A Brief Discussion on CitcomS

History:

1) Cartesian multigrid Citcom [Moresi, 1995]

2) Parallelized with NX/MPI [1996]

3) Stokes flow in spherical geometry [1997]

4) Thermal convection and FMG [1999]

5) On the Geoframework [2002]

6) On the CIG [2005]

Some General Features (Well, Citcom)

***Element type*: brick element (8 velocity nodes and constant pressure per element).**

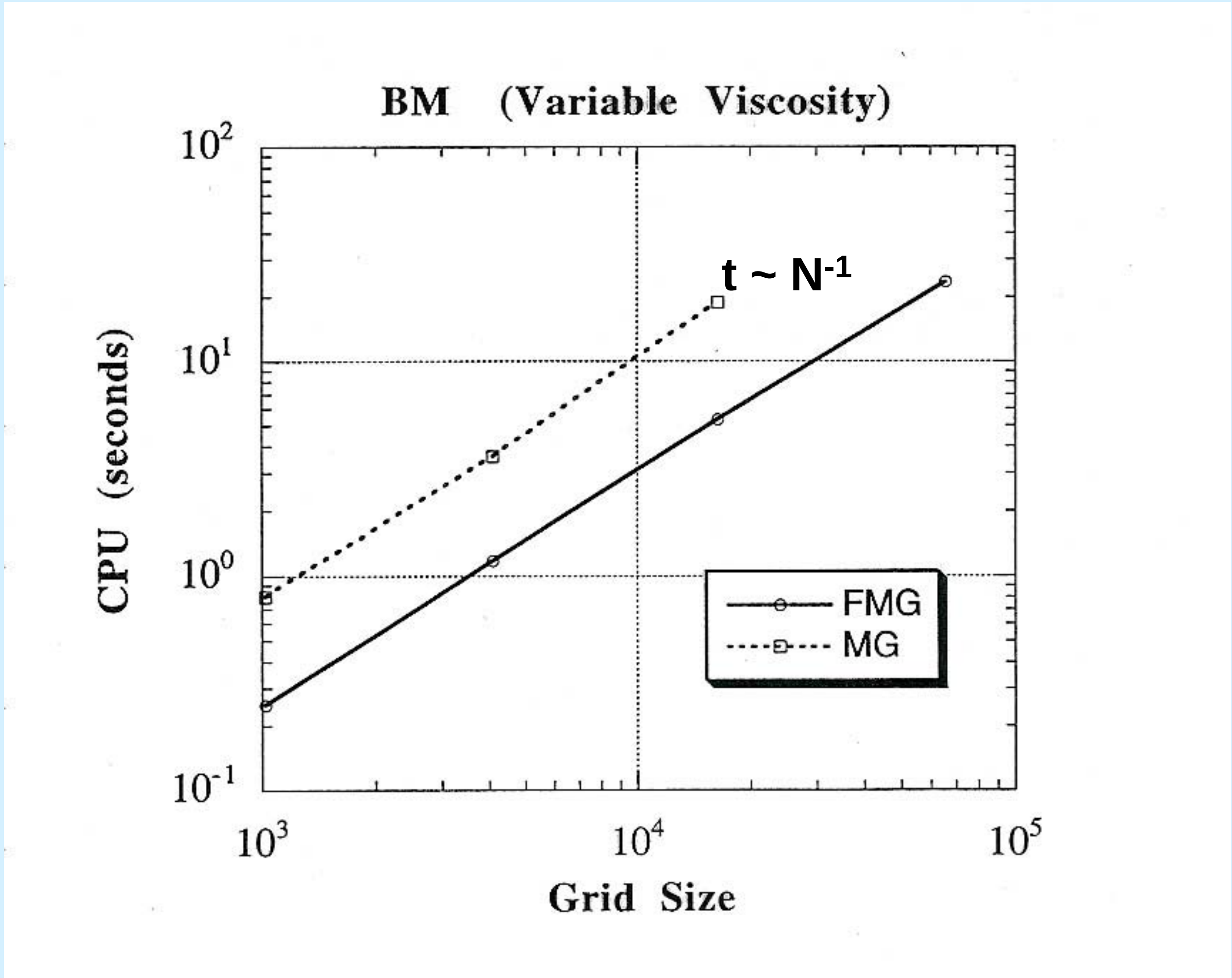
***Energy equation solver*: SUPG [same as in Conman]**

