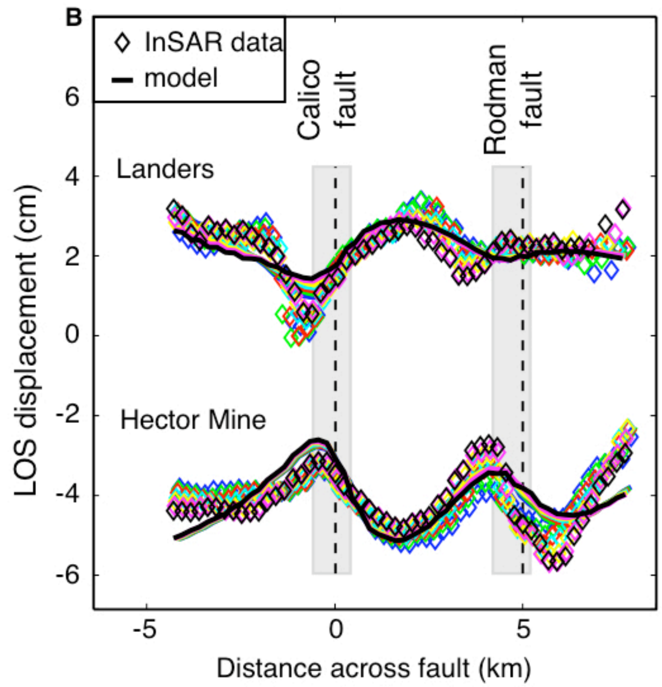
991020-990113
cm
2
1
0
-1
-2
Calico
Rodman
Pisgah
Ludlow
Emerson
Pinto Mountain
34˚ 45'
34˚ 30'
34˚ 15'
34˚ 00'
243˚ 15'
243˚ 30'
243˚ 45'
244˚ 00'



Pinto Mountain
B'
B
1.5
1
0.5
0
−0.5
−1
−1.5
LOS displacements (cm)
A
Pinto Mountain
0
5
10
−1
−2
−3
LOS displacements, cm
Pinto Mountain
ascending
Topography, km
1.5
1
0.5
0
5
10
15
Distance along profile B−B´ (km)

Calico fault





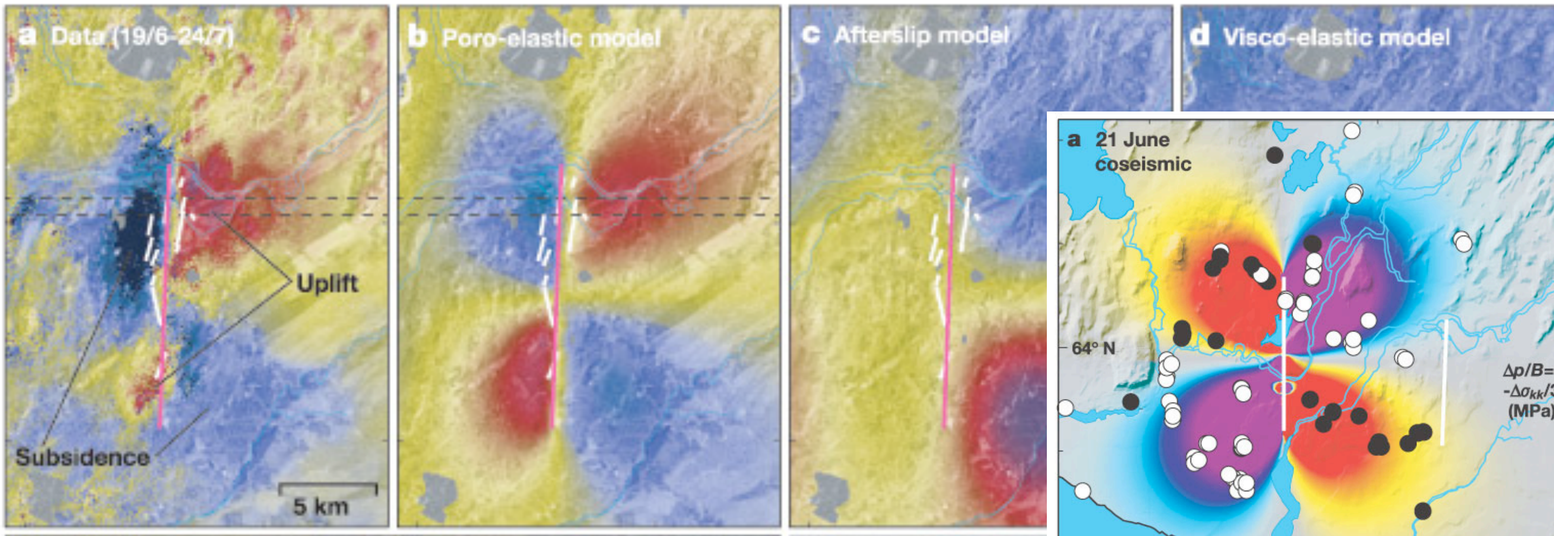
seismic tomograpy

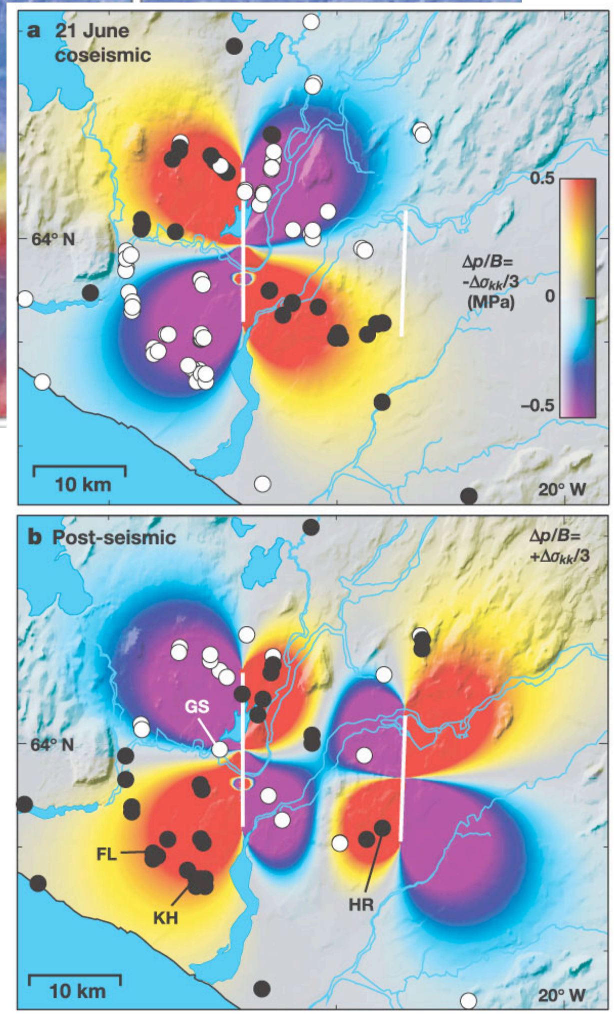
Time-dependent deformation following earthquakes: Common suspects

