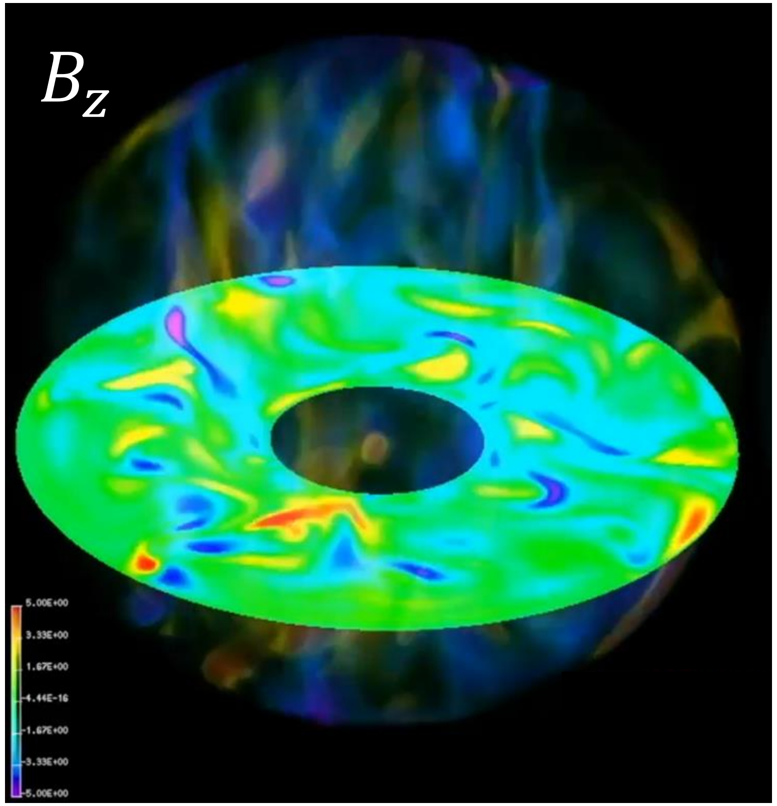
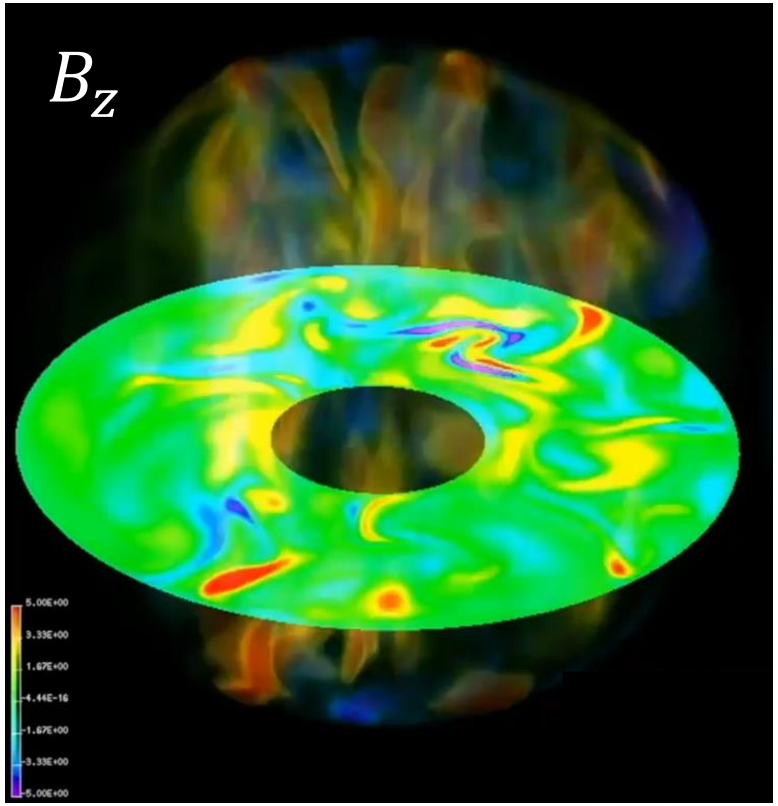
Energy transfer among flow and magnetic fields with different equatorial symmetry during the dipole reversal in a geodynamo simulation





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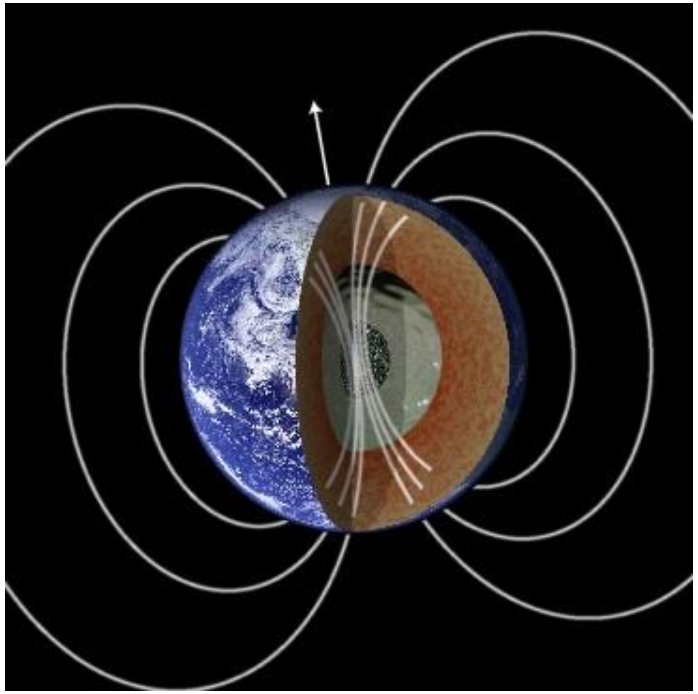
[1]Tohoku University, [2]University of California, Davis, [3]Tokyo Institute of Technology

Introduction

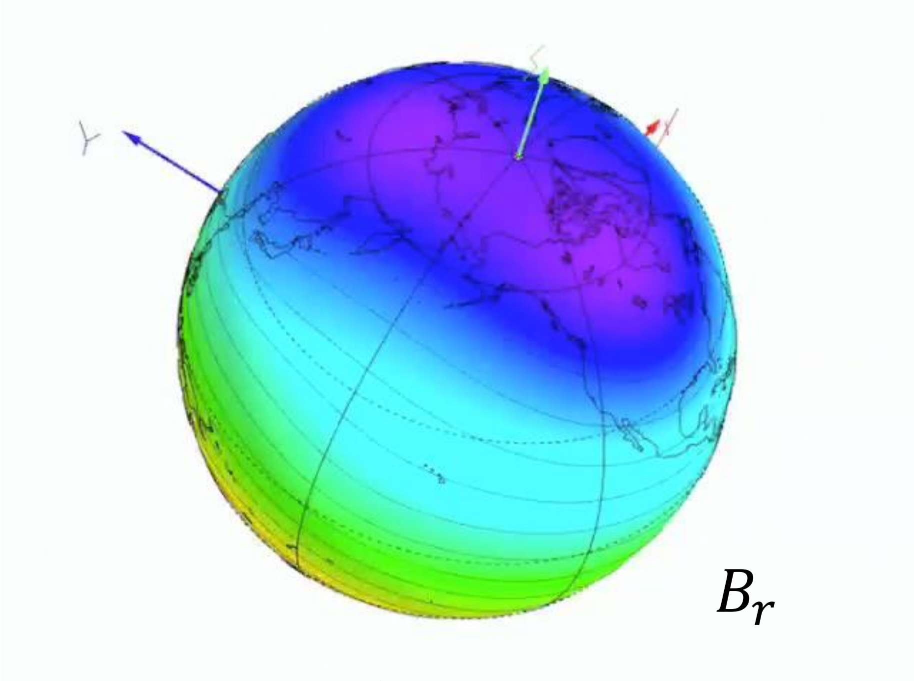
Model

Results

Spatial structure of geomagnetic field



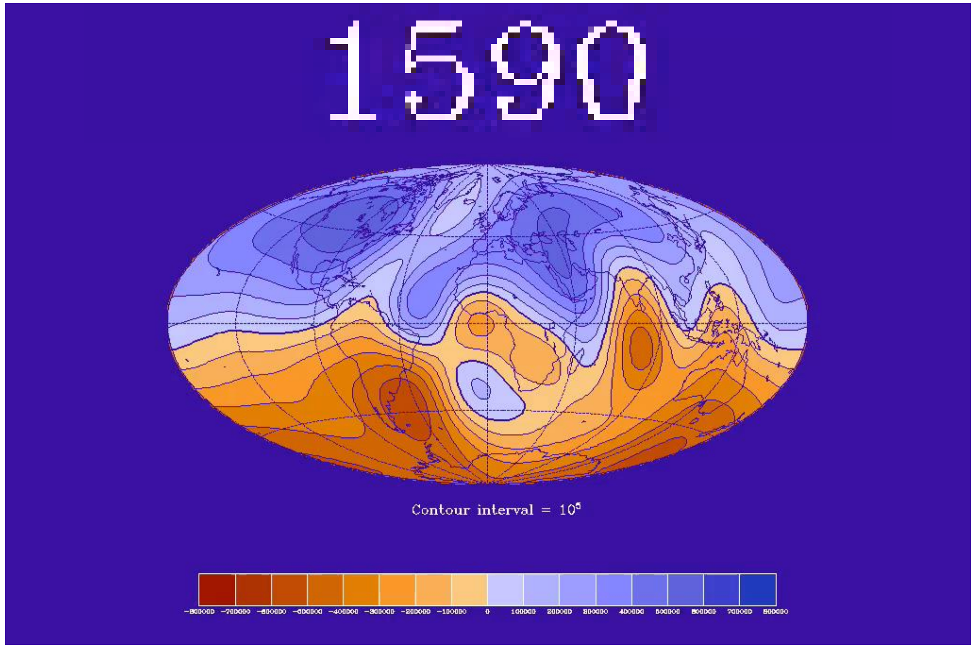
Nearly axisymmetric dipole field



Radial magnetic field on ground and CMB
CMB: Core-Mantle Boundary

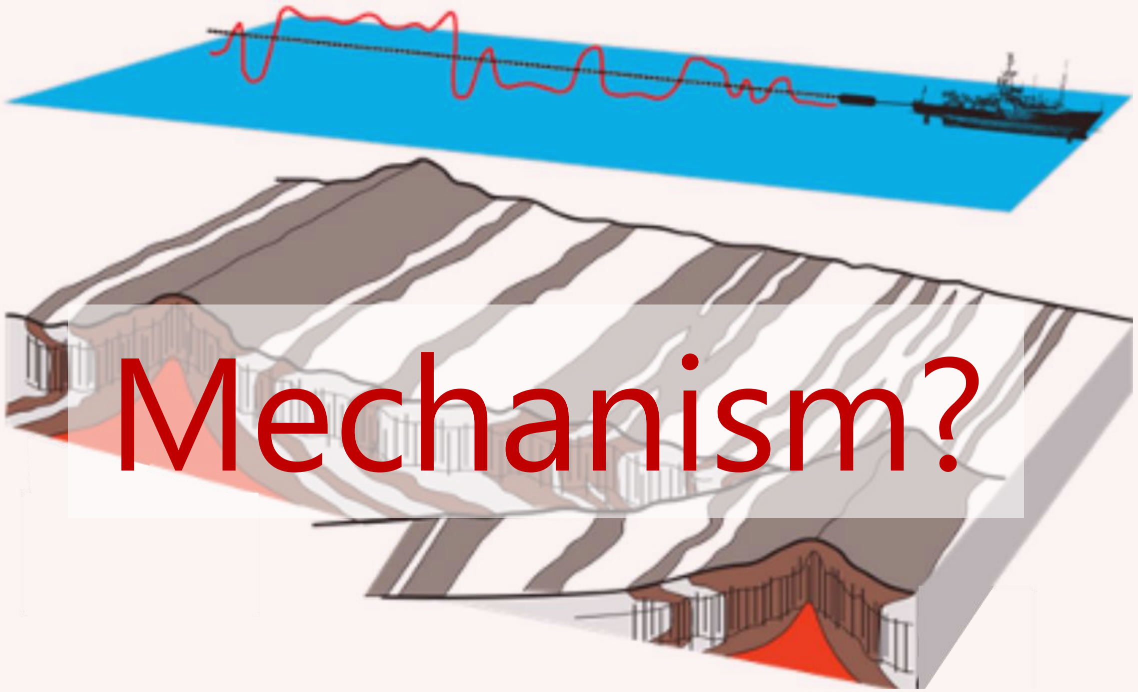
Includes multipolar components

Evolution of geomagnetic field



Evolution of the radial magnetic field at CMB
(Finlay and Jackson)

Secular variations



(Gee and Kent, n.d.)

Polarity reversal

What is the mechanism for reversals ?

➢ We don't understand very well.

