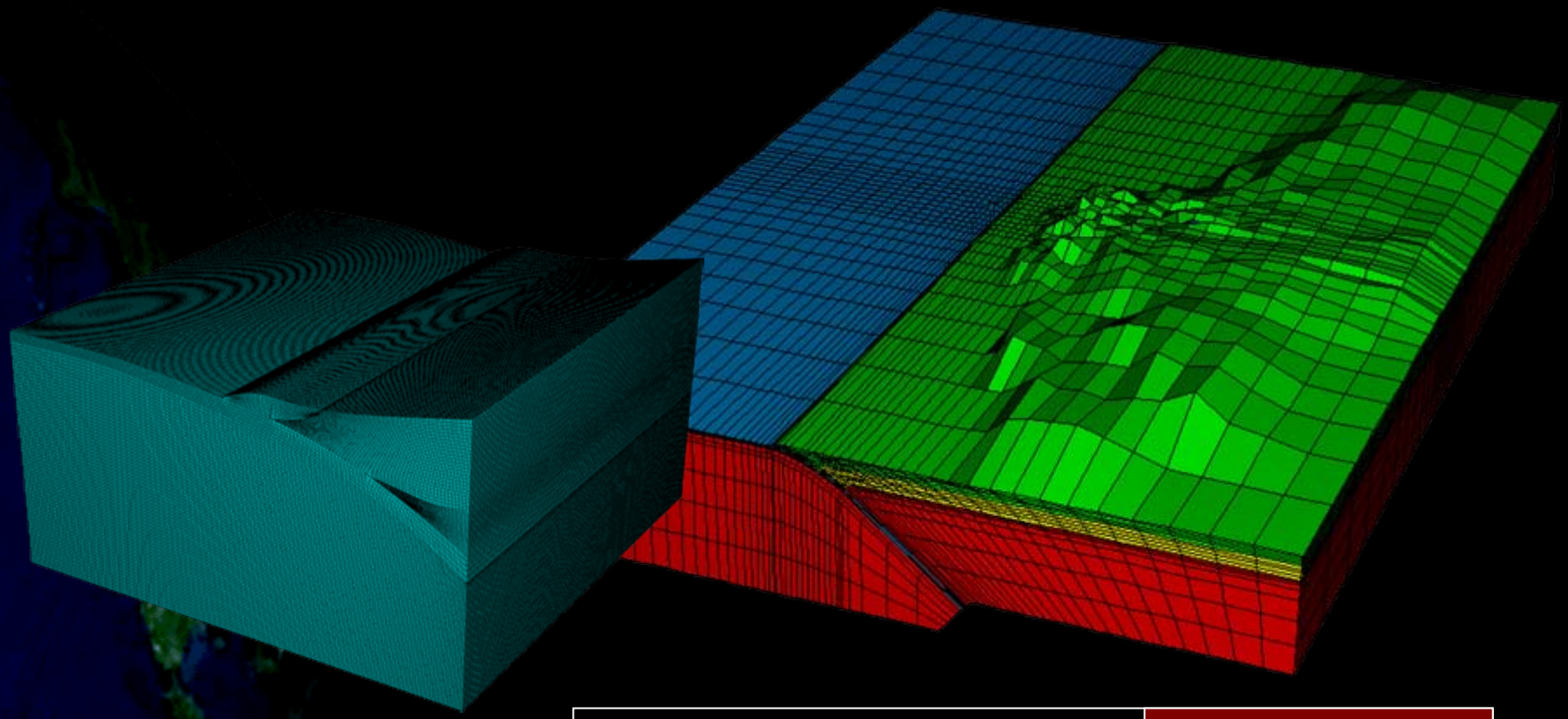
FEMs of dislocations within a subduction zone

Simulating complex structures with both forward and inverse models



Tim Masterlark
Geodynamics & Active Deformation

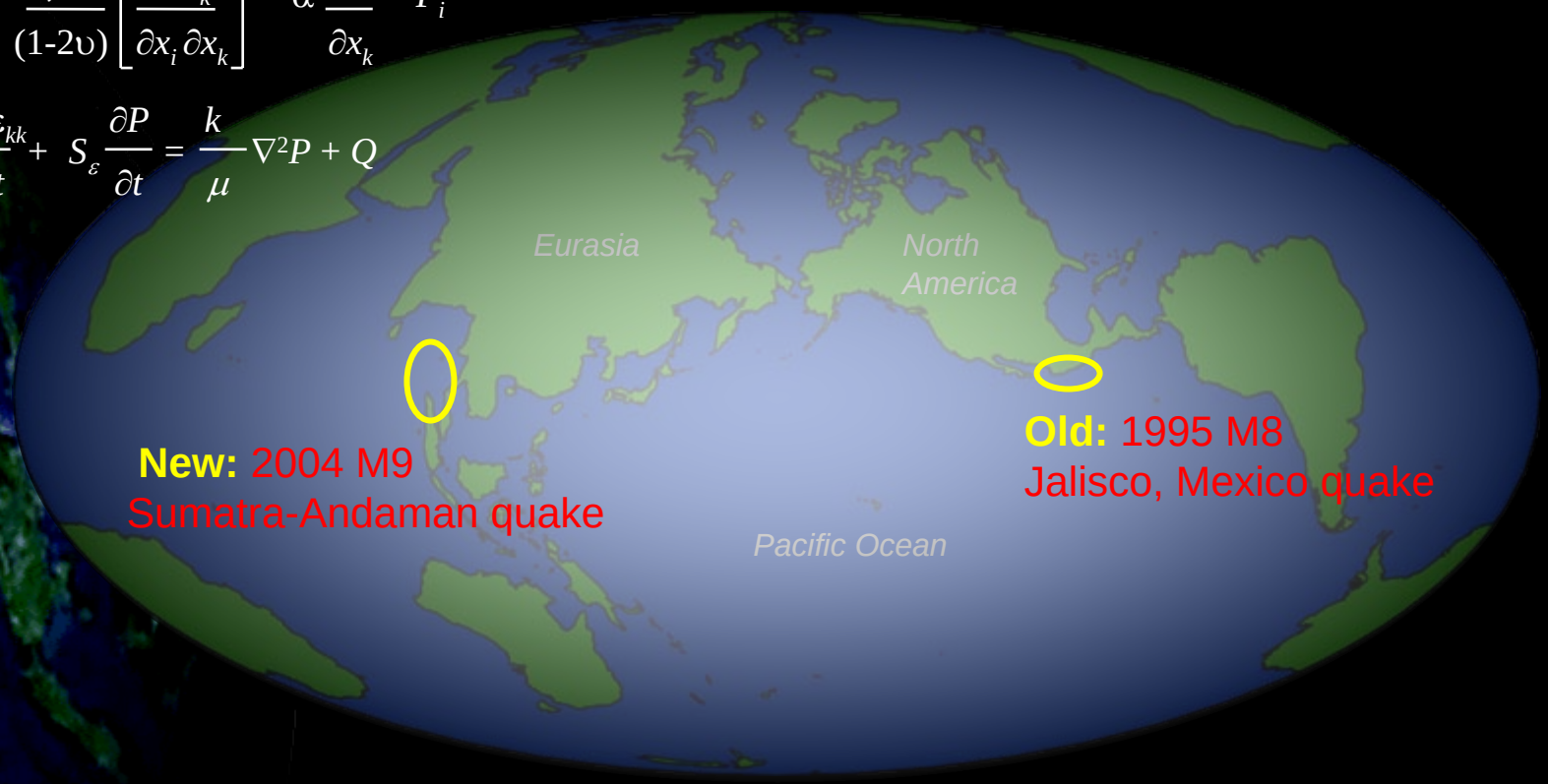
The University of Alabama
Geological Sciences

Virtual field trips via FEMs

Deformation of big subduction quakes

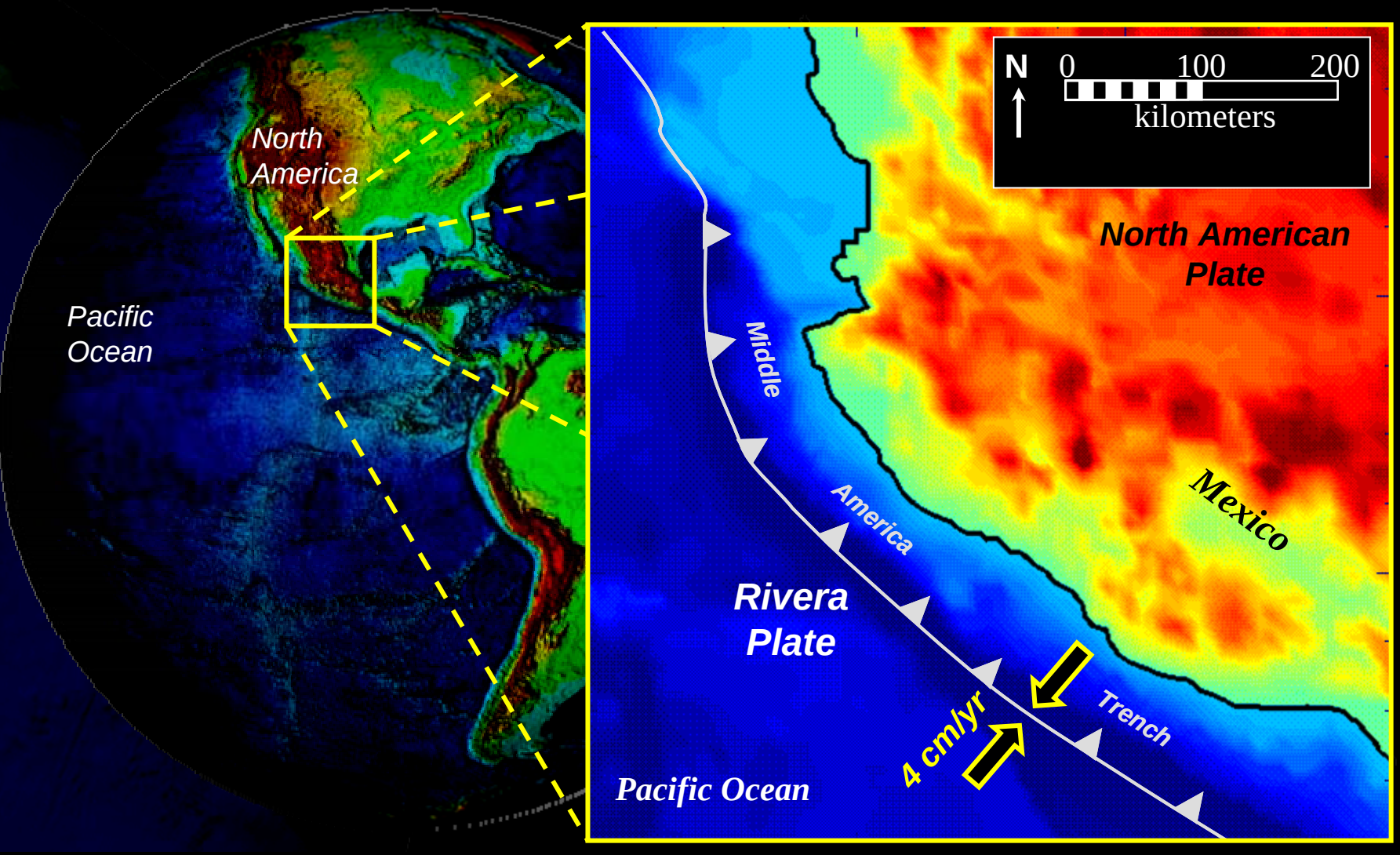
$$\mu \nabla^2 u_i + \frac{\mu}{(1\text{-}2\upsilon)}\left[\frac{\partial^2 u_k}{\partial x_i \partial x_k}\right] = \alpha \frac{\partial P}{\partial x_k} - F_i$$

$$\alpha \frac{\partial \varepsilon_{kk}}{\partial t} + S_\varepsilon \frac{\partial P}{\partial t} = \frac{k}{\mu} \nabla^2 P + Q$$



