Two phase theory of compaction and damage

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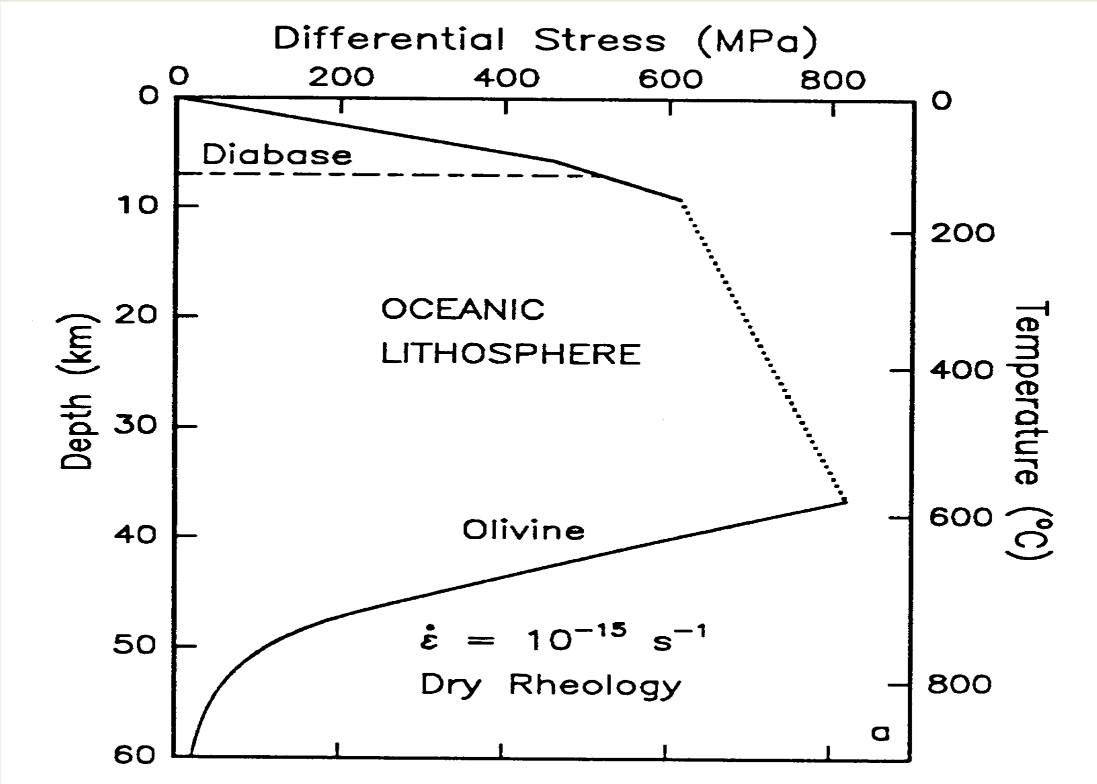
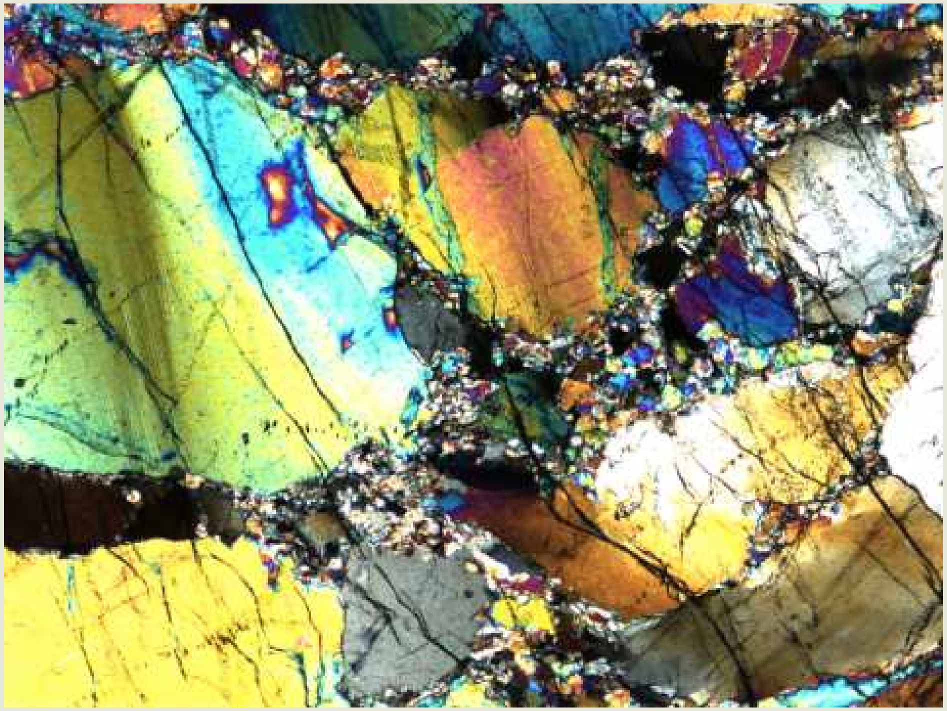
Damage 1: Microcrack and void generation



Brittle-ductile behavior in lithosphere connects the pure brittle/frictional-sliding regime and the viscous/ductile regimes

Damage 2: Grainsize reduction



- Mylonites indicate that grainsize reduction causes shear localization in lithosphere during creep such as through dynamic recrystalization.
- Fault gauge involves grainsize reduction by cataclastic processes

Two-Phase Damage Theory*

Basic Hypothesis:

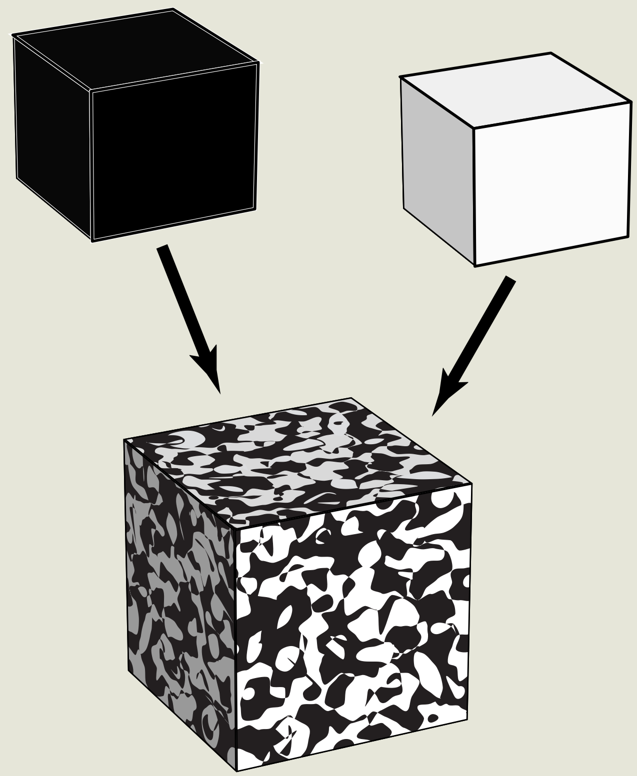
- Cracks, fractures = voids .... implies 2 phases:
  - "matrix" (host rock)
  - "fluid" (void-filling medium, e.g., water, or air)
- Deformational work goes into making voids or cracks
- Energy to make voids/cracks:

  ≈ surface energy on fracture surface

  ≈ surface energy on interface between phases

Approach (mild tutorial)

- Start with two *simple* viscous materials called **matrix** (= host) and **fluid** (= void filler)
  - Basic properties: densities (ρ_m, ρ_f), viscosities (μ_m, μ_f), etc.
- Mix them "simply" (isotropic, no phase changes)



Mixture's additional properties

- Location of **fluid pores** and **matrix grains**:

$$\Theta = \begin{cases} 1, \text{in pores} \\ 0, \text{in grains} \end{cases}$$

such that fluid and matrix volumes within total volume δV are

$$\delta V_f = \int_{\delta V} \Theta dV, \qquad \delta V_m = \int_{\delta V} (1 - \Theta) dV$$

- Location and orientation of **interface** $\boldsymbol{\nabla}\Theta$ and interface area:

$$\delta A_i = \int_{\delta V} |\boldsymbol{\nabla}\Theta| dV$$

- Interfacial surface tension (energy): γ

Continuum theory

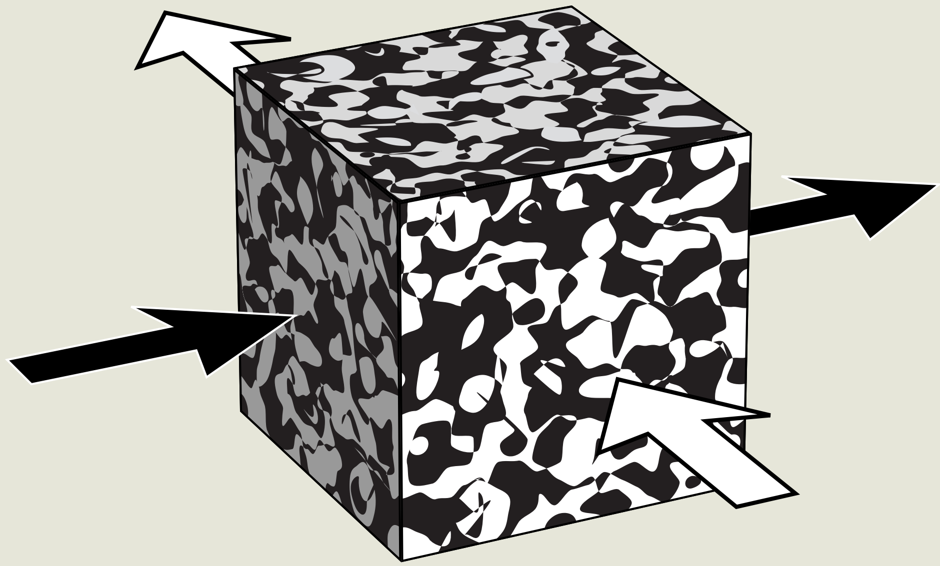
- Can't track individual pores, grains and interfaces: use quantities that are **volume-averaged, continuous** (i.e., exist at all points):
  - Porosity (fluid volume fraction) $\phi = \frac{1}{\delta V} \int_{\delta V} \Theta dV$
  - Interface area per volume $\alpha(\phi) = \frac{\delta A_i}{\delta V} = \mathcal{A}\phi^a(1-\phi)^b$ where $\mathcal{A} \sim (\mathrm{grain/pore-size})^{-1};\ a, b \leq 1$ and interface curvature $\sim d\alpha/d\phi$