***Momentum equation solver*: Uzawa with two level iterations**

[outer level: pressure, CG; inner level: velocity, FMG]

All the equations are explicitly written in spherical geometry and then coded up in stiffness matrix, force,...

Execution Time vs Grid Size *N* for Multi-grid Solvers in *Citcom*



Some General Features (Well, Citcom)

***Element type*: brick element (8 velocity nodes and constant pressure per element).**

***Energy equation solver*: SUPG [same as in Conman]**

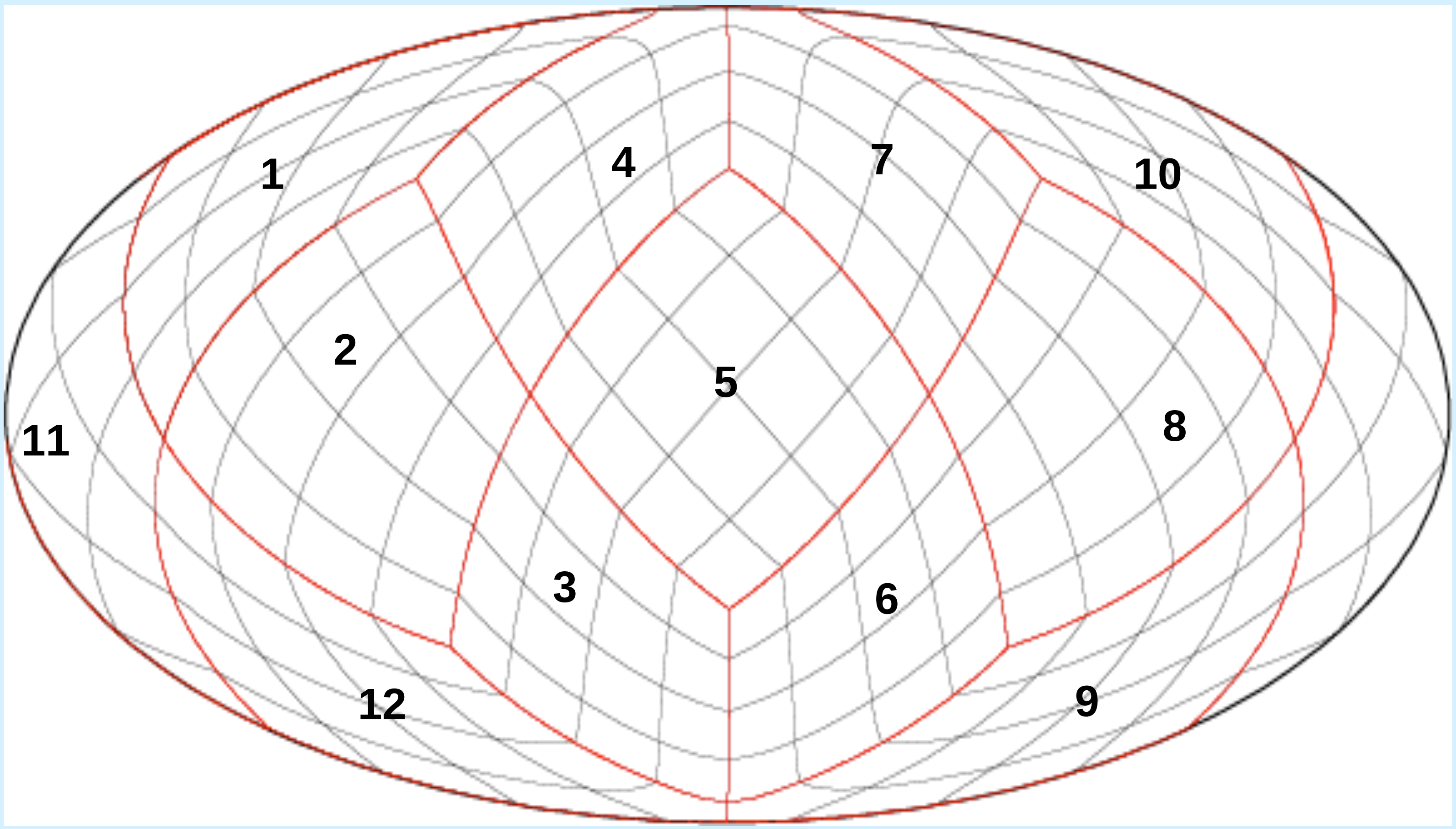
***Momentum equation solver*: Uzawa with two level iterations**