Very limited information
from paleomagnetic observation

- Difficulties in simultaneous & multiple point observations
- Complexity of the magnetization process

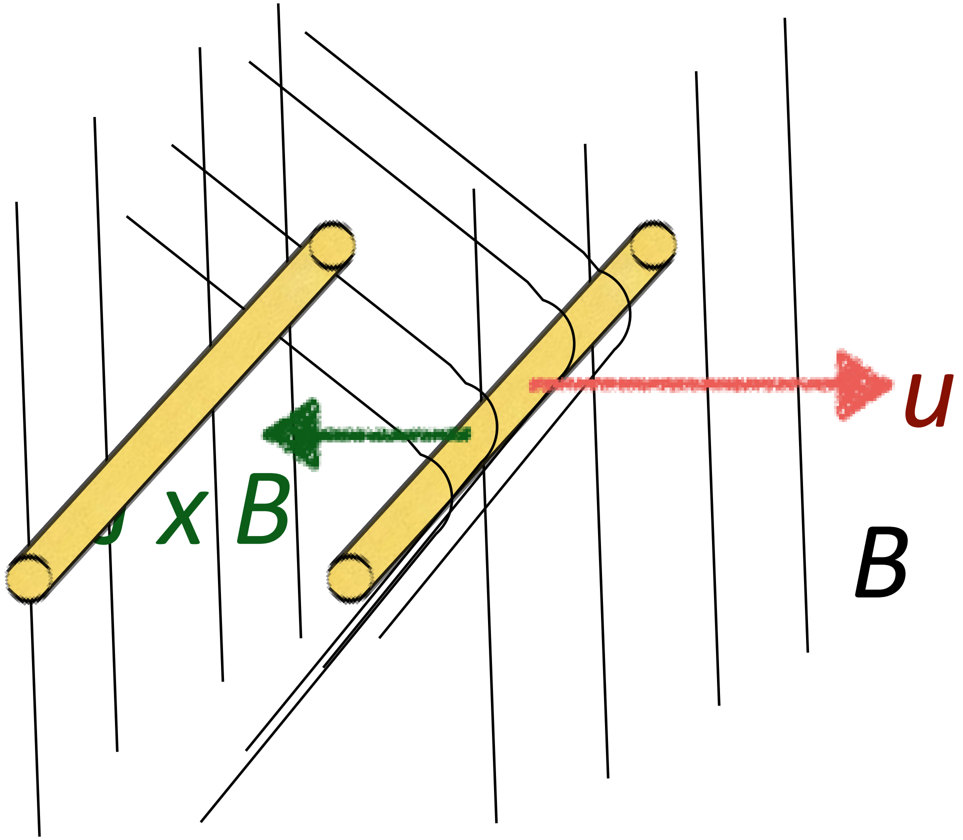
In the first place…

- It is impossible to obtain direct observation of the flow in the outer core

The generator of the geomagnetic field

Dynamo model

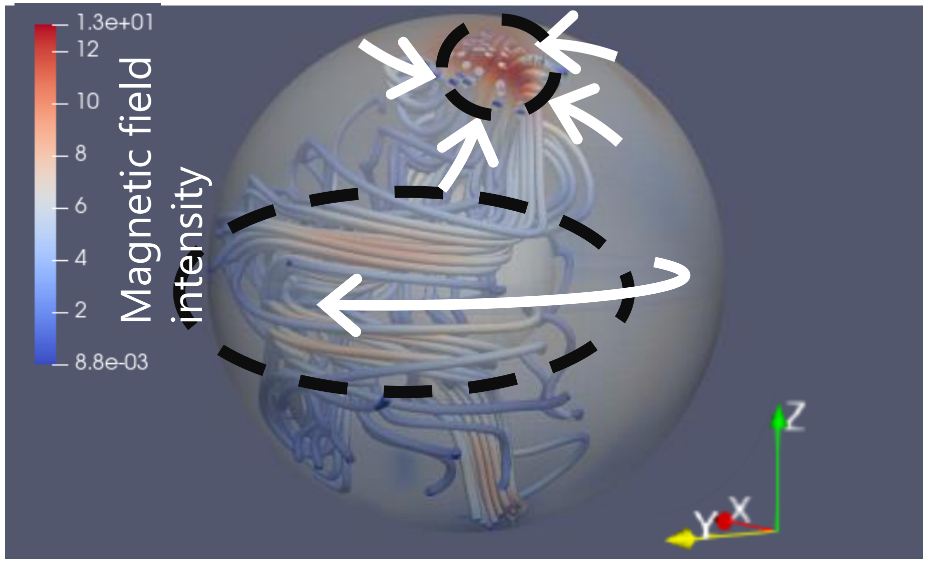
-The simplest magnetic induction-



If there is
no magnetic diffusivity,
magnetic field line moves
with the fluid motion

Dynamo model

-Induction in a conductive fluid-

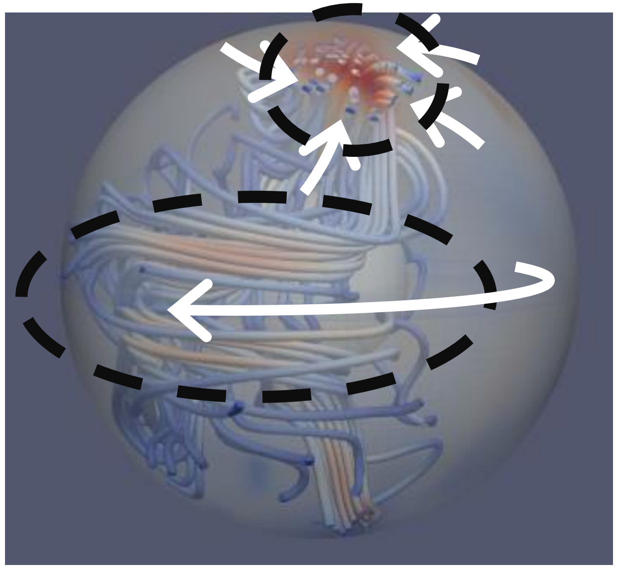


Magetic field lines are…

- Twisted
- Stretched
- Concentrated

by fluid motion

Numerical dynamo model



Represents

- Twists
- Stretch
- Concentration

by fluid motion

Governing equation

- Momentum equation

Coliolis

$$E\left(\frac{\partial \boldsymbol{u}}{\partial t}+(\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u}-\boldsymbol{\nabla}^2\boldsymbol{u}\right)+2\boldsymbol{e}_z\times\boldsymbol{u}+\boldsymbol{\nabla}P=Ra_f\frac{\boldsymbol{r}}{r_o}T+\frac{1}{Pm}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}$$

Inertia Viscous Buoyancy Lorentz

- Magetic induction equation

$$\frac{\partial \boldsymbol{B}}{\partial t}=\nabla\times(\boldsymbol{u}\times\boldsymbol{B})+\frac{1}{Pm}\nabla^2\boldsymbol{B}$$

- Heat conduction equation

$$\frac{\partial T}{\partial t}+\boldsymbol{u}\cdot\nabla T=\frac{1}{Pr}\nabla^2 T$$

- Equation of continuity

$$\nabla\cdot\boldsymbol{u}=0$$

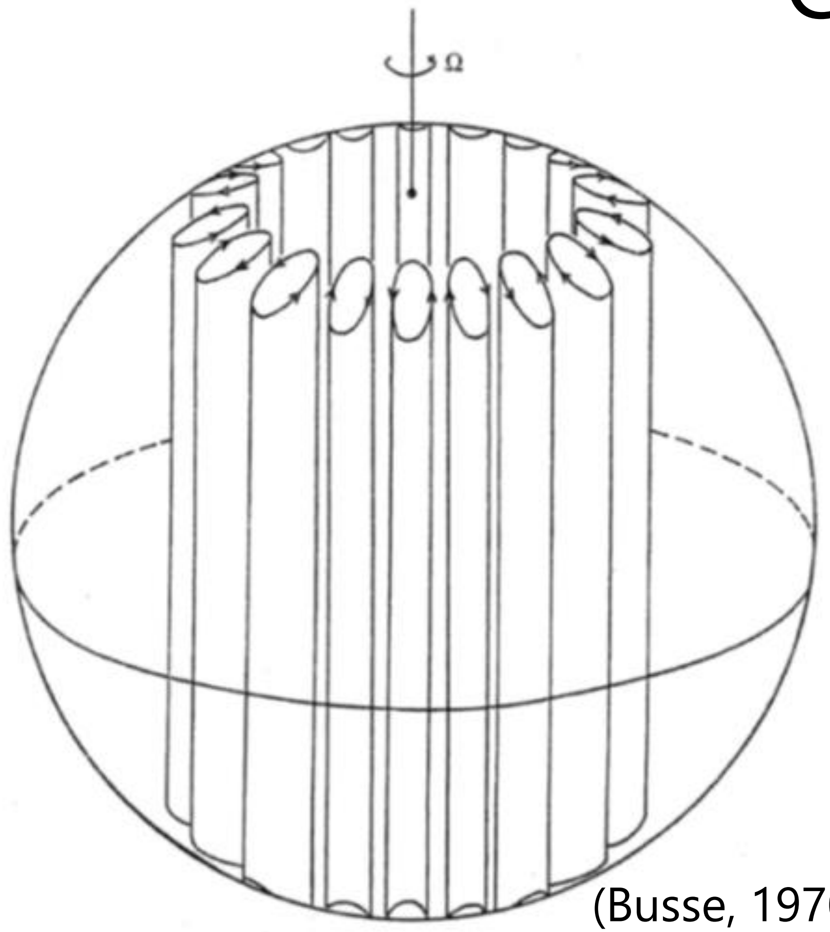
- Gauss's law

$$\nabla\cdot\boldsymbol{B}=0$$

- Non dimensional parameters

$$Ra_f=\frac{\alpha g_o\beta_o D^2}{\nu\Omega},\ Pm=\frac{\nu}{\eta},\ E=\frac{\nu}{\Omega D^2},\ Pr=\frac{\nu}{\kappa}$$

Convective structure
-Columnar convection-

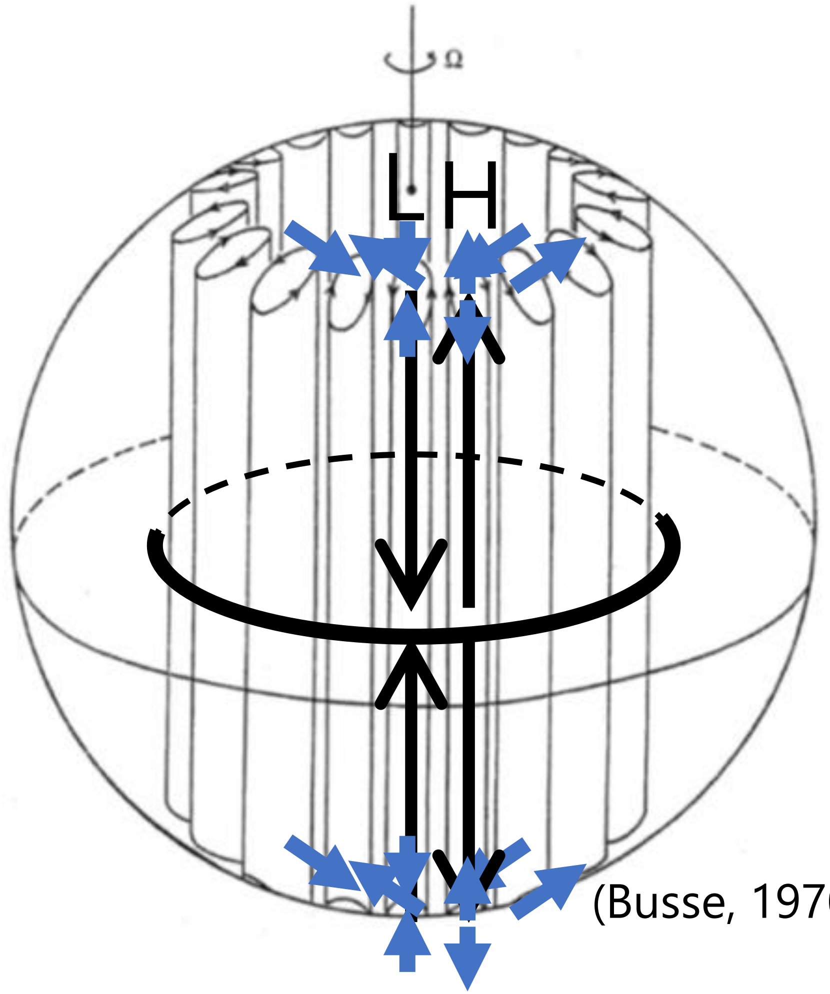


(Busse, 1970)

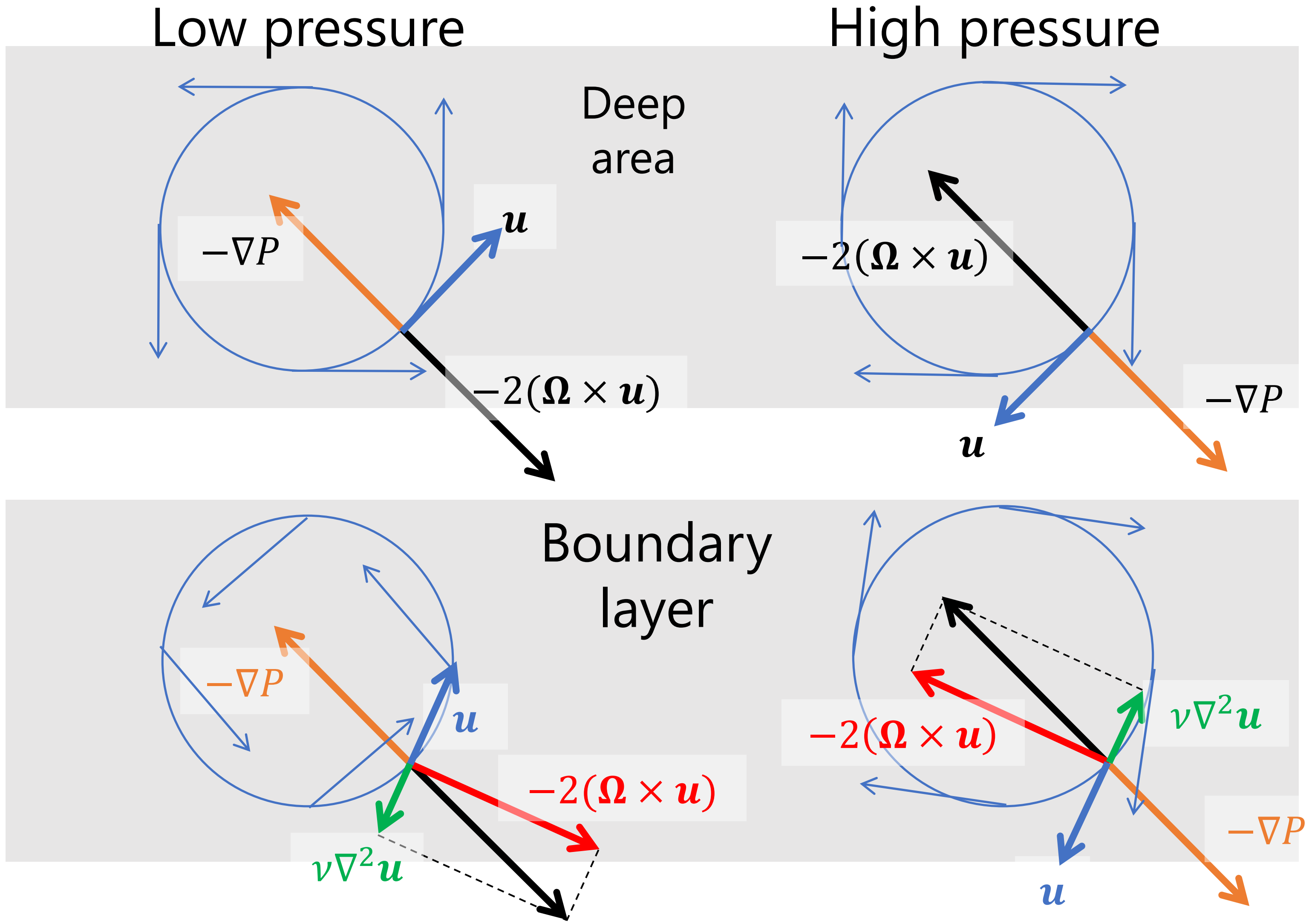
The flow cannot change easily
along rotaion axis in rotating system