- Get governing equations in terms of **averaged** quantities, e.g., velocities

$$\mathbf{v}_f = \frac{1}{\phi\delta V}\int_{\delta V} \mathbf{v}_f^{true}\Theta dV \quad , \quad \mathbf{v}_m = \frac{1}{(1-\phi)\delta V}\int_{\delta V} \mathbf{v}_m^{true}(1-\Theta) dV$$

- Until symmetry breaking assumption is made (regarding difference between phases), equations should be invariant to a switch of indices f and m (and ϕ with $1-\phi$).

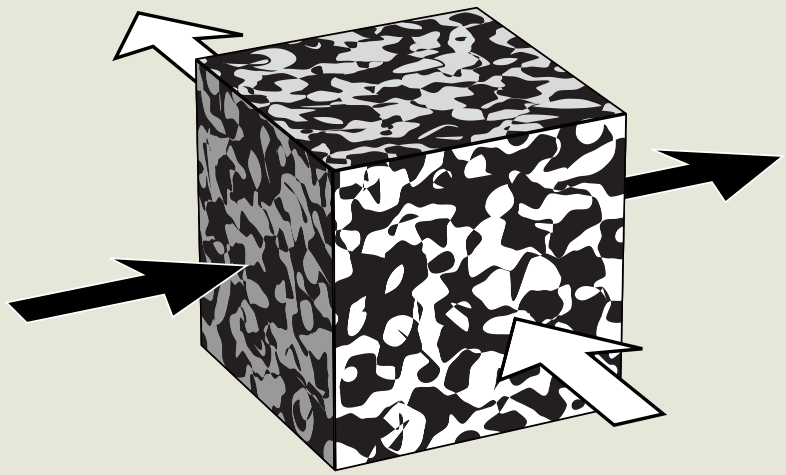
Mass conservation



- Growth in fluid volume governed by influx/efflux of fluid through surface exposure of pores on control volume; likewise for matrix volume:
- Result: equations for volume-fraction of pores and grains:

$$\frac{\partial \phi}{\partial t} + \boldsymbol{\nabla} \cdot [\phi \mathbf{v}_f] = 0 \qquad \frac{\partial (1-\phi)}{\partial t} + \boldsymbol{\nabla} \cdot [(1-\phi)\mathbf{v}_m] = 0$$

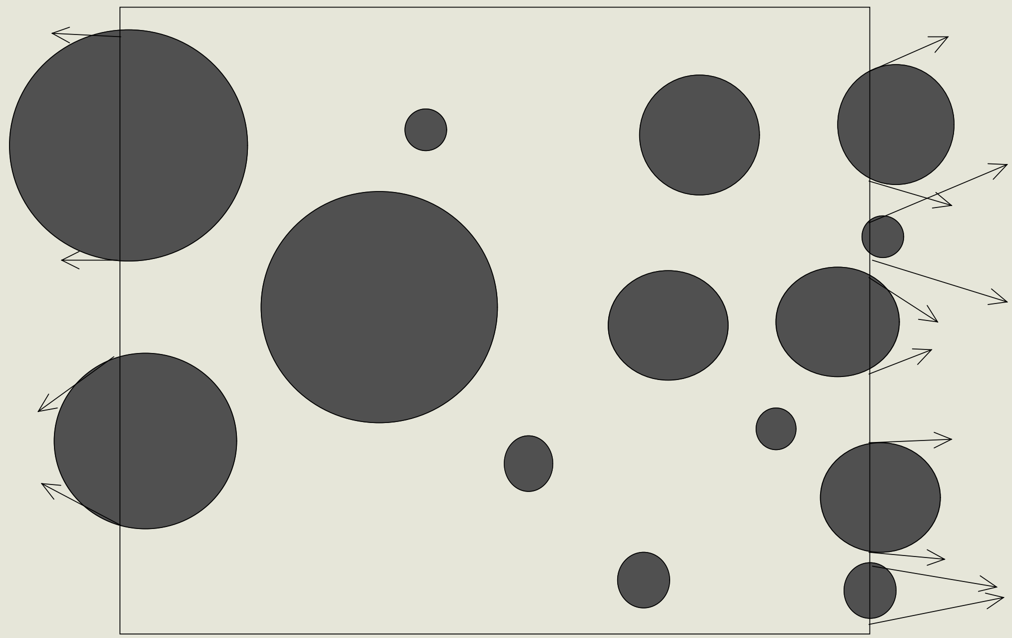
Momentum conservation (force balance)



- Body force, e.g., gravity $\boxed{g}$, acts on pores and grains
- Fluid and matrix pressures $\boxed{P_f, P_m}$ and stresses $\boxed{\underline{\boldsymbol{\tau}}_f, \underline{\boldsymbol{\tau}}_m}$ act on surface exposures of pores and grains
- Interaction force: fluid surface forces (e.g., drag) acting on matrix through their interface and vice versa

Surface energy in two-phase theory

- Surface tension $\boxed{\gamma}$ acts as line force on intersection of interface with surface



- Surface energy exists at interface
  - Interface area per volume $\alpha = \mathcal{A}\phi^a(1-\phi)^b$ where $\mathcal{A} \sim \frac{1}{\text{grain/pore-size}};\ a, b \leq 1$ and ϕ is fluid volume fraction
  - Interface curvature $\sim d\alpha/d\phi$

Interaction (body) force

- Forces acting on fluid through interface (by matrix + interface)
- ... and on matrix through interface (by fluid + interface).
- Includes:
  - Common pressure force
  - Common viscous drag: $\boxed{\pm c(\mathbf{v}_m - \mathbf{v}_f)}$

    where $c \sim \frac{\text{viscosity}}{\text{permeability}}$
  - Interface surface tension

Resulting momentum equations

- Fluid:

$$0 = -\phi\left[\nabla P_f + \rho_f g\hat{\mathbf{z}}\right] + \nabla\cdot\left[\phi\underline{\boldsymbol{\tau}}_f\right]$$

$$+c\Delta\mathbf{v} + \omega\left[(P_m - P_f)\nabla\phi + \nabla(\gamma\alpha)\right]$$

- Matrix:

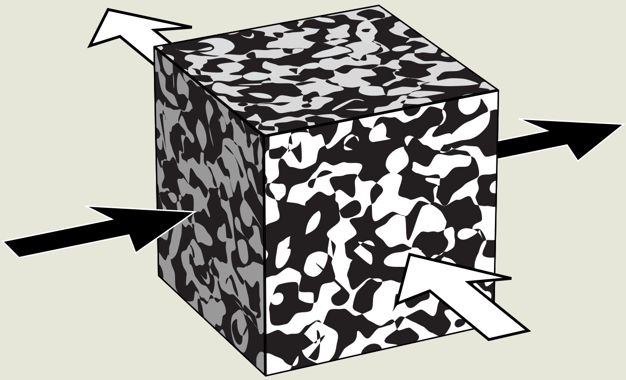
$$0 = -(1-\phi)\left[\nabla P_m + \rho_m g\hat{\mathbf{z}}\right] + \nabla\cdot\left[(1-\phi)\underline{\boldsymbol{\tau}}_m\right]$$

$$-c\Delta\mathbf{v} + (1-\omega)\left[(P_m - P_f)\nabla\phi + \nabla(\gamma\alpha)\right]$$

- where stress is $\underline{\boldsymbol{\tau}}_j = \mu_j\left(\nabla\mathbf{v}_j + [\nabla\mathbf{v}_j]^t - \frac{2}{3}(\nabla\cdot\mathbf{v}_j)\underline{\boldsymbol{I}}\right)$ with $j = f$ or m.
- average and difference quantities are $\bar{q} = \phi q_f + (1-\phi)q_m$ and $\Delta q = q_m - q_f$.
- ω represents extent to which surface tension/energy is embedded in one phase or the other; for solid matrix and liquid fluid $\omega \approx 0$.

