

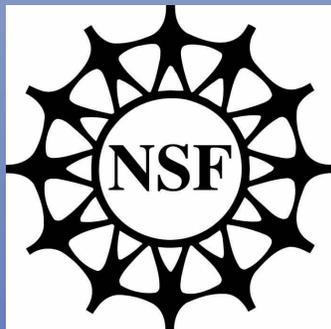
Bayesian Uncertainty Quantification of Subduction Zone Rheology from the Geoid

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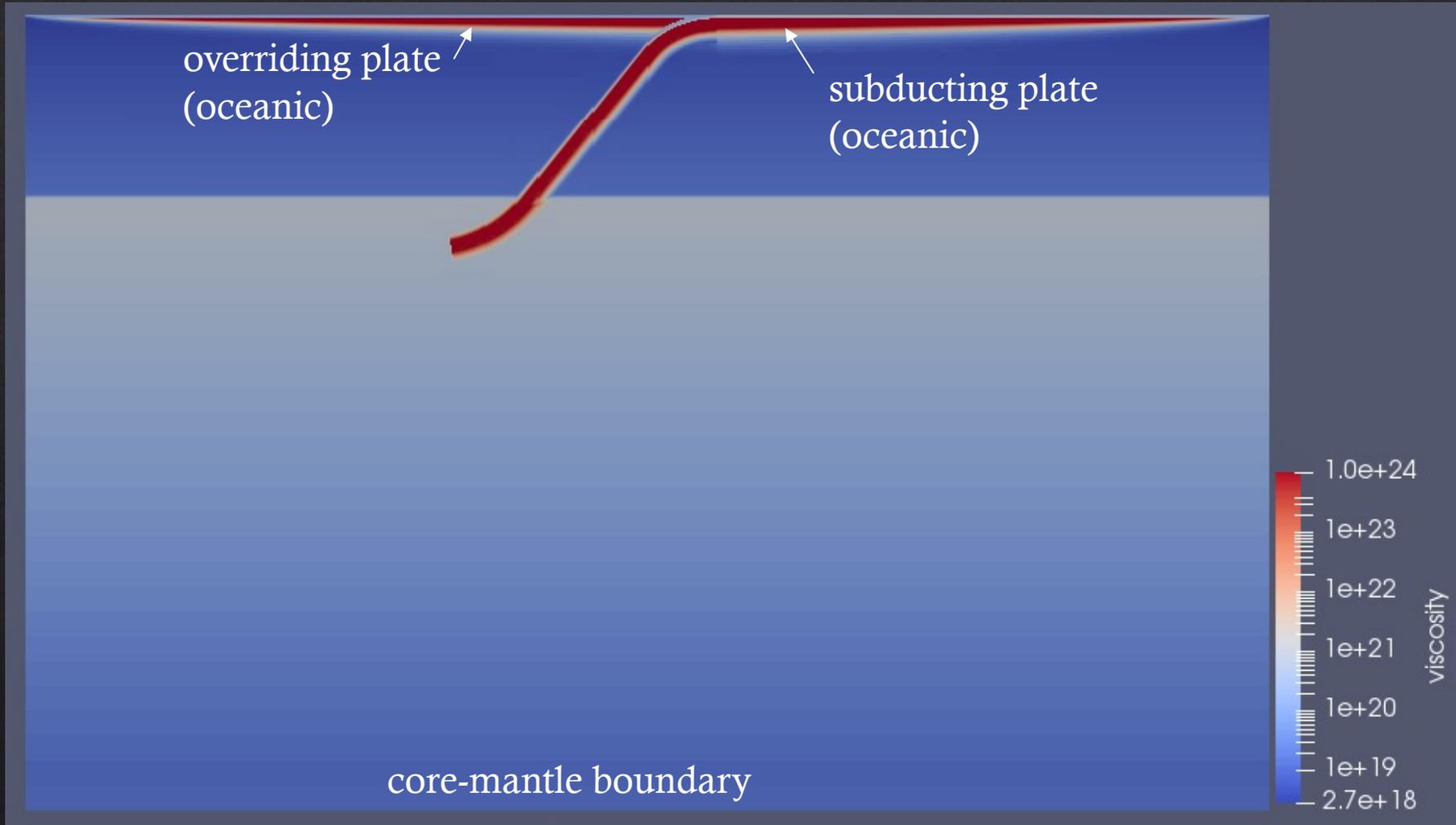
Central Question:

Can we recover rheological parameters in a subduction zone from Bayesian inversion of the geoid?