Study Site

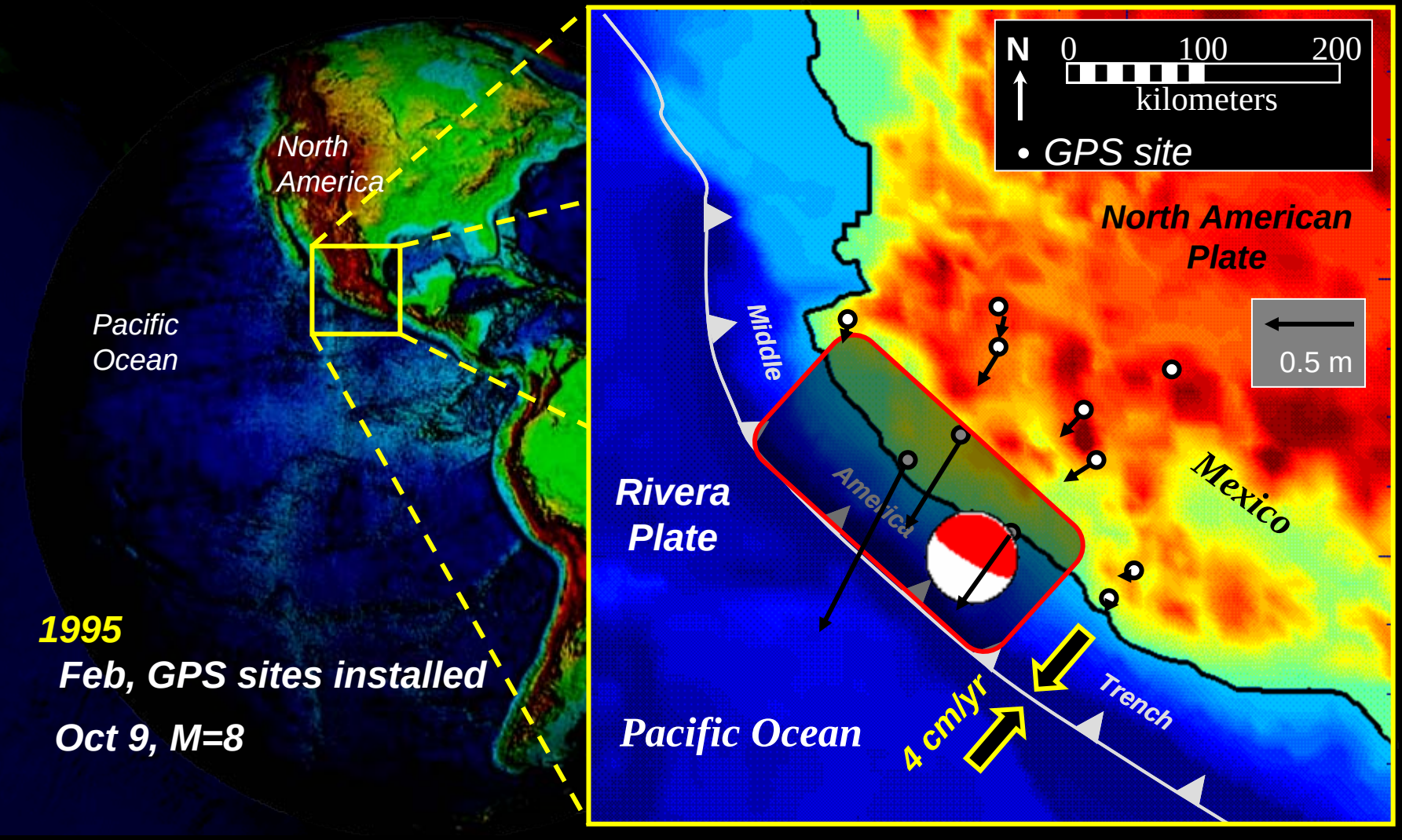
West-central Mexico



1995 M8 Jalisco quake

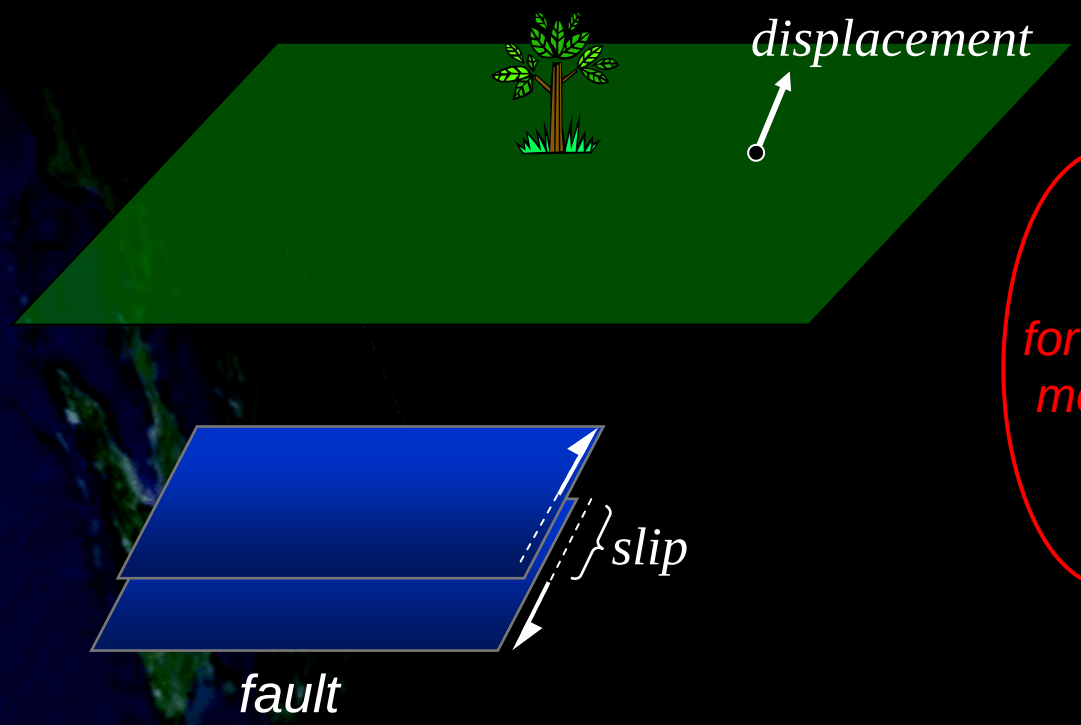
Study Site

1995 Jalisco earthquake



Deformation models

observed deformation ↔ causes of deformation



deformation

what we see (InSAR,GPS)

forward models

inverse models

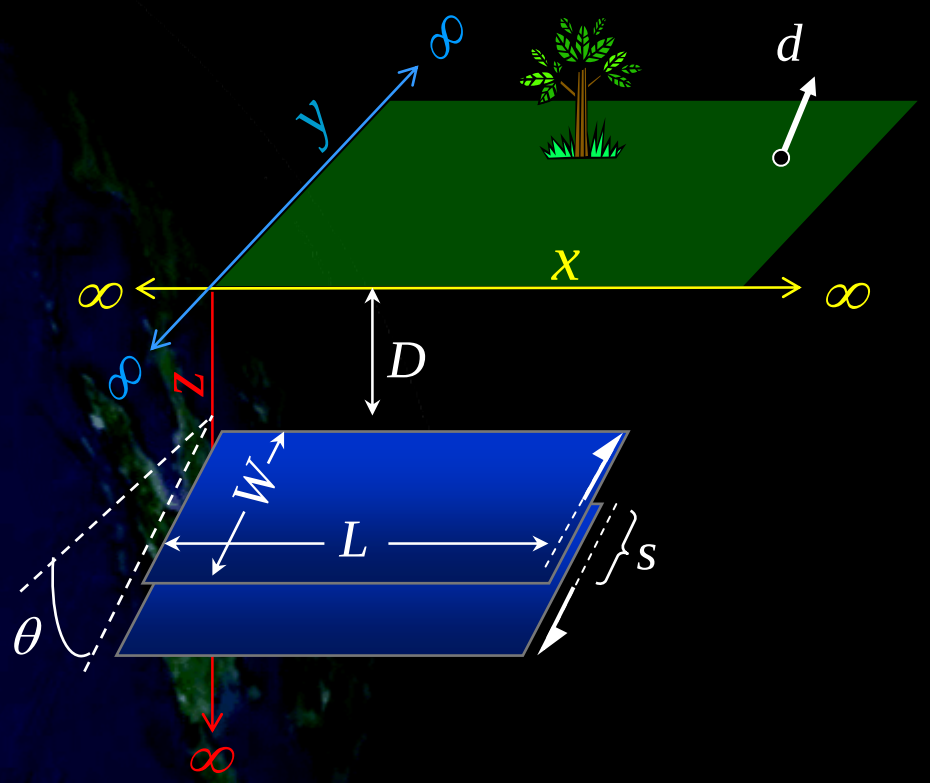
slip along a fault

what we want to know!

1995 M8 Jalisco quake

Forward Model

Predict deformation caused by dislocation



elasto-static behavior

analytical solution

$$\boldsymbol{G}\ \boldsymbol{s} = \boldsymbol{d}$$

slip

vertical displacement

unit impulse response function

→ *geometry*

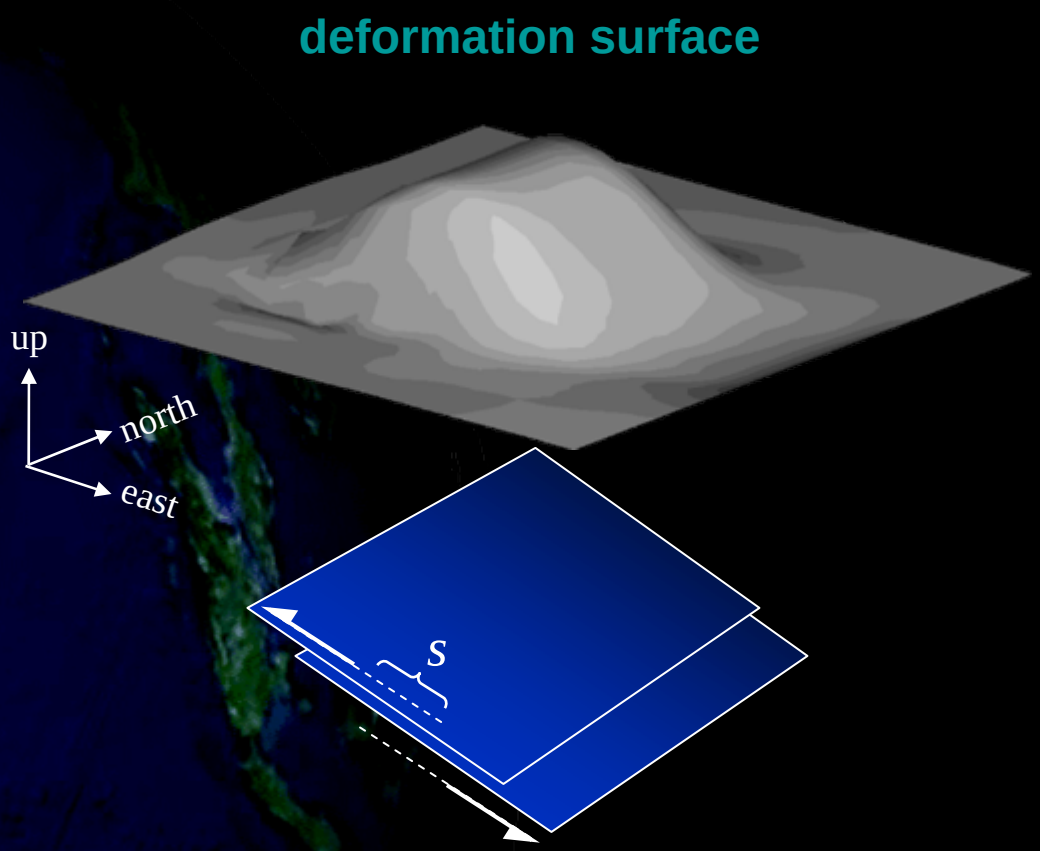
$$\boldsymbol{G} = f\,(\,D, W, L, \theta, x, y, z\,)$$

e.g., *Okada* [1992]

1995 M8 Jalisco quake

Inverse Model

Estimate dislocation to account for observed deformation



forward model

model (vector)

$$\mathbf{G}\,\mathbf{s} = \mathbf{d}$$

slip

displacement (vector)

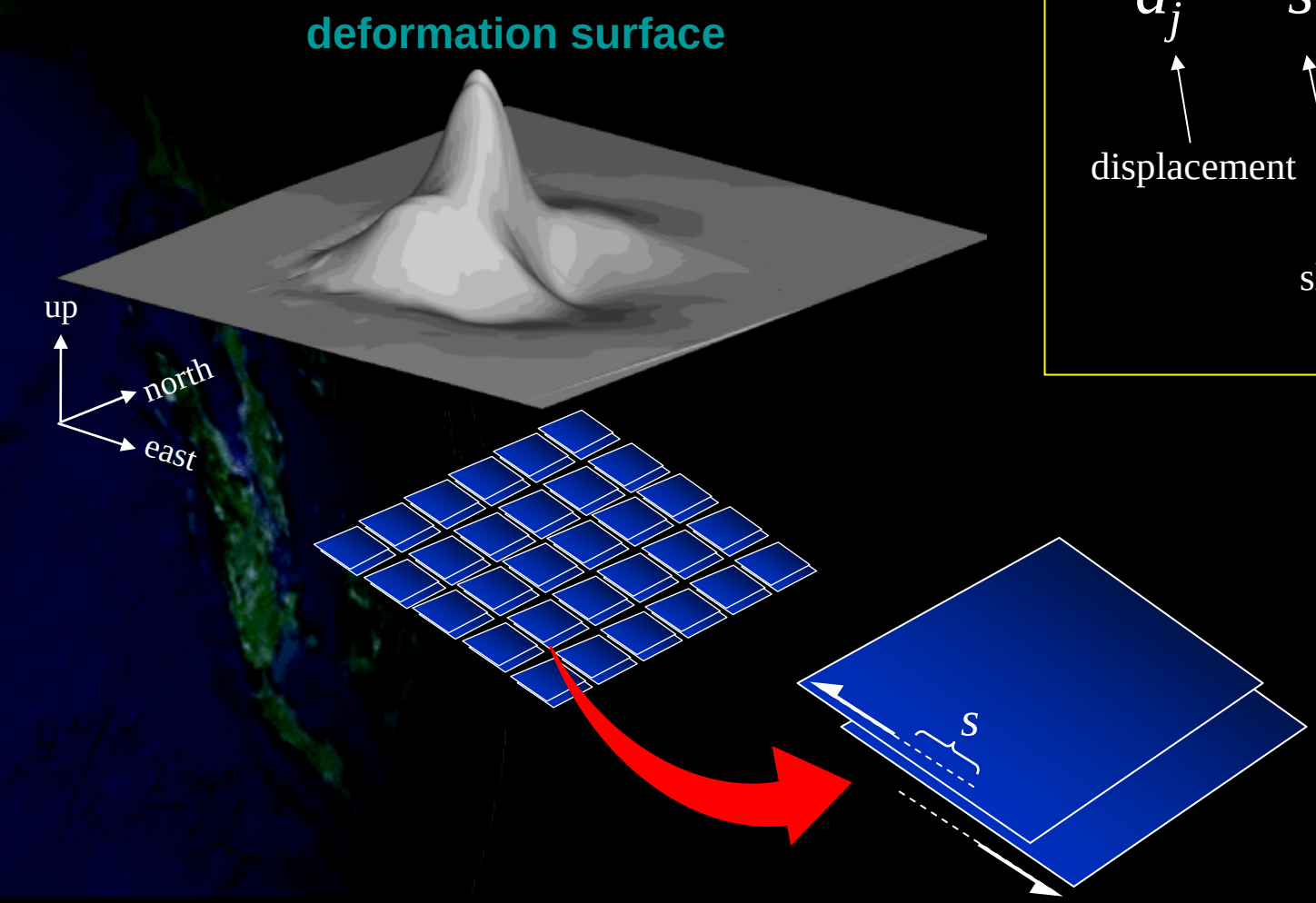
inverse model

$$\mathbf{s}^{est.} = \left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\,\mathbf{d}$$

1995 M8 Jalisco quake

Complicated Deformation Pattern

Array of dislocation patches



$$d_j = s_i G_{ij}$$

displacement

slip

unit impulse response function (geometry)

1995 M8 Jalisco quake

Matrix Assembly

Array of dislocation patches

$$G_{ij}\ s_i = d_j$$

system of linear equations

$$\begin{aligned} G_{11}\, s_1 + \ldots G_{n1}\, s_n &= d_1 \\ G_{12}\, s_1 + \ldots G_{n2}\, s_n &= d_2 \\ &\vdots \\ G_{1m}\, s_1 + \ldots G_{nm}\, s_n &= d_m \end{aligned}$$

matrix expression

$$\begin{pmatrix} G_{11} & G_{21} & G_{31} & \ldots & G_{n1} \\ G_{12} & G_{22} & G_{32} & \ldots & G_{n2} \\ G_{13} & G_{23} & G_{33} & \ldots & G_{n3} \\ \vdots & \vdots & \vdots & & \vdots \\ G_{1m} & G_{2m} & G_{3m} & \ldots & G_{nm} \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_n \end{pmatrix} = \begin{pmatrix} d_1 \\ \vdots \\ d_m \end{pmatrix}$$

$$\overset{n\times m}{\mathbf{G}}\ \overset{1\times n}{\mathbf{s}} = \overset{1\times m}{\mathbf{d}}$$

unknown

- *we can solve for **s** using linear matrix inverse methods*

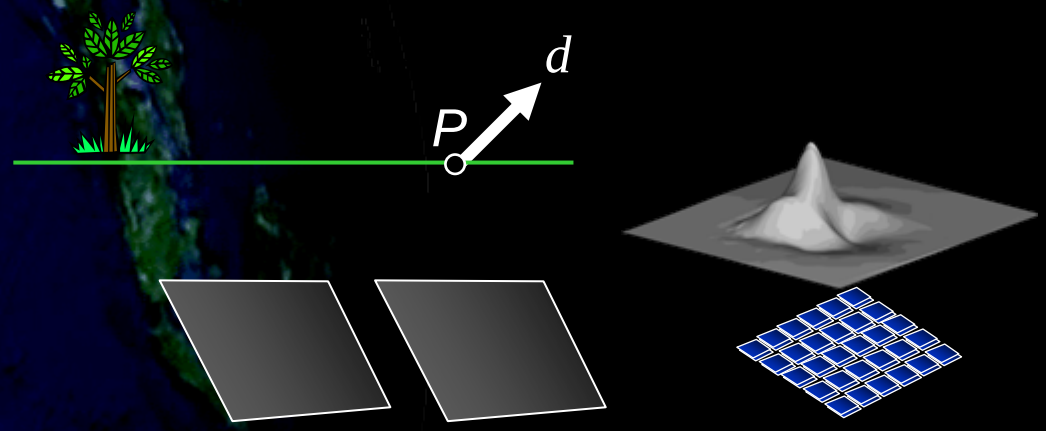
Solution Constraints

Laplacian operator → smoothing

matrix expression: $\mathbf{G}\,\mathbf{s} = \mathbf{d}$

unknown

additional constraints: avoid large variations between neighboring dislocations



Solution Constraints

Laplacian operator → smoothing

matrix expression: $\mathbf{G}\,\mathbf{s} = \mathbf{d}$

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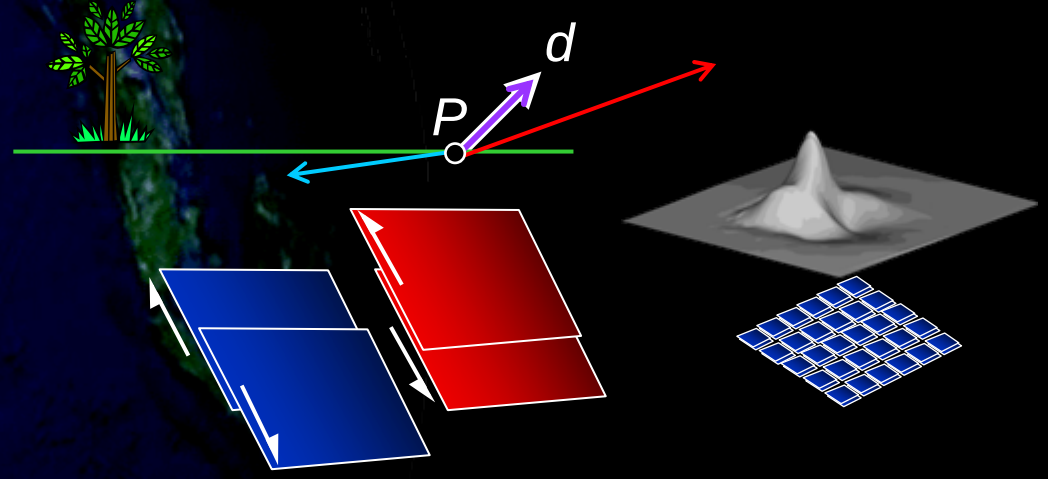