Energy Equations: Heating and Damage

- Consider all input and growth of energy in fluid and matrix, and on interface:



- Heat (entropy related):

$$\overline{\rho c}\frac{\overline{\overline{D}}T}{Dt} - T\frac{\widetilde{D}}{Dt}\left(\alpha\frac{d\gamma}{dT}\right) - T\alpha\frac{d\gamma}{dT}\boldsymbol{\nabla}\cdot\tilde{\mathbf{v}} = Q - \boldsymbol{\nabla}\cdot\mathbf{q} + B\left(\frac{\widetilde{D}\phi}{Dt}\right)^2 + (1-f)\Psi$$

where "$\sim$" means frame of reference of interface (i.e., $\tilde{\mathbf{v}} = \omega\mathbf{v}_f + (1-\omega)\mathbf{v}_m$)

Interface Work and Damage

- Equilibrium:

$$P_m - P_f + \gamma \frac{d\alpha}{d\phi} = 0$$

- Quasi-equilibrium:

$$P_m - P_f + \gamma \frac{d\alpha}{d\phi} = -B \frac{\widetilde{D}\phi}{Dt}$$

- Far from equilibrium (assume for now 1/grainsize $\mathcal{A}$ is constant):

$$\left(P_m - P_f + \gamma \frac{d\alpha}{d\phi} \right) \frac{\widetilde{D}\phi}{Dt} = -B \left(\frac{\widetilde{D}\phi}{Dt} \right)^2 + f\Psi$$

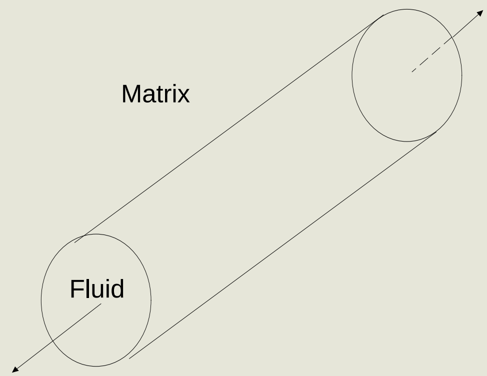
where the deformational work is

$$\Psi = c\Delta v^2 + \phi \boldsymbol{\nabla} \mathbf{v}_f : \underline{\boldsymbol{\tau}}_f + (1-\phi) \boldsymbol{\nabla} \mathbf{v}_m : \underline{\boldsymbol{\tau}}_m$$

Partitioning argument: $1 - f =$ fraction of deformational work going into dissipative heating. $f =$ remainder "stored" on interface, leads to **damage**

Pressure jump

- Micro-mechanical model:



$$B = K\frac{\mu_m + \mu_f}{\phi(1-\phi)}$$

where K is a dimensionless constant of $O(1)$

McKenzie 1984 Limit

- For $\gamma = 0$ and $\mu_f \ll \mu_m$ (hence $\omega \approx 0$)

$$\Delta P = \frac{K\mu_m}{\phi}\boldsymbol{\nabla}\cdot\mathbf{v}_m$$

  since in this case $\frac{\widetilde{D}\phi}{Dt} = \frac{D_m\phi}{Dt} = (1-\phi)\boldsymbol{\nabla}\cdot\mathbf{v}_m$

- Recovers the McKenzie 1984 momentum equations exactly, assuming a “bulk viscosity” of $K\mu_m/\phi$.

Recent application: Source-sink driven 2D flow (BR2005)

- Velocity now given by

$$\mathbf{v}_m = \text{“compressible” potential flow} + \text{toroidal flow} + \text{poloidal flow}$$

$$\mathbf{v}_m = \boldsymbol{\nabla}\theta + \boldsymbol{\nabla} \times (\psi\hat{\mathbf{z}}) + \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times (W\hat{\mathbf{z}})$$

or

$$\mathbf{v}_h = \boldsymbol{\nabla}(\theta + \xi) + \boldsymbol{\nabla} \times (\psi\hat{\mathbf{z}}) \quad \text{where } \xi = \frac{\partial W}{\partial z}$$

- Source-sink S prescribes poloidal flow; vertical vorticity Ω determines toroidal flow; dilation rate G determines “compressible” dilational/compactive potential:

$$\nabla^2\xi = S, \quad \nabla^2\psi = -\Omega, \quad \nabla^2\theta = G$$

- S imposed; but equations for Ω and G are given by combined force and damage equations and are ugly:

$$\nabla^2\Omega = [\textsf{ugly mess}]$$

$$\nabla^2 G = [\textsf{hideous mess}]$$

Void generating vs grain/void size reducing damage

- Recall interface area density defined as

$$\alpha = \mathcal{A}\eta(\phi) \quad \text{where} \;\; \eta(\phi) = \phi^a(1-\phi)^b$$

  and $a, b \leq 1$.

- $\mathcal{A}$ is effectively the inverse of the average grain and/or void size.
- If now we consider $\mathcal{A}$ as a variable, and allow damage to incur non-void interface growth then

$$\gamma\mathcal{A}\frac{d\eta}{d\phi}\frac{\overline{D}\phi}{Dt} + \gamma\eta\frac{\overline{D}\mathcal{A}}{Dt} = -(P_m - P_f)\frac{\overline{D}\phi}{Dt} - B\left(\frac{\overline{D}\phi}{Dt}\right)^2 + f\Psi$$

- Assuming deformational work partitions between void and non-void interface growth, $f = f_\phi + f_\mathcal{A}$ we have

$$\gamma \mathcal{A} \frac{d\eta}{d\phi} \frac{\overline{D}\phi}{Dt} = -(P_m - P_f) \frac{\overline{D}\phi}{Dt} - B \left(\frac{\overline{D}\phi}{Dt} \right)^2 + f_\phi \Psi$$

$$\gamma \eta \frac{\overline{D}\mathcal{A}}{Dt} = f_\mathcal{A} \Psi$$

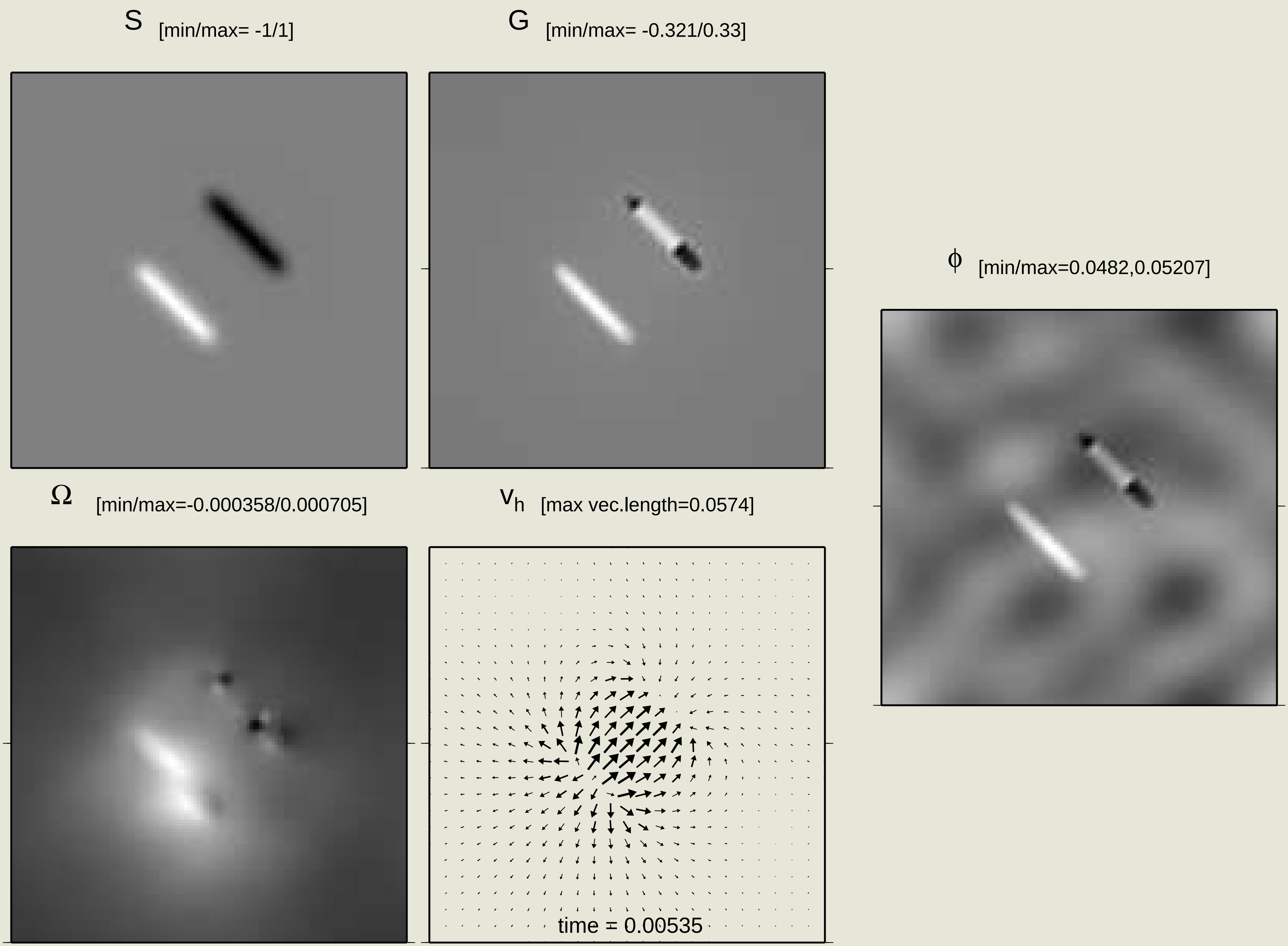
- Also allow for grain-size dependent viscosity (using diffusion creep model for a general relation):

$$\mu_m = \mu_0 (\mathcal{A}_0 / \mathcal{A})^m$$

2-D source-sink flow tests...