- Localized slip on or below the seismic rupture (“afterslip”)
- Visco-elastic relaxation (lower crust/upper mantle; various stress-strain relationships)
- Poro-elastic rebound (incapable of large horizontal displacements; mostly vertical deformation)
- …or a combination of the above

2000 M6 Iceland earthquakes

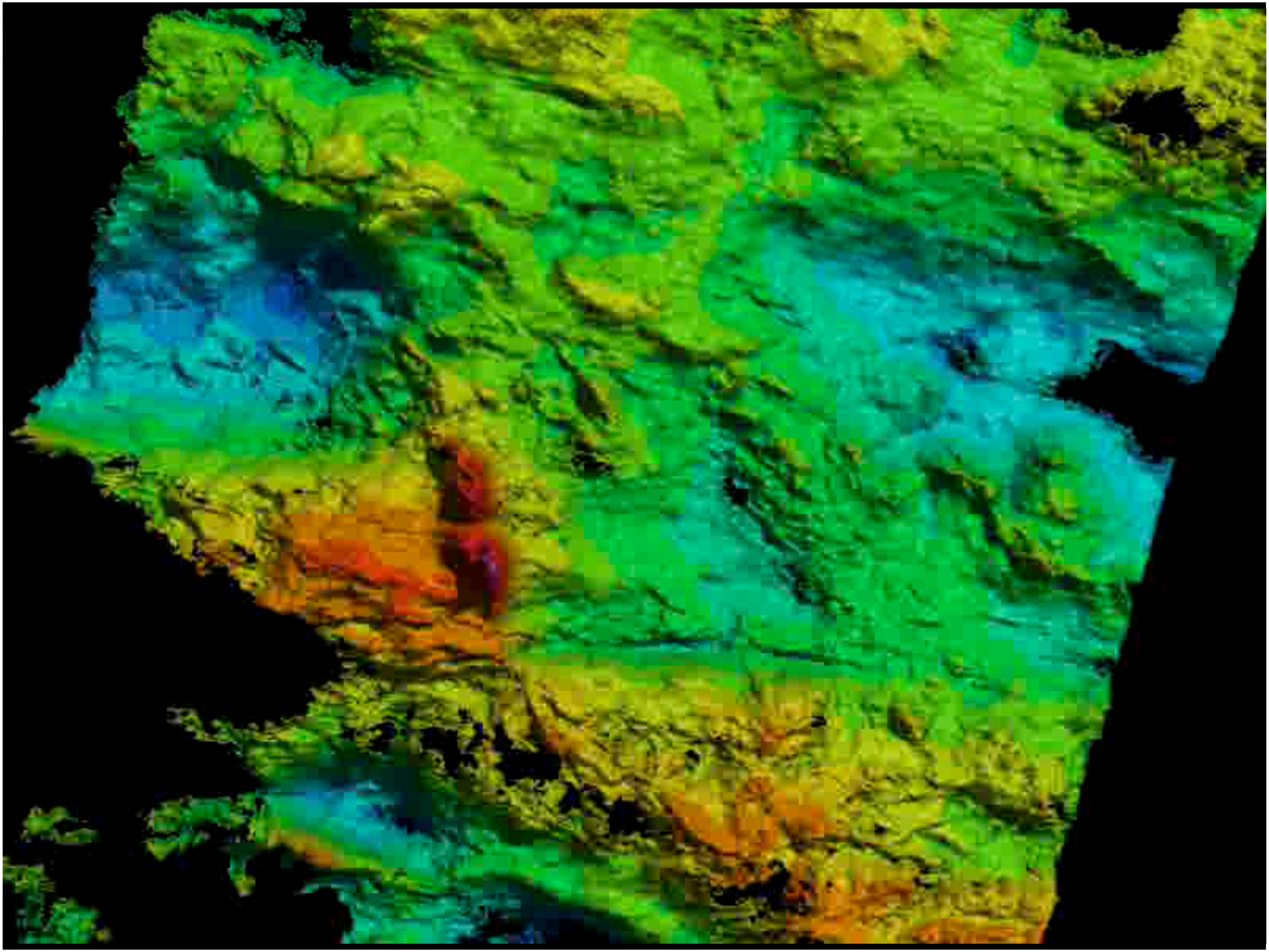
Jonsson et al., Nature 2003



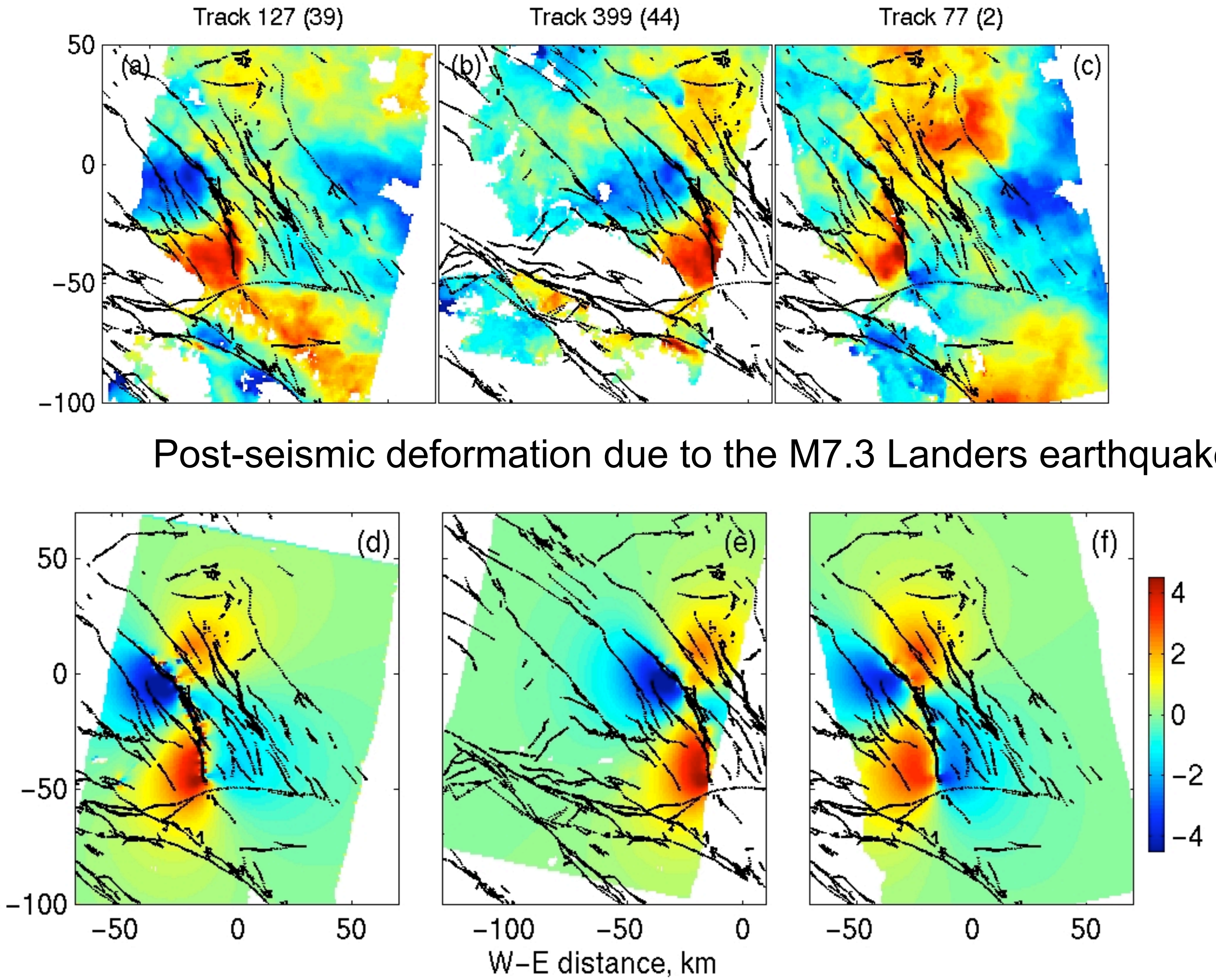


$$\sigma_{ij} = 2G\varepsilon_{ij} + \left(K - \frac{2G}{3}\right)\delta_{ij}\varepsilon_{kk} - \alpha\delta_{ij}p$$