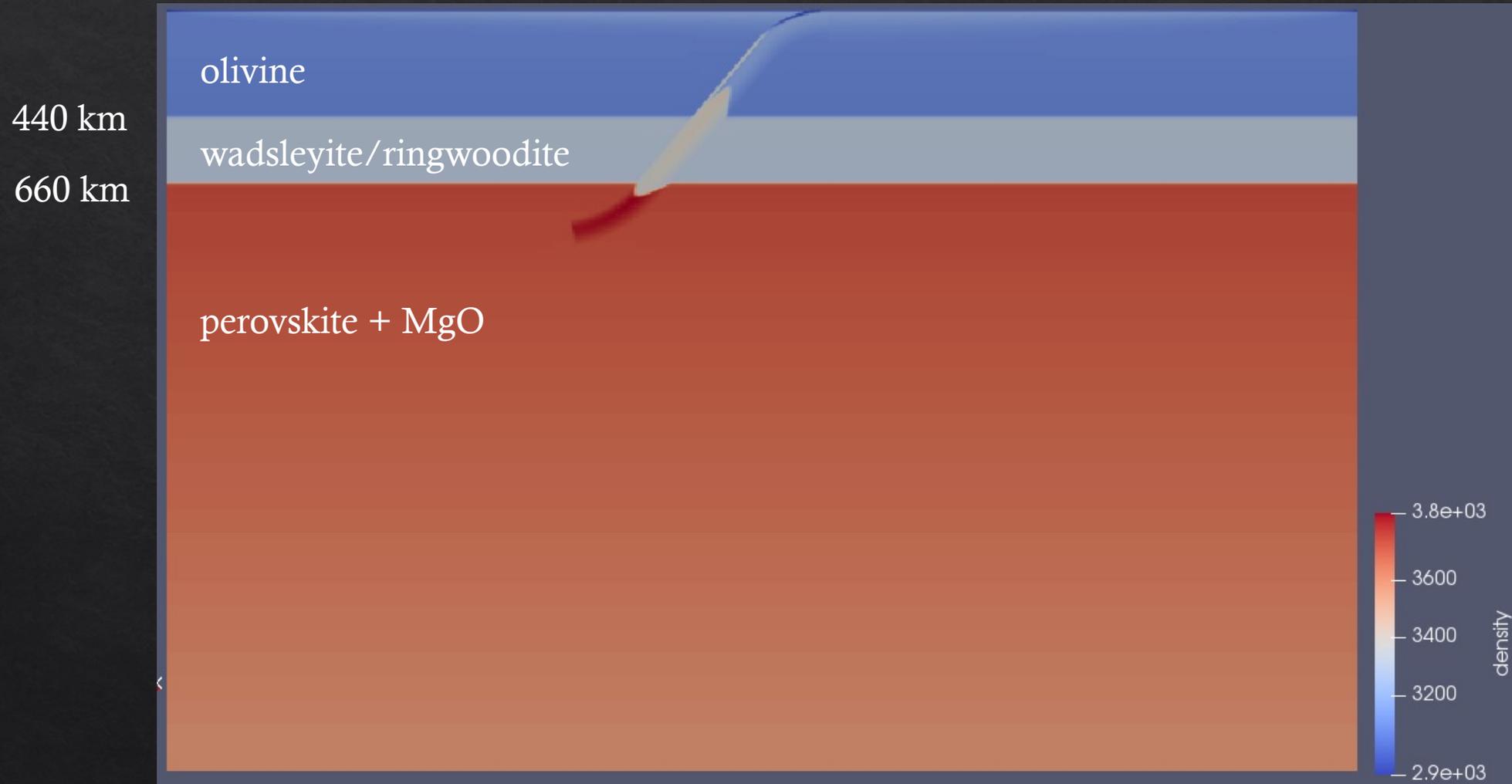
Model Subduction Zone: Viscosity

4500 km

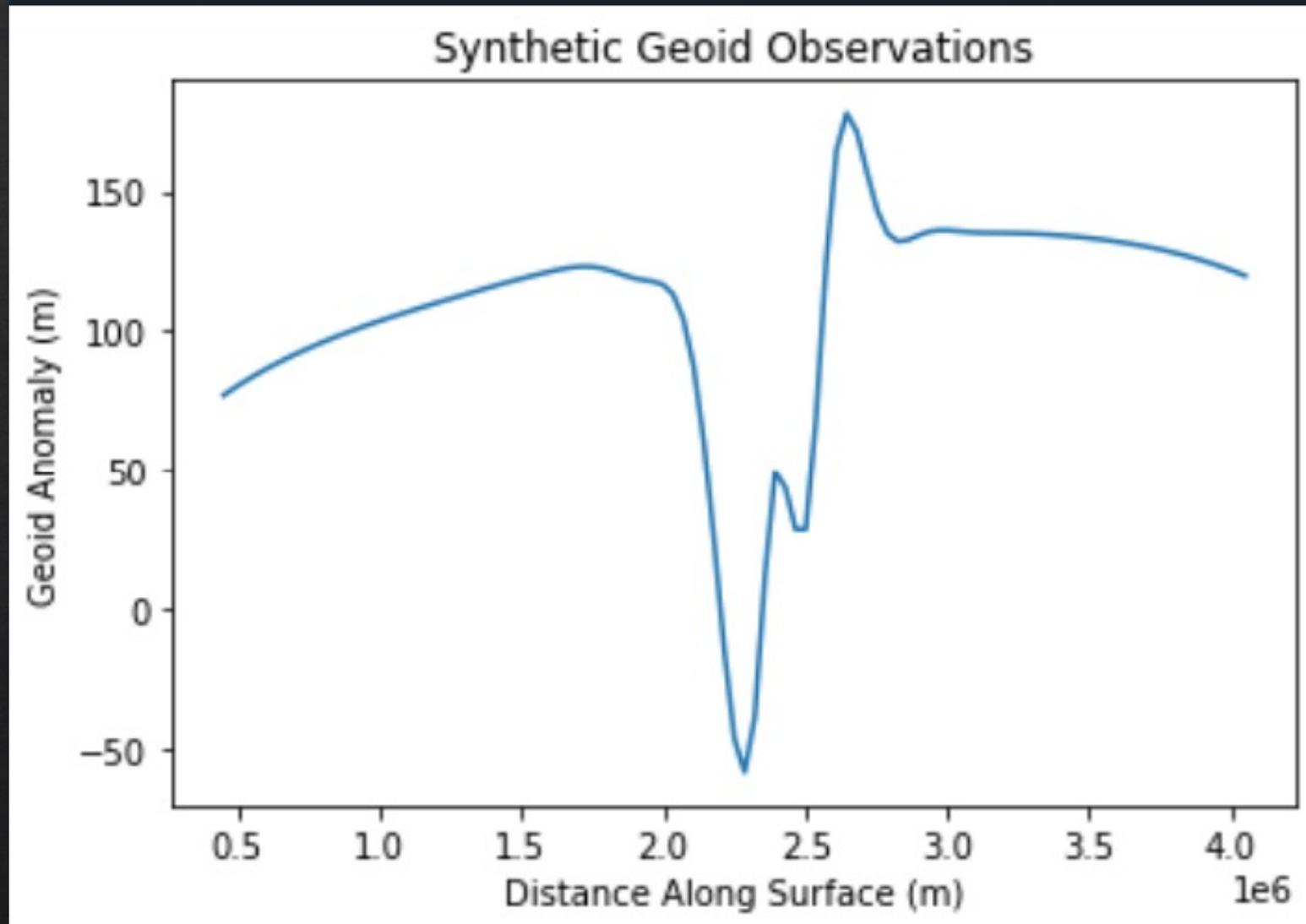
2890 km



Model Subduction Zone: Density



Synthetic “Observed” Geoid



Visco-Plastic Flow Law

$$\eta = \frac{1}{2} A^{-\frac{1}{n}} d^{-\frac{m}{n}} \dot{\epsilon}_{ii}^{\frac{1-n}{n}} \exp\left(\frac{E + PV}{nRT}\right)$$

η : viscosity

A : prefactor

n : stress exponent

$\dot{\epsilon}_{ii}$: square root of second invariant of deviatoric strain rate tensor