- S = driving source-sink flow field
- G = void-generating dilational flow field
- Ω = toroidal (strike-slip) vorticity field
- $\mathbf{v}_h$ = velocity
- ϕ = void fraction (porosity)
- $\mathcal{A}$ = inverse grain/void-size

Void-generating damage: $f_{\mathcal{A}} = 0$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G [min/max= -0.321/0.33]
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Ω [min/max=-0.000358/0.000705]
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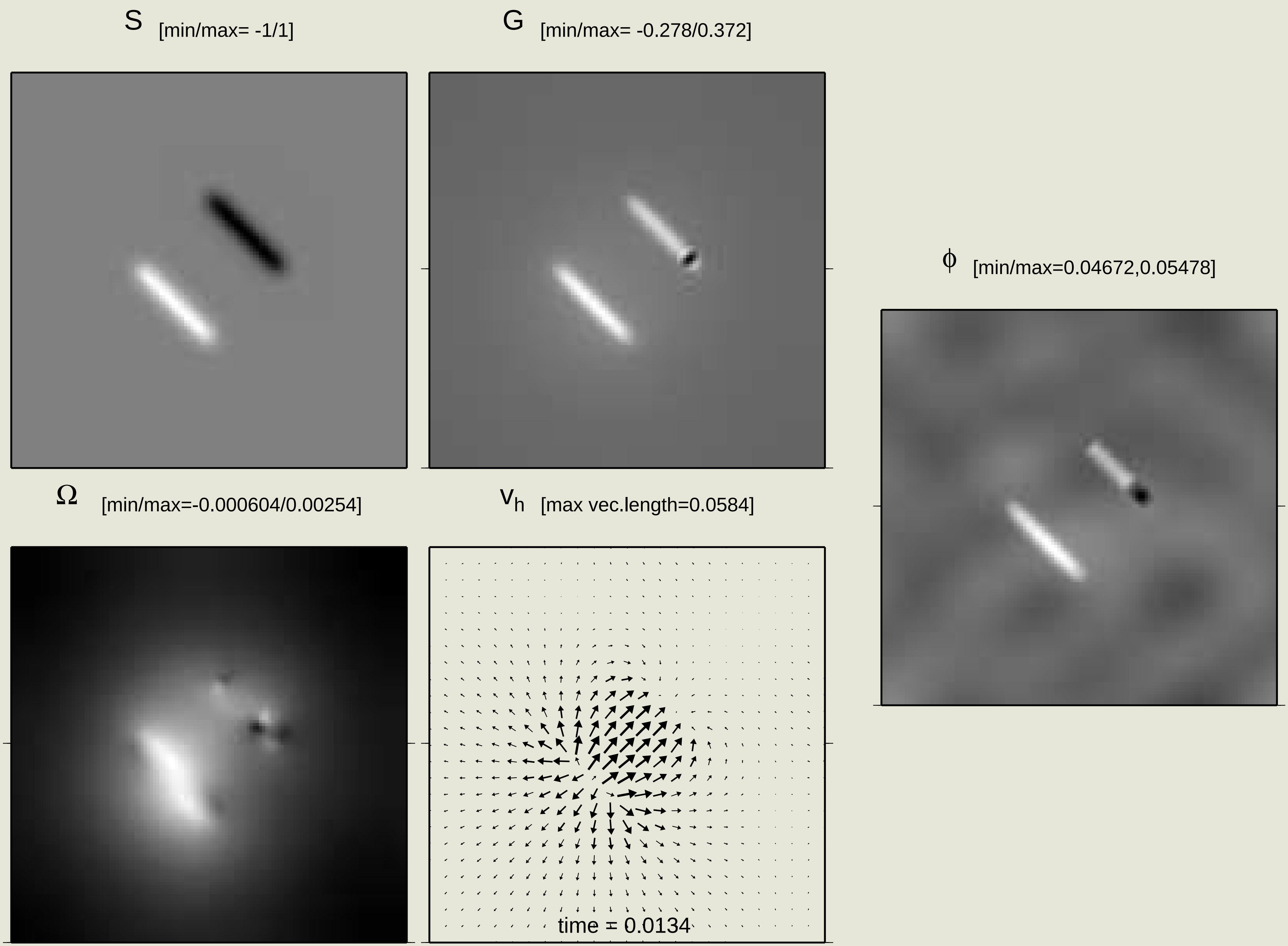