$$\frac{k}{\alpha\eta_f}\nabla^2 p = \frac{\partial}{\partial t}\left(\varepsilon_{kk} + \frac{\alpha}{K_u - K}p\right)$$



Fialko, JGR 2004



Post-seismic deformation due to the M7.3 Landers earthquake

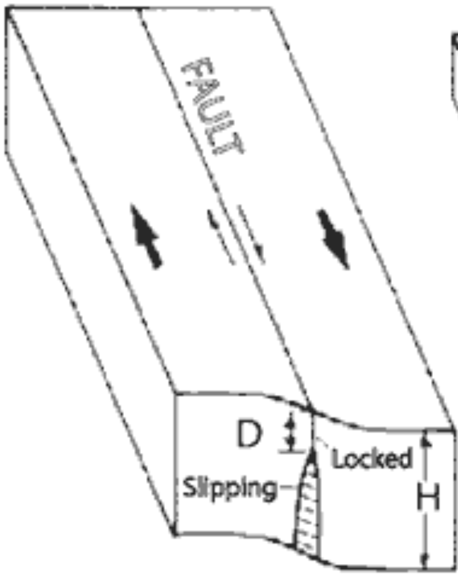
Thick Lithosphere Model D/H << 1

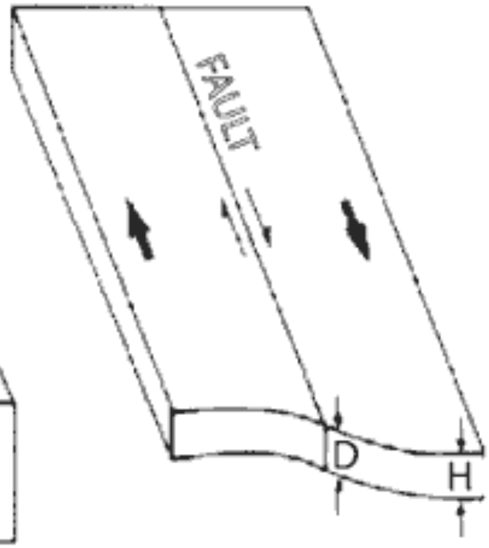
vs

Thin Lithosphere Model D/H = 1

"Thin viscous sheet vs Fault-block"

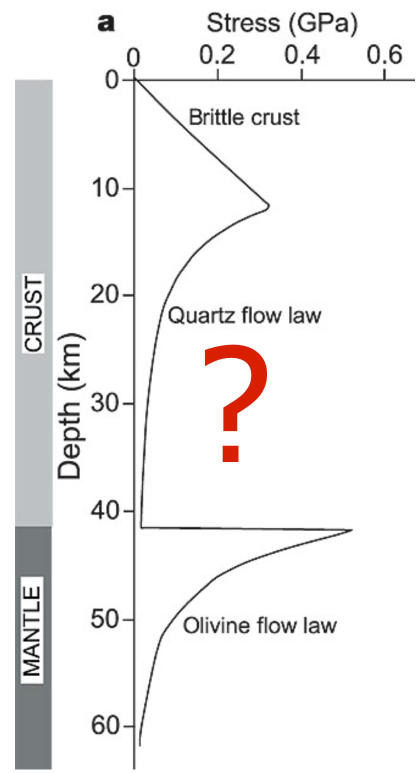
"Jelly Sandwich vs Crème Brule"