Convective structure

-Pumping in columns-

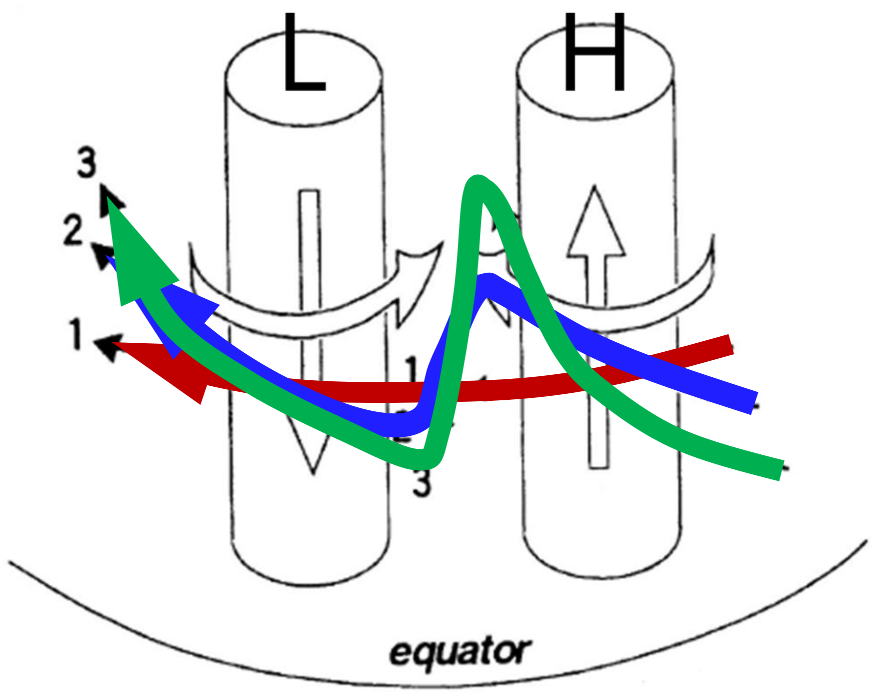


(Busse, 1970)

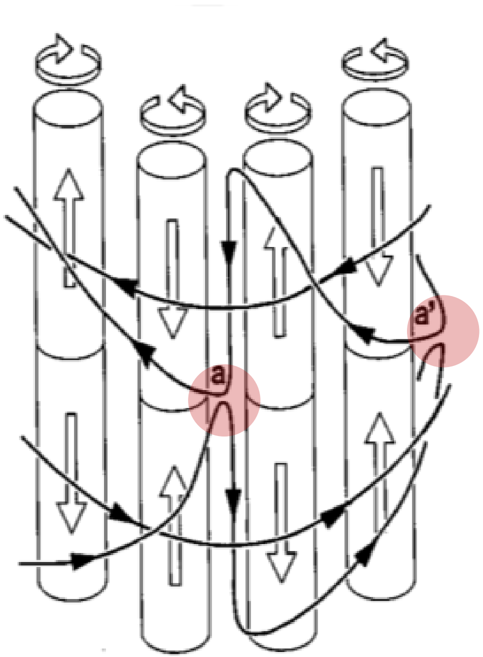


How does this flow generate magnetic field ?

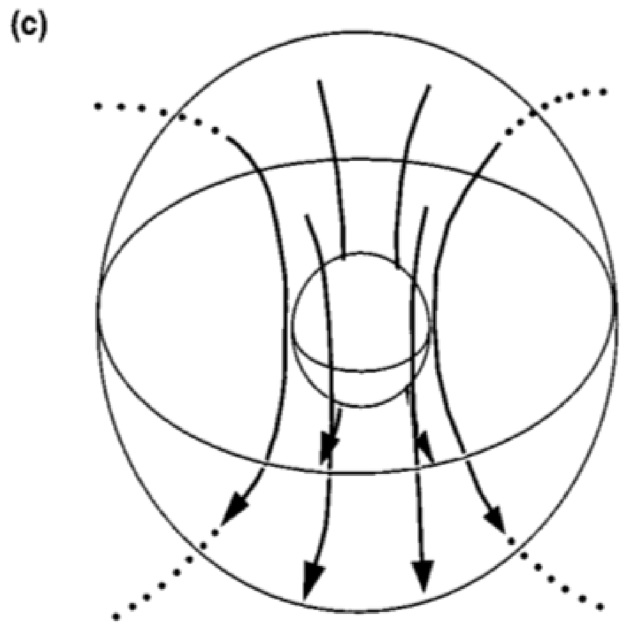
Dipolar field generation



1. Zonal field in northern hemispher
Distorted by **2. Inward flow**
3. Axial flow



Northern&
Southern hemispher
→ Reconection



Dipolar field
(Kageyama and Sato, 1997)

Columnar & equatorially symmetric convection effectively generates a dipole magnetic field.

What happens during the dipolar reversal?

What happens during the dipolar reversal?

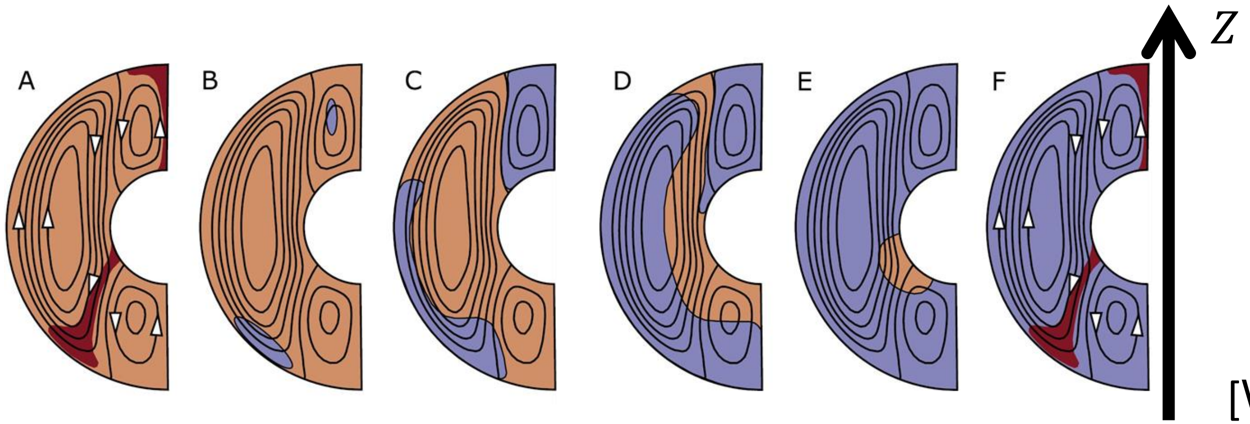
Equatorially antisymmetric flow is important.

[Wicht & Olson, 2004] [Takahashi et al., 2007]

Reversed magnetic field generation

Growth of antisymmetric flow

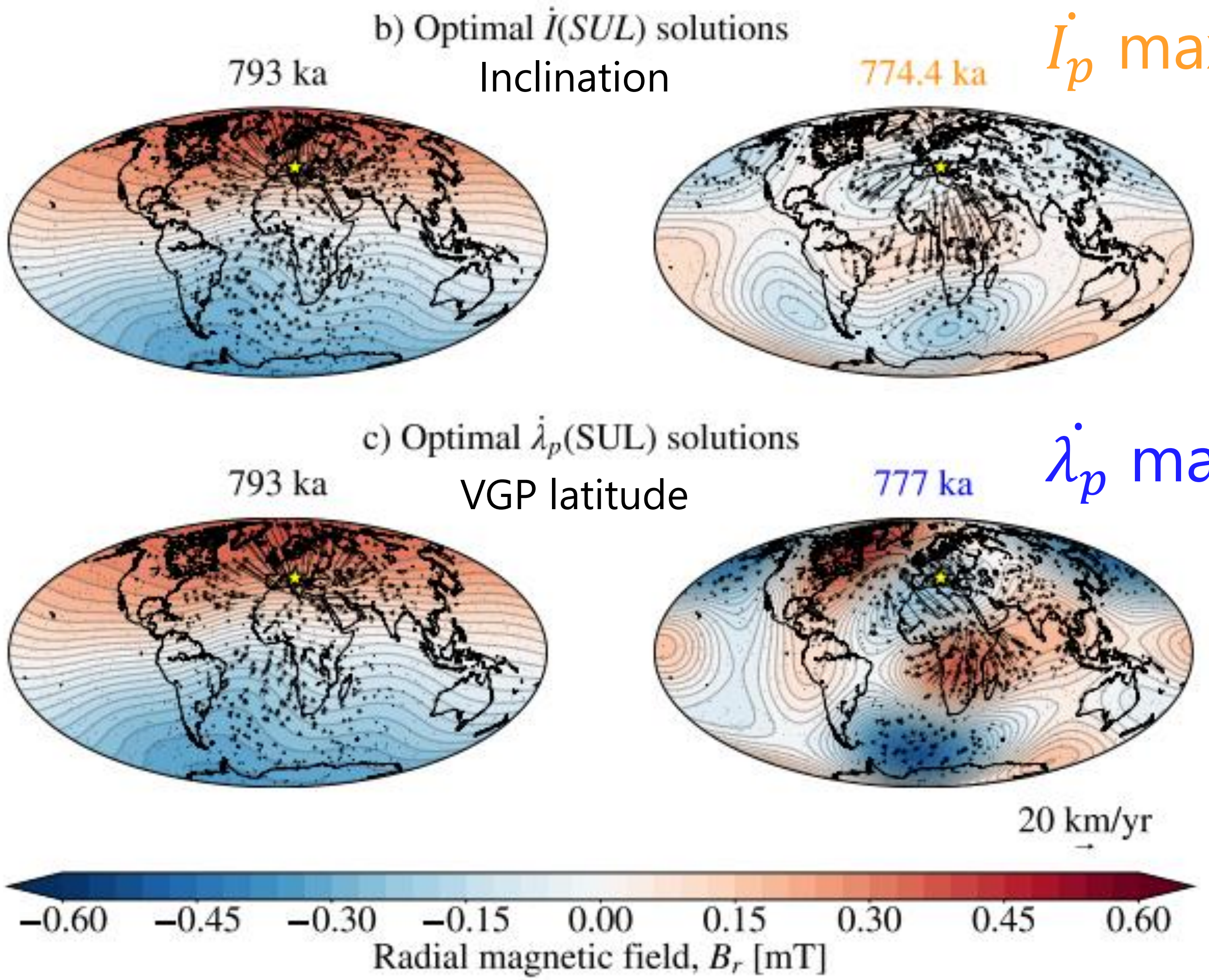
Transport by antisymmetric flow

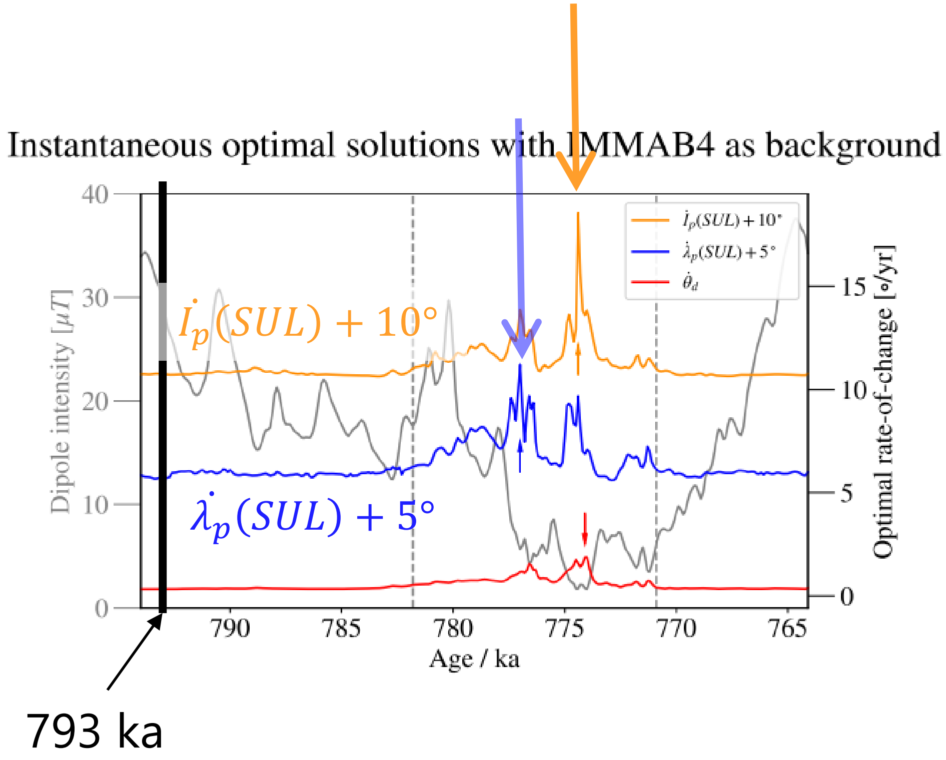


[Wicht & Olson, 2004]

Core-surface flows during the M-B reversal

(Maffei et al., 2021)





Flows crossing the equator are stronger during the reversal.