S [min/max= -1/1]
G [min/max= -0.278/0.372]
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v h [max vec.length=0.0584]
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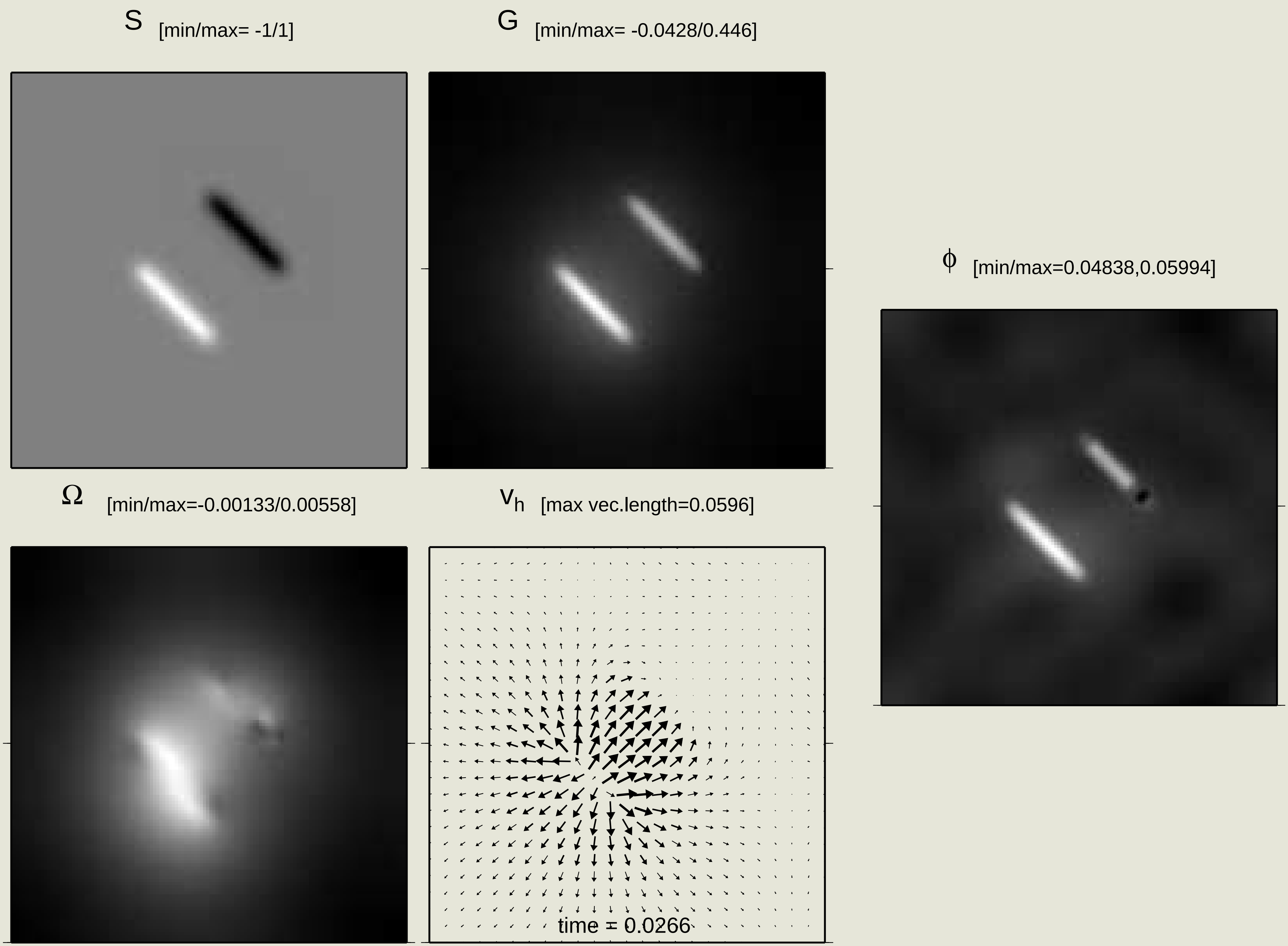
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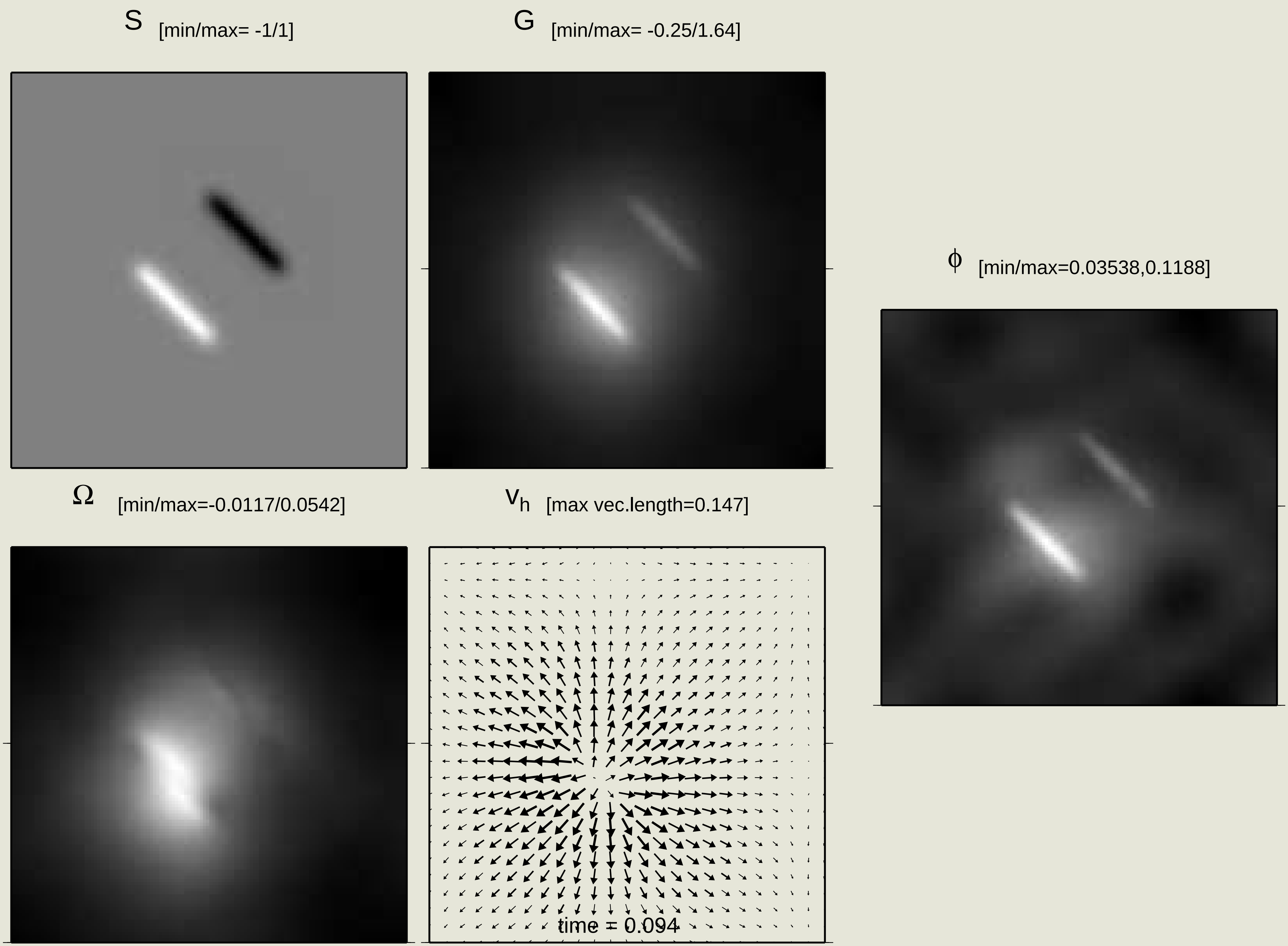
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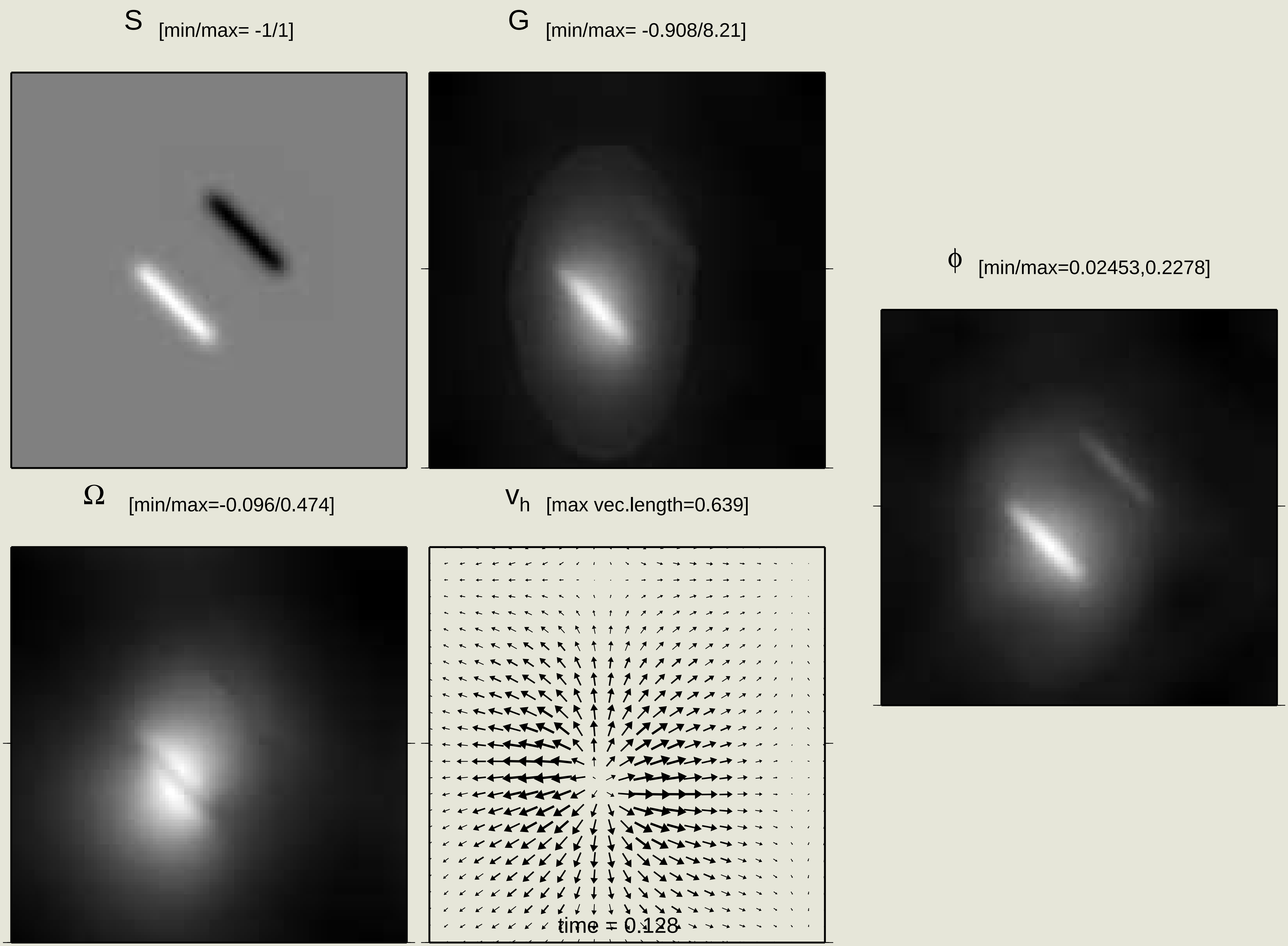