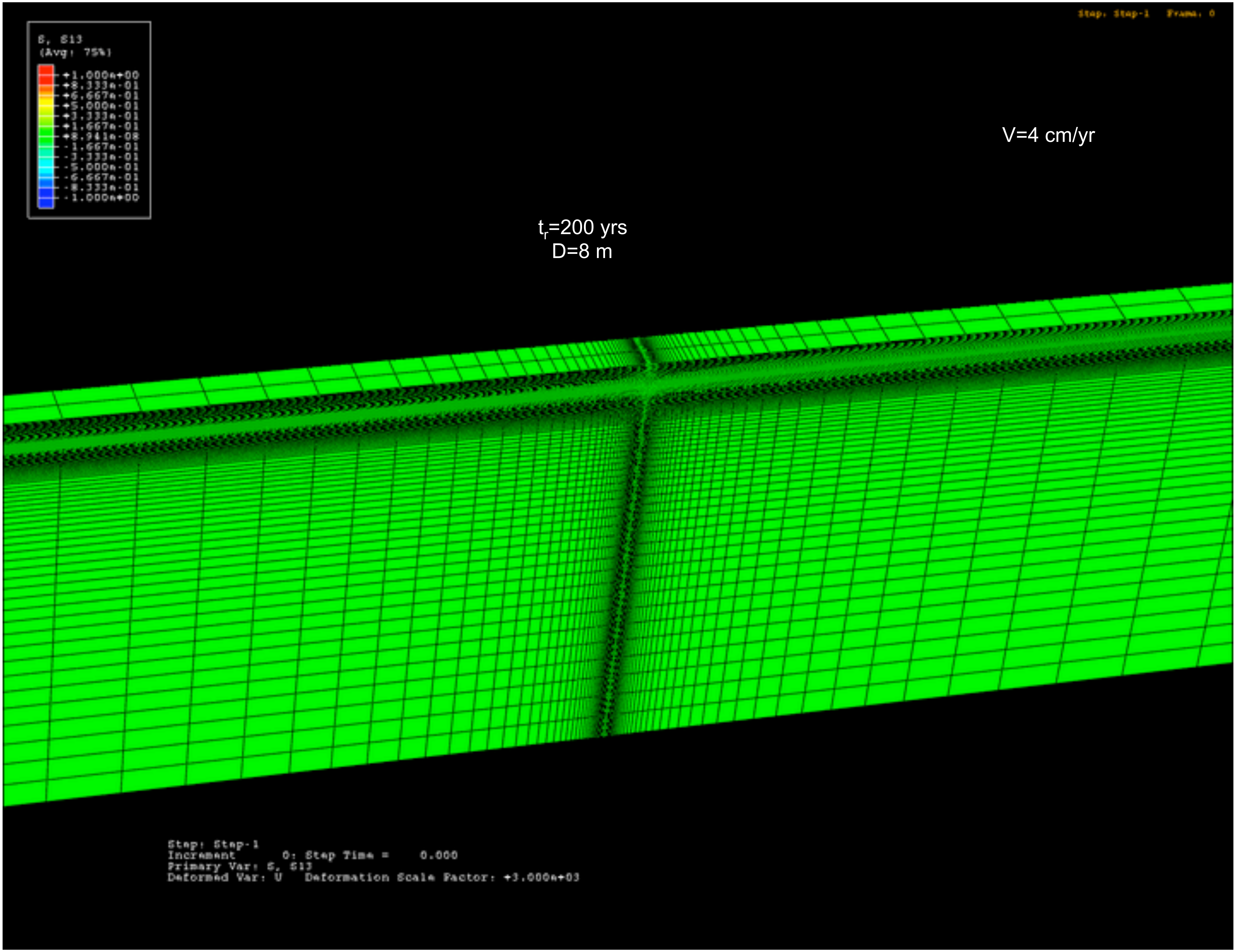


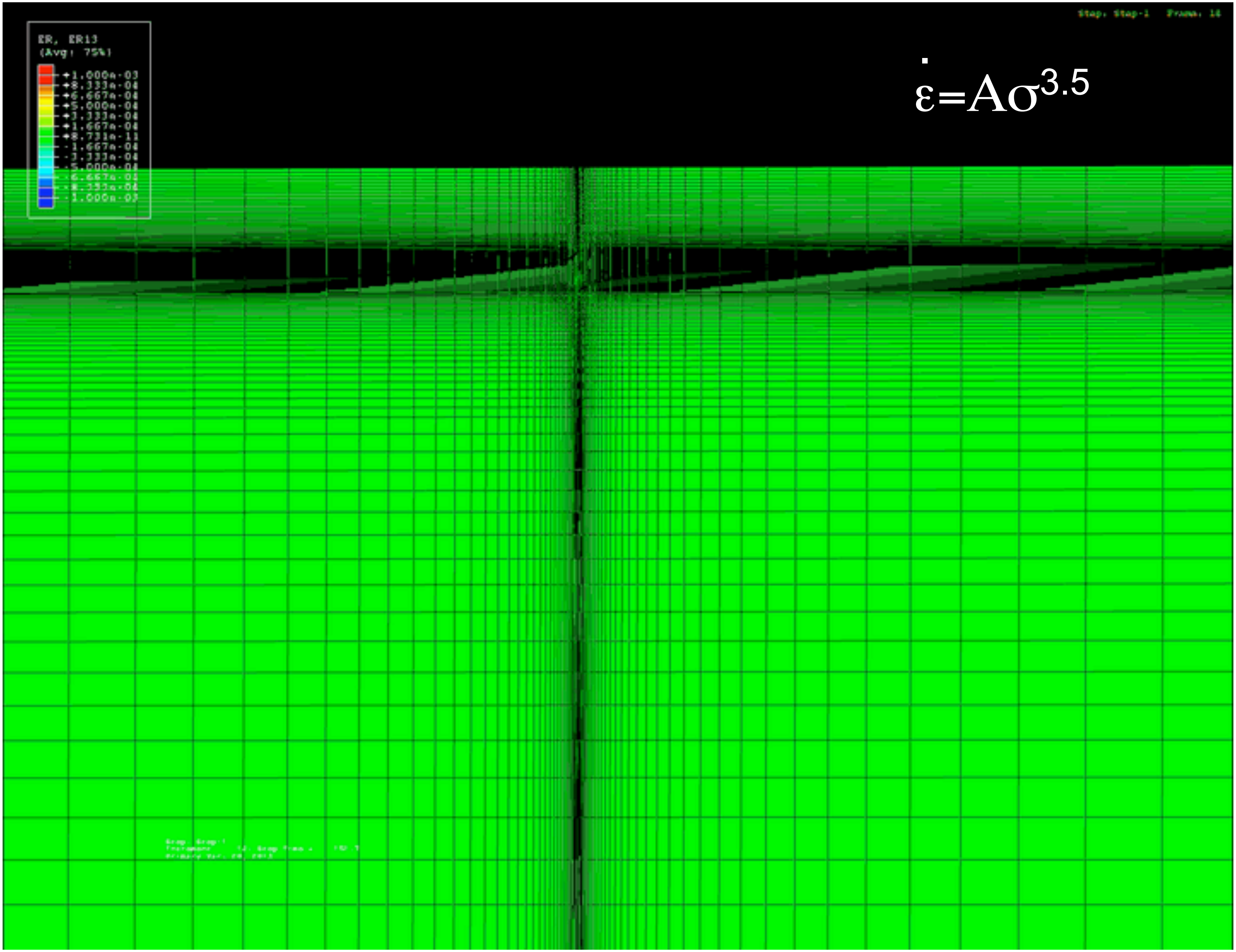


Savage and Burford, 1970
Thatcher, 1983

Elsasser, 1969
Savage and Prescott, 1978



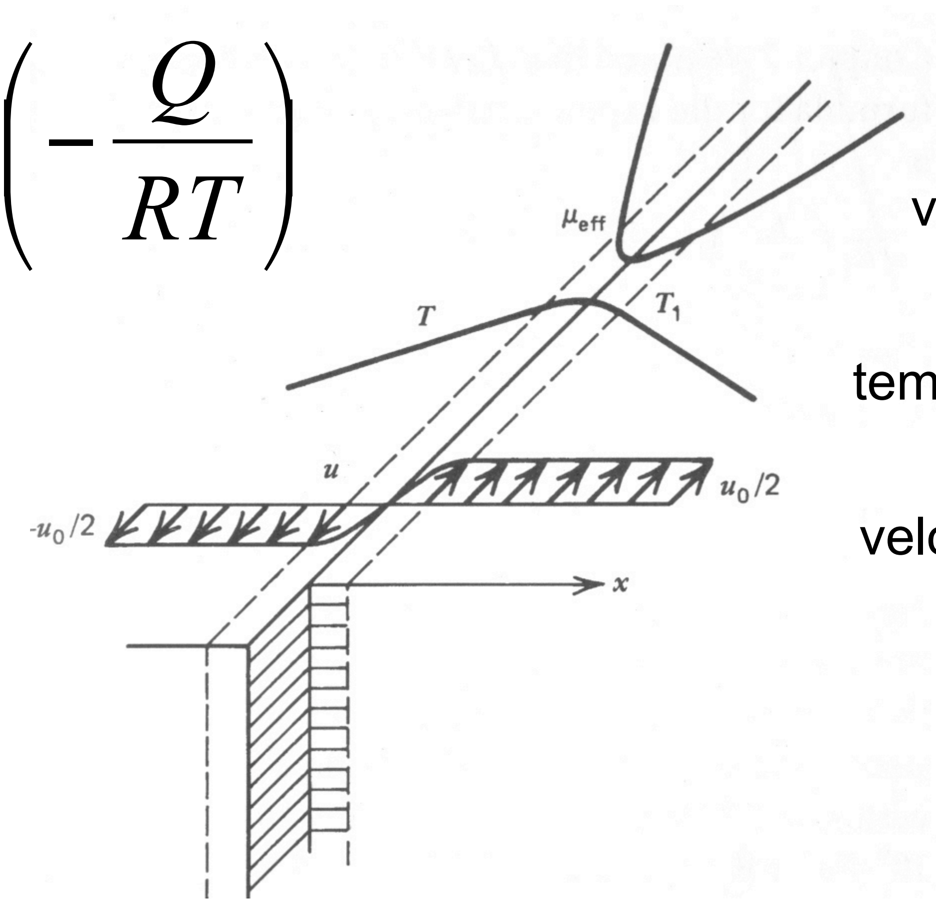

S, S13
(Avg: 75%)
+1.000e+00
+8.333e-01
+6.667e-01
+5.000e-01
+3.333e-01
+1.667e-01
+8.941e-08
-1.667e-01
-3.333e-01
-5.000e-01
-6.667e-01
-8.333e-01
-1.000e+00
Step: Step-1 Frame: 0
V=4 cm/yr
t_r=200 yrs
D=8 m
Step: Step-1
Increment 0: Step Time = 0.000
Primary Var: S, S13
Deformed Var: U Deformation Scale Factor: +3.000e+03


ER, ER13
(Avg: 75%)
+1.000e-03
+8.333e-04
+6.667e-04
+5.000e-04
+3.333e-04
+1.667e-04
+8.731e-11
-1.667e-04
-3.333e-04
-5.000e-04
-1.000e-03
$\dot{\varepsilon}=A\sigma^{3.5}$

Thermo-mechanical coupling

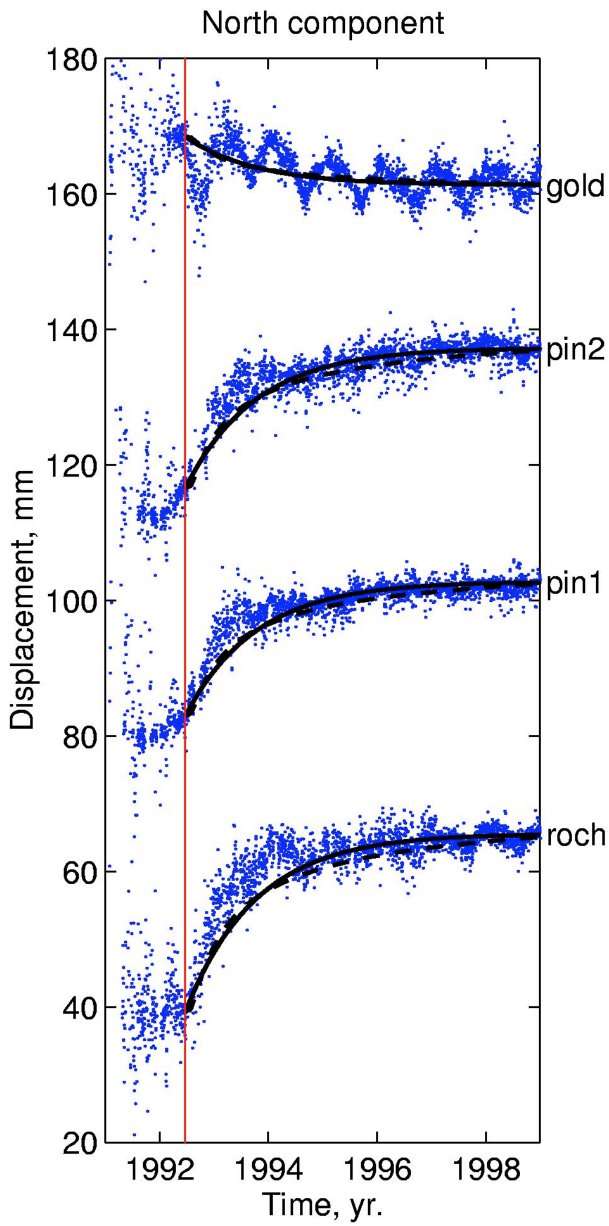
$$\dot{\varepsilon} = C\sigma^n \exp\left(-\frac{Q}{RT}\right)$$

$$T \propto \dot{\varepsilon}\sigma$$