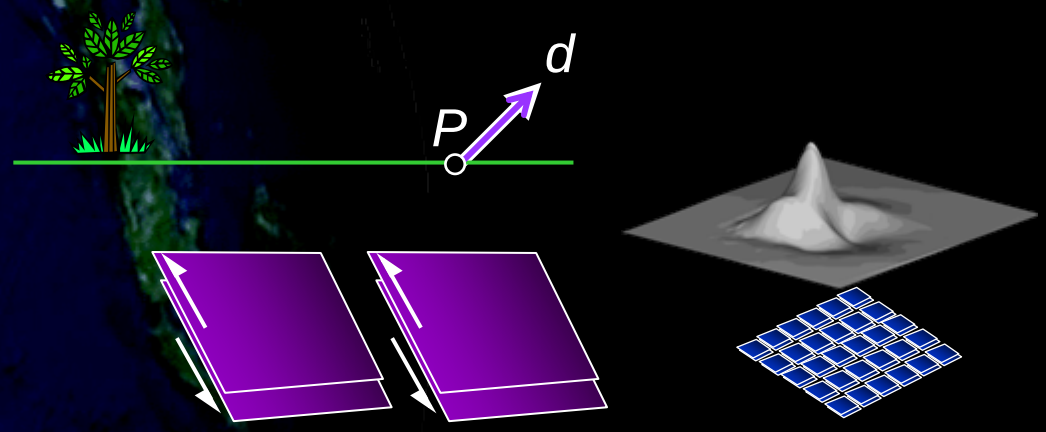
Solution Constraints

Laplacian operator → smoothing

matrix expression: $\mathbf{G}\,\mathbf{s} = \mathbf{d}$

unknown

additional constraints: avoid large variations between neighboring dislocations



Solution Constraints

Laplacian operator → smoothing

matrix expression: $\mathbf{G}\,\mathbf{s} = \mathbf{d}$

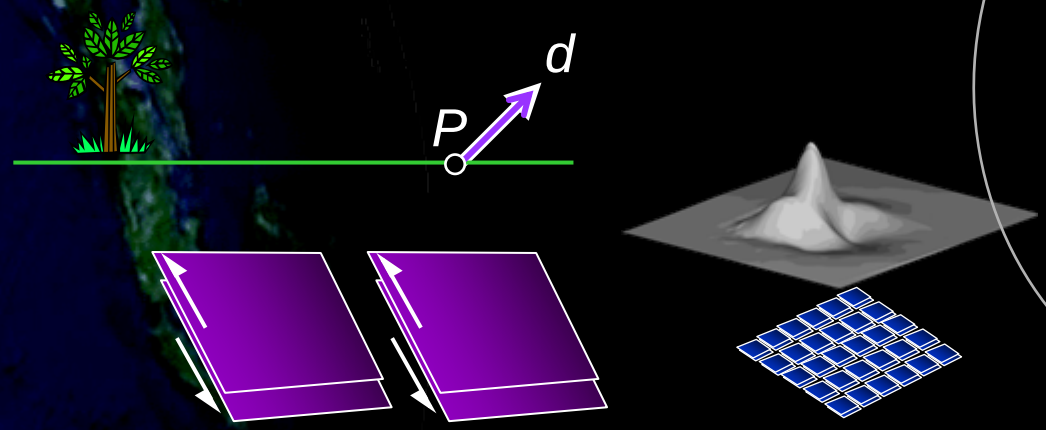
unknown

additional constraints: avoid large variations between neighboring sources

∇^2 → $\overset{n\times n}{\mathbf{L}}\ \overset{1\times n}{\mathbf{s}} = \mathbf{0}$

$$\begin{bmatrix} \mathbf{G} \\ \beta\mathbf{L} \end{bmatrix} \mathbf{s} = \begin{bmatrix} \mathbf{d} \\ \mathbf{0} \end{bmatrix}$$

β *scalar, controls amount of smoothing*



matrix expression: forward solution

$$\hat{\mathbf{G}}\ \mathbf{s} = \hat{\mathbf{d}}$$

unknown

1995 M8 Jalisco quake

Inverse Solution

Damped least-squares

matrix expression
– forward solution

$$\hat{\mathbf{G}}\,\mathbf{s} = \hat{\mathbf{d}}$$

unknown

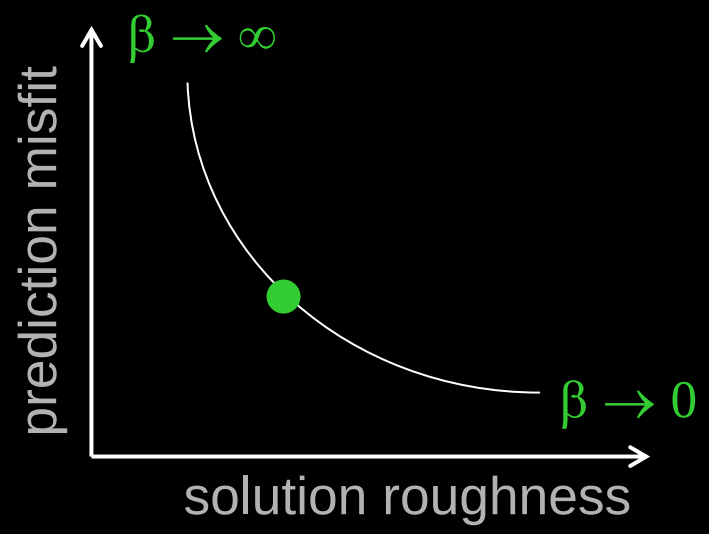
matrix expression
– inverse solution

$$\mathbf{s}^{\text{est.}} = \left(\hat{\mathbf{G}}^{T}\hat{\mathbf{G}}\right)^{-1}\hat{\mathbf{G}}^{T}\hat{\mathbf{d}}$$

recall…

$$\begin{pmatrix}\mathbf{G}\\ \beta\mathbf{L}\end{pmatrix}\mathbf{s} = \begin{pmatrix}\mathbf{d}\\ \mathbf{0}\end{pmatrix}$$

trade-off: misfit vs roughness



Inverse Solution

Over-smoothing

matrix expression
– forward solution

$$\hat{\mathbf{G}}\,\mathbf{s} = \hat{\mathbf{d}}$$

unknown

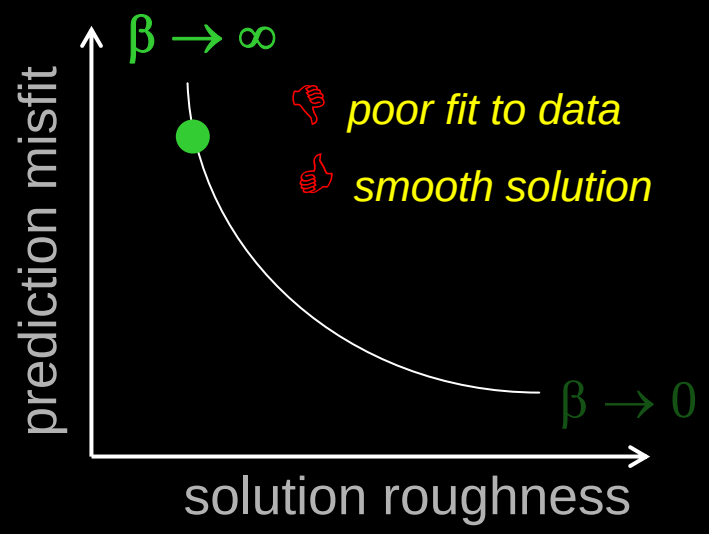
matrix expression
– inverse solution

$$\mathbf{s}^{\text{est.}} = \left(\hat{\mathbf{G}}^{T}\hat{\mathbf{G}}\right)^{-1}\hat{\mathbf{G}}^{T}\hat{\mathbf{d}}$$

recall…

$$\begin{pmatrix}\mathbf{G}\\ \beta\mathbf{L}\end{pmatrix}\mathbf{s} = \begin{pmatrix}\mathbf{d}\\ \mathbf{0}\end{pmatrix}$$

trade-off: misfit vs roughness



Inverse Solution

Under-smoothing

matrix expression
– forward solution

$$\hat{\mathbf{G}}\,\mathbf{s} = \hat{\mathbf{d}}$$

unknown

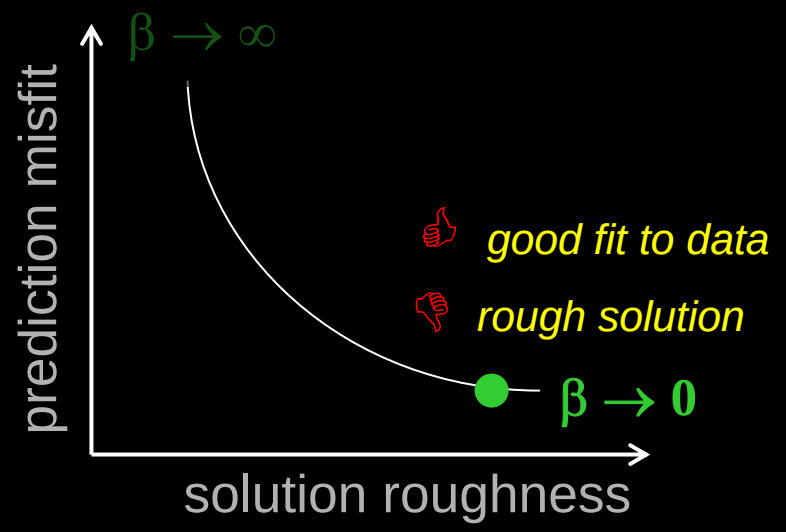
matrix expression
– inverse solution

$$\mathbf{s}^{\text{est.}} = \left(\hat{\mathbf{G}}^{T}\hat{\mathbf{G}}\right)^{-1}\hat{\mathbf{G}}^{T}\hat{\mathbf{d}}$$

recall…

$$\begin{pmatrix}\mathbf{G}\\ \beta\mathbf{L}\end{pmatrix}\mathbf{s} = \begin{pmatrix}\mathbf{d}\\ \mathbf{0}\end{pmatrix}$$

trade-off: misfit vs roughness



Inverse Solution

Balance of fit and smoothing

matrix expression
– forward solution

$$\hat{\mathbf{G}}\,\mathbf{s} = \hat{\mathbf{d}}$$

unknown

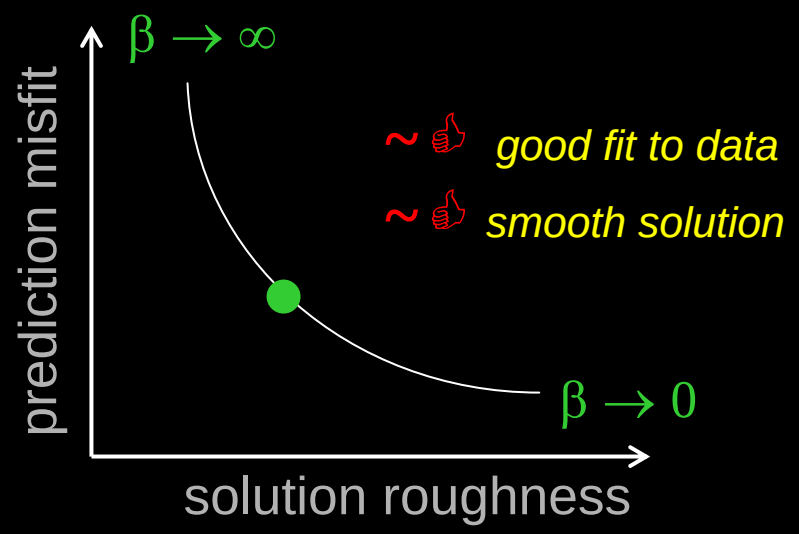
matrix expression
– inverse solution

$$\mathbf{s}^{est.} = \left(\hat{\mathbf{G}}^T \hat{\mathbf{G}}\right)^{-1} \hat{\mathbf{G}}^T \hat{\mathbf{d}}$$

recall…

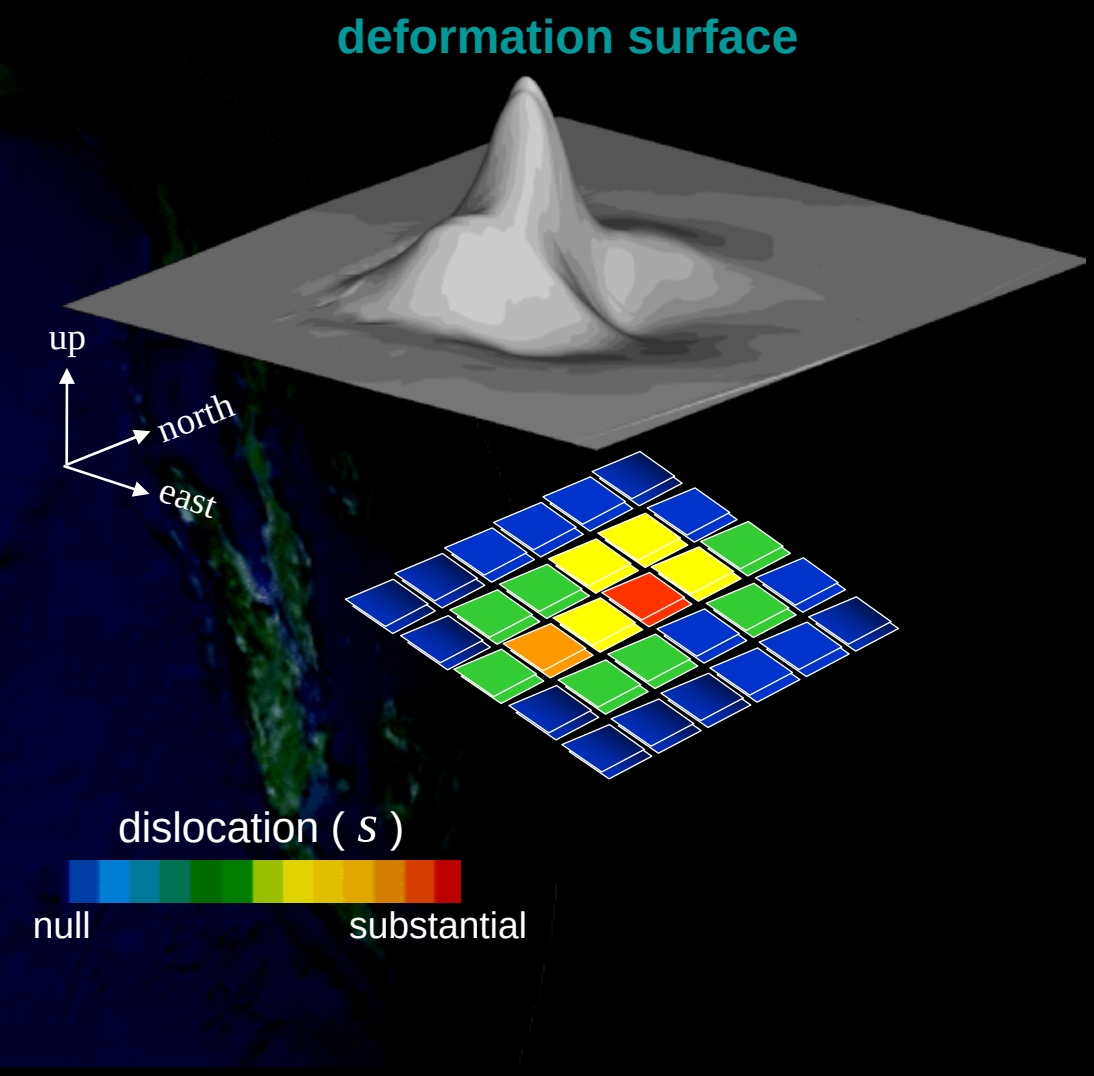
$$\begin{pmatrix} \mathbf{G} \\ \beta\mathbf{L} \end{pmatrix} \mathbf{s} = \begin{pmatrix} \mathbf{d} \\ \mathbf{0} \end{pmatrix}$$