[outer level: pressure, CG; inner level: velocity, FMG]

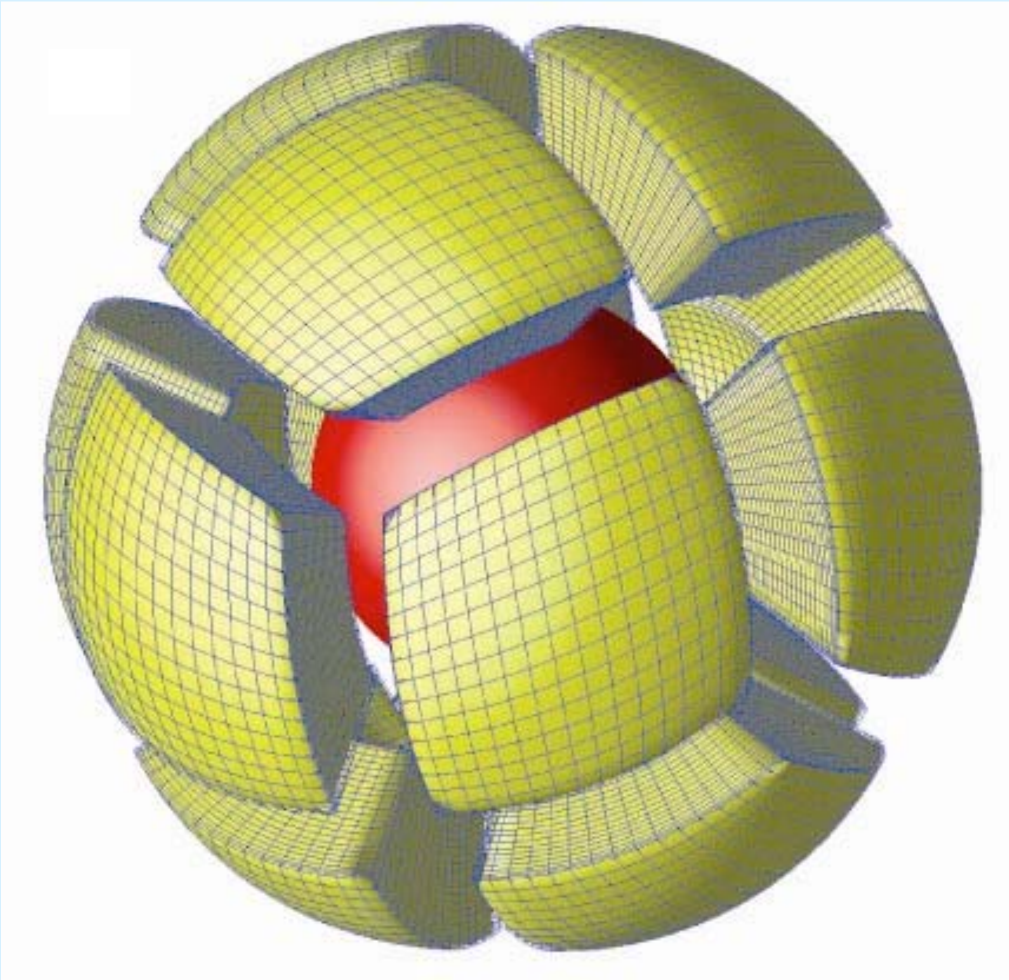
All the equations are explicitly written in spherical geometry and then coded up in stiffness matrix, force,...

