Using multi-cycle earthquake simulations to understand crustal dynamics

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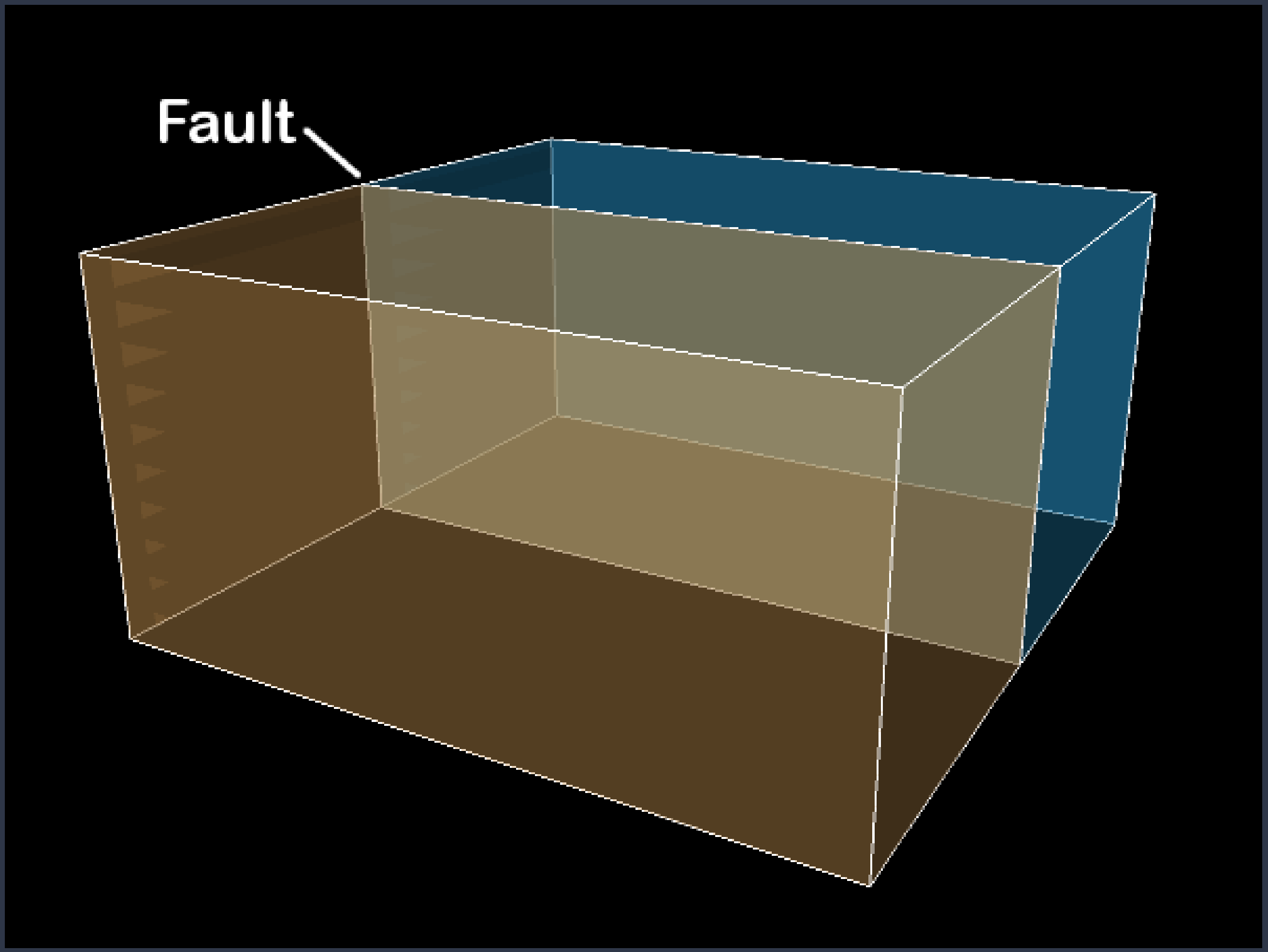
Contributors: Charles Williams, Matt Knepley among many others

Overview

- Use modeling of the earthquake cycle to illustrate principal obstacles we face in short-term tectonics research
  - Geoscience obstacles (physics, boundary conditions)
  - Computational obstacles
- Outline one approach for overcoming geoscience obstacles
  - Doesn’t alleviate current computational obstacles
  - Adds new ones