trade-off: misfit vs roughness



“Fit” Complicated Deformation Patterns

Geometric distribution of dislocation patches

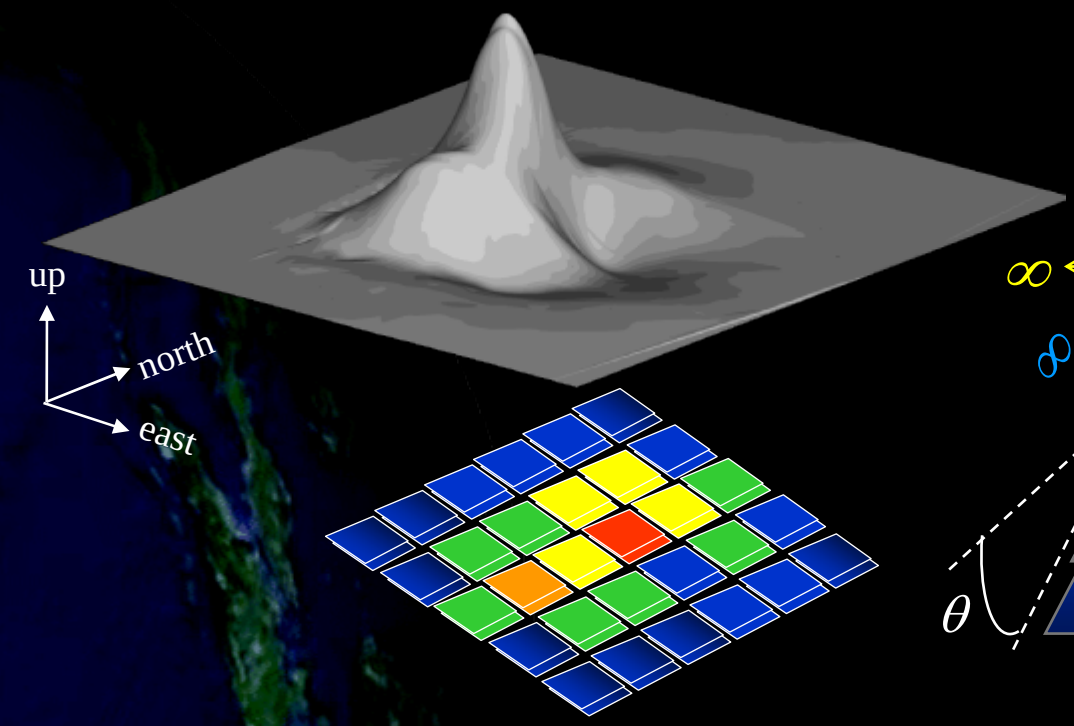


1995 M8 Jalisco quake

Wait a minute…

How well does the model represent the natural system?

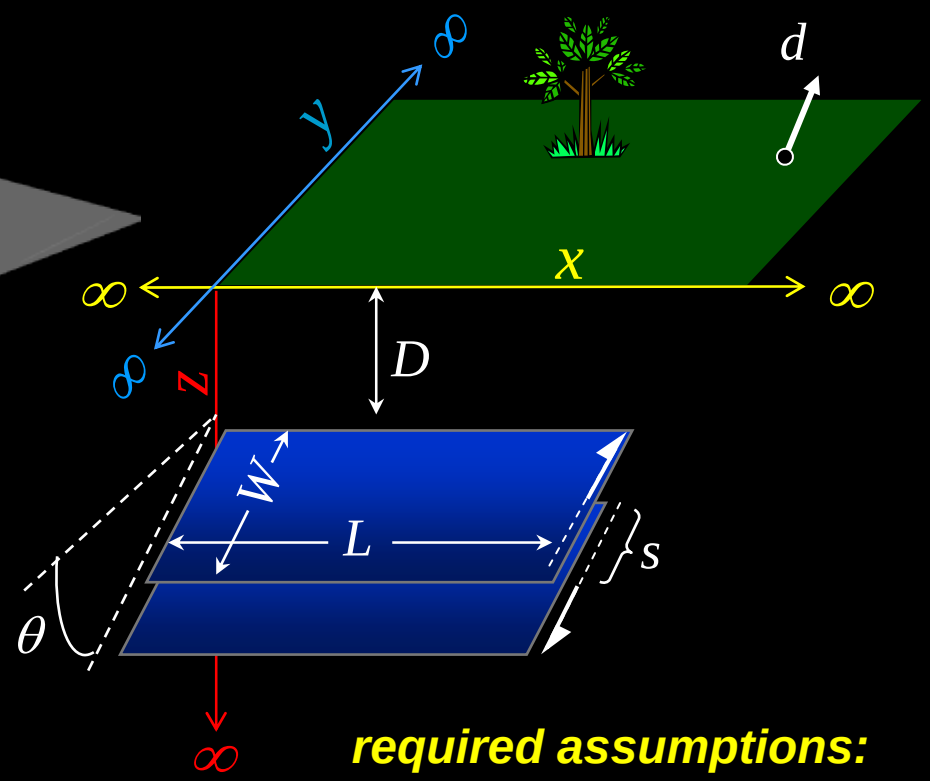
deformation surface



dislocation (s)

null substantial

analytical solution



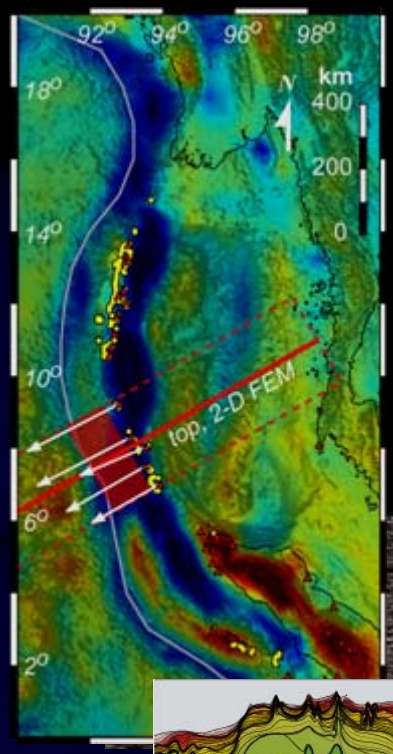
required assumptions:

- ***homogeneous***
- ***isotropic***
- ***Poisson-solid***
- ***half-space***

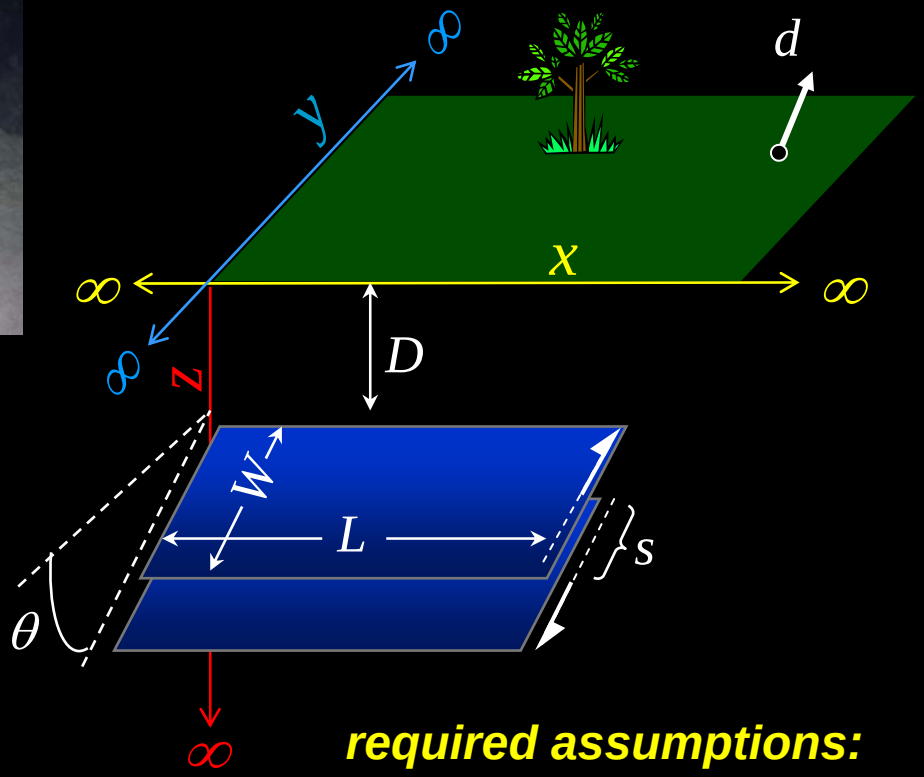
1995 M8 Jalisco quake

Wait a minute…

How well does the model represent the natural system?



analytical solution



required assumptions:

- ***homogeneous***
- *isotropic*
- *Poisson-solid*
- *half-space*

1995 M8 Jalisco quake

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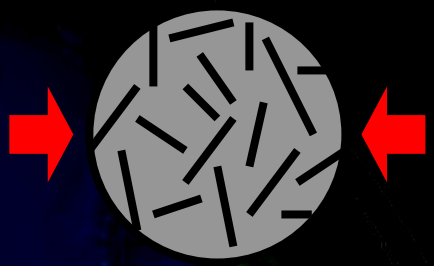
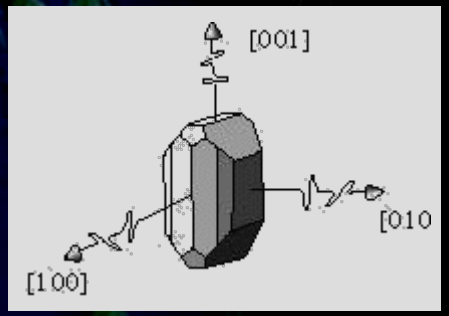
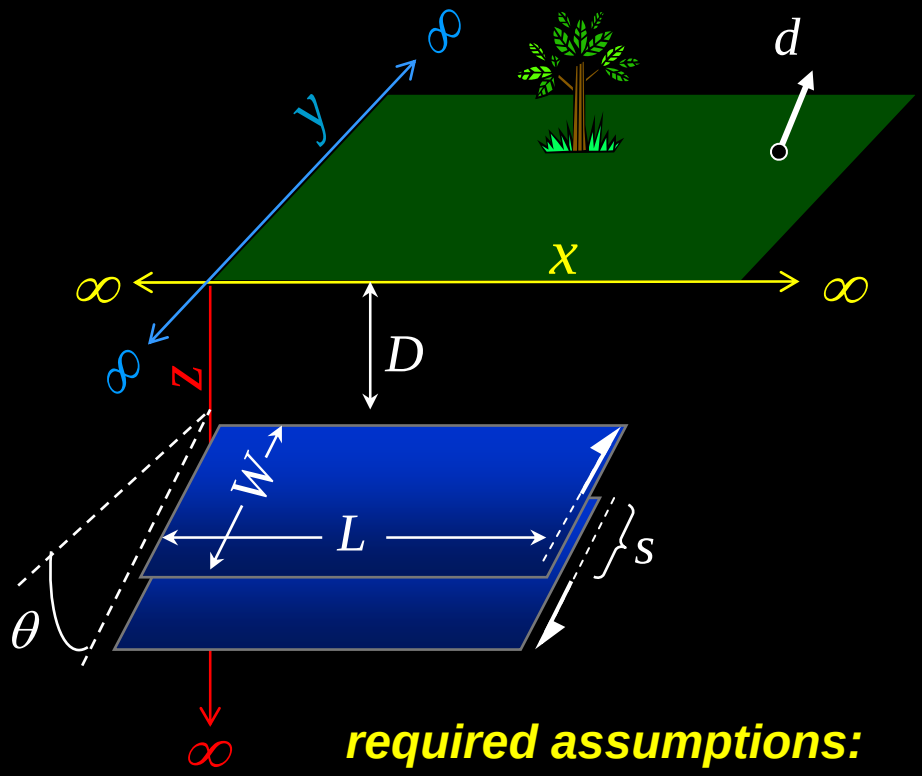
How well does the model represent the natural system?





analytical solution



required assumptions:

- *homogeneous*
- ***isotropic***
- *Poisson-solid*
- *half-space*

1995 M8 Jalisco quake

Wait a minute…

How well does the model represent the natural system?

$$V_p / V_s \Rightarrow 0.25 \leq \nu \leq 0.30$$

lower limit

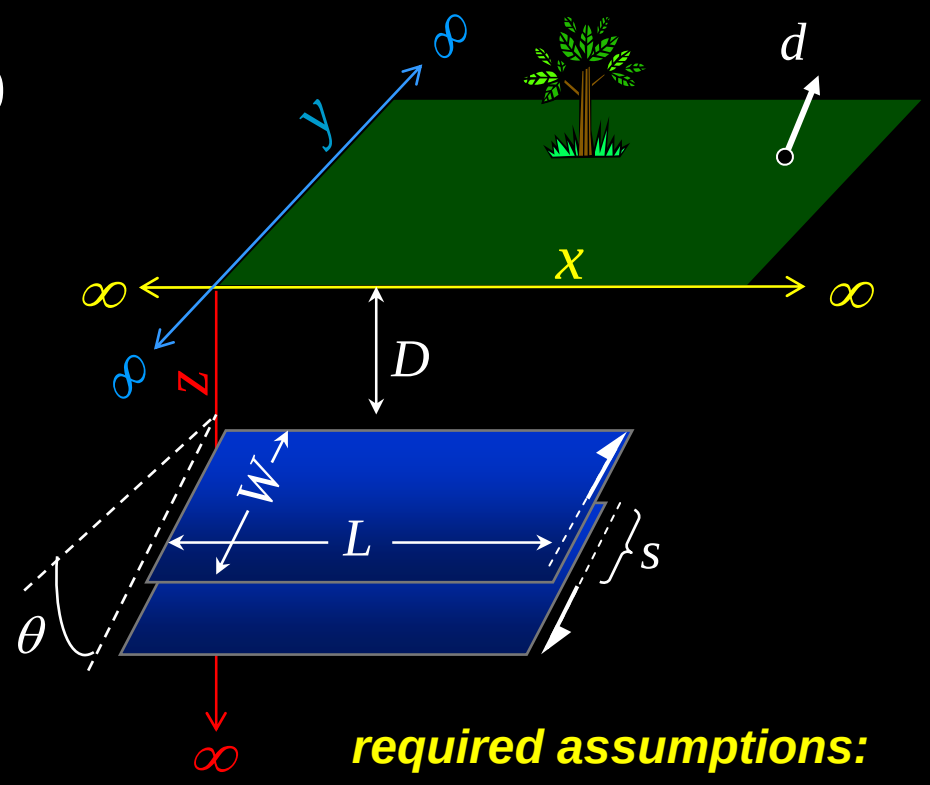
drained ***undrained***

$$\nu < \nu_u$$

$$\nu = \frac{3\nu_u - \alpha\beta(1+\nu_u)}{3 - 2\alpha\beta(1+\nu_u)}$$

$$\nu_u = \frac{3\nu - \alpha\beta(1-2\nu)}{3 - \alpha\beta(1-2\nu)}$$

analytical solution



required assumptions:

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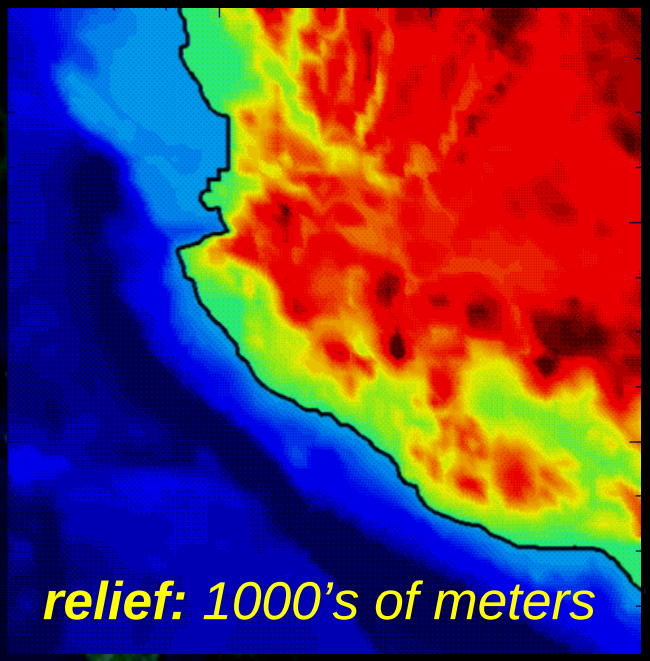
- *isotropic*
- ***Poisson-solid***
- *half-space*

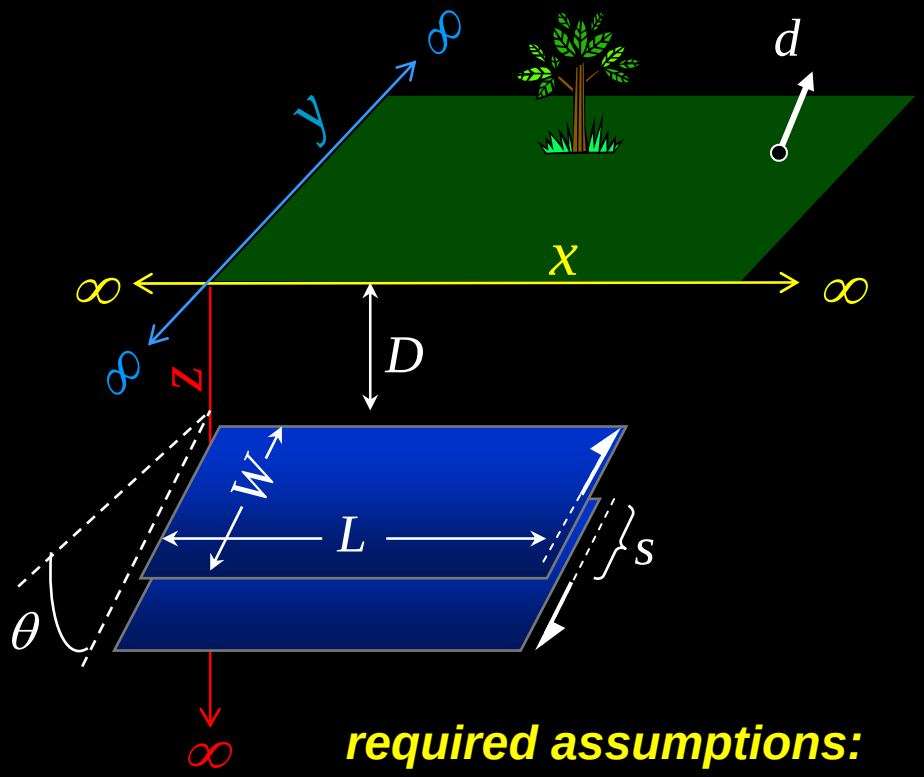
1995 M8 Jalisco quake

Wait a minute…

How well does the model represent the natural system?



analytical solution



required assumptions:

- *homogeneous*
- *isotropic*
- *Poisson-solid*
- ***half-space***

1995 M8 Jalisco quake

Test sensitivity to HIPSHS assumptions

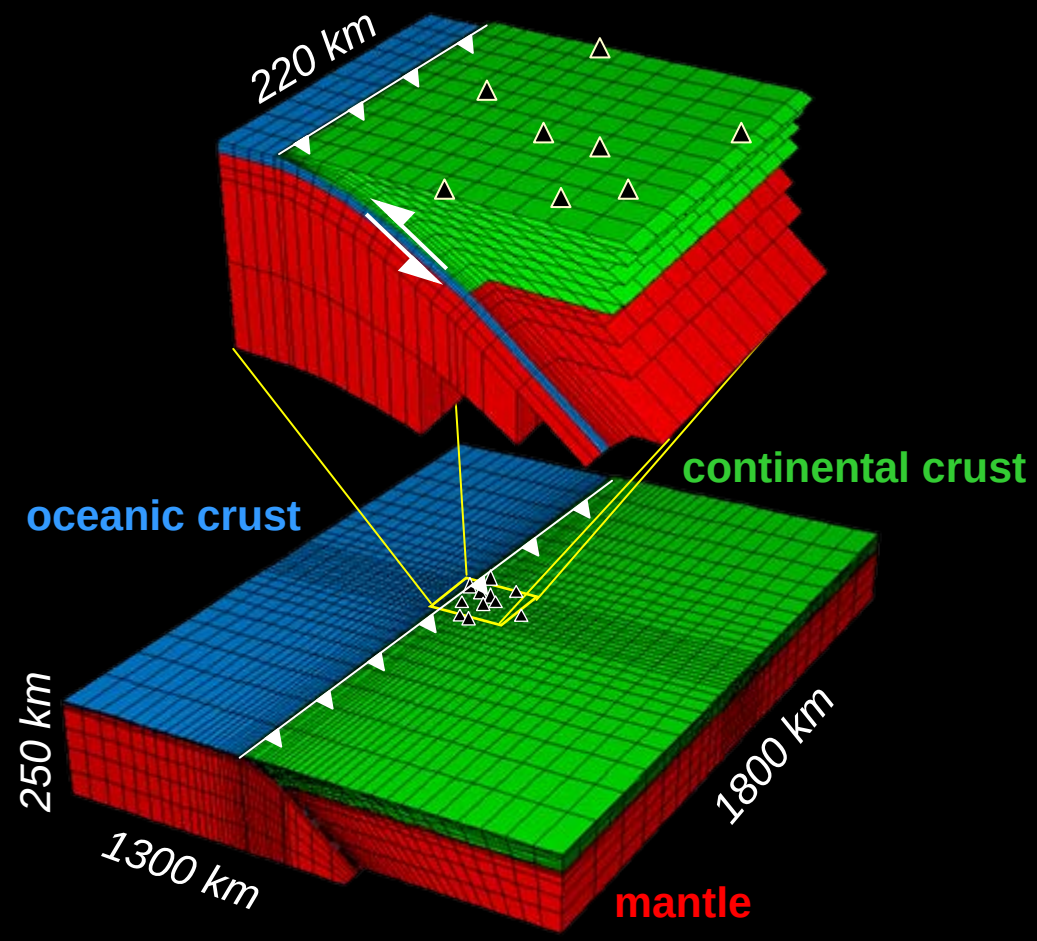
Systematically relax and isolate each assumption

definition

Finite Element Model (FEM):
numerical model; predicts deformation, stress, and pore pressure

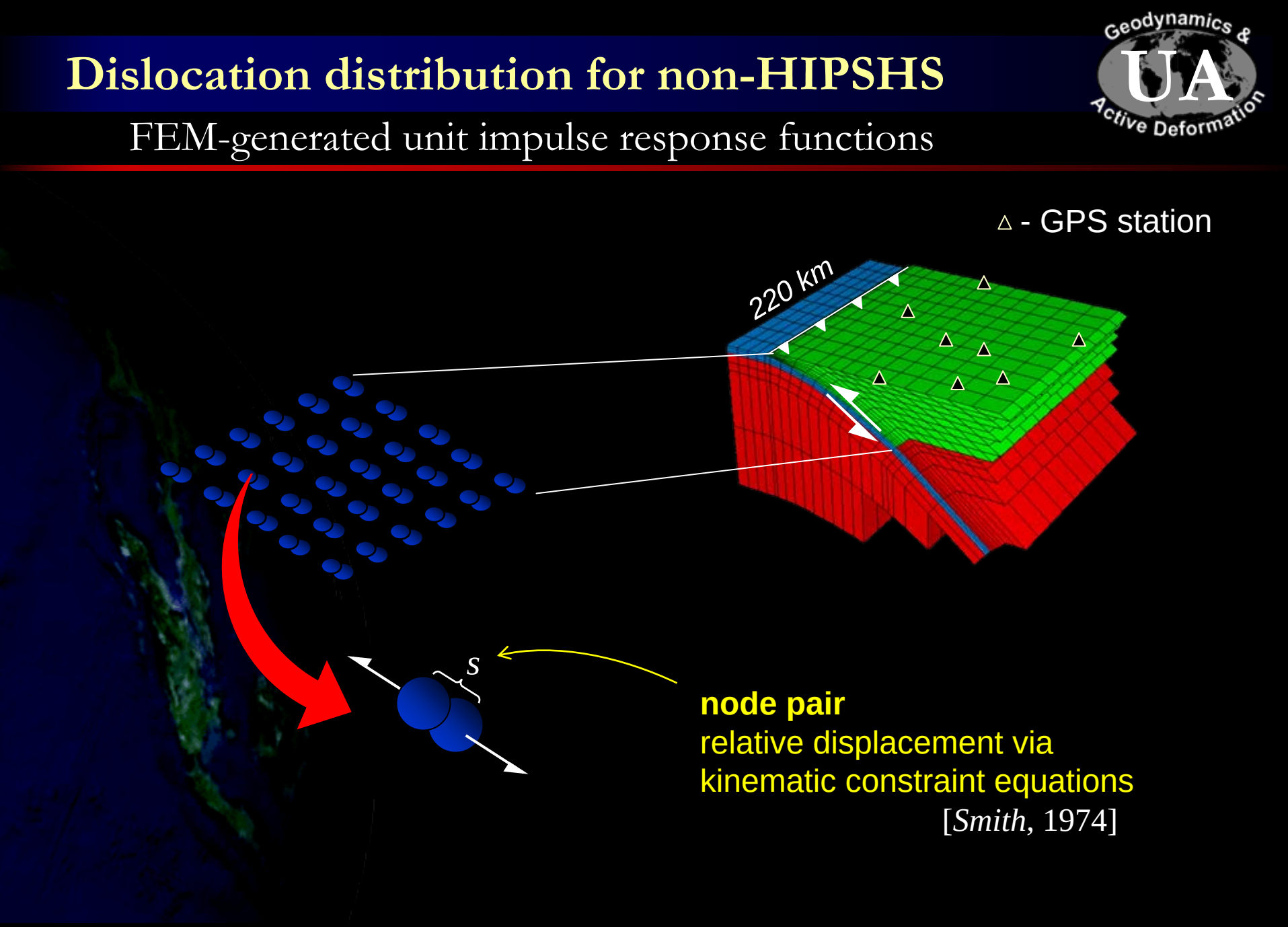
$$\mu \nabla^2 u_i + \frac{\mu}{(1-2\upsilon)}\left[\frac{\partial^2 u_k}{\partial x_i \partial x_k}\right] = \alpha \frac{\partial P}{\partial x_k} - F_i$$

$$\alpha \frac{\partial \varepsilon_{kk}}{\partial t} + S_\varepsilon \frac{\partial P}{\partial t} = \frac{k}{\mu}\nabla^2 P + Q$$



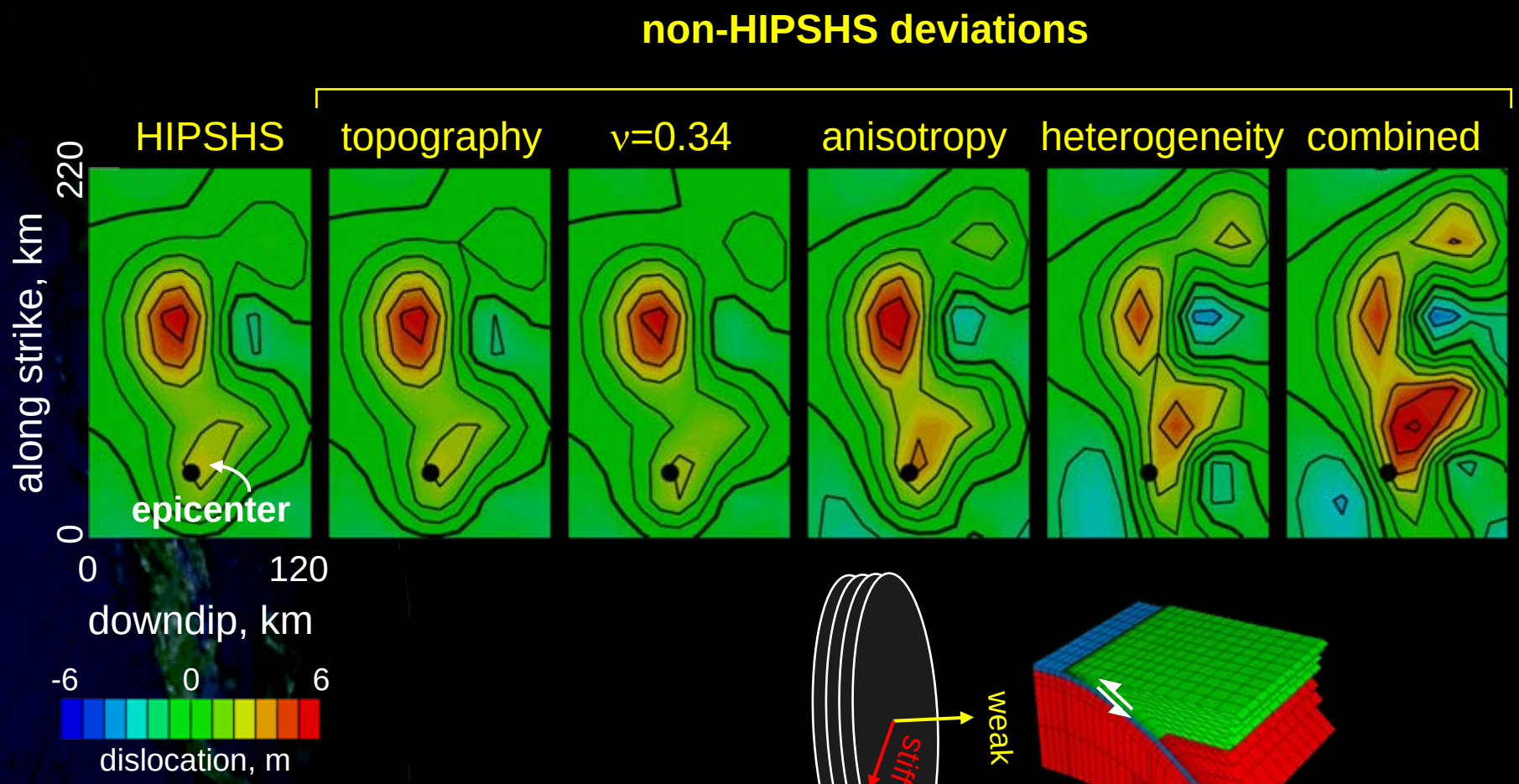
Dislocation distribution for non-HIPSHS

FEM-generated unit impulse response functions



Dislocation distributions

Non-HIPSHS configurations



1995 M8 Jalisco quake

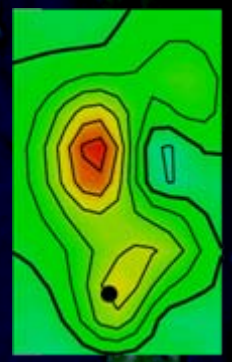
Sensitivity to HIPSHS assumptions

Loading a non-HIPSHS FEMs with HIPSHS loads

$$\Delta = \mathbf{G}^H \mathbf{s}^H - \mathbf{G}\,\mathbf{s}^H$$

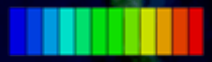
$\Delta \rightarrow 0$, not sensitive
$\Delta \rightarrow \infty$, sensitive