Some General Features (continued)

Grids: **First divide a spherical shell into 12 caps, and then further divide each cap into elements with roughly uniform size. Each cap has the same number of elements.**



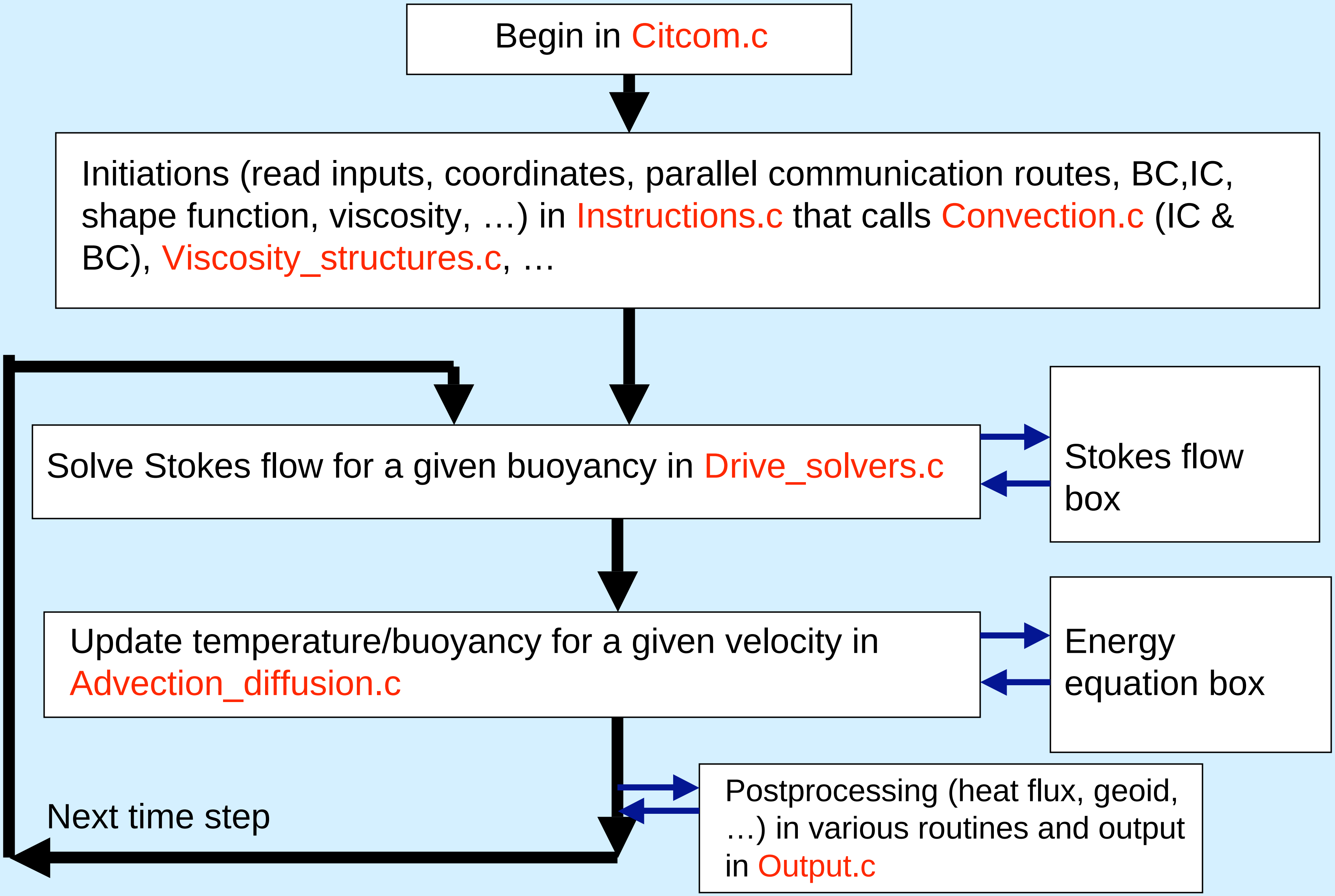
3D View of Grids