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ϕ [min/max=0.02453,0.2278]
Ω [min/max=-0.096/0.474]
v_h [max vec.length=0.639]
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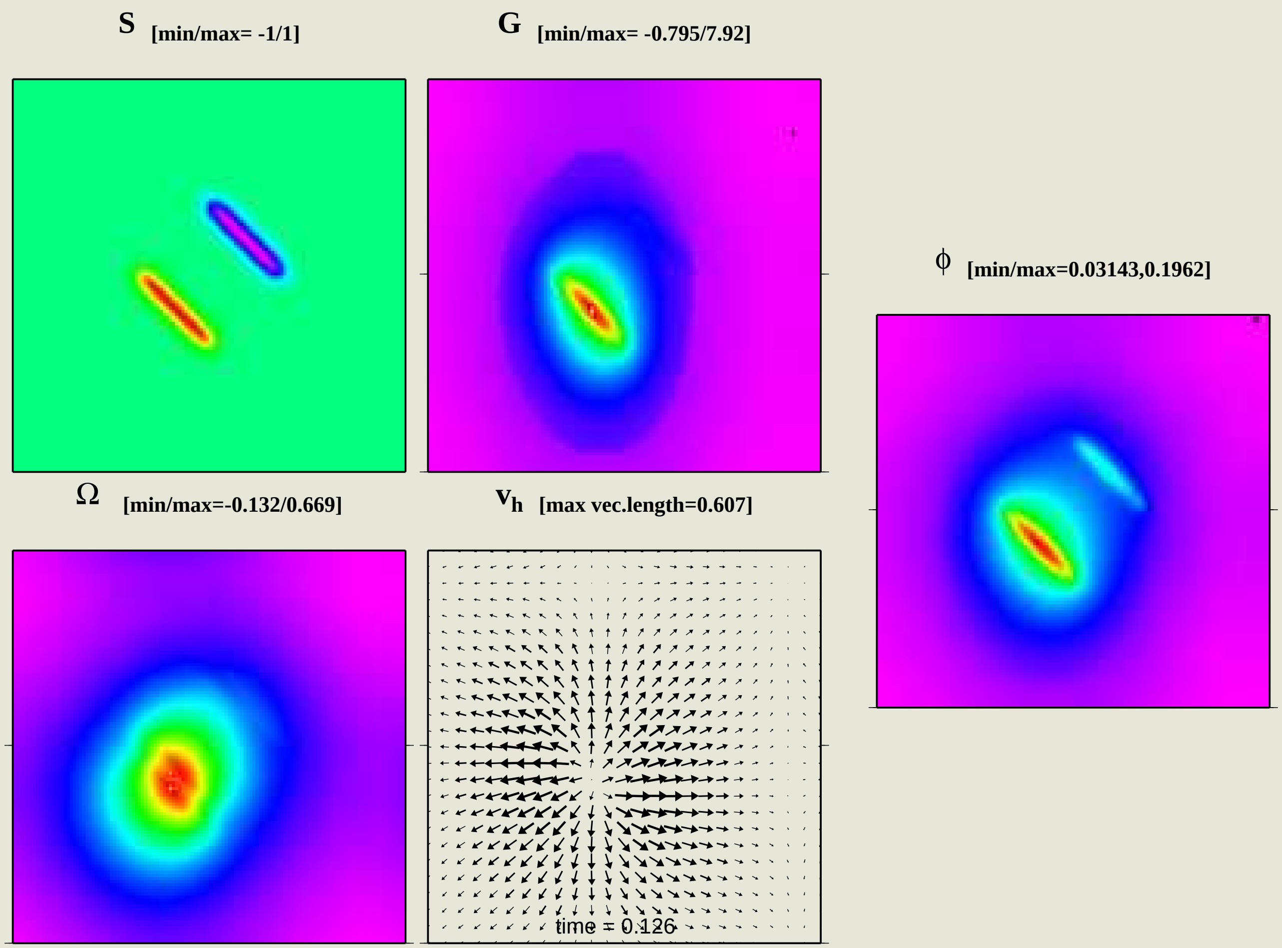
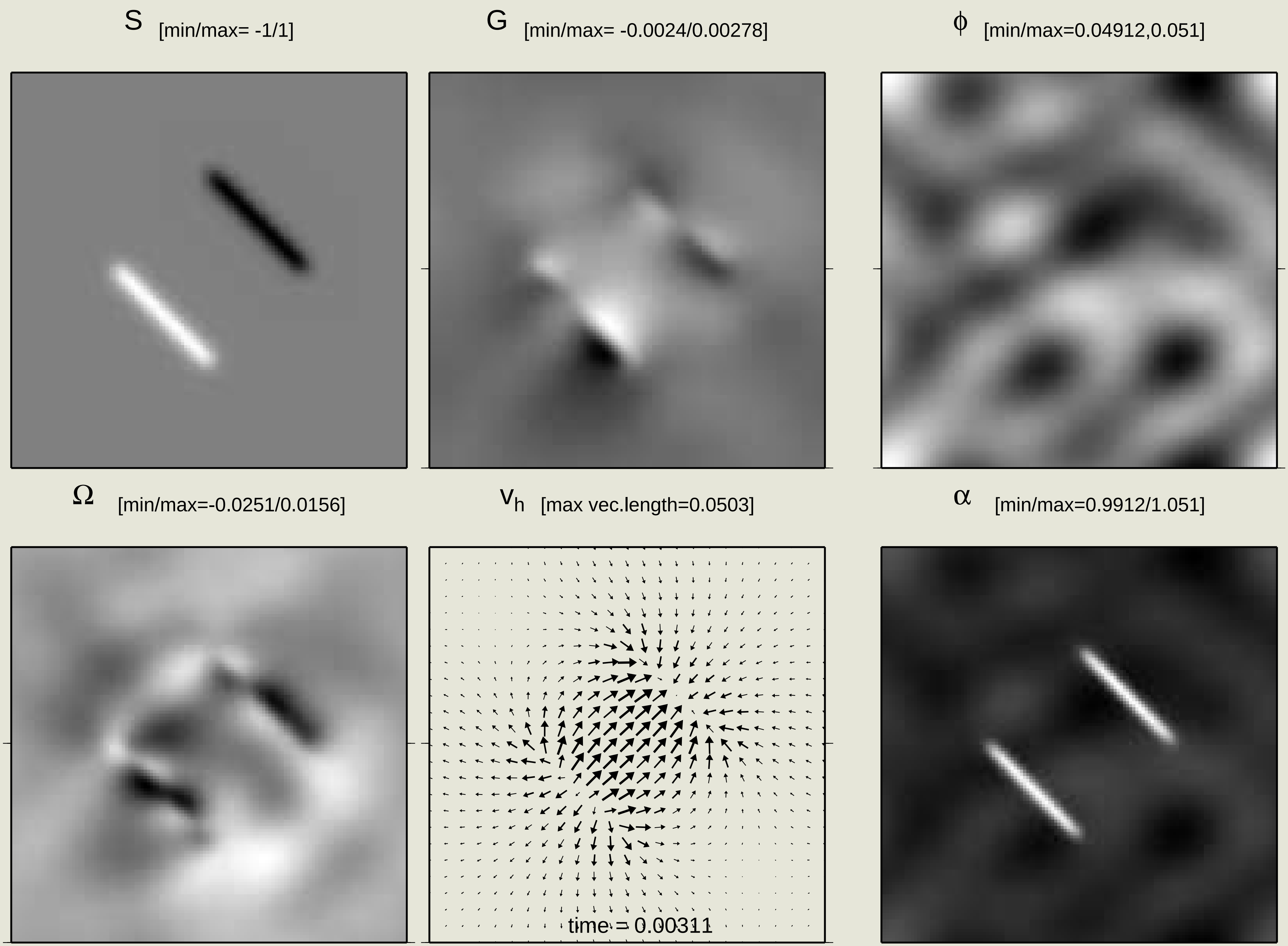
Void-generating damage



Void-generating damage generates strong dilational field that enhances apparent poloidal flow (even causes monopolar flow) and inhibits strike-slip/toroidal flow

Fineness generating or grainsize reducing damage: $f_\phi = 0$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Ω [min/max=-0.0251/0.0156]
v_h [max vec.length=0.0503]
α [min/max=0.9912/1.051]
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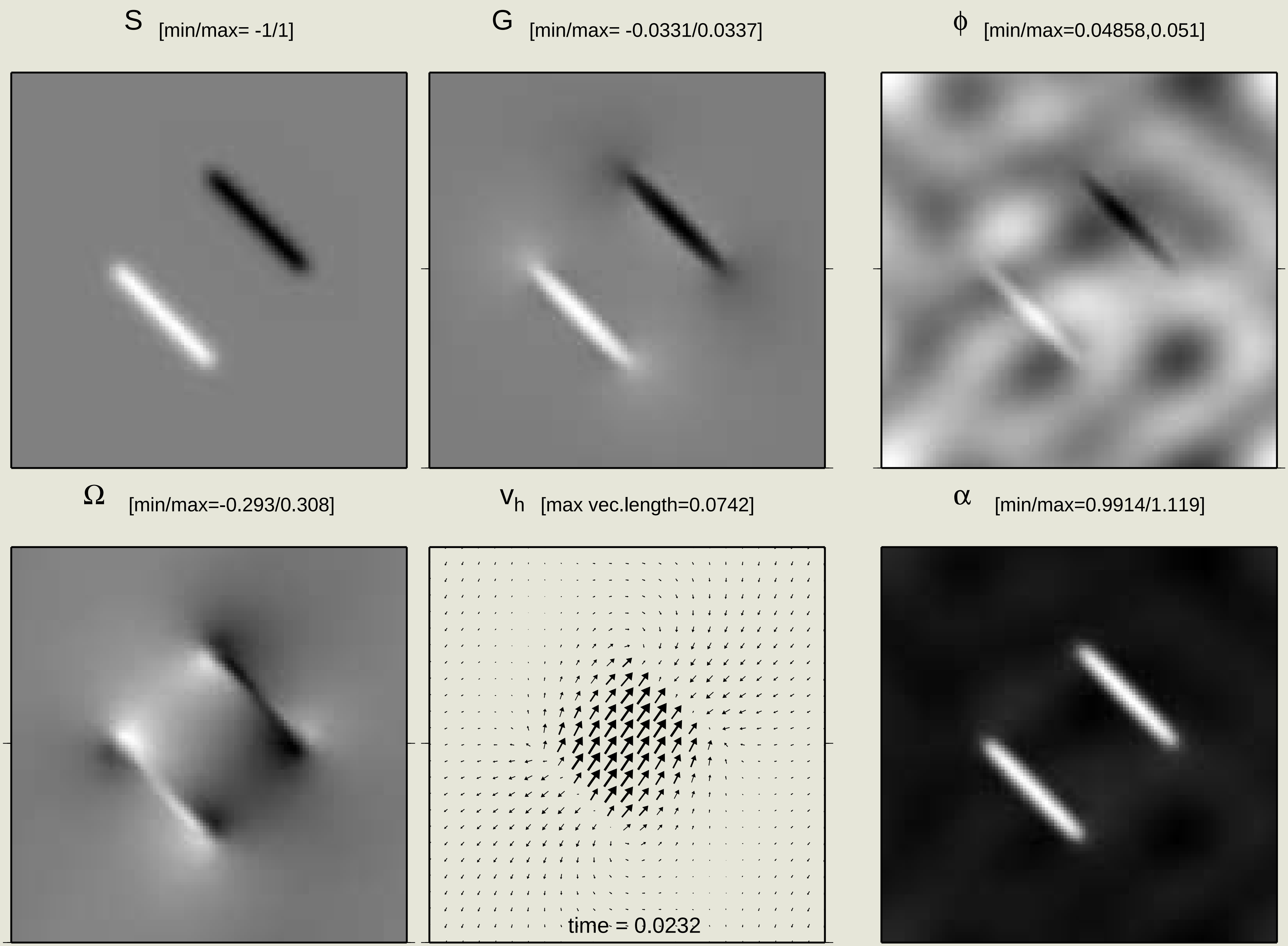
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vh [max vec.length=0.0742]
α [min/max=0.9914/1.119]
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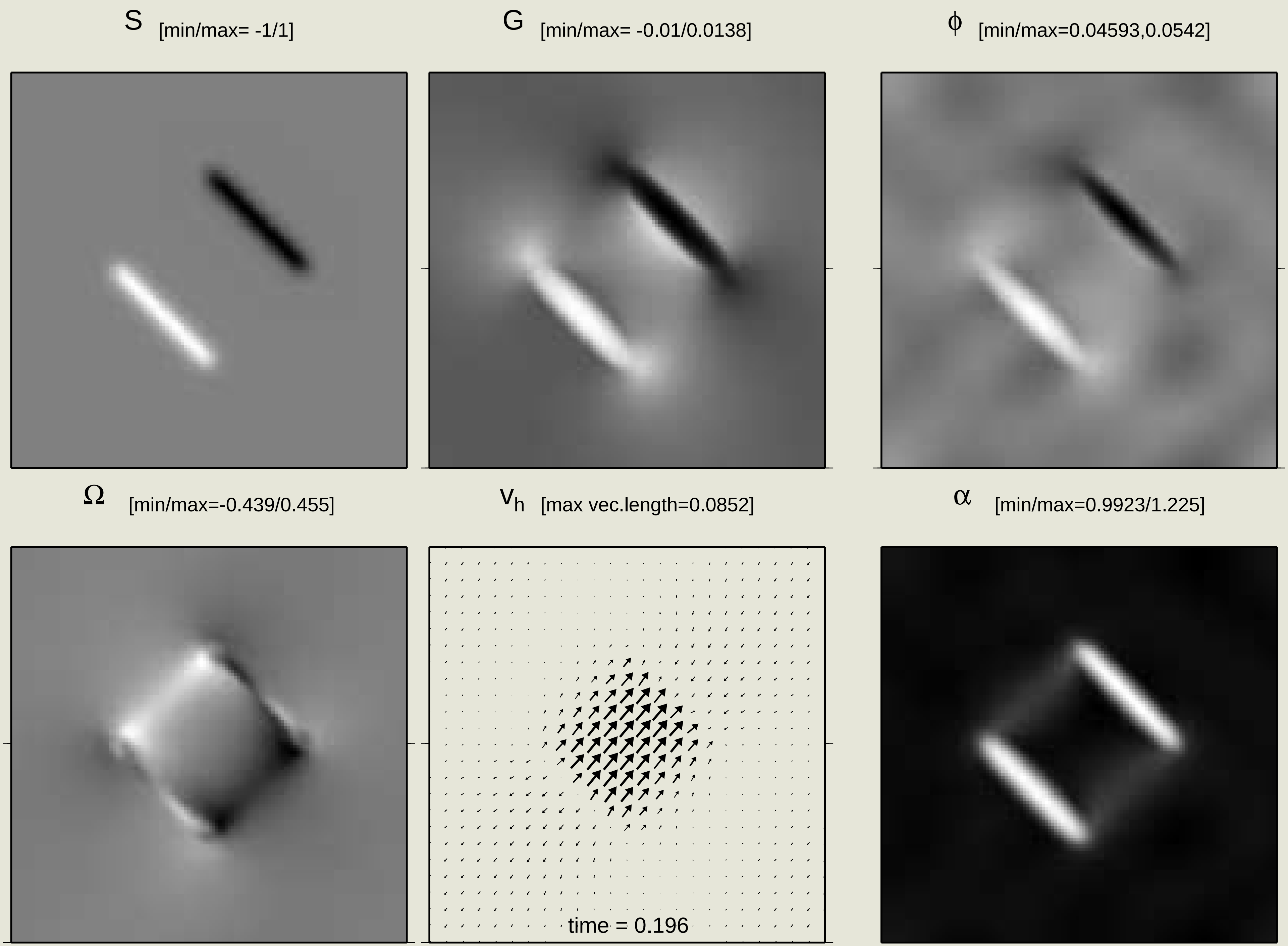