HIPSHS load
(inverted from GPS)



Forward FEM

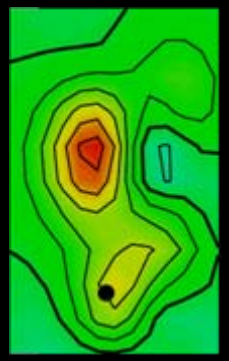
HIPSHS



—

Forward FEM
(relaxed assumption)

HIPSHS load



- half-space
- Poisson-solid
- isotropy
- homogeneity
- combined

1995 M8 Jalisco quake

Sensitivity to HIPSHS assumptions

Loading a non-HIPSHS FEMs with HIPSHS loads

$$\Delta = \mathbf{G}^H \mathbf{s}^H - \mathbf{G}\,\mathbf{s}^H$$

$\Delta \rightarrow 0$, not sensitive
$\Delta \rightarrow \infty$, sensitive

assumption	*sensitivity*
half-space	
Poisson-solid	
isotropy	
homogeneity	
combined	

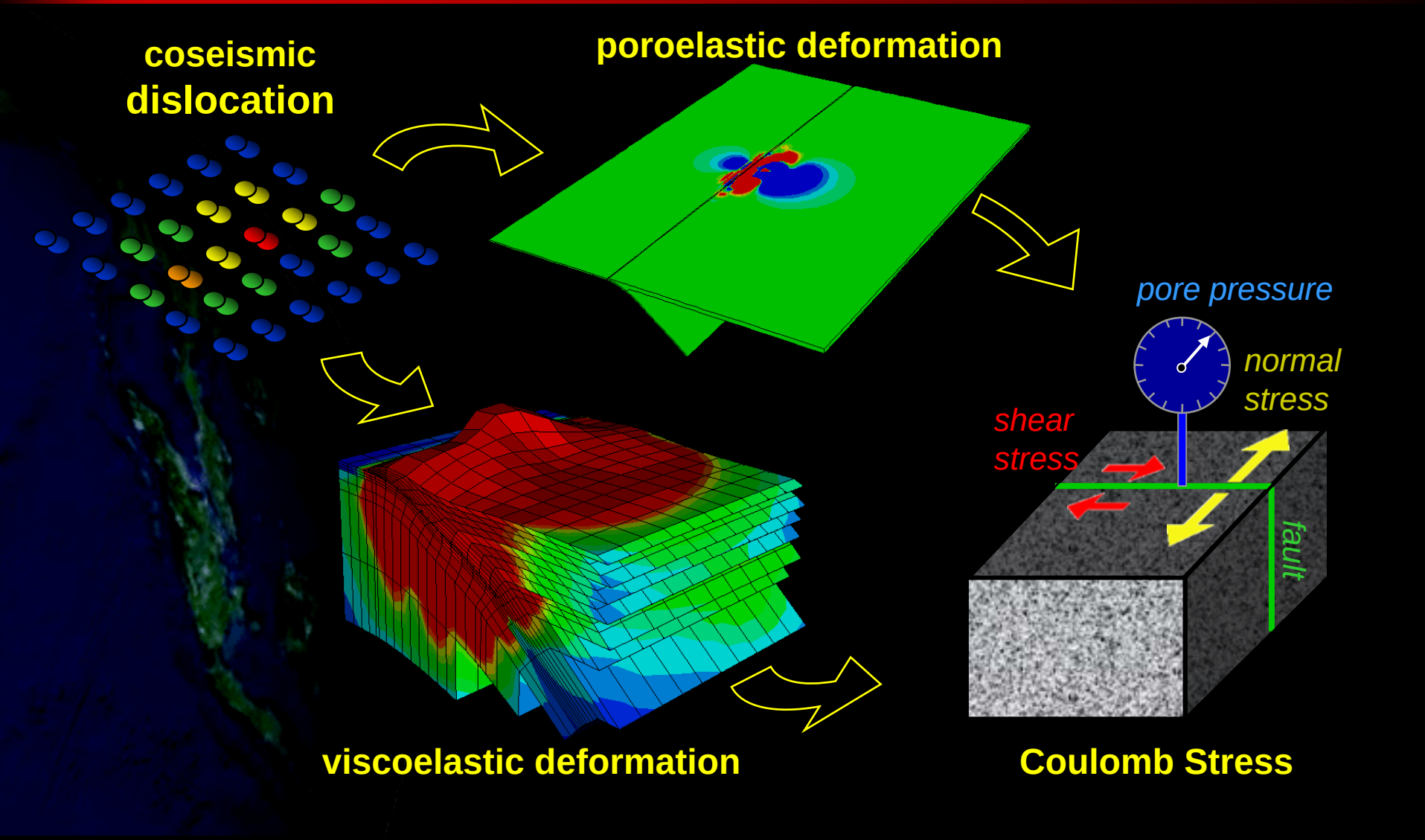
ellipses are 1σ uncertainty, centered on GPS site locations

0 100 200
kilometers

1995 M8 Jalisco quake

Why worry about the dislocation distribution?

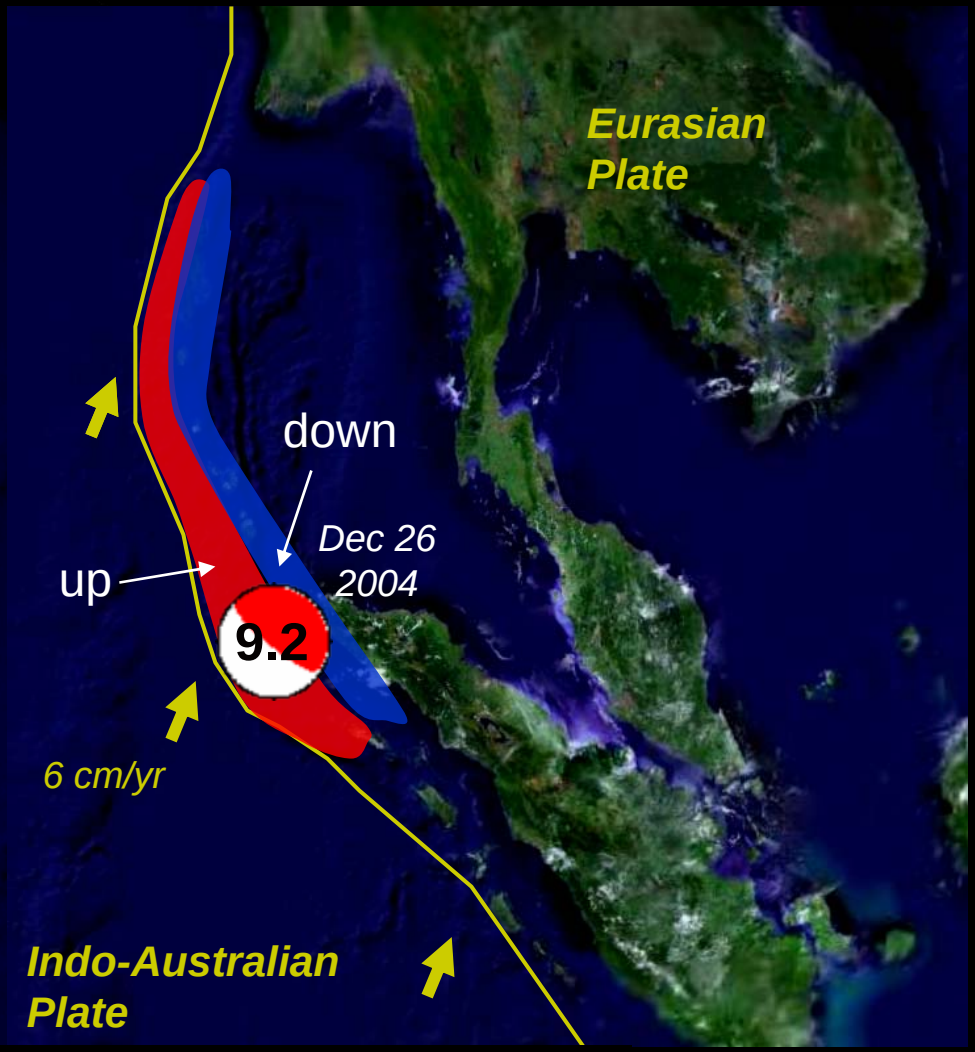
Dislocation drives postseismic processes



1995 M8 Jalisco quake

Rupture & Deformation

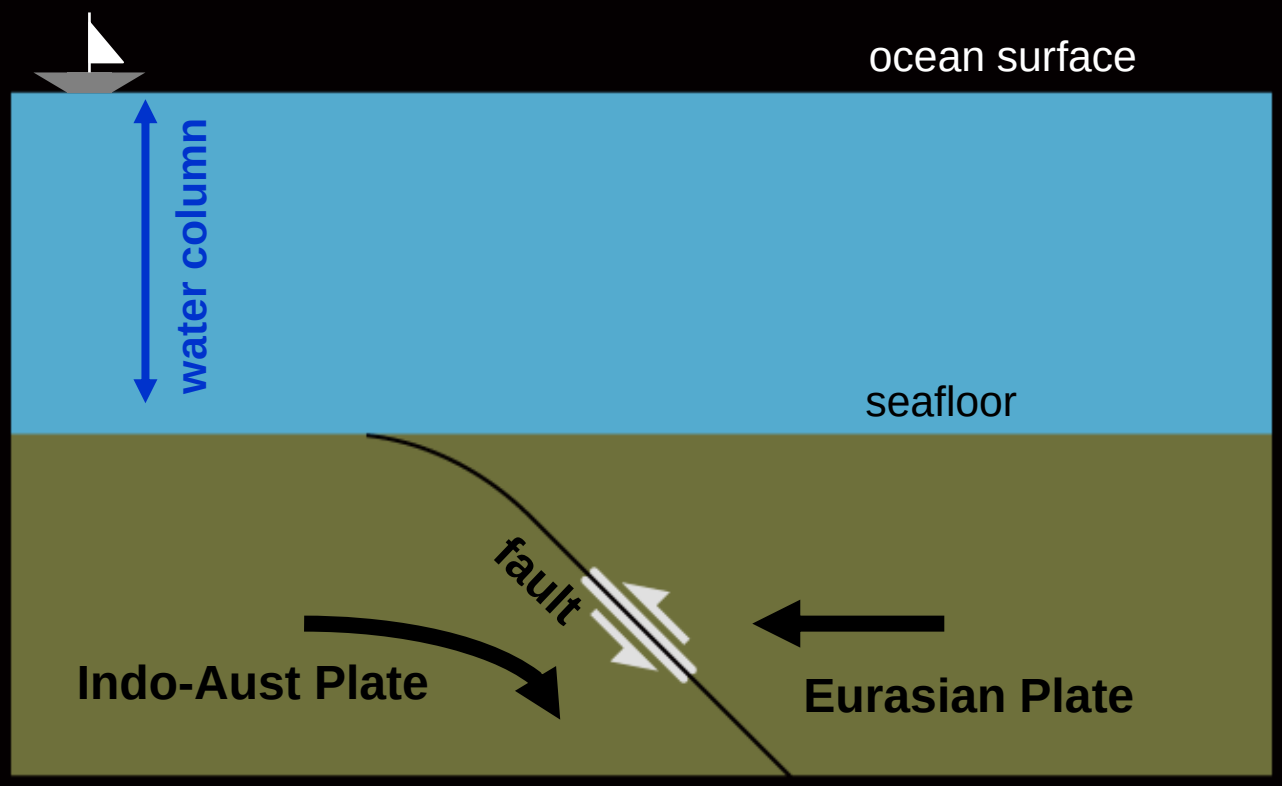
2004 M9 Sumatra-Andaman earthquake & tsunami