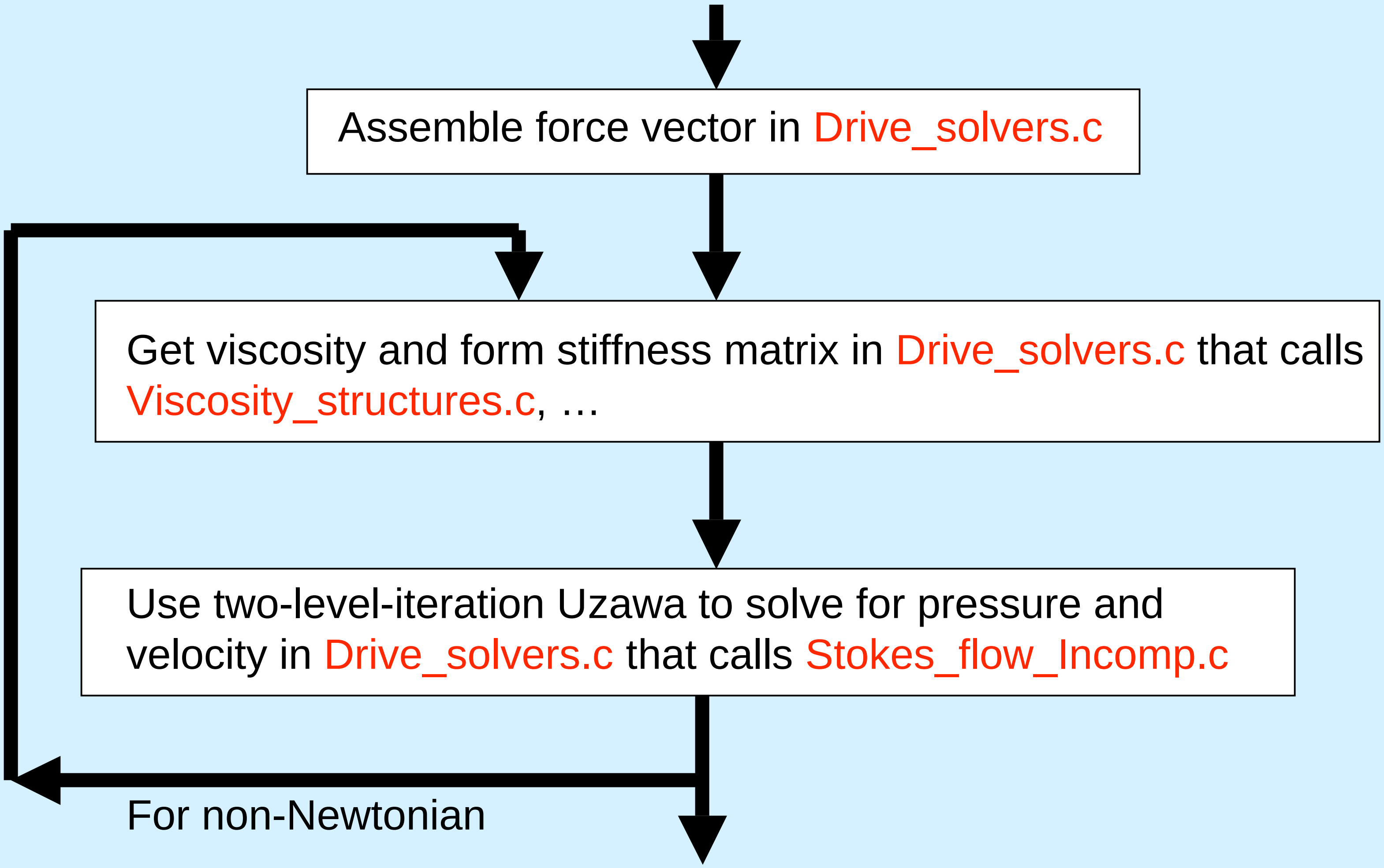
Structure and Organization

- ***Source code: 33 *.c files.***
- **8 files: Citcom.c, Instructions.c, Convection.c, Viscosity_structures.c, Drive_solvers.c, Stokes_flow_Incomp.c, Advection_diffusion.c, and Output.c.**