d : grain size

m : grain size exponent

E : activation energy

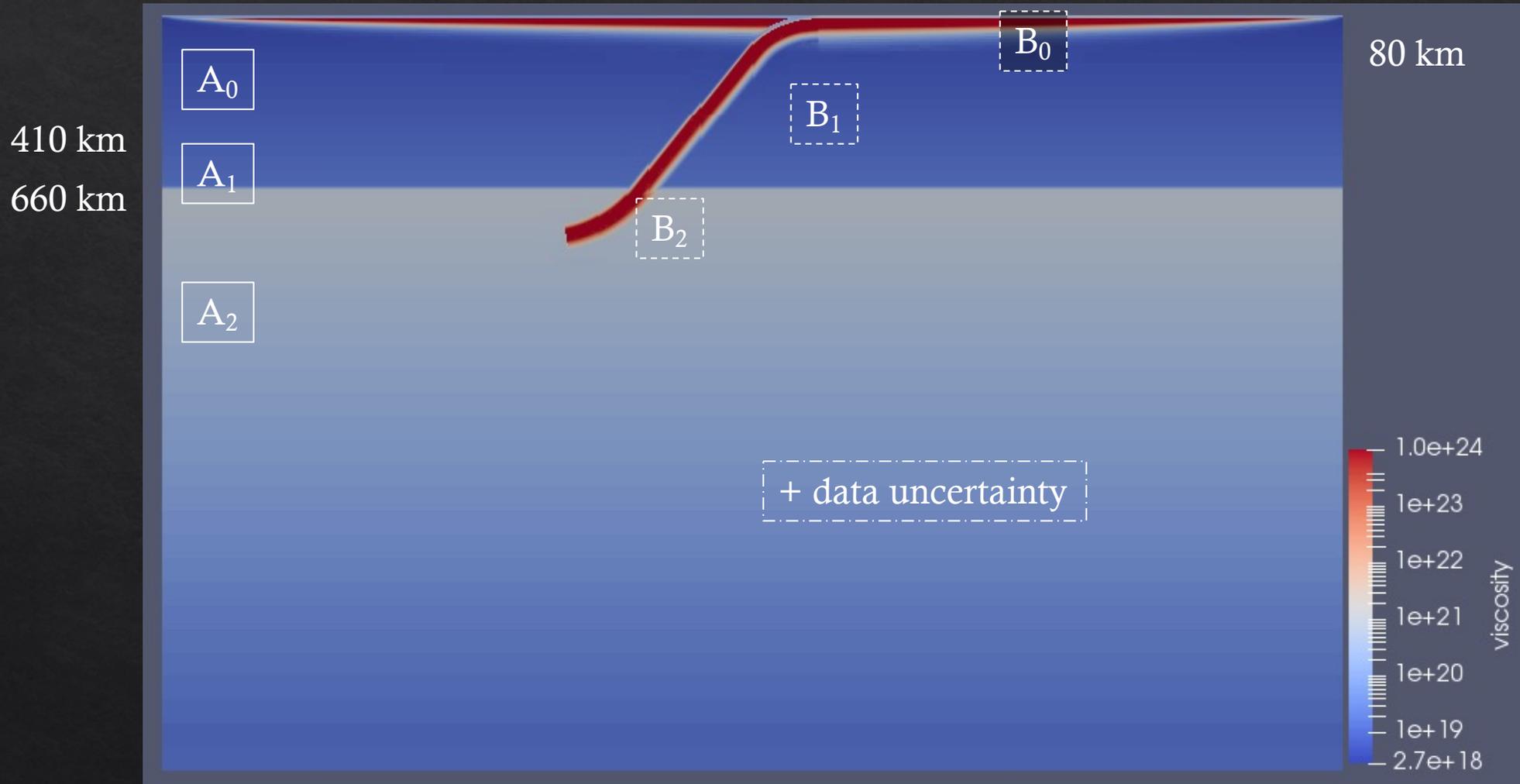
V : activation volume

P : pressure

R : gas exponent

T : temperature

Unknowns in Inversion

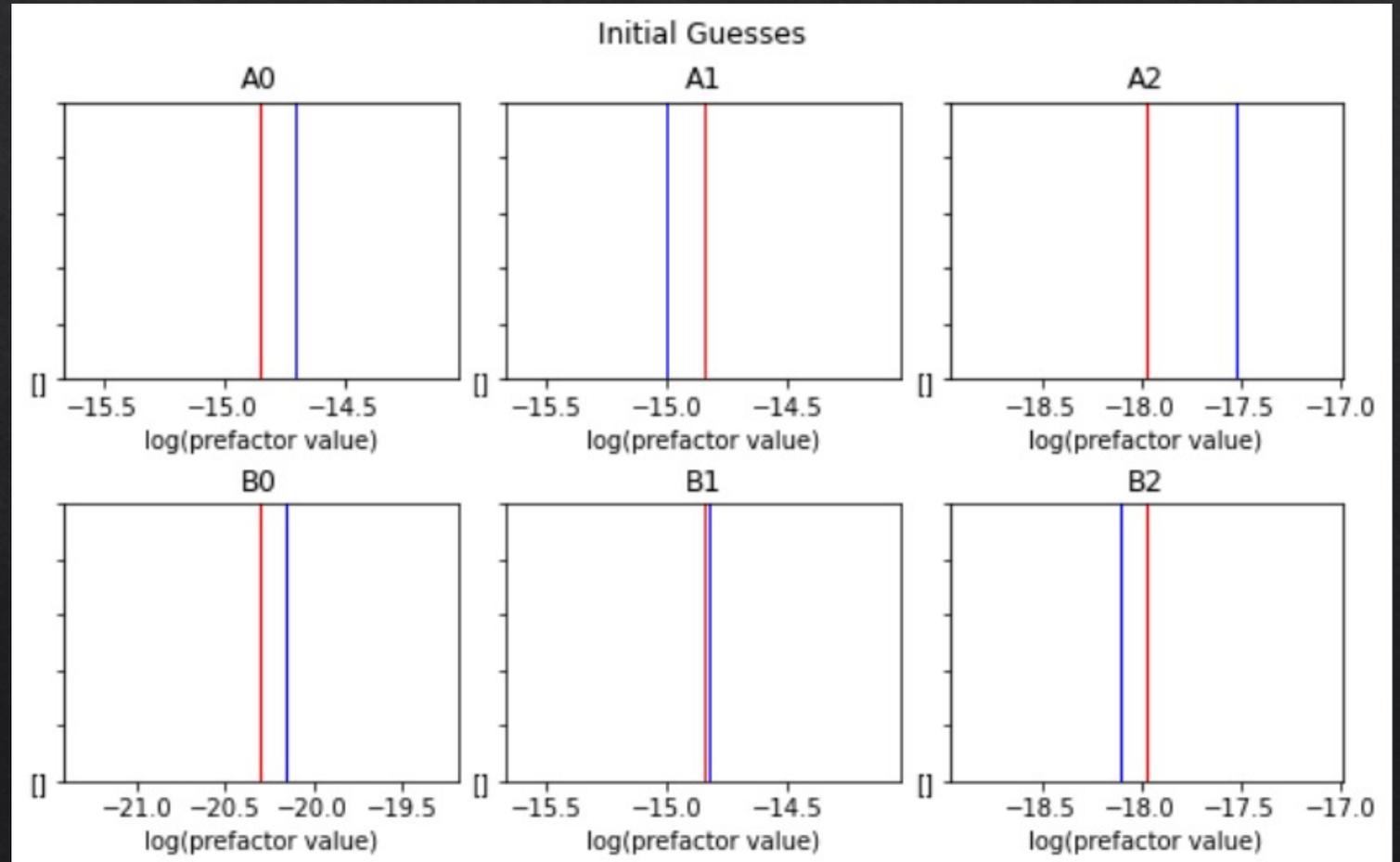


Bayesian Inversion Steps