Yuen et al., 1978; Fleitout and Frodivaux, 1980;
Turcotte and Schubert, 2002

Evidence for and implications from non-linear rheologies



Post-Landers CGPS data

-Rapid initial transient followed by a more gradual decay

-Difficult to fit assuming exponential dependence (not consistent with linear Maxwell viscoelastic behavior)

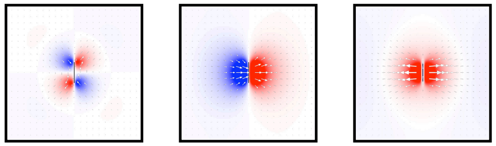
- Possible explanations:

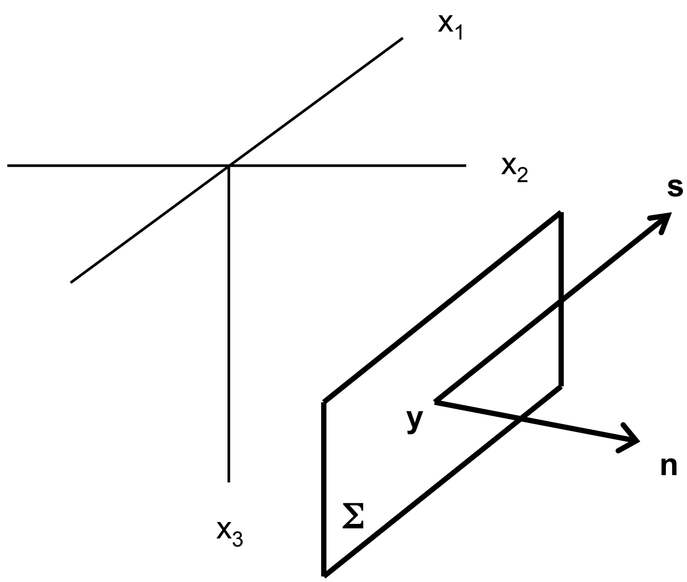
  Bi- (or multi-) viscous rheology

  Power-law rheology

  Rate-and-state friction (or some other form of non-linear localized creep)

Equivalence between dislocations and body force couples (point-source solution)





Potency Tensor (Eigenstrain)

$$\boldsymbol{\varepsilon}^i(\mathbf{x},\mathbf{y}) = \frac{1}{2}(\hat{\mathbf{n}} \otimes \mathbf{s} + \mathbf{s} \otimes \hat{\mathbf{n}})\ \delta(\mathbf{x}-\mathbf{y})$$