S [min/max= -1/1]
G [min/max= -0.01/0.0138]
ϕ [min/max=0.04593,0.0542]
Ω [min/max=-0.439/0.455]
v_h [max vec.length=0.0852]
α [min/max=0.9923/1.225]
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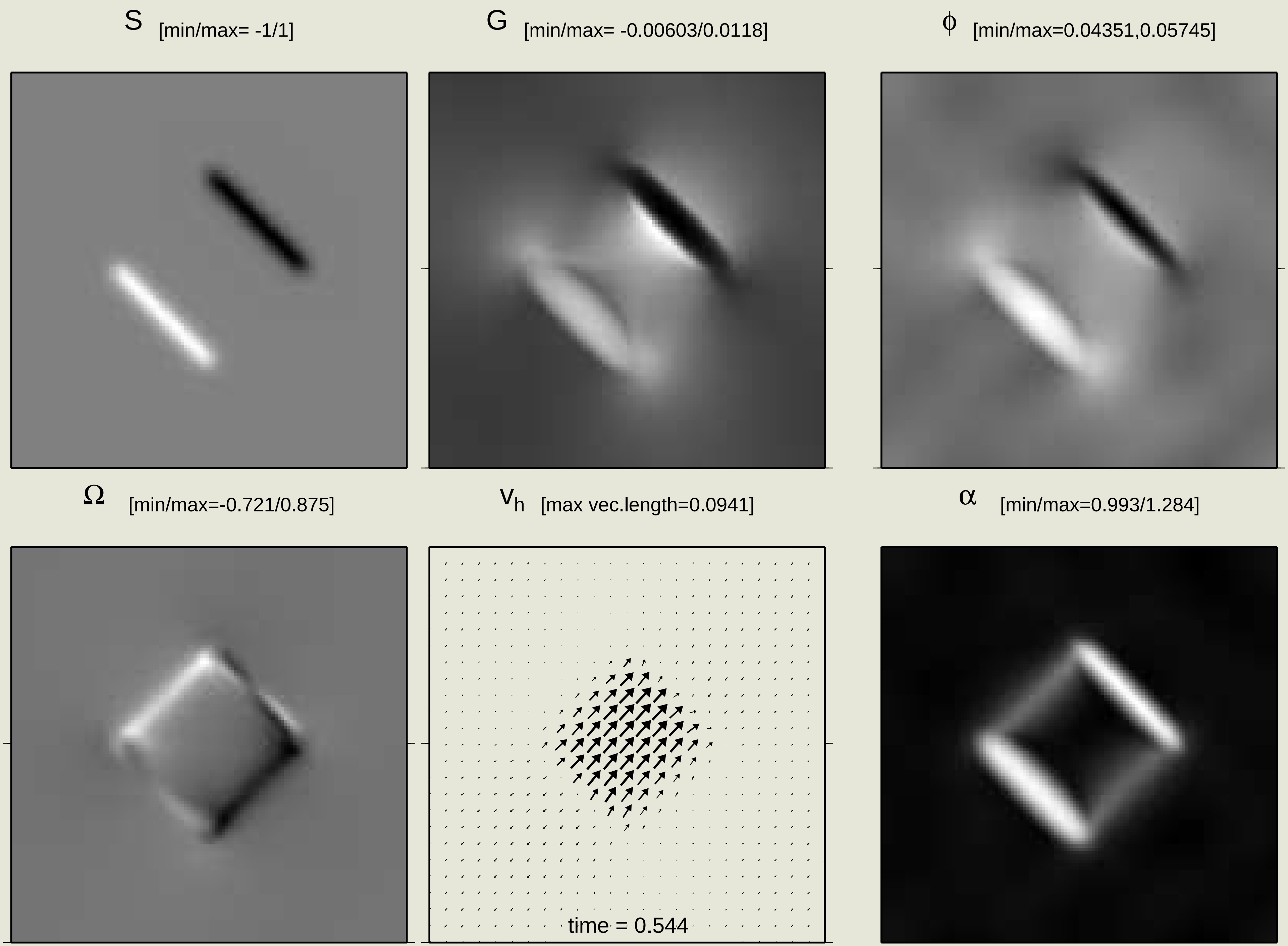
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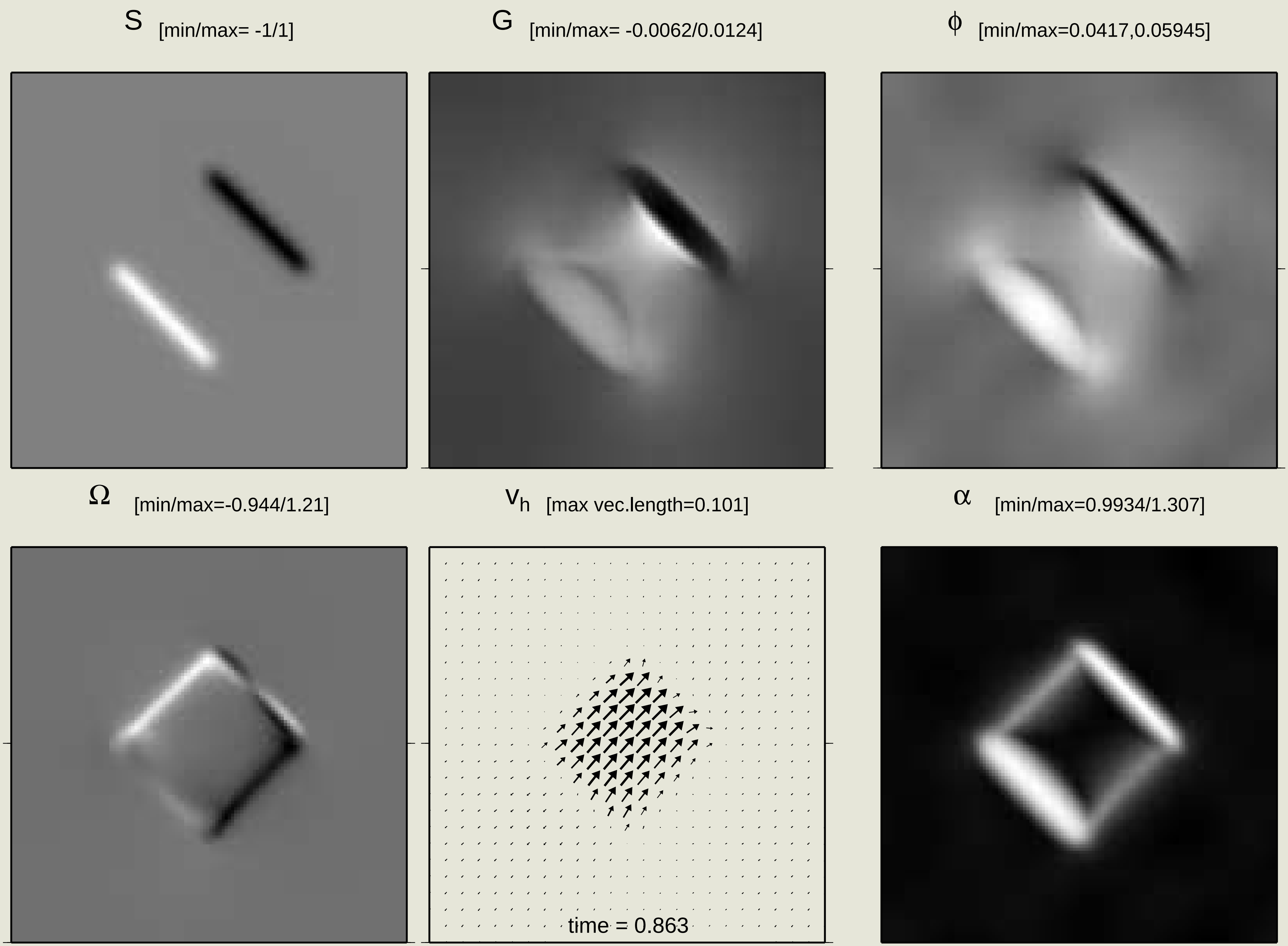
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Ω [min/max=-0.944/1.21]
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α [min/max=0.9934/1.307]
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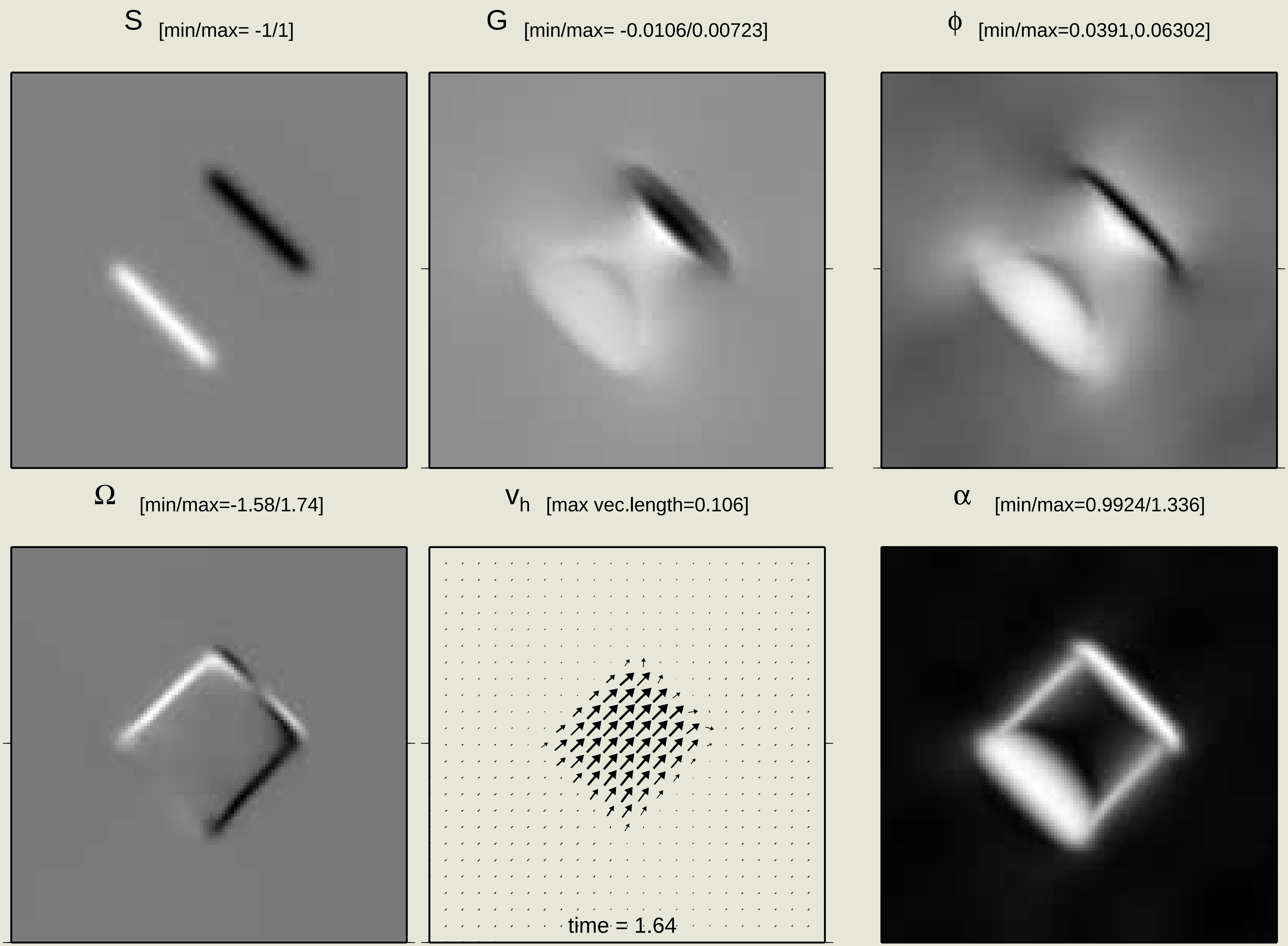