Try to recover the parameter values that produced the observation data

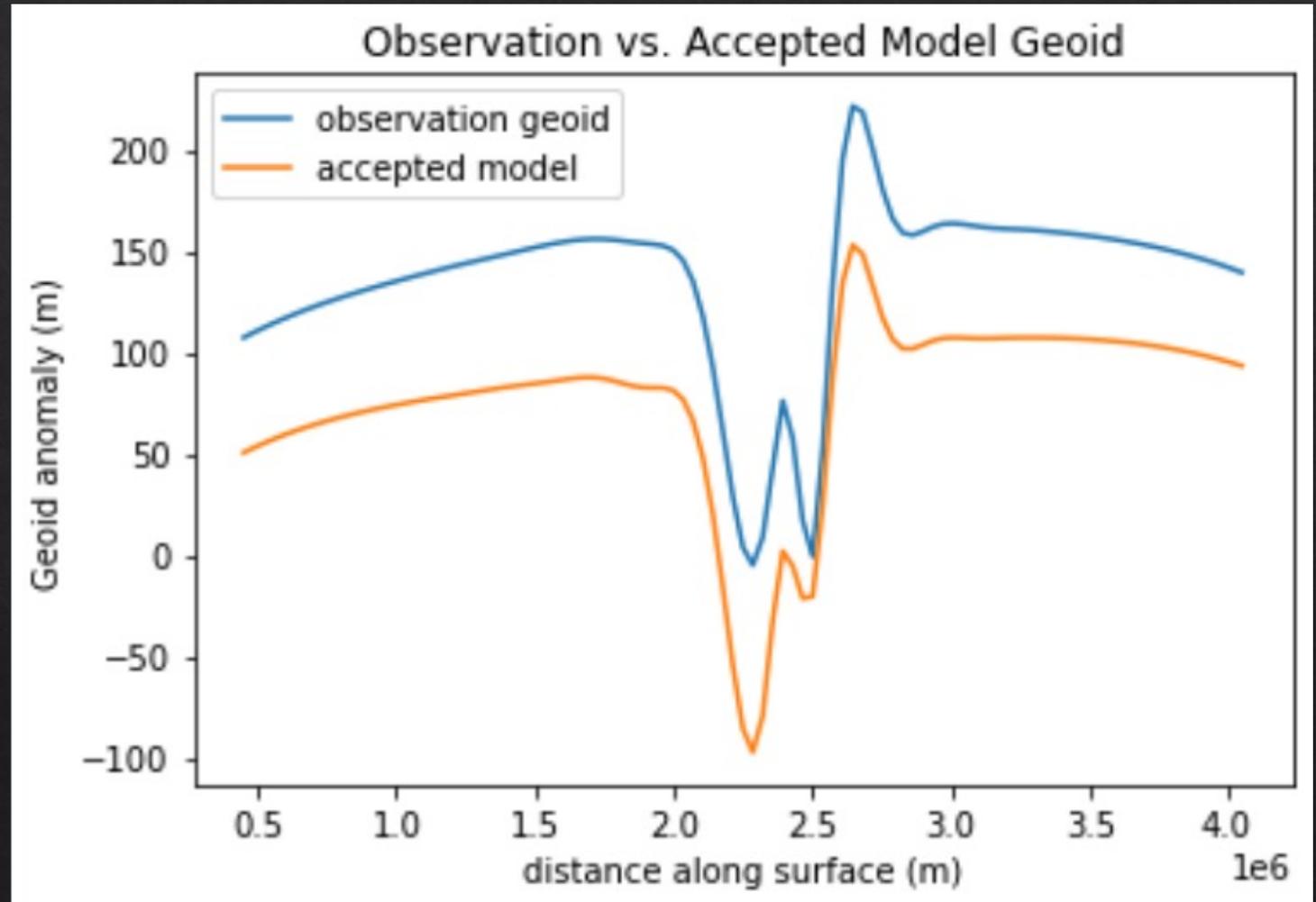
Generate Starting Model: Initial Guess

- ◇ Set parameter bounds
- ◇ Begin with random prefactor values within bounds → first “Accepted Model”