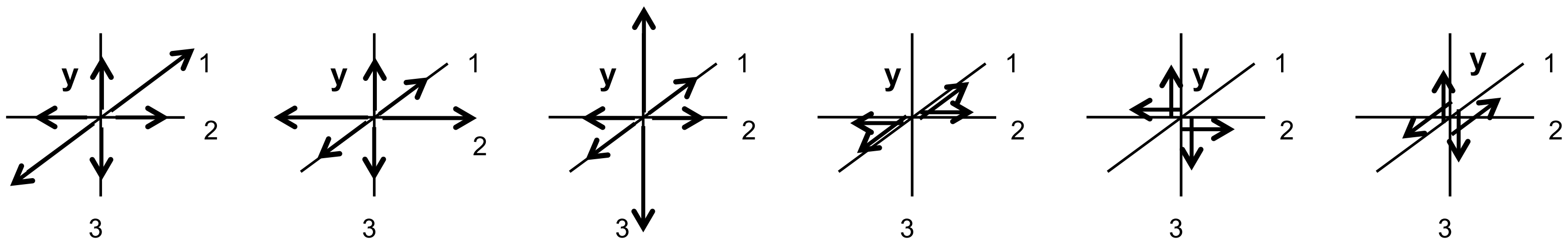
Moment Density Tensor

$$\mathbf{m}(\mathbf{x},\mathbf{y}) = \mathbf{C} : \boldsymbol{\varepsilon}^i = \mathbf{C} : \mathbf{s} \otimes \hat{\mathbf{n}}\ \delta(\mathbf{x}-\mathbf{y})$$

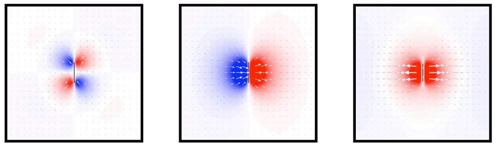
Equivalent Body Forces

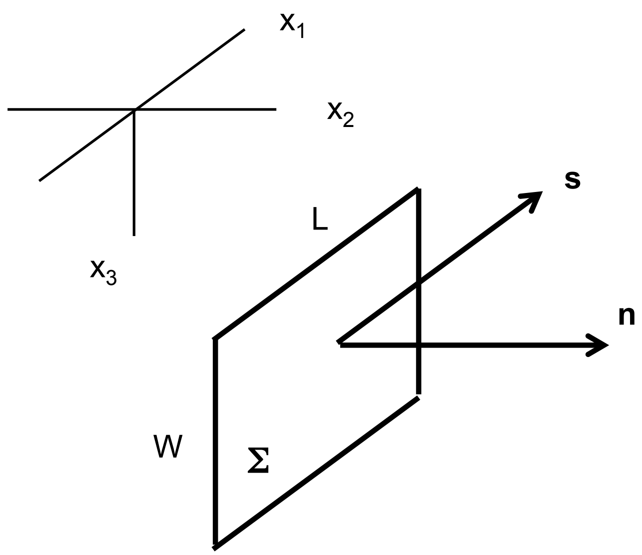
$$\mathbf{f}(\mathbf{x},\mathbf{y}) = -\nabla \cdot \mathbf{m} = -\nabla \cdot (\mathbf{C} : \boldsymbol{\varepsilon}^i)$$

Equivalent Body Forces Are Linear Combination of 6 Double Couples



Finite Fault Source





Potency Tensor (Eigenstrain)

$$\boldsymbol{\varepsilon}^i(\mathbf{x}) = \int_\Sigma \boldsymbol{\varepsilon}^i(\mathbf{x},\mathbf{y})\ d\mathbf{y}$$

Moment Density Tensor

$$\mathbf{m}(\mathbf{x}) = \int_\Sigma \mathbf{C} : \boldsymbol{\varepsilon}^i(\mathbf{x},\mathbf{y})\ d\mathbf{y}$$

Example for Uniform Rectangular Fault

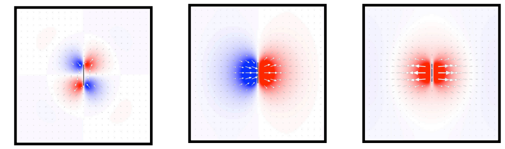
$$\boldsymbol{\varepsilon}^i(\mathbf{x}) = \frac{1}{2}(\hat{\mathbf{e}}_1 \otimes \hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_2 \otimes \hat{\mathbf{e}}_1)\ \Pi\left(\frac{x_1 - y_1}{L}\right)\ \delta\left(x_2 - y_2\right)\ \Pi\left(\frac{x_3 - y_3}{W}\right)$$

Eigenstrain characterizes:
- slip system (tensor part)
- location
- dimension

$$\mathbf{f}(\mathbf{x}) = -2\mu\ \nabla \cdot \boldsymbol{\varepsilon}^i = -\mu \begin{pmatrix} \Pi\left(\frac{x_1 - y_1}{L}\right)\ \frac{\partial}{\partial x_2}\delta\left(x_2 - y_2\right)\ \Pi\left(\frac{x_3 - y_3}{W}\right) \\ \frac{\partial}{\partial x_1}\Pi\left(\frac{x_1 - y_1}{L}\right)\ \delta\left(x_2 - y_2\right)\ \Pi\left(\frac{x_3 - y_3}{W}\right) \\ 0 \end{pmatrix}$$

analytic expression for
equivalent body forces allows:
- numerical sampling & processing
- analytic Fourier transform
- continuum representation of a discontinuous field

Greens' Function in Fourier Domain



Navier's Equation in Space Domain

$$\nabla \cdot \left(\mathbf{C} : \nabla \otimes \mathbf{u} \right) + \mathbf{f} = 0$$

or, for isotropic elasticity

$$(\lambda + \mu)\nabla\nabla \cdot \mathbf{u} + \mu \nabla^2 \mathbf{u} + \mathbf{f} = 0$$

solution is

$$\mathbf{u} = \int_\Omega \mathbf{G}(\mathbf{x}, \mathbf{x}_0) \cdot \mathbf{f}(\mathbf{x}_0) \; dV$$

Navier's Equation in Fourier Domain

$$\mathbf{k} \cdot \left(\mathbf{C} : \mathbf{k} \otimes \mathbf{u} \right) = \mathbf{f} / 4\pi$$

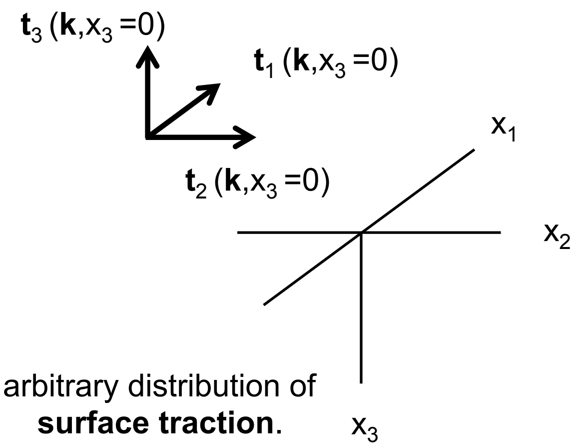
or

$$(\lambda + \mu)\mathbf{k} \otimes \mathbf{k} \cdot \mathbf{u} + \mu \, \mathbf{k} \cdot \mathbf{k} \otimes \mathbf{u} = \mathbf{f} / 4\pi$$

or simply, with full space elastic Greens' function

$$\mathbf{G}^{-1}(\mathbf{k}; \lambda, \mu) \cdot \mathbf{u}(\mathbf{k}) = \mathbf{f}(\mathbf{k})$$