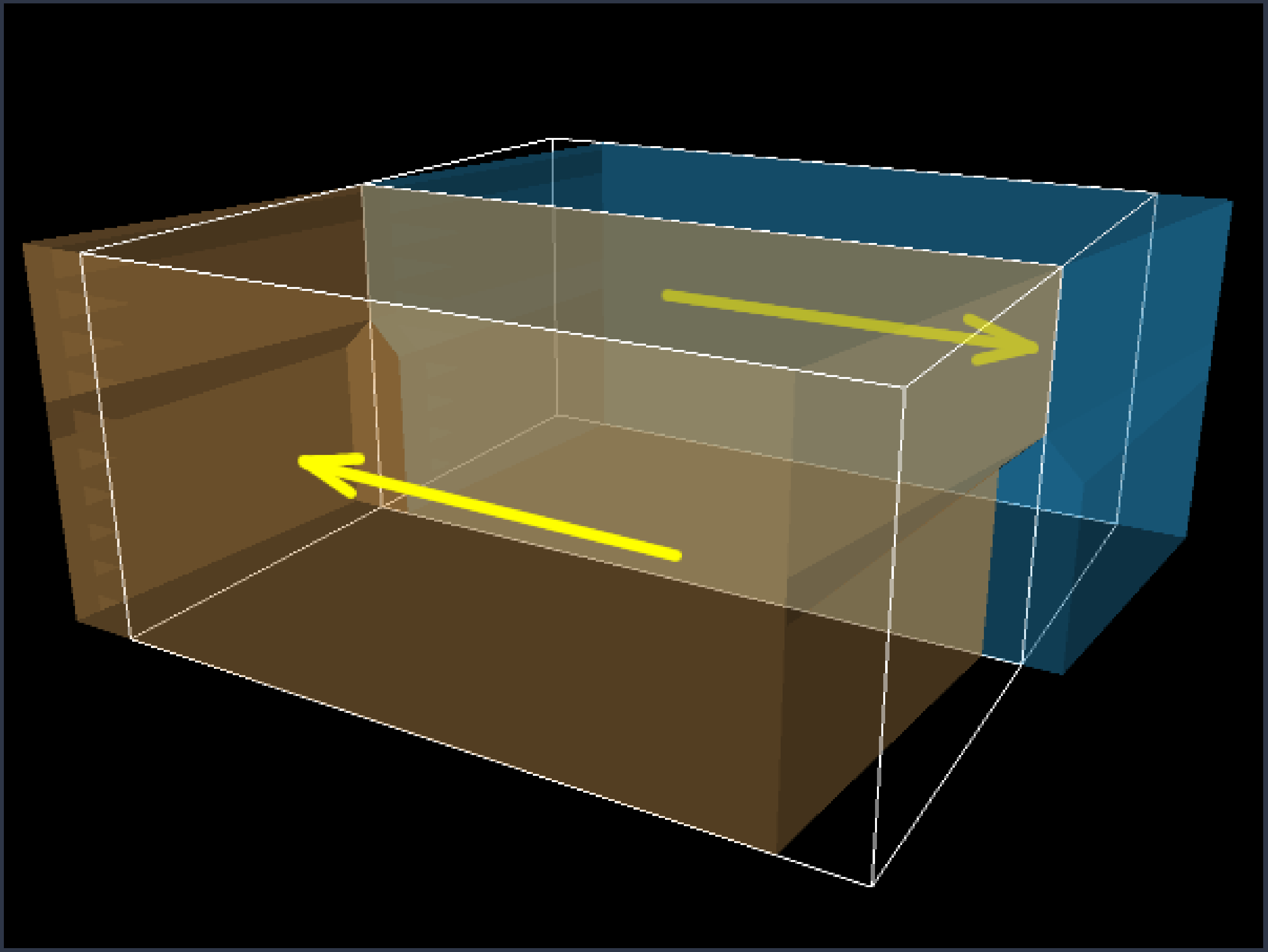
The Earthquake Cycle

1. Start with two tectonic plates separated by a fault



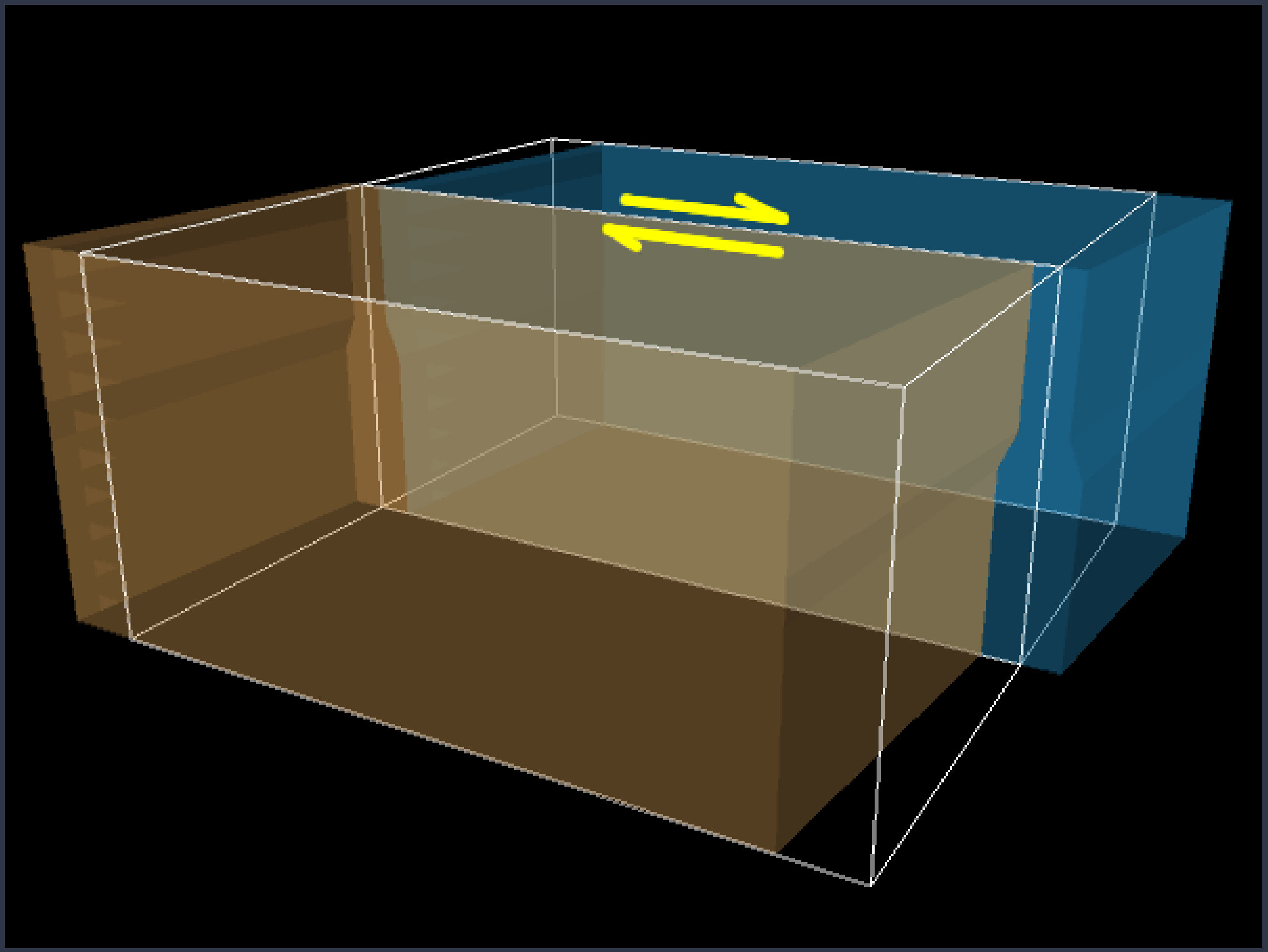
The Earthquake Cycle

2. Plate motion drives strain accumulation in elastic crust



The Earthquake Cycle

3. Rapid slip on fault = earthquake, releasing & redistributing strain



The Earthquake Cycle

4. Post-earthquake deformation

Earthquake triggers:

- Aftershocks (lots of smaller earthquakes in surrounding region)
- Slow slip (on portions of fault dominated by viscous behavior)
- Viscoelastic relaxation (diffusion of high stress/strain concentrations)

Post-earthquake deformation decays and the cycle starts again . . .

Modeling the Earthquake Cycle

Goal: Understand seismic behavior & hazard by studying entire cycle

- Multi-scale problem
  - Temporal scales: milliseconds to thousands of years
  - Spatial scales: tens of meters to thousands of kilometers

- Encompasses two fundamental processes
  - Seismic
    - Earthquake rupture
    - Milliseconds to minutes
  - Interseismic
    - Strain accumulation and post-earthquake deformation
    - Minutes to thousands of years

- Seismic and interseismic processes have very different computational requirements

Components of Short-Term Tectonics Modeling

Processes are different but basic components are the same

- Geologic structure
  - Geometry of fault surfaces
  - Spatial variations in material properties
- Constitutive models
  - Solid materials (stress/strain)
  - Fault surfaces (friction/slip)
- Initial conditions/boundary conditions
  - State of system (e.g., state of stress in Earth's crust)
  - Rate of deformation (e.g., how are the plates moving/colliding)

Modeling the Seismic Process

Physics based approach to forward modeling of earthquake rupture

For a given set of faults, we want to know:

- How does rupture behavior influence strong ground shaking?
- Do faults fail together or independently?
- Do ruptures start at predictable locations?
- What controls rupture speed and when/where they stop?