VGP: Virtual Geomagnetic Pole

How does the antisymmetric flow grow?

How does the antisymmetric flow grow?

- Momentum equation

$$E\left(\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\nabla})\boldsymbol{u} - \boldsymbol{\nabla}^2\boldsymbol{u}\right) + 2\boldsymbol{e}_z \times \boldsymbol{u} + \boldsymbol{\nabla}P = Ra_f \frac{\boldsymbol{r}}{r_o} T + \frac{1}{Pm}(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}$$

Inertia Buoyancy Lorentz force

Terms that may contribute to the reversal

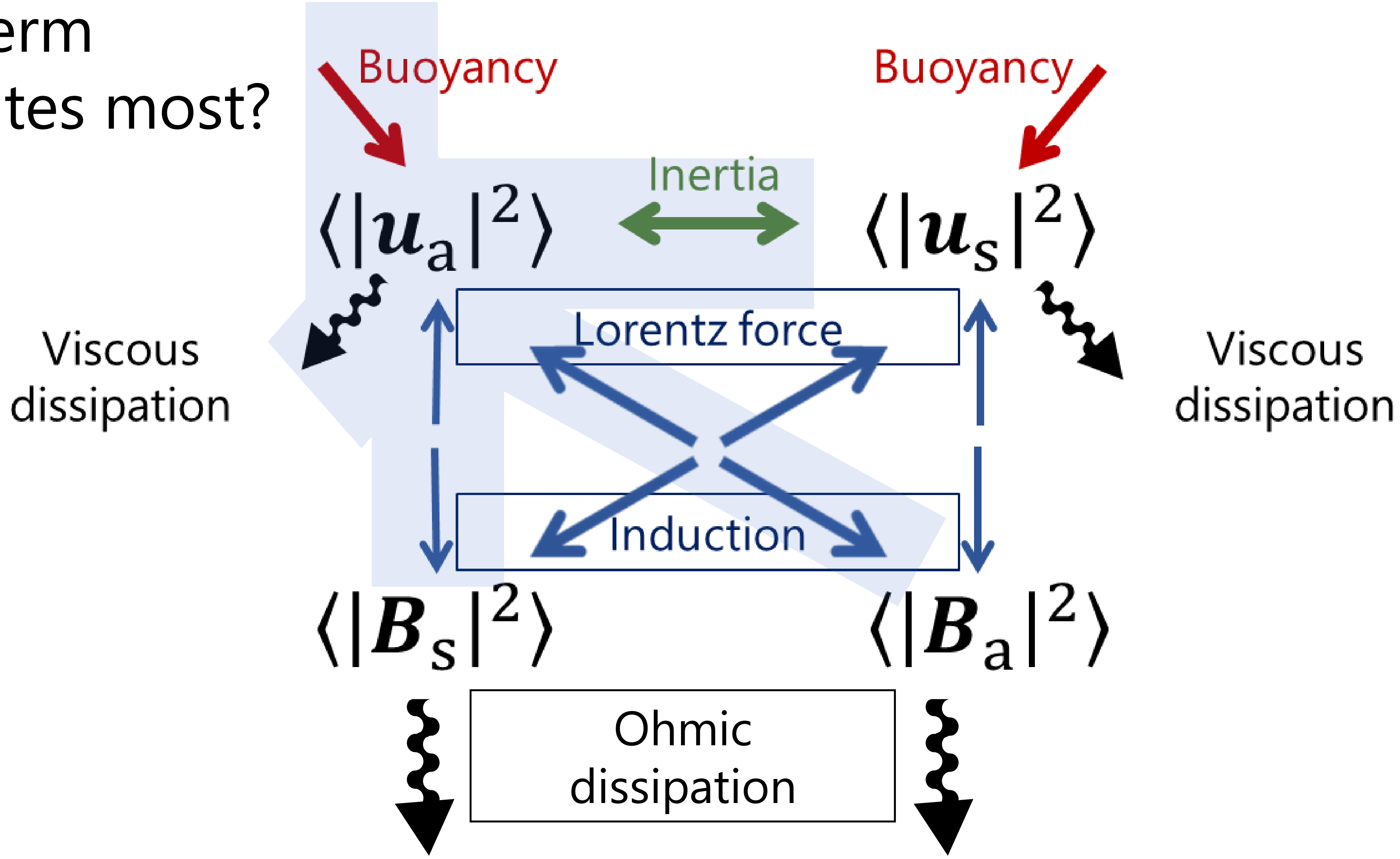
Term	Basis	Author
Lorentz force	Energy conversion	[Nishikawa & Kusano, 2007]
Inertia	Rossby number	[Olson & Christensen, 2006]
Buoyancy	Rayleigh number	[Sreenivasan et al., 2014]

※Rossby number: Inertia / Coriolis

Relative assessment is not reported.

Approach: Energy flux analysis

Which term contributes most?



◆Purpose:

To clarify the drivers of antisymmetric flow.

◆Approach:

To calculate the energy flux into the antisymmetric flow.

In addition …

- ◆ To make sure that
  the antisymmetric flow contributes to reversals.

Introduction

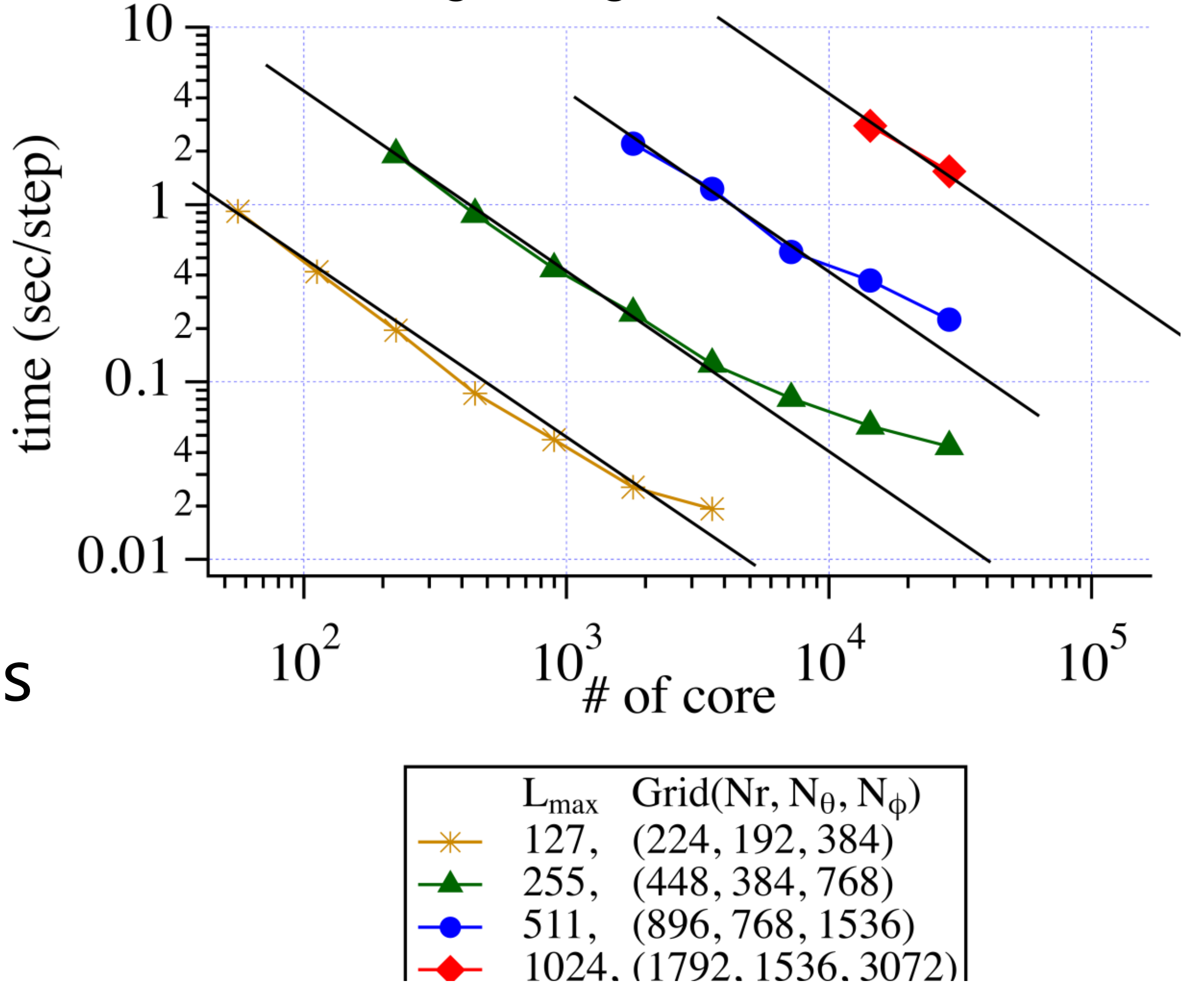
Model

Results

Dynamo simulation code “Calypso”

- Open source code for
  a numerical dynamos
- Released and maintained by CIG
- 90 pages of documentation
- Confirmed scaling to 3×10^4 cores
- Support various boundary conditions

Strong scaling on TACC Frontera



https://geodynamics.org/cig/software/calypso/

Dynamo simulation code “Calypso”

Numerical method

- Radial discretization:

  Second order Finite difference method

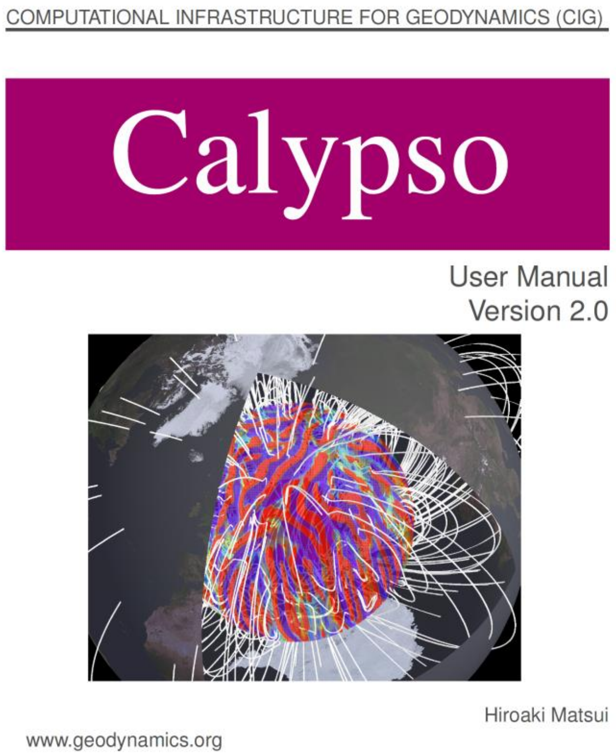
- Horizontal discretization:

  Spherical harmonics expansion

- Time stepping:

  the linear diffusive terms … the Crank-Nicolson method

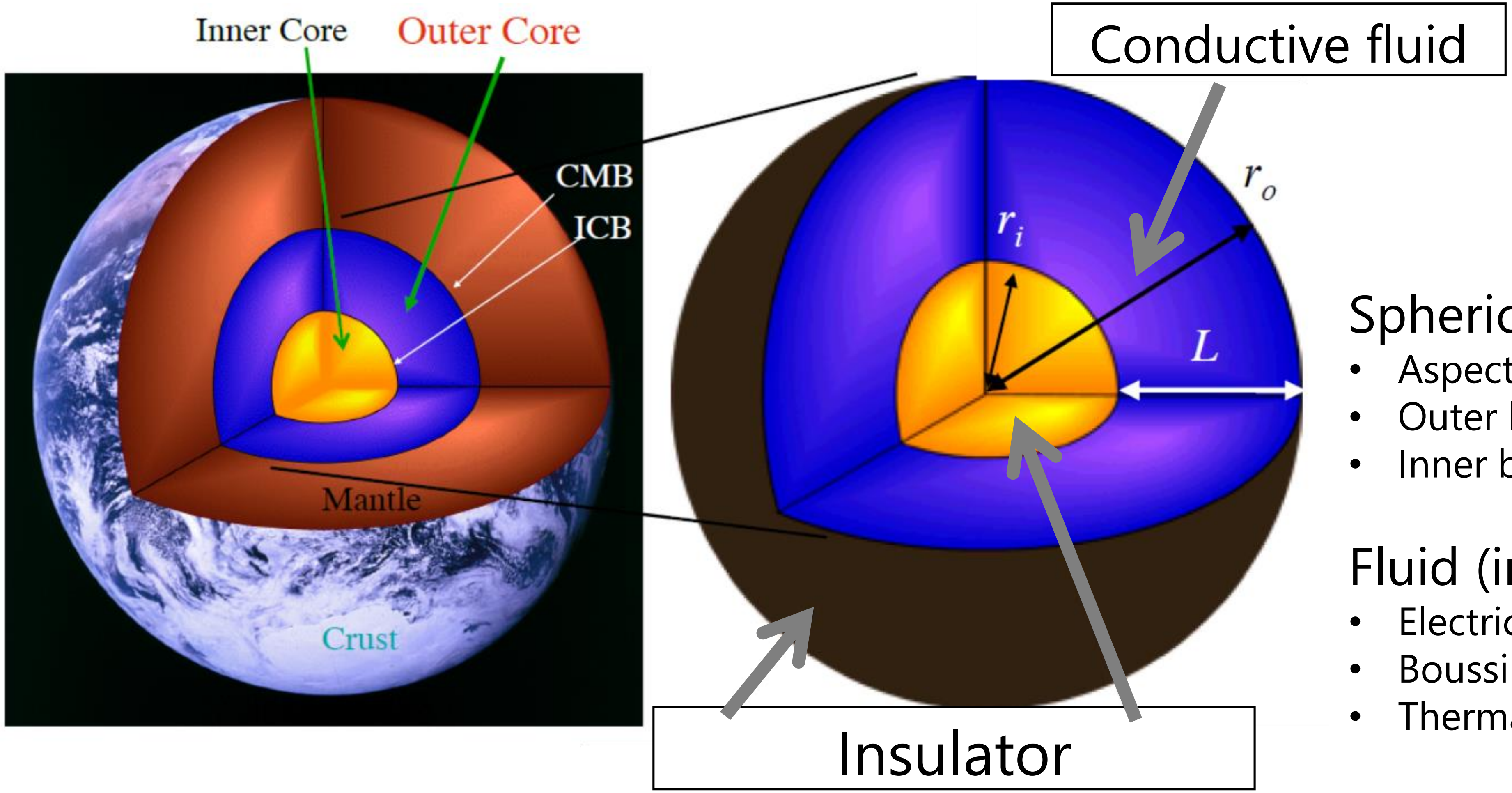
  the other terms … second order Adams-Bashforth method



Ver. 2.0 will be released soon.

https://geodynamics.org/cig/software/calypso/

Domain & Fluid model



Spherical shell (outer core)

- Aspect ratio $r_i/r_o = 0.35$
- Outer boundary (CMB) } corotating
- Inner boundary (ICB) }

Fluid (iron liquid)

- Electrically conductive
- Boussinesq
- Thermally driven

Settings

◆Boundary conditions

- Velocity: Non-slip ($\boldsymbol{u} = 0$)
- Magnetic Field: Insulating ($(\nabla \times \boldsymbol{B})_r = 0$)
- Temperature: Fixed heat flux

  at CMB $\beta_o = -\frac{dT}{dr}|_{r=r_o} = 0.4225$

  at ICB so that the heat flow is balanced.

◆Initial conditions

- Velocity: $\boldsymbol{u} = 0$
- Magnetic Field: A seed dipole ($Y_1^0(\theta, \phi) + Y_1^1(\theta, \phi)$)
- Temperature: Sectorial mode ($Y_4^4(\theta, \phi)$)

◆Spatial resolution

- Radial grid number: N_r = 225
- Truncation degree: L_{max} = 127

Dimensionless numbers

Paramber		Ratio	Setting	Earth
E	$\frac{\nu}{\Omega D^2}$	Viscous/Coriolis force	6×10^{-4}	10^{-15}
Ra_f	$\frac{\alpha g_o \beta_o D^2}{\nu \Omega}$	Buoyancy/Coriolis force	2000	10^9
Pr	$\frac{\nu}{\kappa}$	Viscous/Thermal diffusivities	1	1
Pm	$\frac{\nu}{\eta}$	Viscous/Magnetic diffusivities	5	10^{-5}

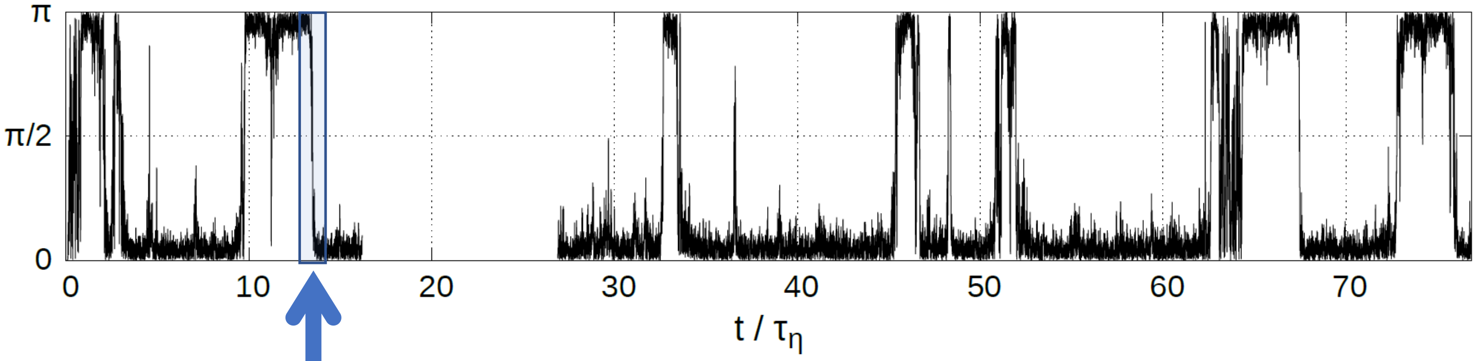
Based on Sreenivasan et al. (2014)

E: Ekman number Ra_f: Modified flux Rayleigh number Pm: Magnetic Prandtl number Pr: Prandtl number

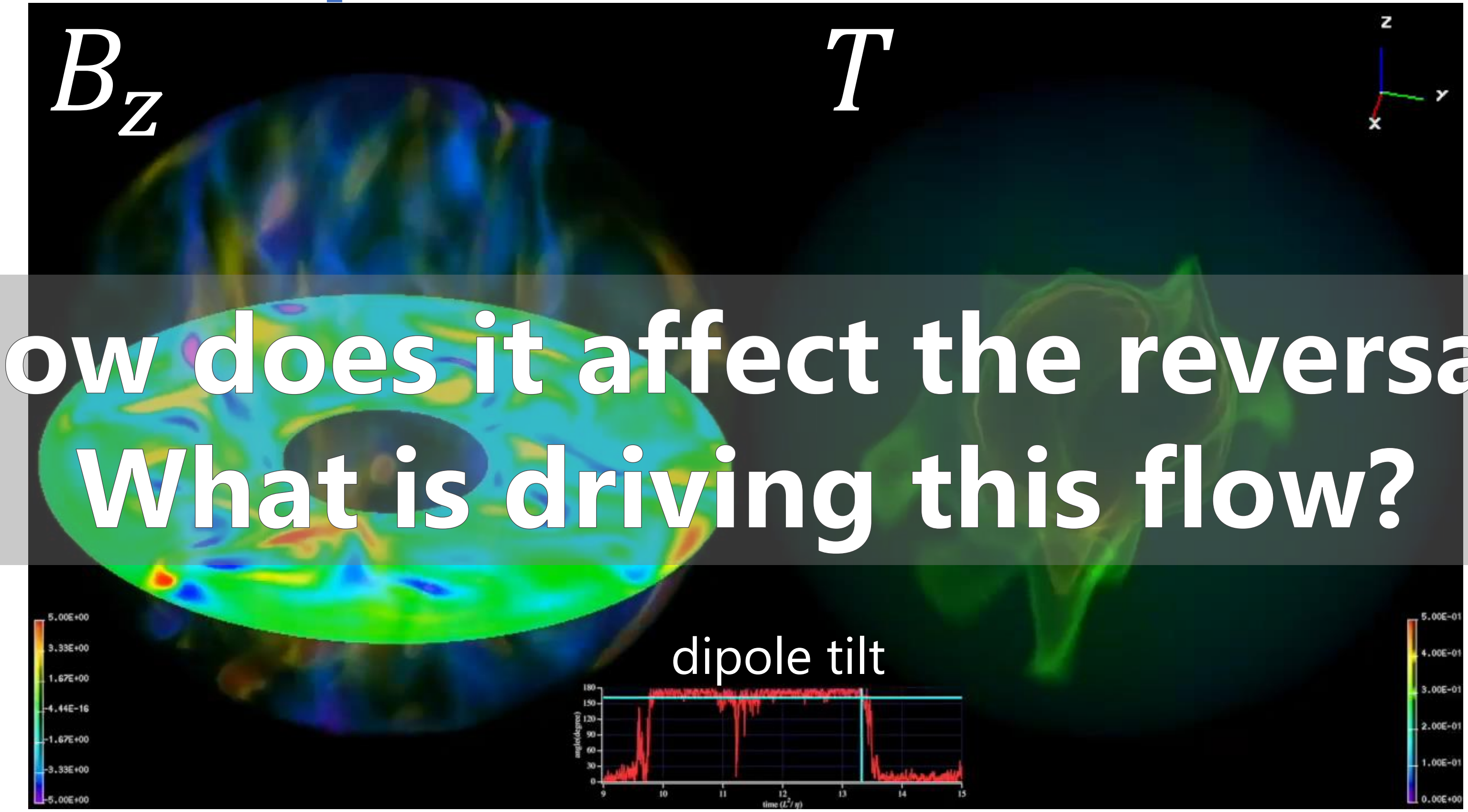
Introduction
Model
Results

Upwelling plume during the reversal

dipole tilt

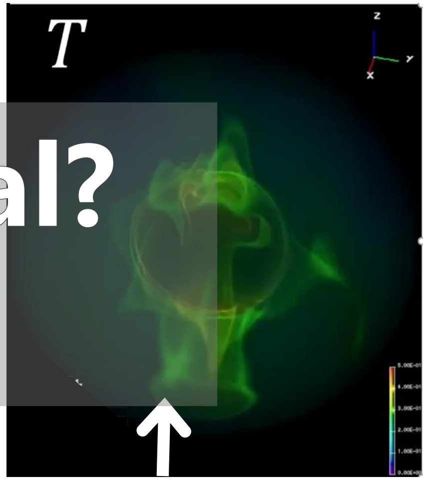


12 reversals



How does it affect the reversal?
What is driving this flow?