Flow Chart


Begin in Citcom.c
Initiations (read inputs, coordinates, parallel communication routes, BC,IC, shape function, viscosity, …) in Instructions.c that calls Convection.c (IC & BC), Viscosity_structures.c, …
Solve Stokes flow for a given buoyancy in Drive_solvers.c
Stokes flow box
Update temperature/buoyancy for a given velocity in Advection_diffusion.c
Energy equation box
Postprocessing (heat flux, geoid, …) in various routines and output in Output.c
Next time step

Flow Chart for Stokes flow box



Some Remarks on Benchmark for Compressible Stokes' flow

- ***2D Cartesian compressible Stokes' flow (not full convection problem which is nonlinear and does not have analytic solutions).***
- ***A collaboration with Joel Ita in 1994, but unpublished.***

Equations

Mass Balance: $\frac{\partial(\rho u_i)}{\partial x_i} = 0,$

Momentum Balance: $\frac{\partial \sigma_{ij}}{\partial x_j} - \rho g \vec{e}_z = 0,$

EOS: $\rho = \rho_o \exp[(1 - z/d) D_i / \Gamma][1 - \alpha(T - T_s) + p / K_t],$

where D_i is a dissipation number and Γ is the Grueneisen parameter.

Constitutive Law:

$$\sigma_{ij} = -(P_h + p)\delta_{ij} + \eta\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \ ,$$

where $\nabla P_h = -\rho_o \exp[(1 - z/d) D_i / \Gamma] g.$

Propagator matrix solutions for layered/columnar viscosity and single harmonic buoyancy force

Define $\gamma = {D_i}/{\Gamma}$, and this system of equations can be written as:

$$\frac{d}{dz}\begin{pmatrix} U \\ V \\ S/(2\eta_0 k) \\ T/(2\eta_0 k) \end{pmatrix} = \begin{pmatrix} 0 & k & 0 & \frac{2k}{\eta} \\ -k & -\gamma & 0 & 0 \\ \eta\gamma & \frac{2\eta\gamma^2}{3k} & \gamma & -k \\ 2k\eta & \eta\gamma & k & 0 \end{pmatrix} \begin{pmatrix} U \\ V \\ S/(2\eta_0 k) \\ T/(2\eta_0 k) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\frac{RaT}{2\eta_0 k} \\ 0 \end{pmatrix}$$

$$\frac{dX}{dz} = AX + B$$

$$X(z) = \exp[A(z - z_0)]X(z_0) + \int_{z_0}^{z} \exp[A(z - \varsigma)]B(\varsigma)d\varsigma$$

Some comparisons with Conman

Benchmark for isoviscous compressible flow in a 1x1 box with free-slip boundary conditions and with Ra=1 and

$T(x,z) = 10^3 \exp[(1-z)\gamma]\sin(\pi z)\cos(\pi x)$ for $0 \leq x, z \leq 1$.