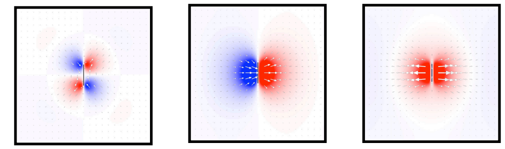
Boussinesq's & Cerruti's Problems: Elastic Deformation for Surface Traction



arbitrary distribution of **surface traction.**

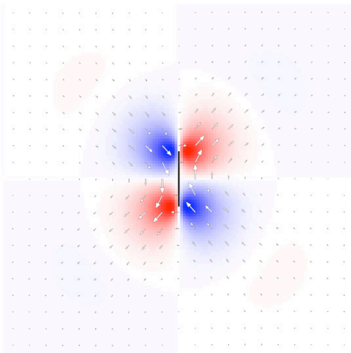
$$\mathbf{u} = \begin{pmatrix} -2B_1\beta^2 + \alpha\omega_1(B_1\omega_1 + B_2\omega_2)(1 + \beta x_3) + \alpha \; i\omega_1\beta \; B_3(1 - \alpha^{-1} + \beta x_3) \\ -2B_2\beta^2 + \alpha\omega_2(B_1\omega_1 + B_2\omega_2)(1 + \beta x_3) + \alpha \; i\omega_2\beta \; B_3(1 - \alpha^{-1} + \beta x_3) \\ \alpha \; \beta^2 \left(i(B_1\omega_1 + B_2\omega_2) \; x_3 - B_3(\alpha^{-1} + \beta x_3) \right) \end{pmatrix} e^{-\beta \; x_3}$$

Use **Boussinesq** and **Cerruti**'s solution
to **remove stress** at the surface

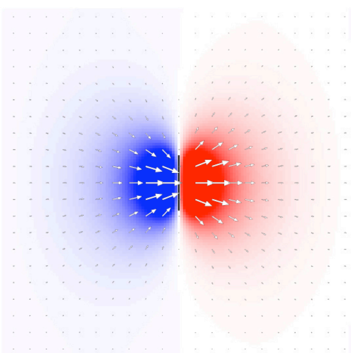


Benchmark

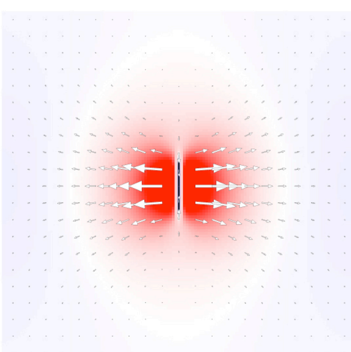
strike slip

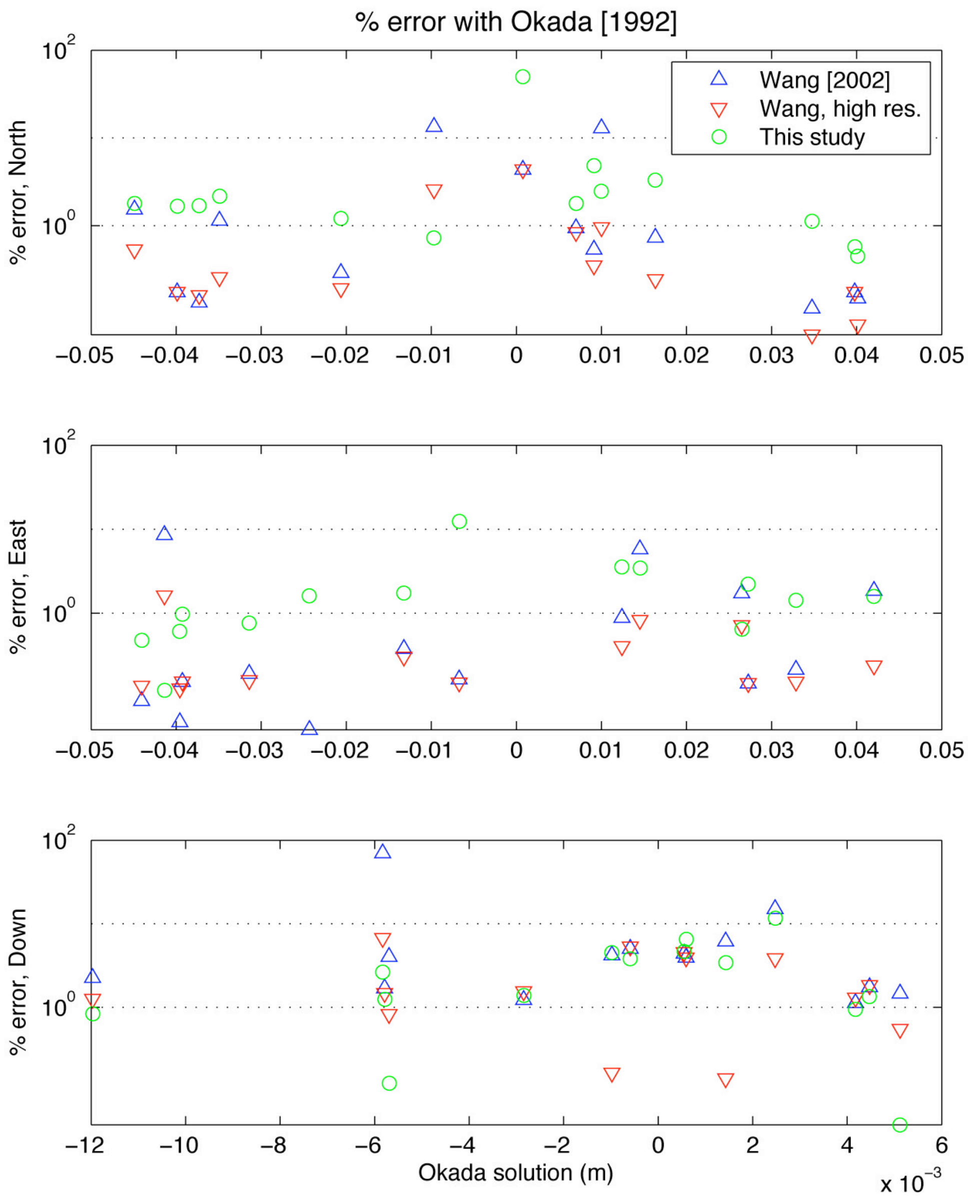


dip slip



tensile dislocation



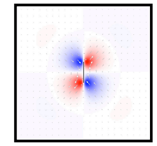


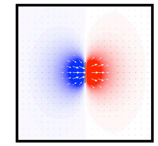
case of strike-slip fault, comparison with Okada [1992] and Wang [2002]:

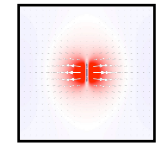
- less than **5% error wrt Okada**
- comparable with Wang
- larger error in the near field (due to discontinuity approximation)