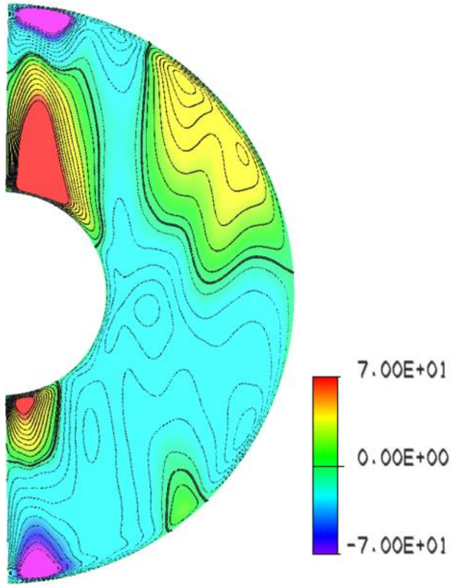
During the reversal



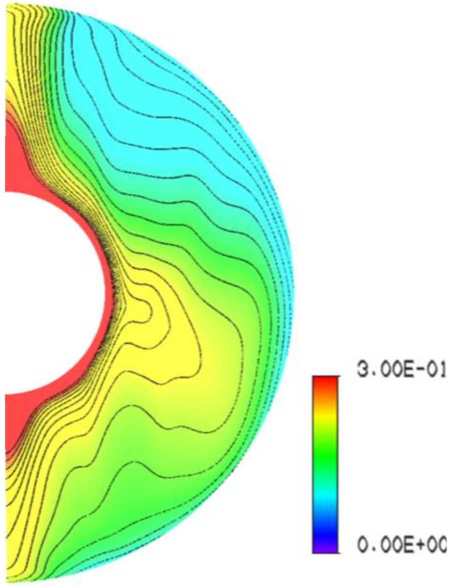
Upwelling plume

Equatorially antisymmetric zonal flow

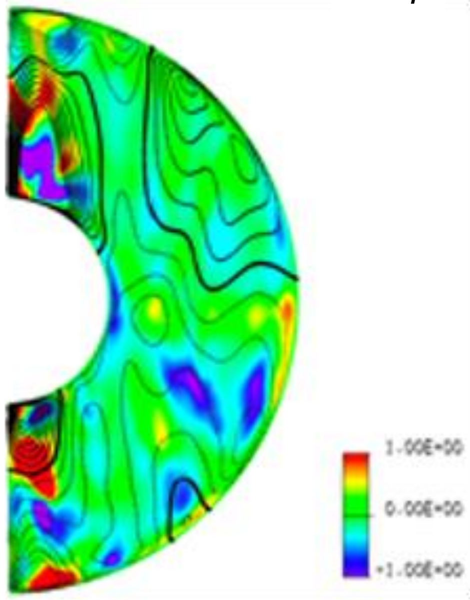
zonal flow u_φ



temperature T



Line: zonal flow u_φ
color: zonal magnetic field B_φ

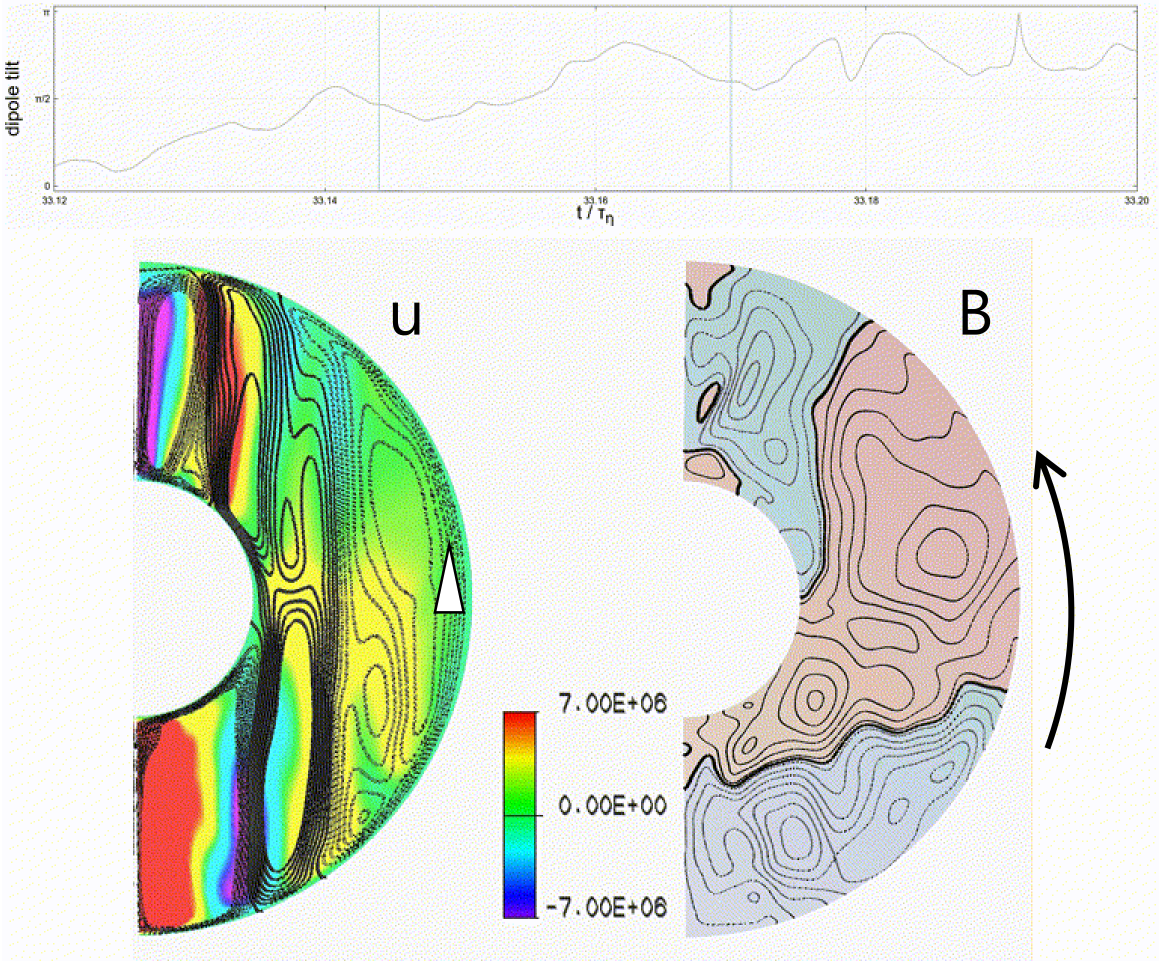


Line: zonal flow u_φ
color: Work of Lorentz force

Antisymmetric zonal flow
by thermal wind

Ω effect by
antisymmetric flow

Advection of reversed magnetic field



Line: streamlines of meridional circulation
color: Buoyancy flux

magnetic field lines of the poloidal field.

What is driving this flow?

Energy flux analysis ~Method~

$\boldsymbol{u}_s$ (Sym) energy equation

$$\frac{B^2}{2\mu_0}$$

$$\frac{\partial}{\partial t}\int \frac{\rho_0 u_s^2}{2}\, dV = \int [\, \rho_0 \alpha g_o T_s\, \boldsymbol{u}_s \cdot \hat{\boldsymbol{r}} + \boldsymbol{u}_s \cdot (\boldsymbol{J}\times\boldsymbol{B})_s - \boldsymbol{u}_s \cdot (\boldsymbol{\omega}_s \times \boldsymbol{u}_a) - \nu \rho_0 \omega_a^2 \,]\, dV$$

$$\frac{B^2}{2\mu_0}$$

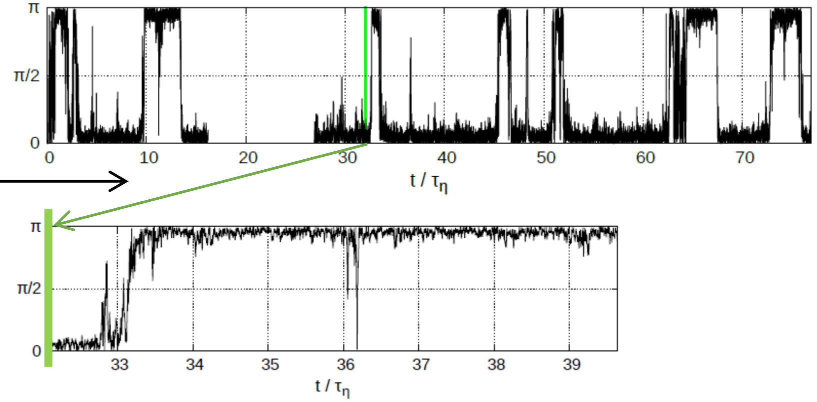
Energy transfer

$\boldsymbol{u}_a$ (Antisym) energy equation

$$\frac{\partial}{\partial t}\int \frac{\rho_0 u_a^2}{2}\, dV = \int [\, \rho_0 \alpha g_o T_a\, \boldsymbol{u}_a \cdot \hat{\boldsymbol{r}} + \boldsymbol{u}_a \cdot (\boldsymbol{J}\times\boldsymbol{B})_a + \boldsymbol{u}_s \cdot (\boldsymbol{\omega}_s \times \boldsymbol{u}_a) - \nu \rho_0 \omega_s^2 \,]\, dV$$

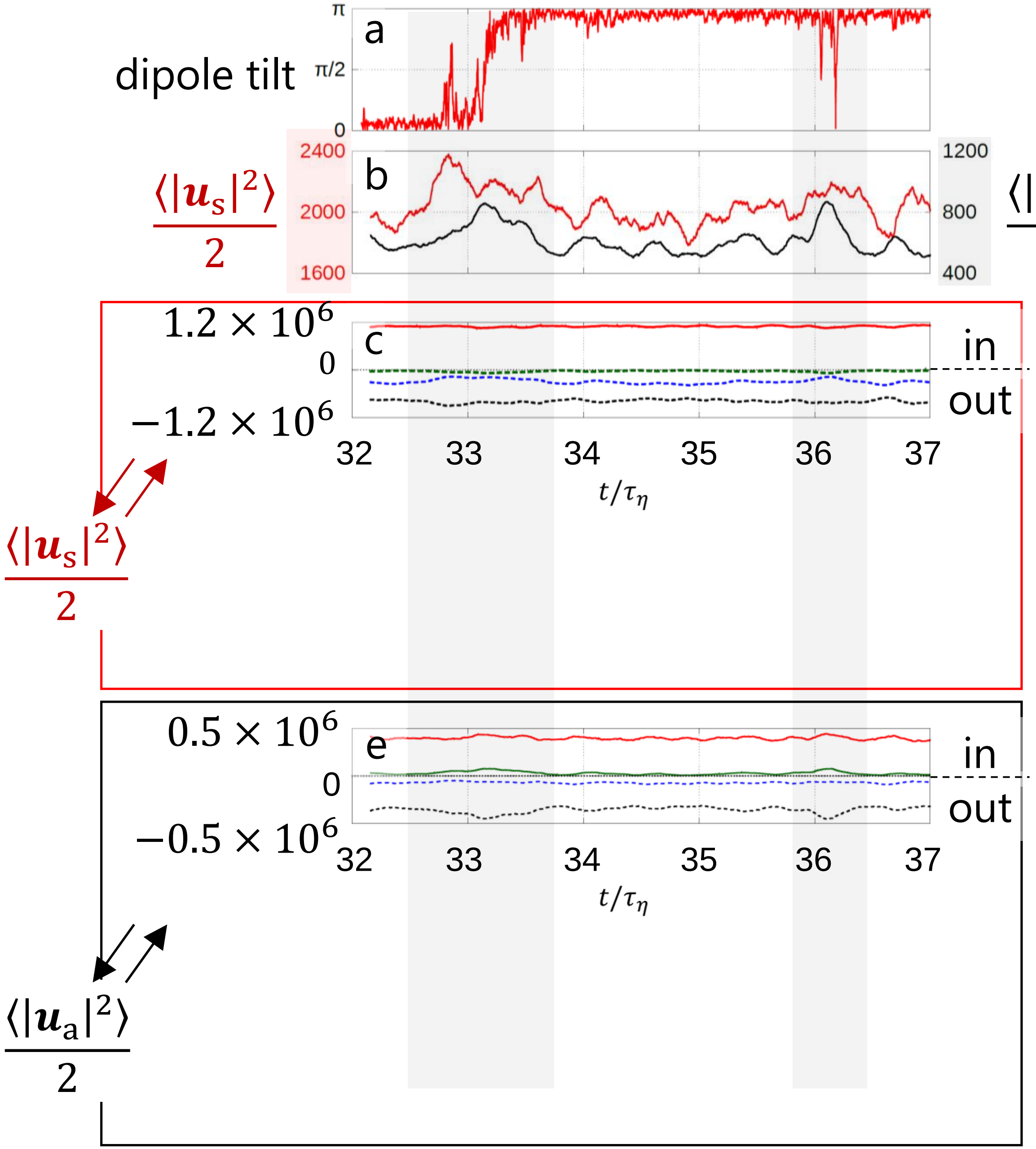
Buoyancy flux | Work of Lorentz force | Inertia | Viscous dissipation