2004 M9 Sumatra-Andaman quake

Predicting seafloor deformation

Changes overlying water column

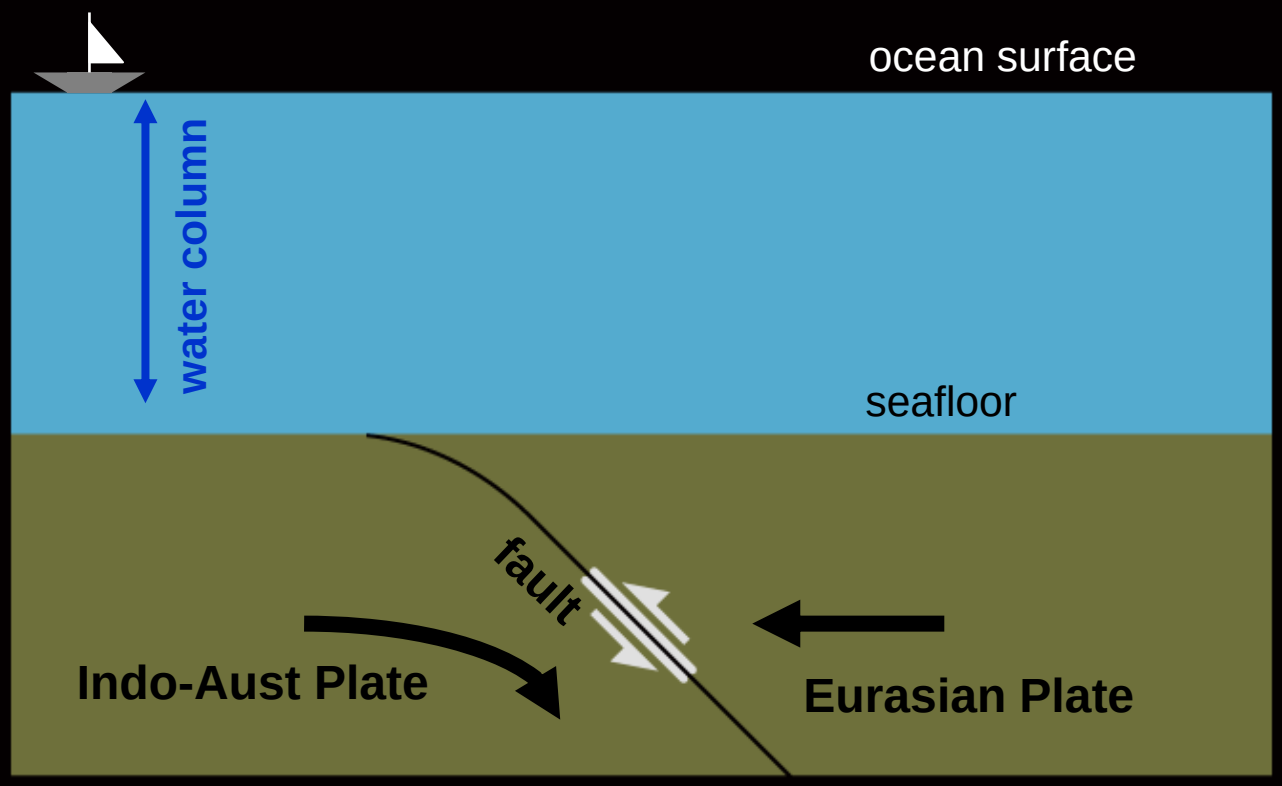


cross-section

2004 M9 Sumatra-Andaman quake

Predicting seafloor deformation

Changes overlying water column

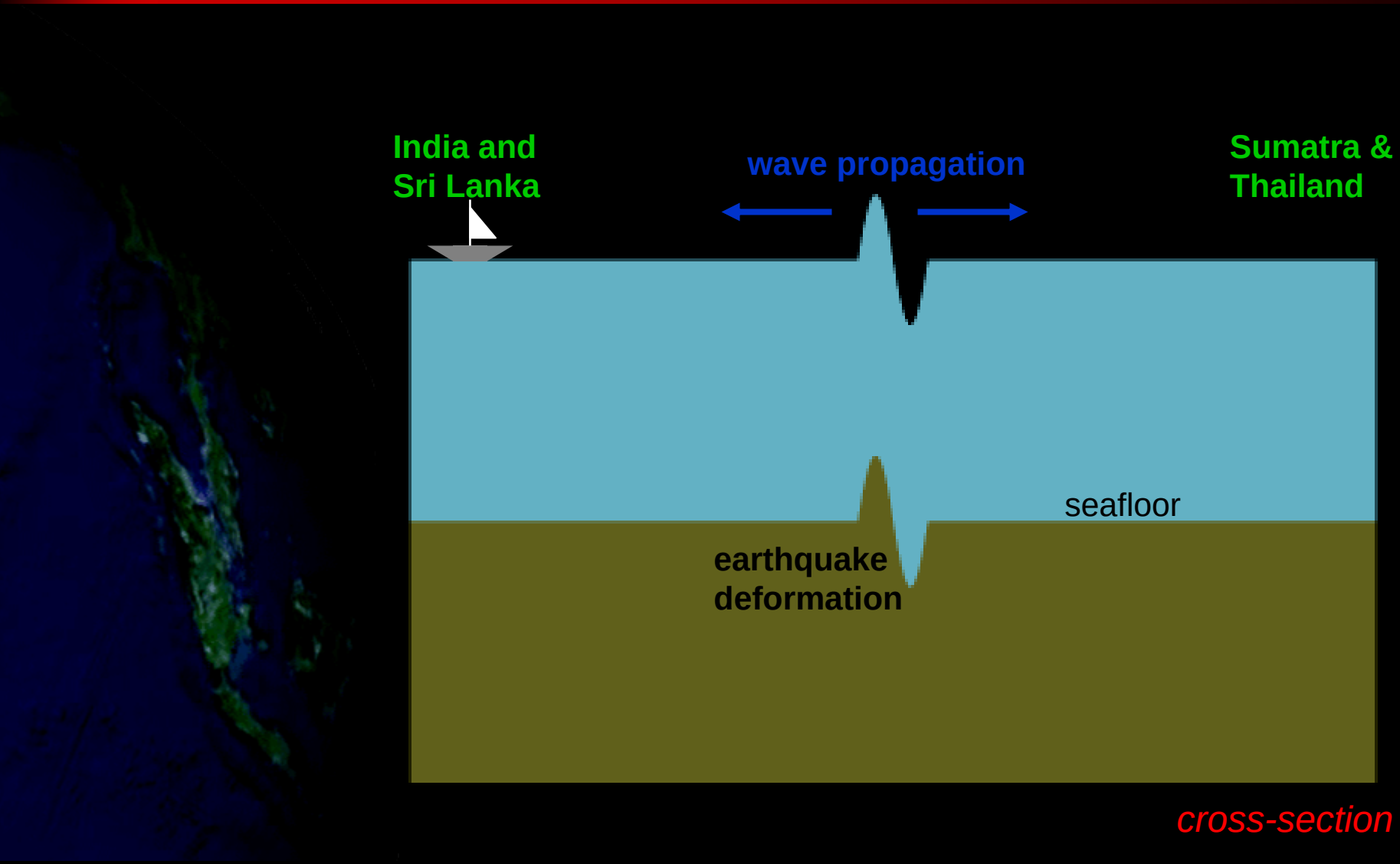


cross-section

2004 M9 Sumatra-Andaman quake

Predicting seafloor deformation

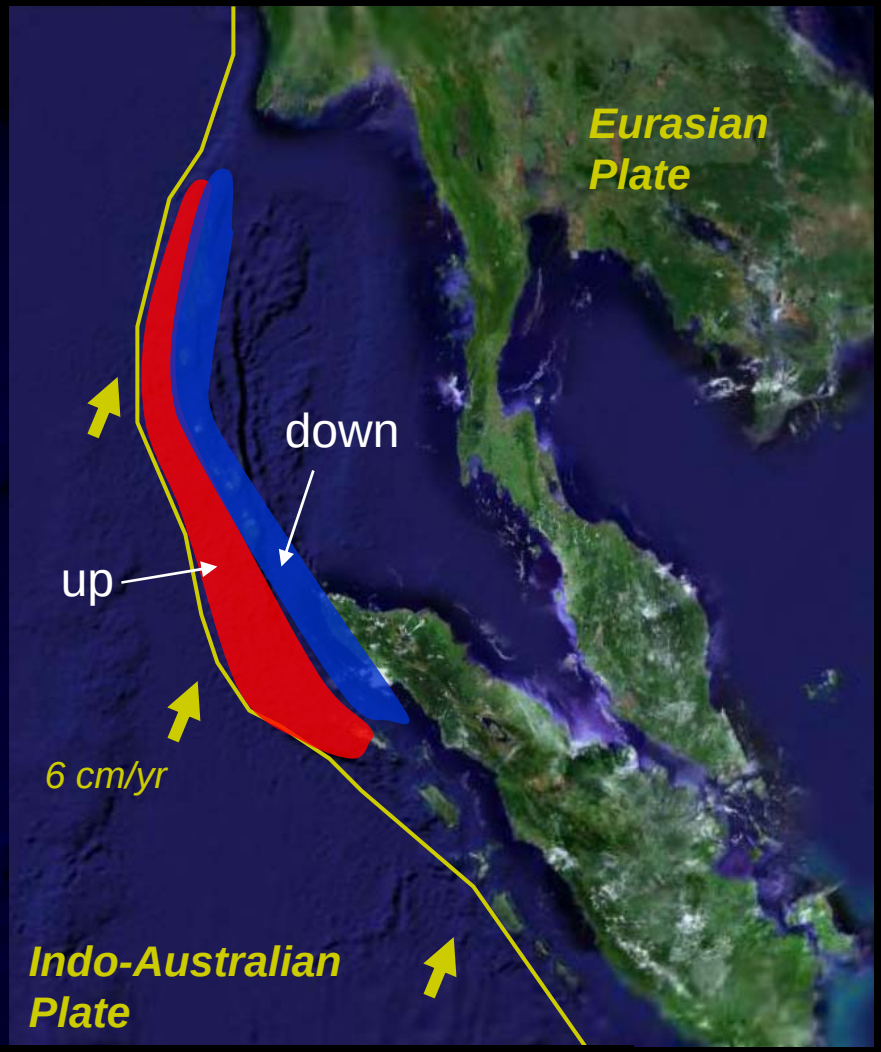
Drives tsunami propagation models



2004 M9 Sumatra-Andaman quake

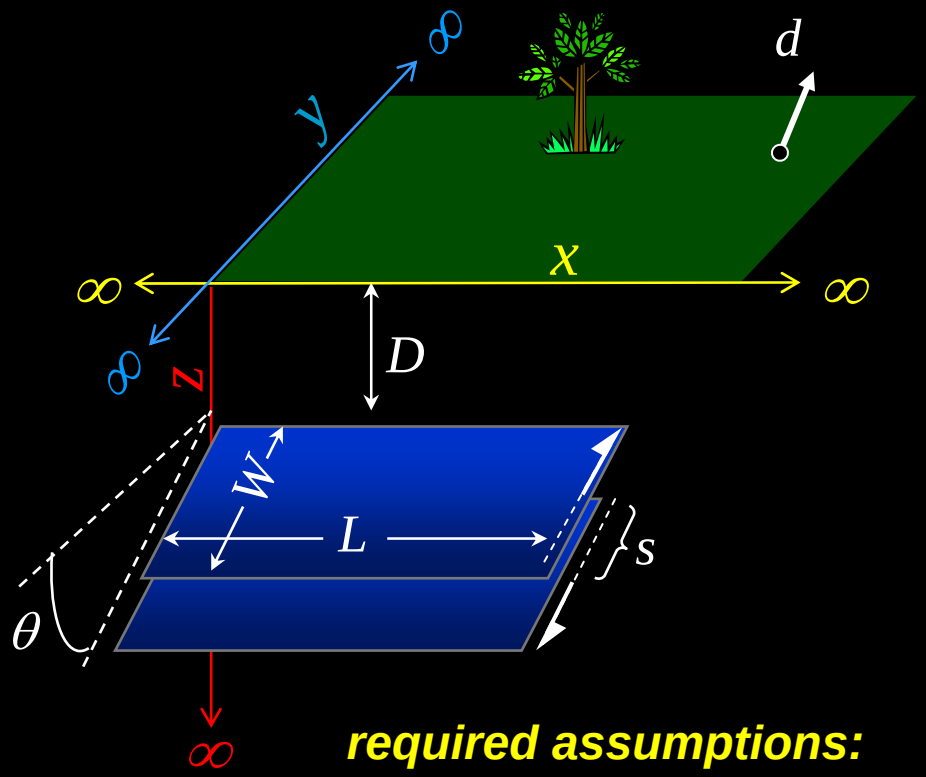
Predicting Seafloor Deformation

The source of the tsunami



2004 M9 Sumatra-Andaman quake

typical deformation model

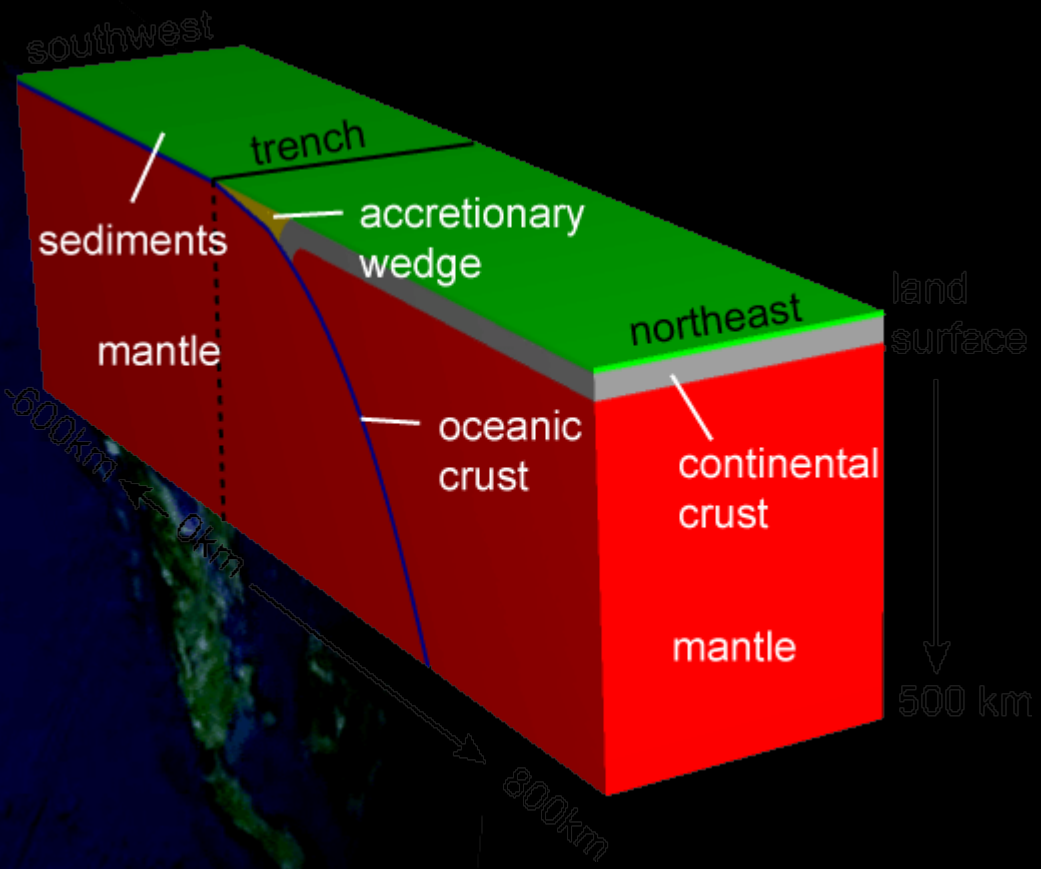


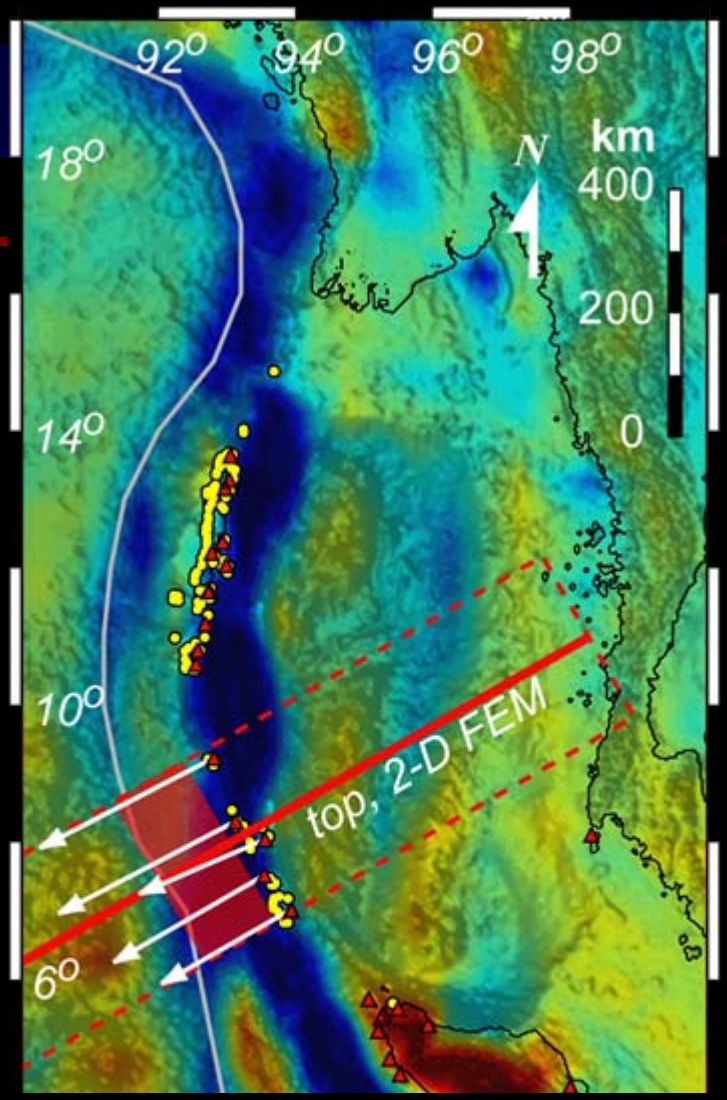
required assumptions:

- ***homogeneous***
- ***isotropic***
- ***Poisson-solid***
- ***half-space***

Predicting Seafloor Deformation

Preliminary model: 2D FEM





-90 90

gravity anomalies, mGal

GPS site

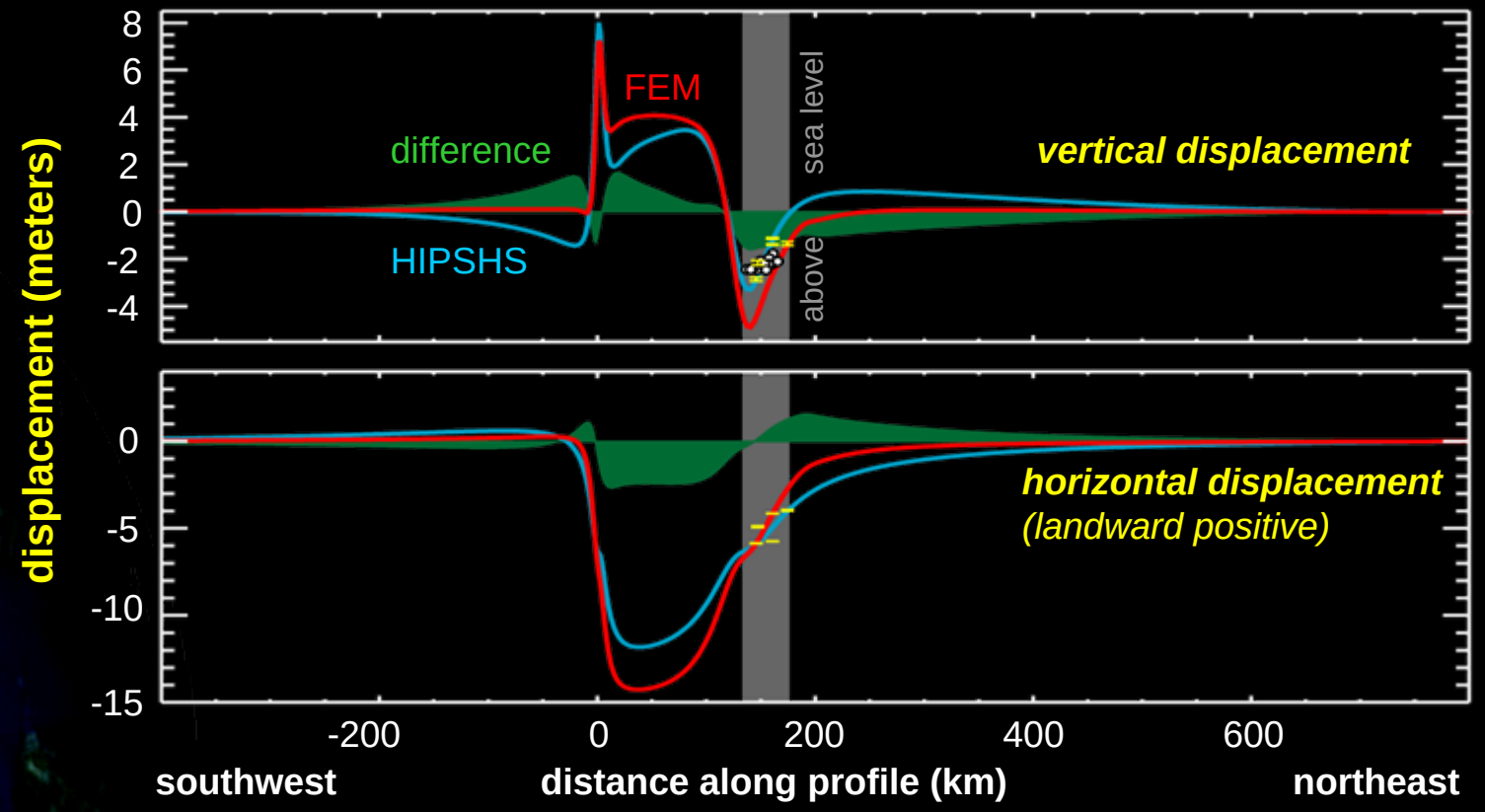
GPS displacement 2 m

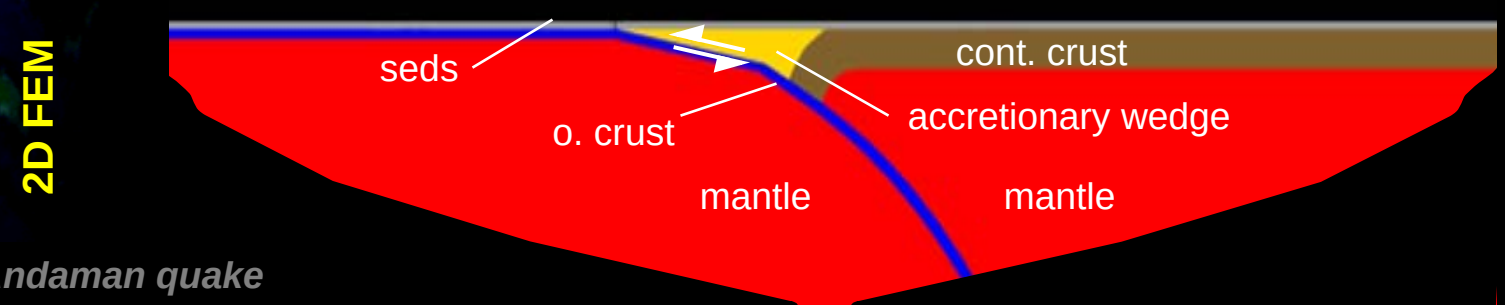
remote sensing targets

2004 M9 Sumatra-Andaman quake

Predicting Seafloor Deformation

Heterogeneity & displacement

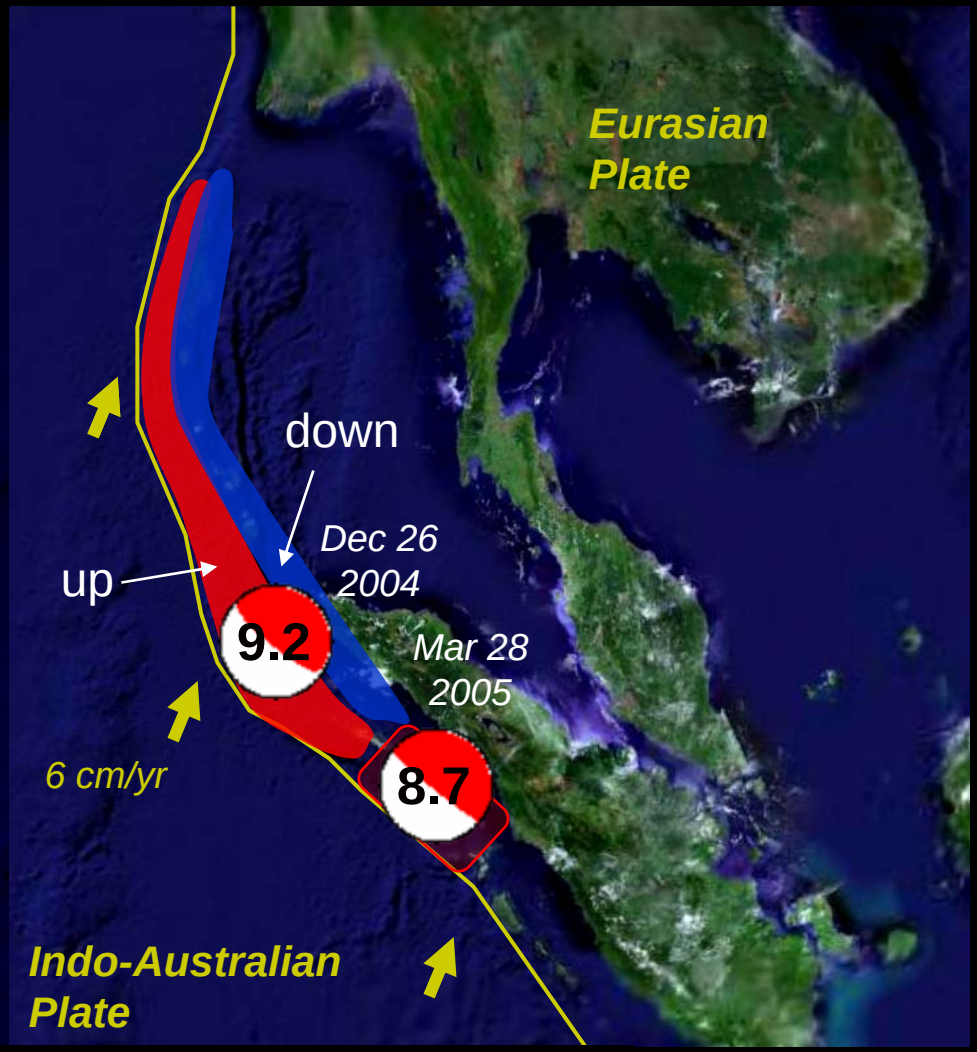


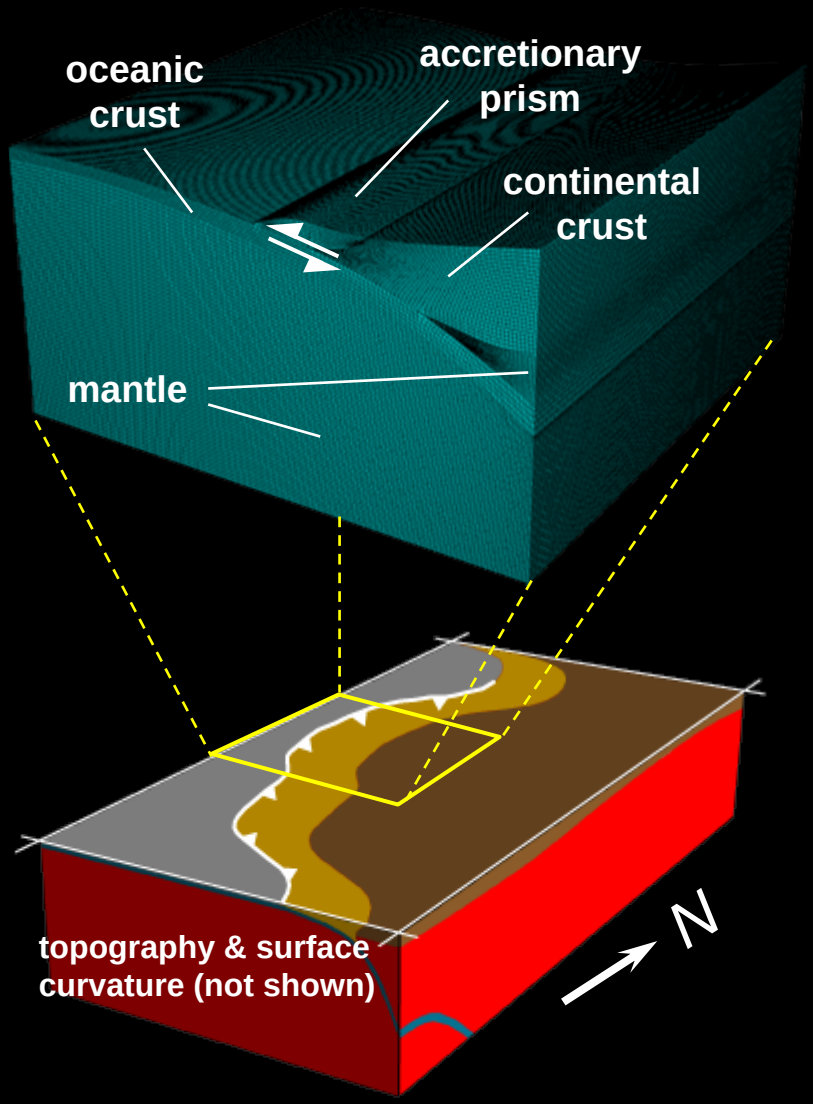


2004 M9 Sumatra-Andaman quake

Coming soon:

3D virtual Sumatra-Andaman subduction zone





2004 M9 Sumatra-Andaman quake

ABAQUS: Comprehensive FEM environment

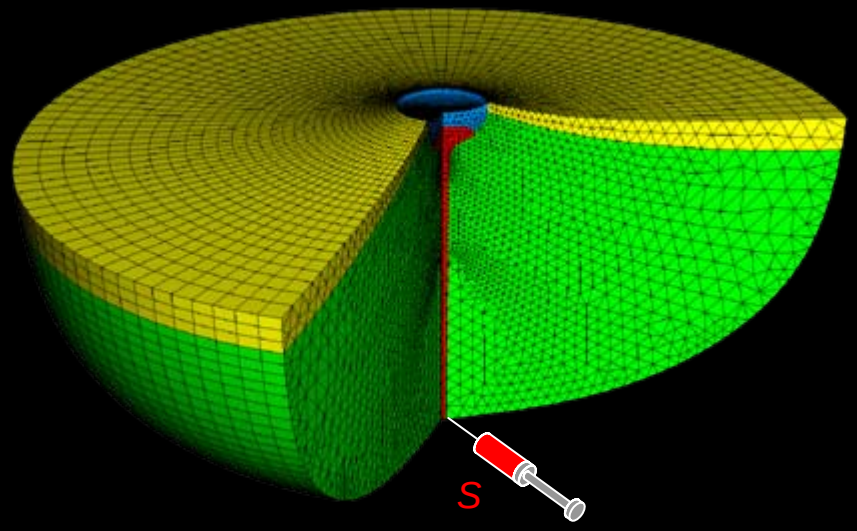
Capabilities

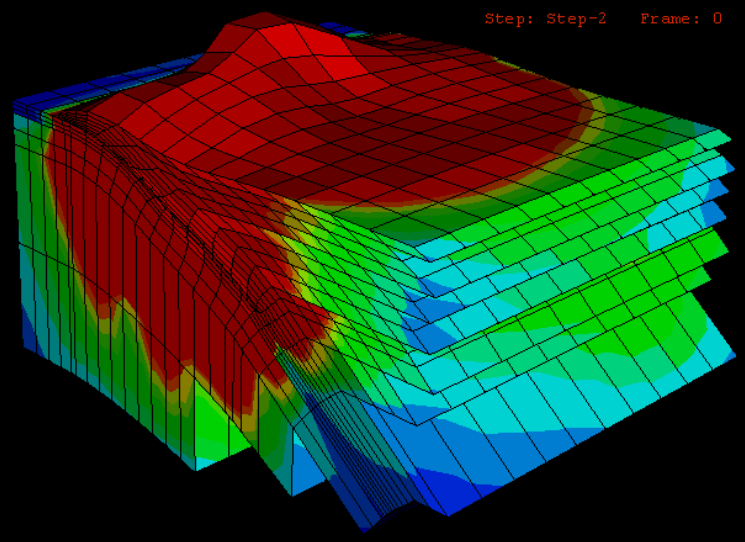
preprocessor
complex geometry
axisym., 2D, 3D

mechanics simulated (partial list)
elastic
poroelastic
viscoelastic (linear & nonlinear)
thermoelastic
plastic

scripts → inverse models
linear GF's
automated geometric perturbations
& remeshing non-linear inverse methods

postprocessor
slicing
time-series extractions
animations
VRML for geowall export





Acknowledgements

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