Compare Geoid to Observation Data

- ◇ Run ASPECT and calculate geoid
- ◇ Compare the observation data to current accepted model
- ◇ Quantify misfit as residual sum



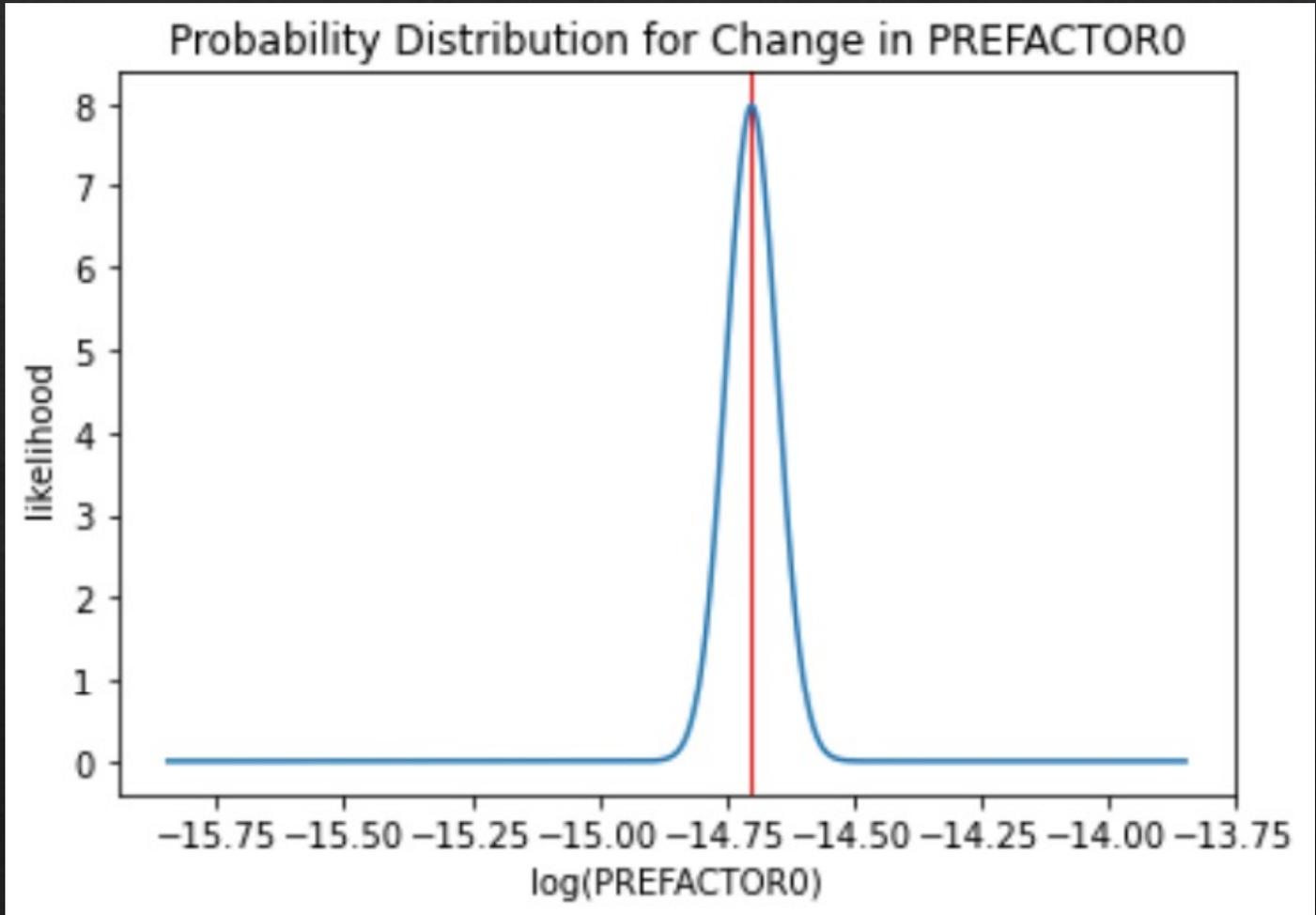
Propose New Model

- ◇ Randomly choose a parameter to change
 - ◇ prefactor or uncertainty
- ◇ Change the current parameter value x to a new value x^* where:

$$P(x^*) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x^*-x}{\sigma}\right)^2}$$

$$\sigma = 0.05$$

→ New “Proposed Model”



Keep Old Model m or Accept New m^* ?