Add calculation routines to Calypso



Analyze the behavior during reversals

Energy flux analysis ~Outline~

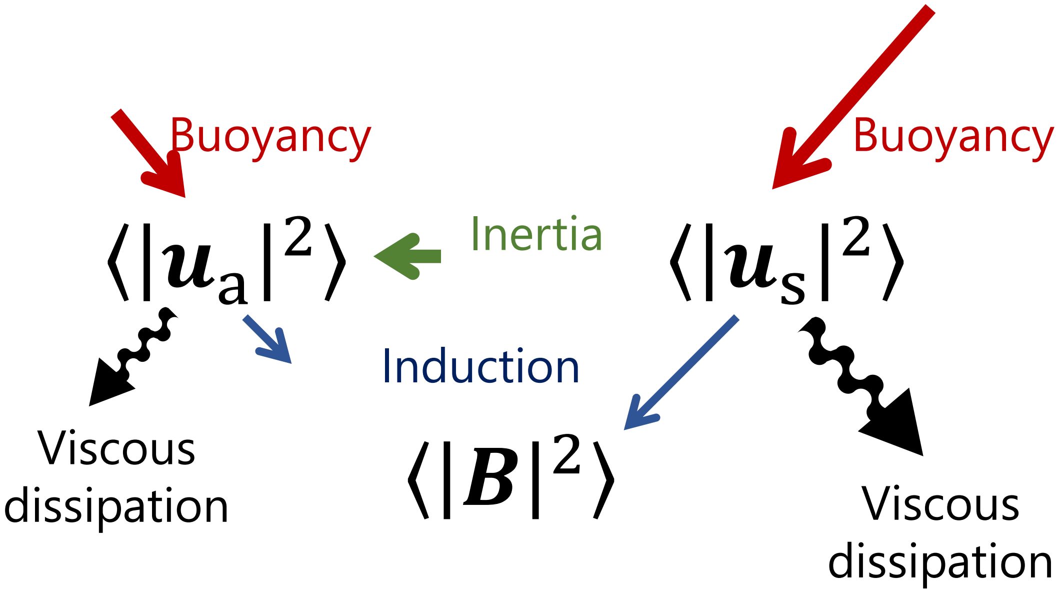


Buoyancy flux

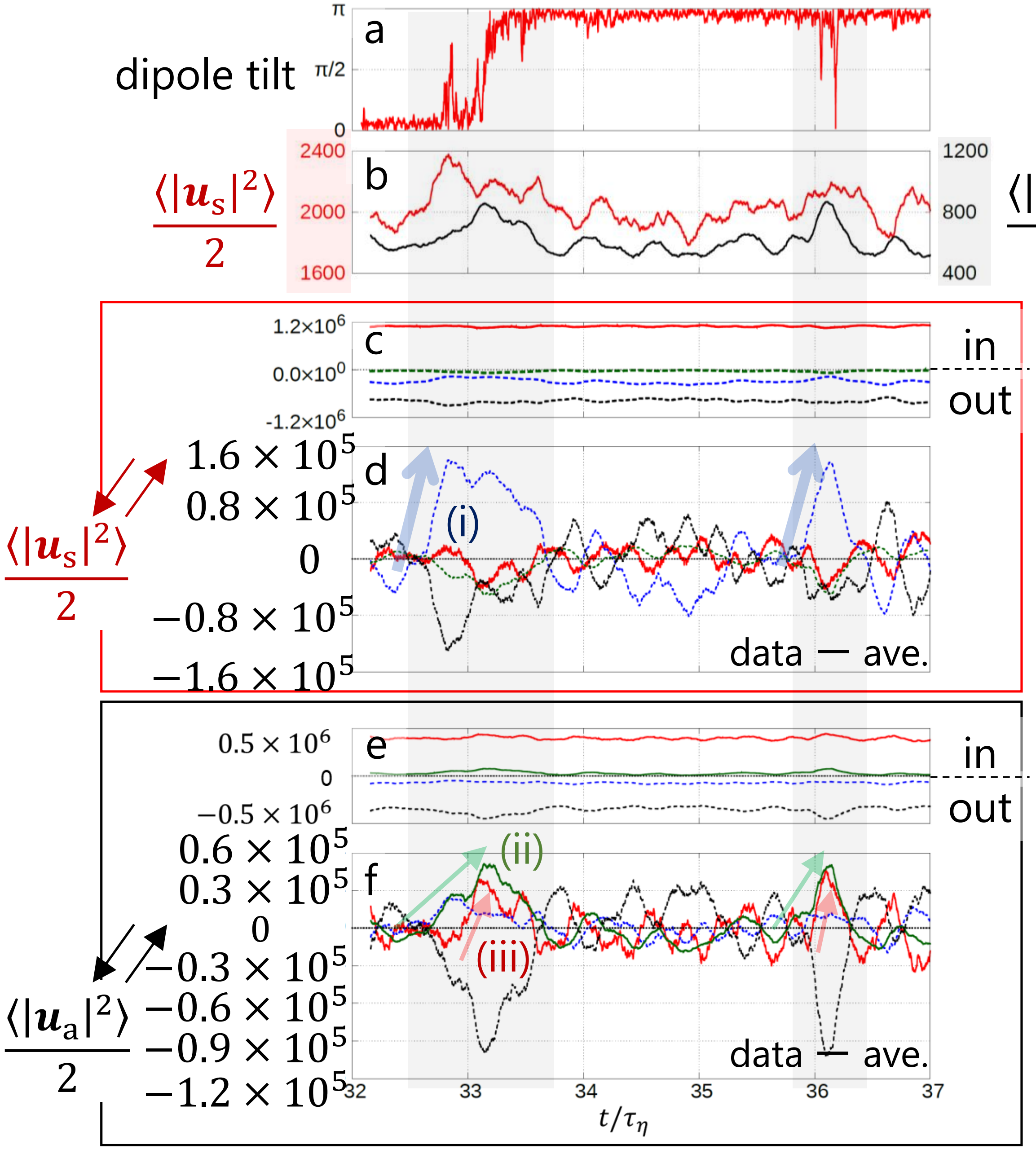
Inertia

Work of Lorentz

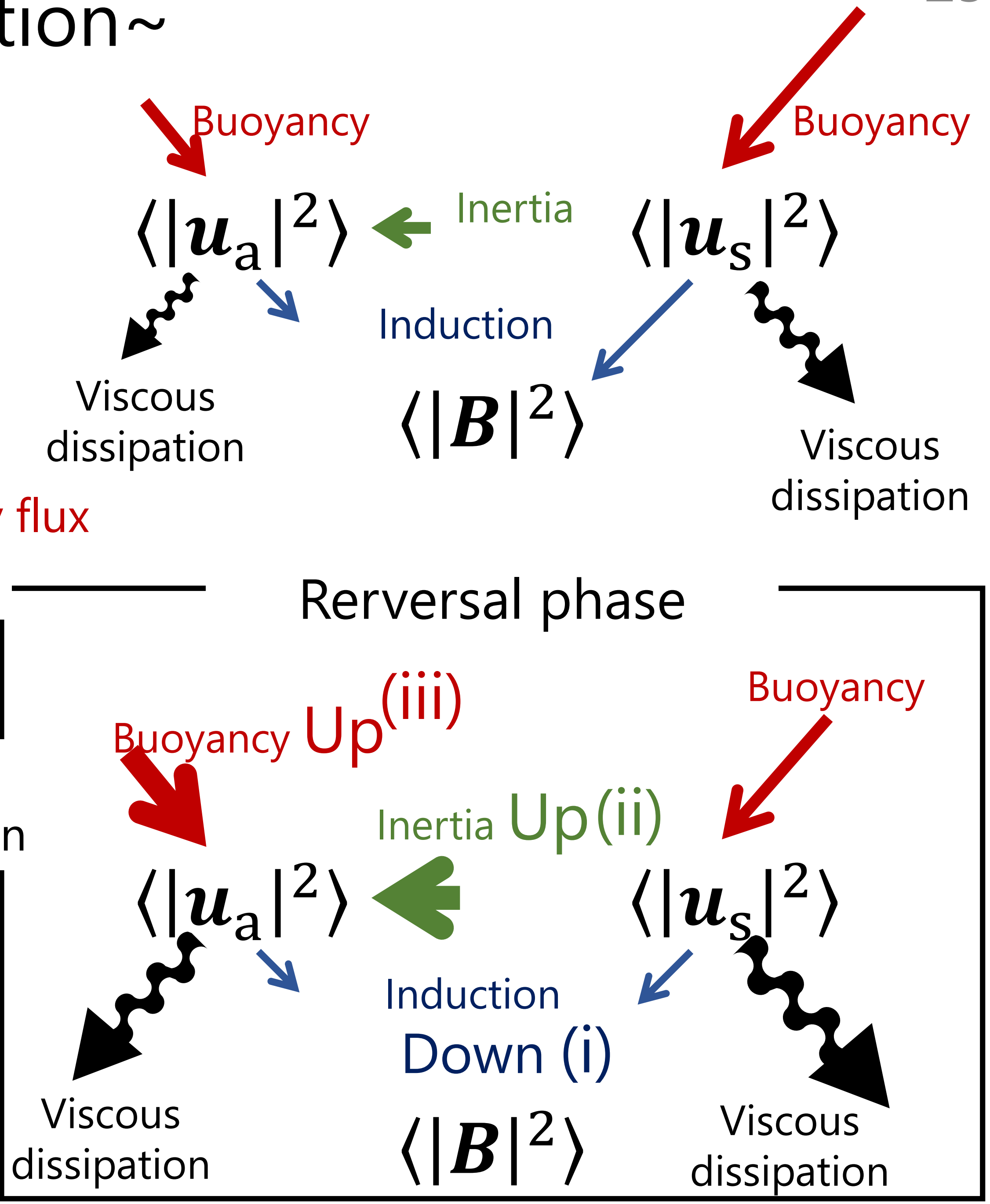
Viscous dissipation



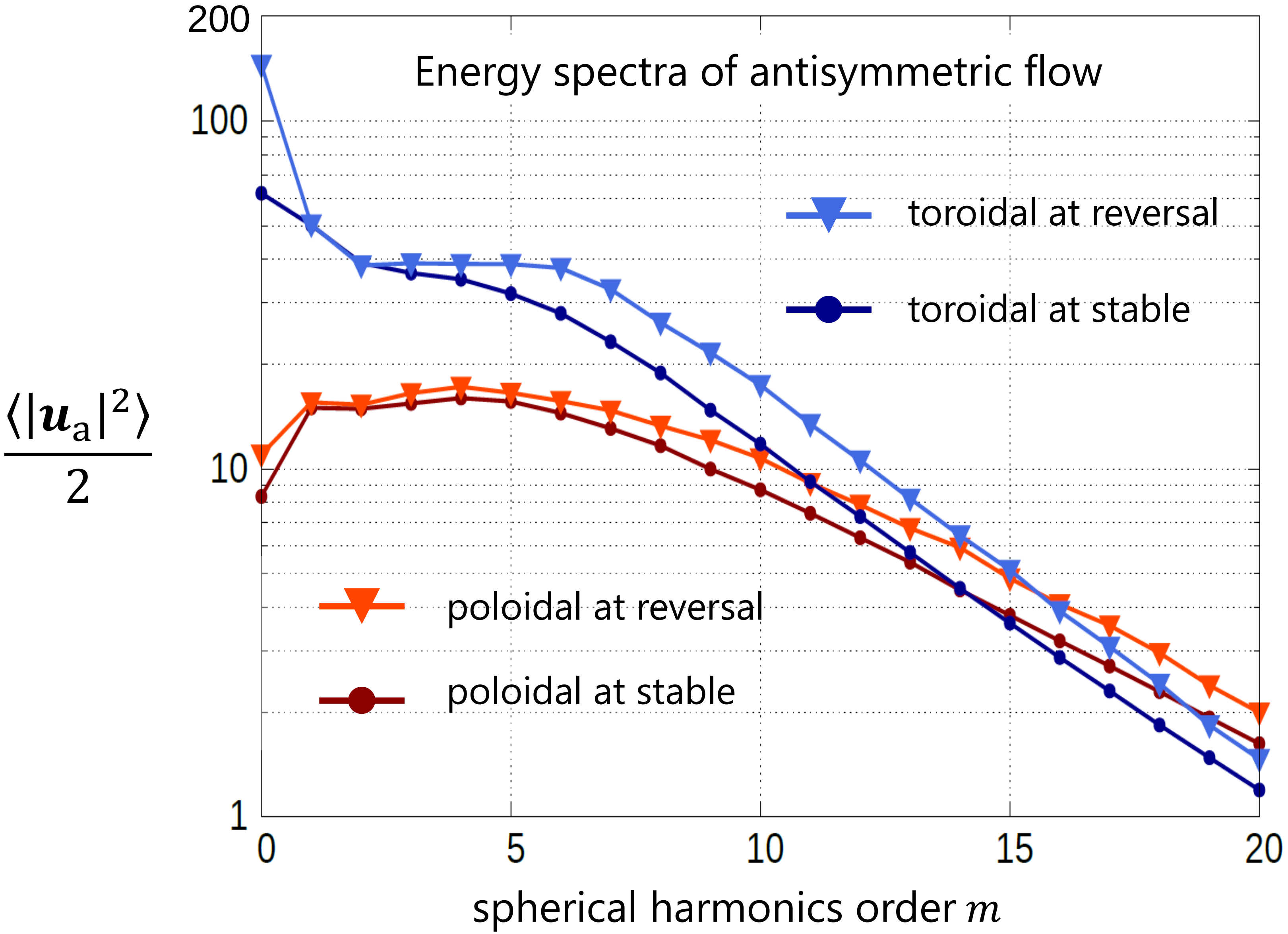
Energy flux analysis ~Perturbation~

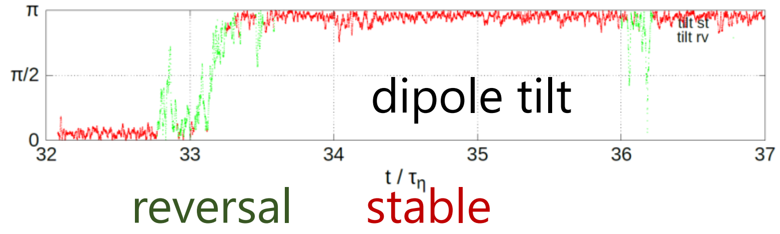


Buoyancy flux
Inertia
Work of Lorentz
Viscous dissipation



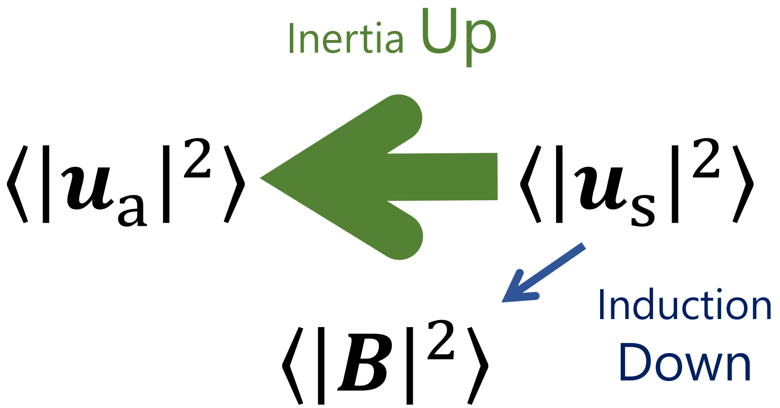
Spatial structure of flow in the reversal phase