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α [min/max=0.9924/1.336]
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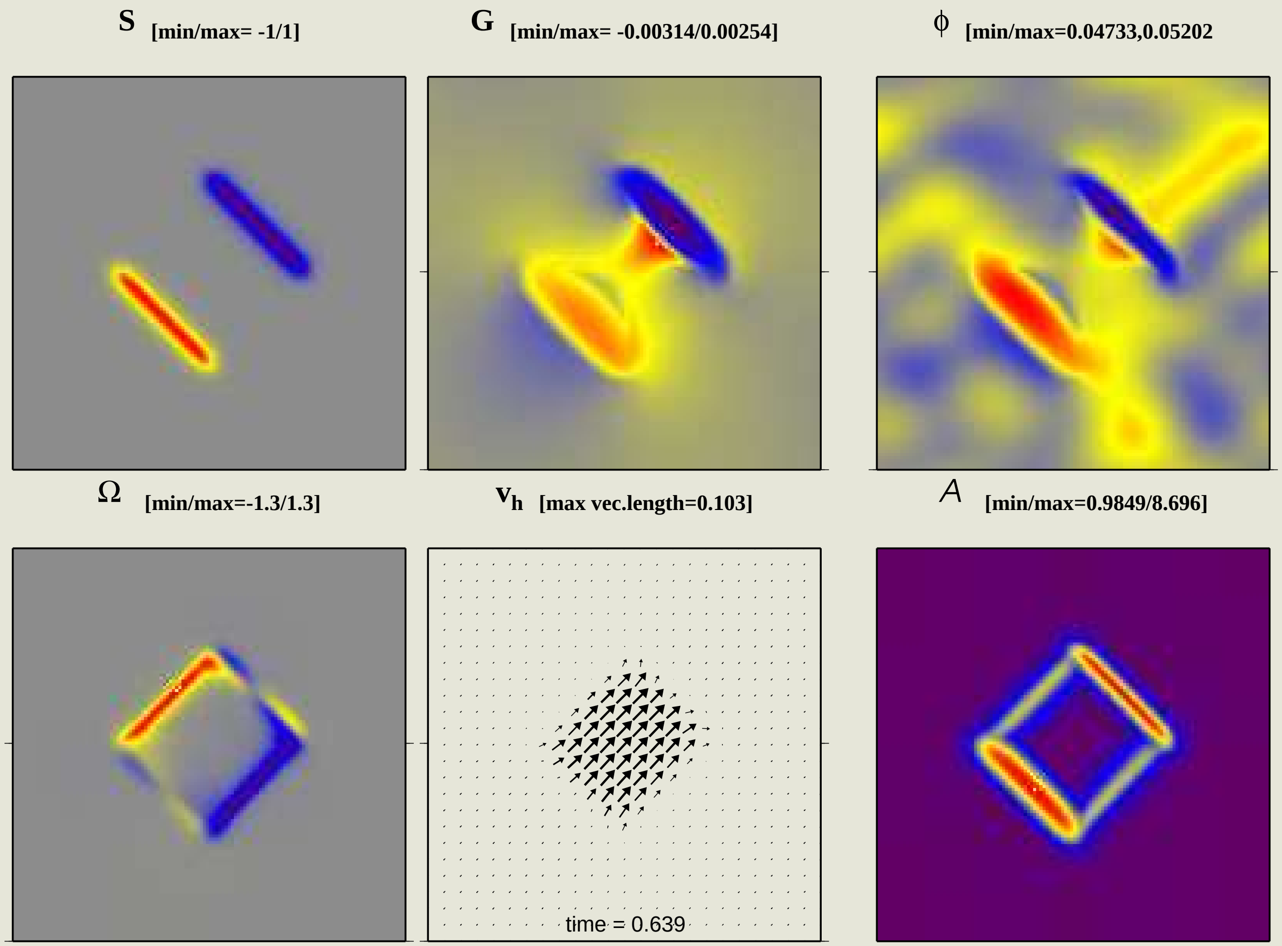
Fineness-generating (“grainsize reducing”) damage



- Fineness-generating (grain-reducing) damage does not involve (even suppresses) dilation and facilitates plate-like strike-slip/toroidal flow
- However, this treats only mean grainsize, not distribution of grainsizes and thus cannot treat healing (graingrowth/coarsening) simultaneously with damage