- ◆ Metropolis Hastings criterion: probability of acceptance

$$\alpha = \frac{p(m^* | d)}{p(m | d)}$$

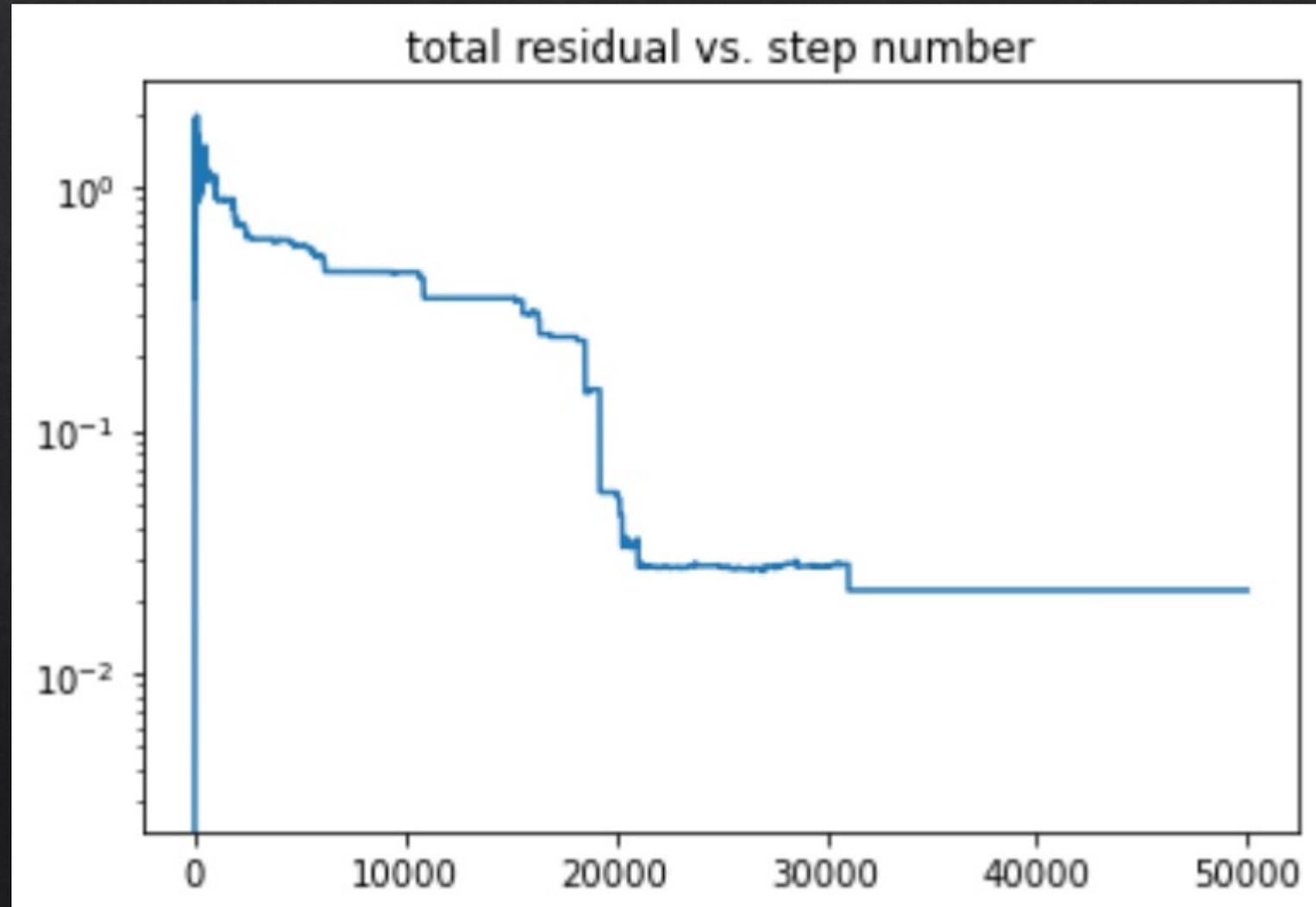
ratio of probabilities that the model is true given the data

$$\log(\alpha) = \frac{N}{2} (\log(\sigma^2) - \log(\sigma^{*2})) - \frac{1}{2\sigma^{*2}} \underline{r}^{*T} \underline{r}^* + \frac{1}{2\sigma^2} \underline{r}^T \underline{r}$$

- ◆ Randomly sample uniform distribution $u \sim U(0,1)$
- ◆ Accept m^* if $u < \min(1, \alpha)$