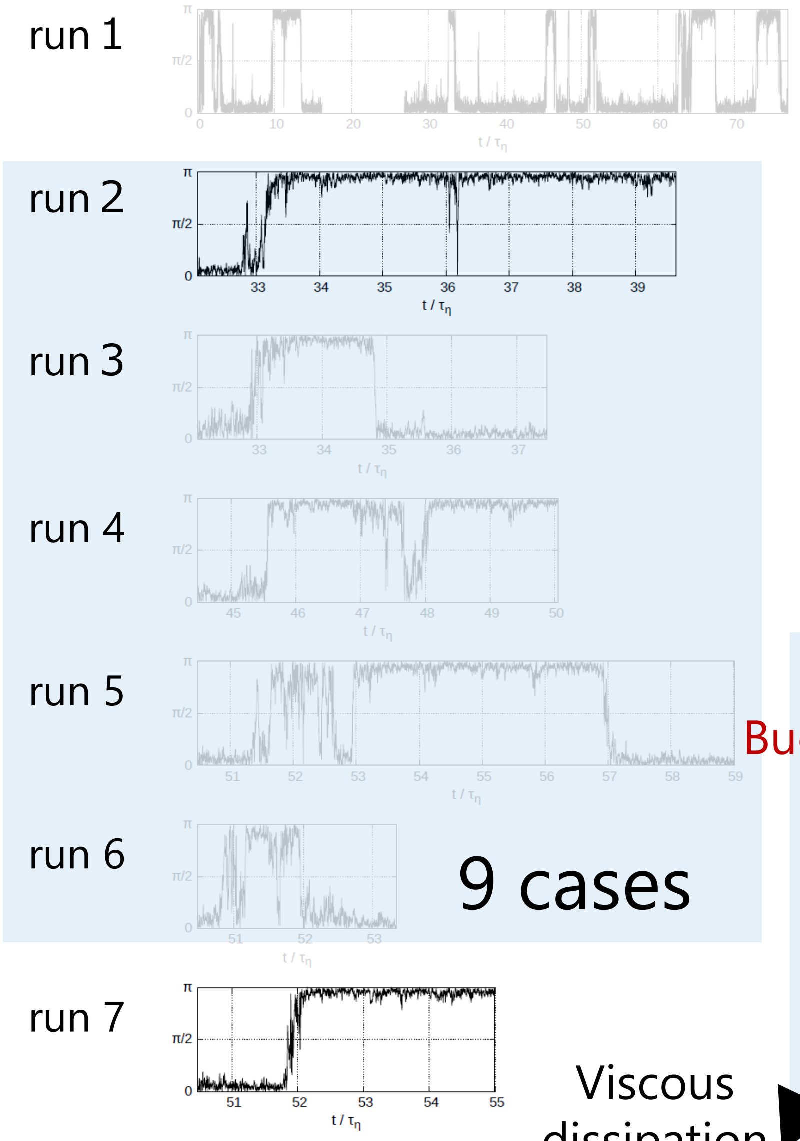


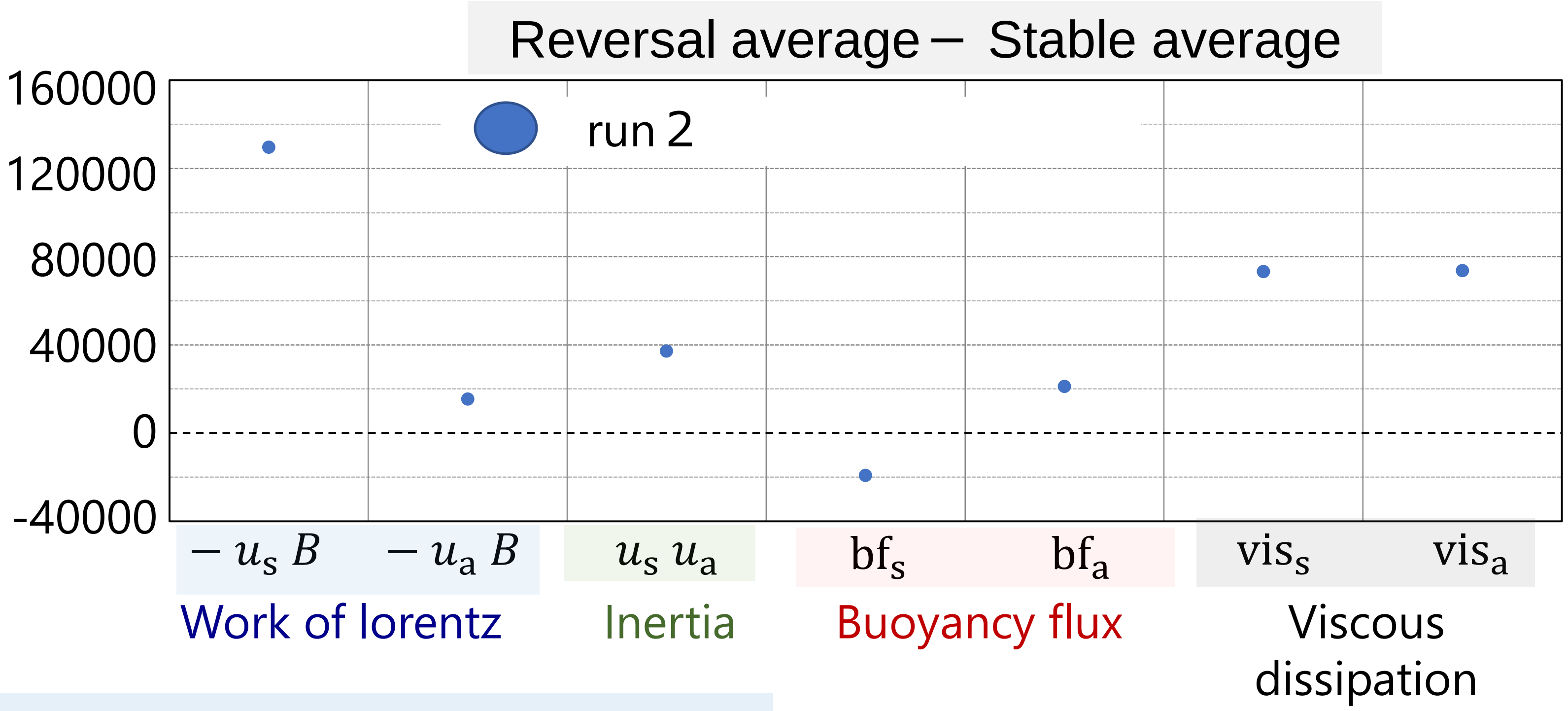
$m \geq 6$ Increase

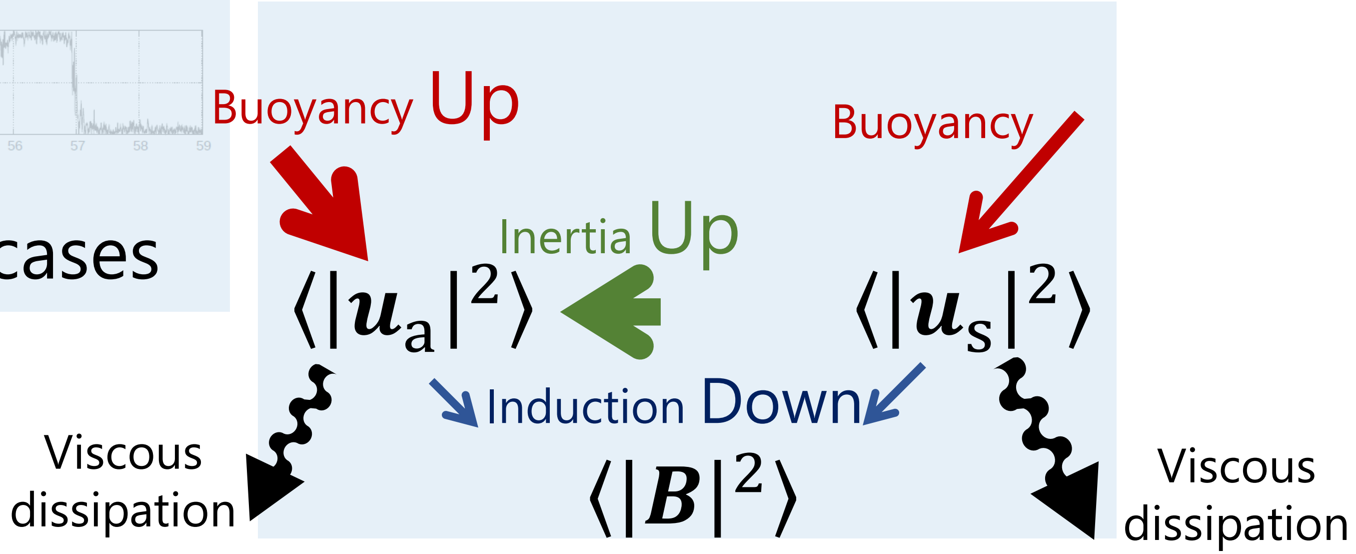
More turbulent in the reversal phase



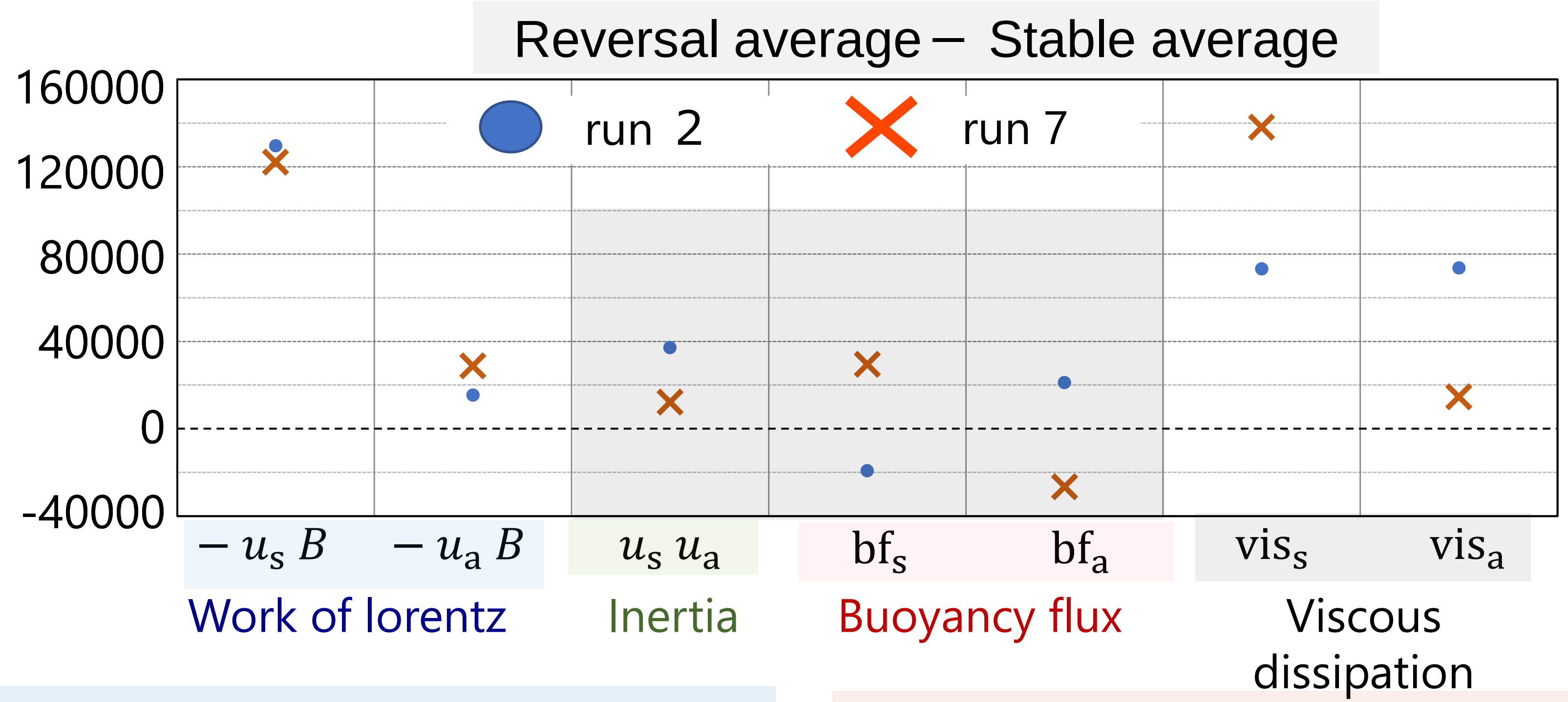
Common for other cases

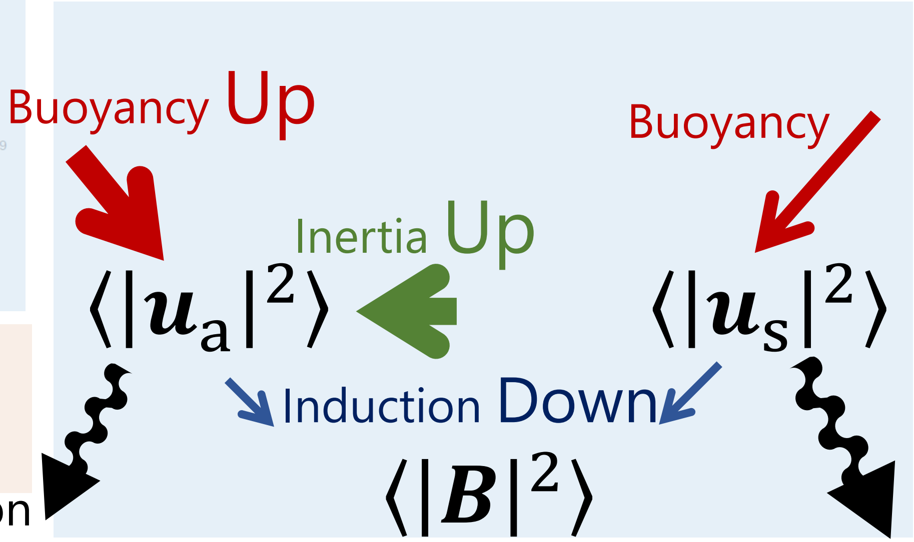


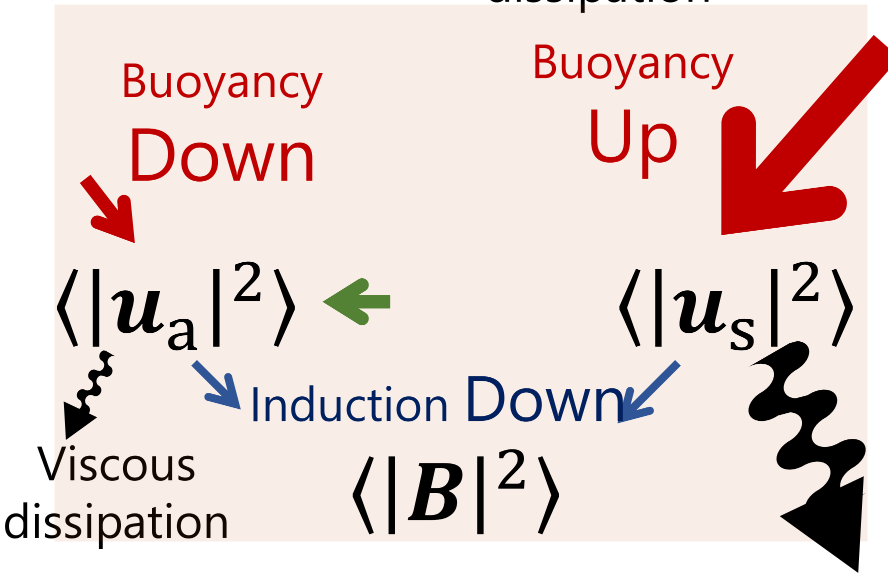




One exception







Conclusion

Inertia grows the antisymmetric flow.

i. **Reduced induction** by symmetric flow
ii. **Inertia converts energy** from symmetric to antisymmetric flow
iii. **Increase in buoyancy** flux into antisymmetric flow

Common for 9 / 10 cases