Earthquake Rupture Modeling

- Overview of approach
  - Load faults in model
  - Initiate rupture at *predetermined* location
  - Allow fault constitutive model to control slip & rupture propagation
  - Dynamic elasticity equation yields earthquake and ground motions

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  - Dynamic elasticity equation yields earthquake and ground motions
- Ingredients
  - Geologic structure
  - Constitutive models
    - Linear elastic isotropic material
    - Friction model for fault surfaces
  - Initial conditions/boundary conditions
    - Initial stresses on fault
    - Absorbing boundaries on lateral edges and bottom of domain

Earthquake Rupture Modeling: Governing Equations

- Solve dynamic elasticity equation for linear elastic material

$$\nabla \cdot \left(\underline{C} \cdot \frac{1}{2} \left(\nabla + \nabla^T\right) \overrightarrow{u}(t) \right) = \rho \frac{\partial^2 \overrightarrow{u}(t)}{\partial t^2}$$

- Subject to friction model on fault surfaces,

$$\underline{\sigma}(t) \cdot \overrightarrow{n} = \overrightarrow{T}(t, slip)$$

- Initial stress field on fault surfaces,

$$\overrightarrow{T}(t = 0, slip = 0) = f(\overrightarrow{x})$$

- Absorbing boundaries (e.g., perfectly matched layers) on boundaries

Earthquake Rupture Modeling: Solution Scheme

Solution scheme strongly influences mesh generation requirements

- Discretize volume with finite elements
- Explicitly include faults as interior surfaces in solid model
- Use explicit time integration (central difference method)
  - Discretization size is proportional to shear wavespeed
  - Time step is controlled by time for wave to pass thru element

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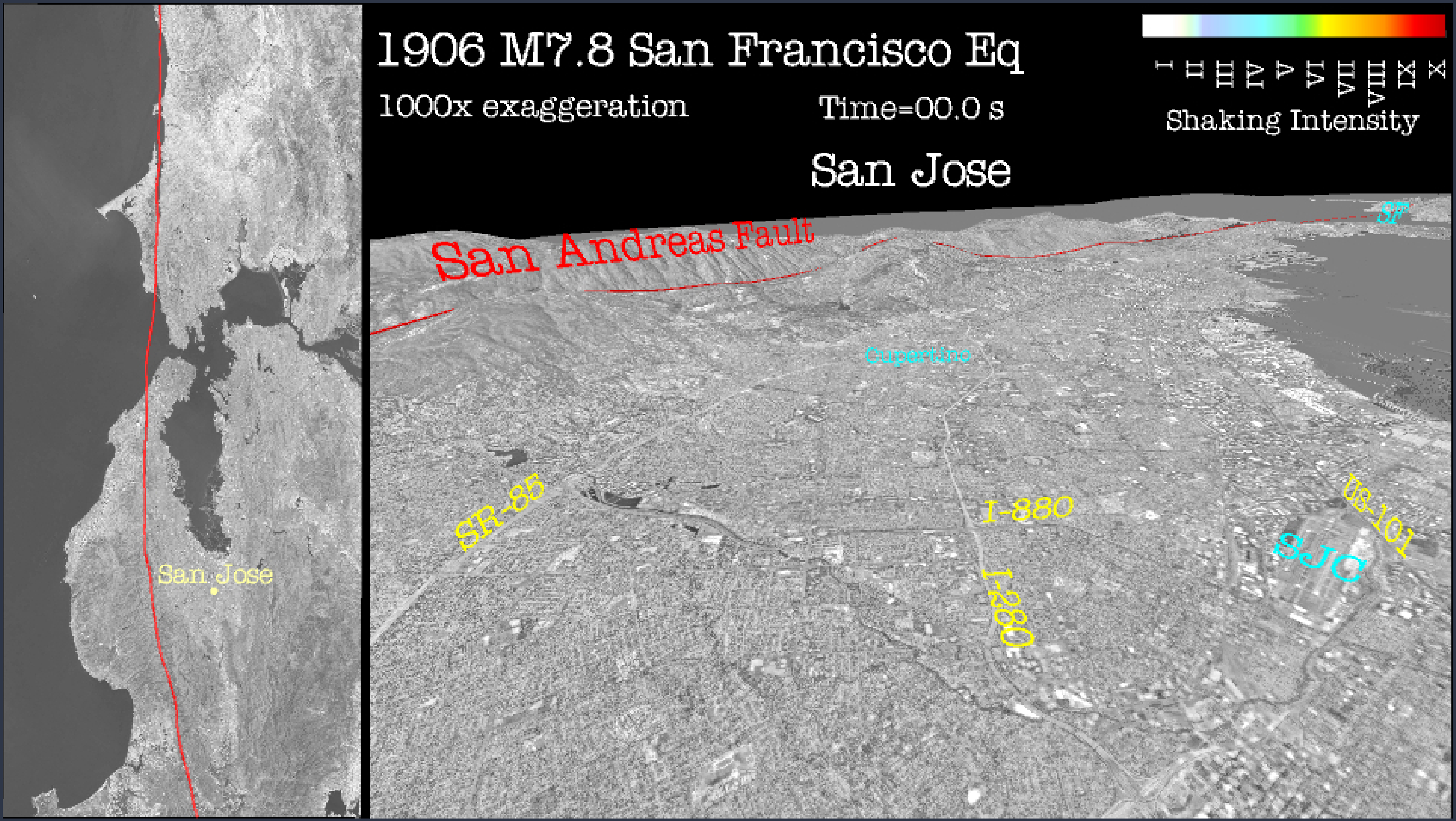
- Computational resources
  - Desktop machine (1–2 processors) - some simple problems
  - Small clusters (4–100 processors) - most real problems
  - Large clusters (200–1000 processors) - only a few problems