Repeat Steps until Convergence

- ◇ Converged after about 30,000 steps
- ◇ New solutions are no longer accepted because no better solutions can be generated

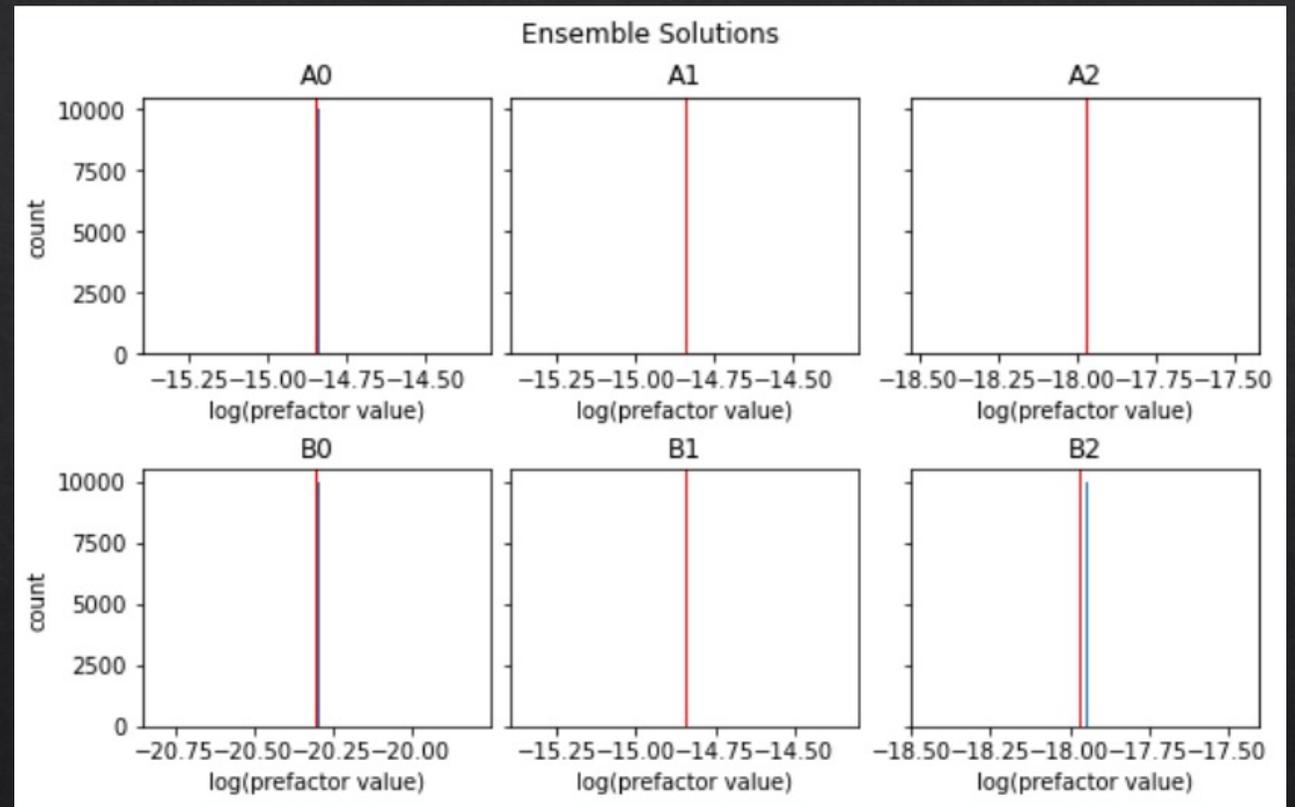


Generate Solution Ensemble

- ◆ After convergence, keep exploring the sample space
- ◆ Collect a random sample of solutions with low misfit
 - **Solution Ensemble:** Set of solutions that are compatible with observations

Ensemble: Viscosity Prefactors

	Accepted Value	Ensemble Solution Value
A0	1.4250e-15	1.4258668271145713e-15
A1	1.4250e-15	1.424265434107675e-15
A2	1.0657e-18	1.0657438291044843e-18
B0	0.5e-20	5.001940957603892e-21
B1	1.4250e-15	1.4226464693482968e-15
B2	1.0657e-18	1.1198936860607321e-18



→ Converged to accepted value within log space step size of 0.05

Next Steps

- ◇ Add noise to “observation” data to better simulate real-world data
- ◇ Investigate other rheological parameters

Acknowledgements

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References

- ◆ Computations were done using the ASPECT code version 2.3.0, see (<https://doi.org/10.5281/ZENODO.5131909>; Kronbichler et al., 2012, Heister et al., 2017)
- ◆ Fraters, M., Thieulot, C., van den Berg, A., and Spakman, W.: The Geodynamic World Builder: a solution for complex initial conditions in numerical modeling, *Solid Earth*, 10, 1785–1807, <https://doi.org/10.5194/se-10-1785-2019>, 2019.