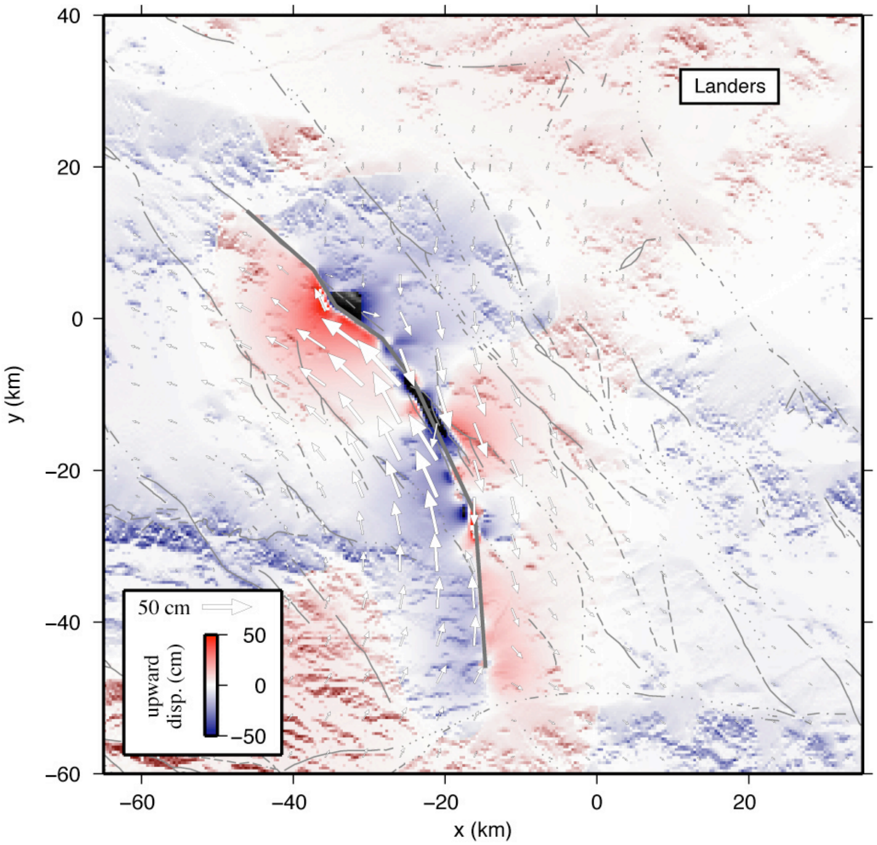
Numerical code implements **strike-slip** and **dip-slip** fault and opening (or closing) **cracks** of **arbitrary orientation**.

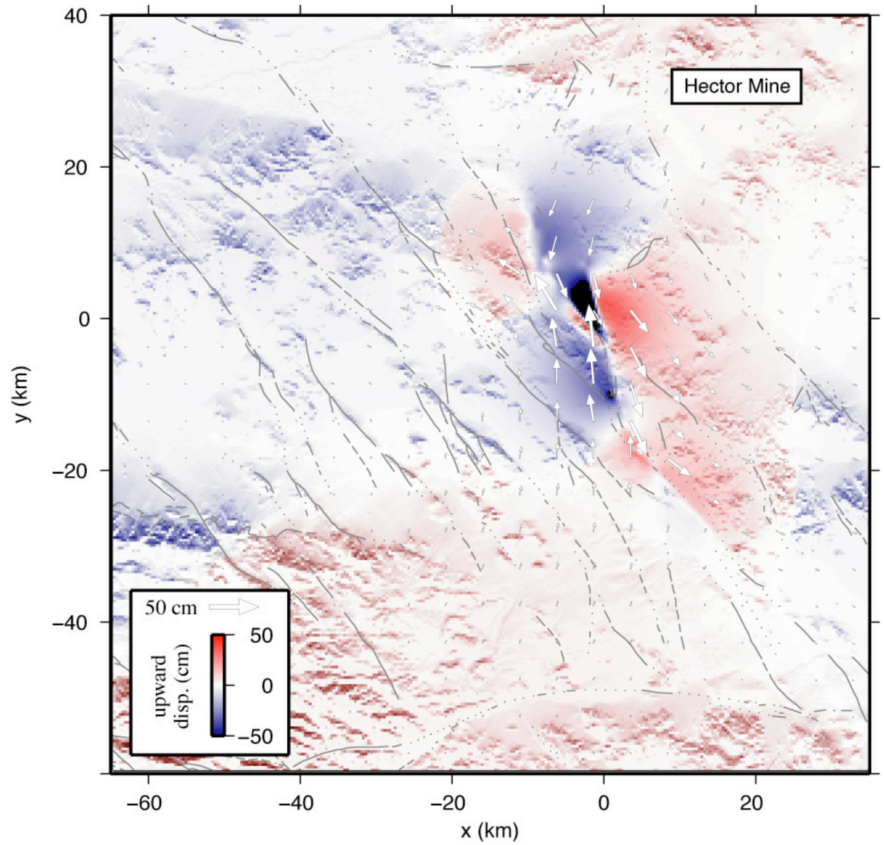
Examples









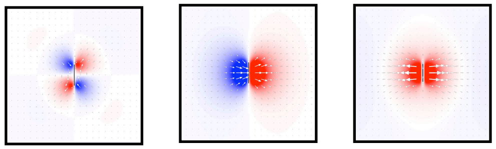


The Fourier domain method is an attractive alternative to FEM in a number of applications:

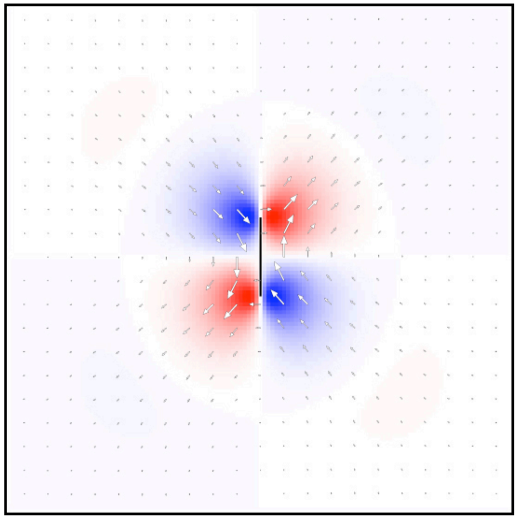
- 3-D static deformation
- nonlinear 3-D viscoelasticity
- rate-and-state fault creep
- poroelasticity
- …

A number of mathematical issues that arise from this formulation (optimal choice of an initial “homogenized” model, convergence, existence, stability analysis, errors, etc)

User Interface Example



```
./static <<EOF
# grid dimension (sx1,sx2,sx3)
128 128 128
# sampling (dx1,dx2,dx3), beta
0.05 0.05 0.05 0.3
# origin position (x0,y0)
0 0
# observation plane depth
0
# output directory
output
# elastic moduli (lambda,mu)
1 1
# observation points
1
1 GPS1 0.5 0.1 0
# shear dislocations
1
# index slip   x1 x2 x3 length width strike dip rake
      1    1 -0.5  0  0      1     1      0  90     0
# tensile cracks
0
EOF
```



simple interface produces output in:
- prescribed points (GPS)
- map view txt file (x1,x2,u1,u2,u3)
- Generic Mapping Tools (GMT)

code is implemented in Fortran90 and uses DFT fourt

above example runs in 10s on a low-end laptop