Case 1: $\gamma = 0.25$ ***and for 32x32 mesh.***

	U_{top}	S_{zz_top}	U_{bottom}	S_{zz_bottom}
Conman	29.11	-348.1	-28.14	371.2
Propagator	29.14	-349.0	-28.17	372.3

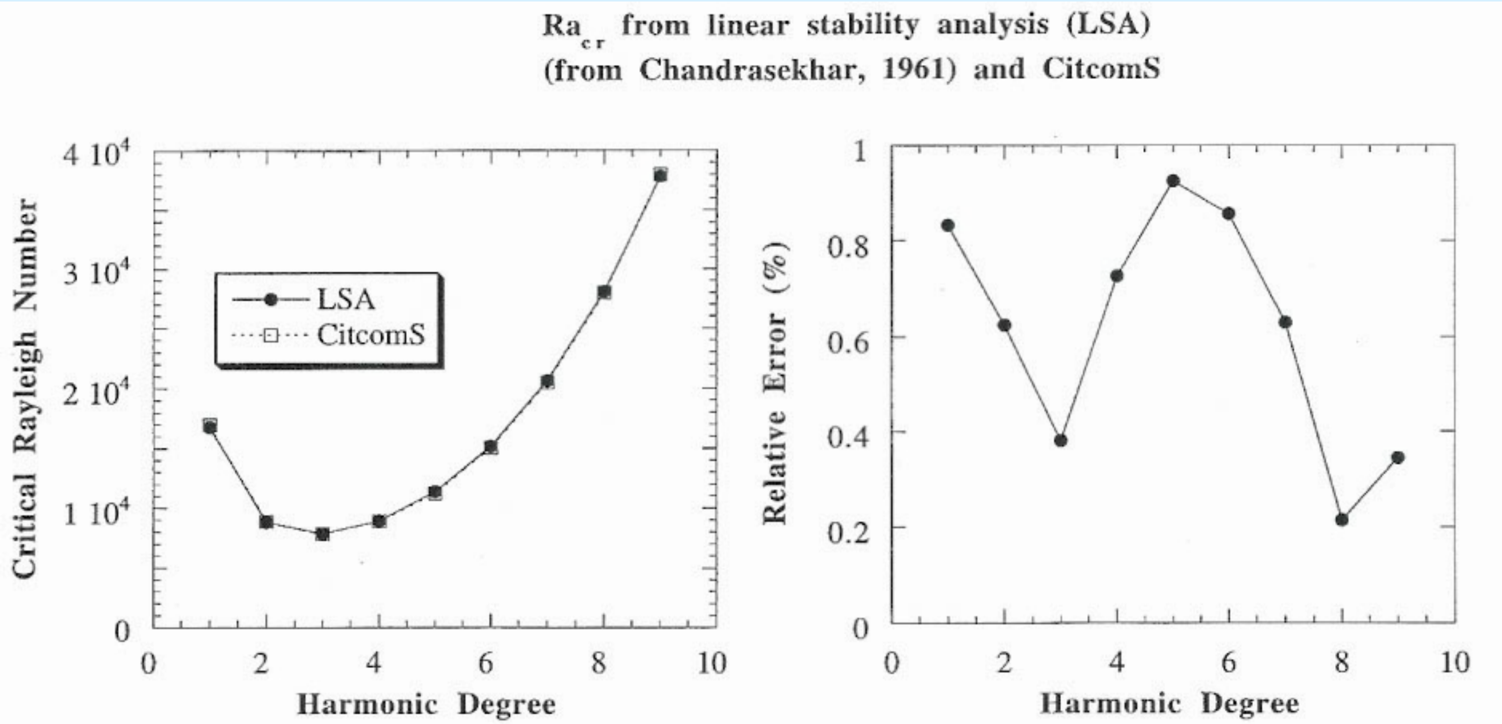
Case 2: $\gamma = 1.25$ ***and for 32x32 mesh.***

	U_{top}	S_{zz_top}	U_{bottom}	S_{zz_bottom}
Conman	49.02	-490.6	-41.77	684.7
Propagator	49.02	-491.2	-41.80	687.0

More comparisons for the Stokes' flow problems

- ***For TALA (no dynamic pressure in the buoyancy).***
- ***Layered or columnar viscosity structure [Zhong, GJI, 1996].***
- ***Spherical geometry (compressible solutions probably already exist, but may need to be fixed for benchmark purposes).***
- ***Finally, compared with marginal stability analysis (i.e., critical Ra).***

Accuracy of CitcomS



Zhong et al., 2000