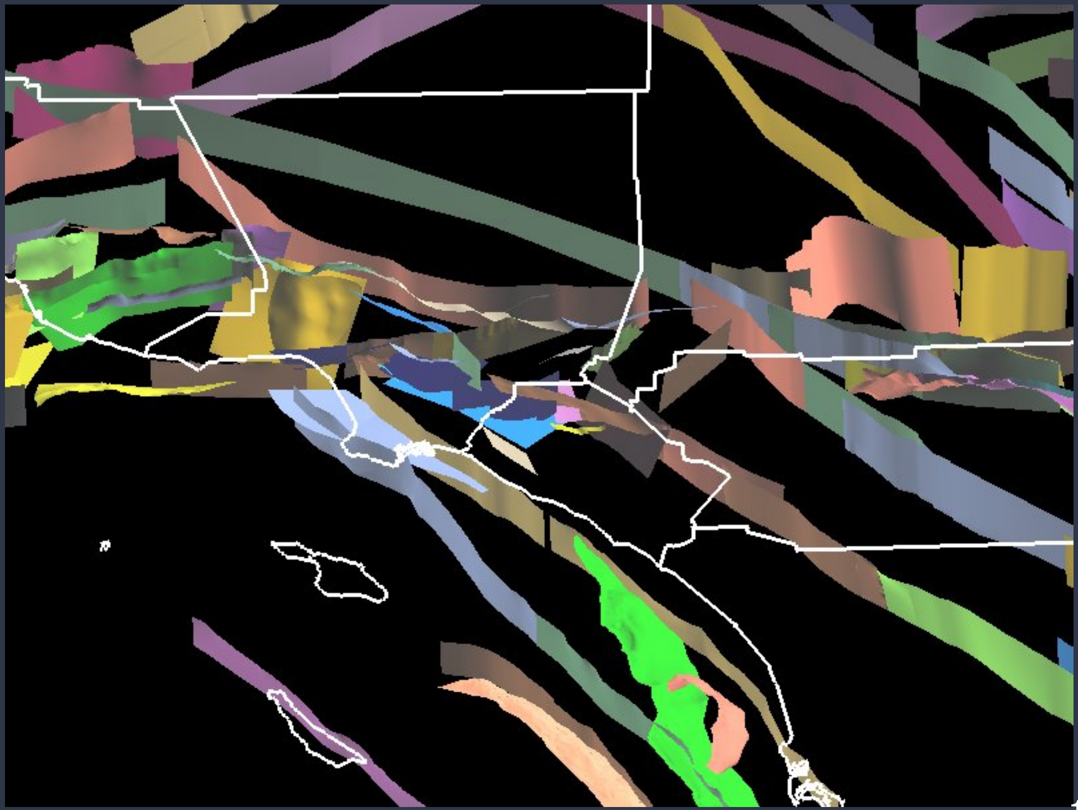
Earthquake Rupture Modeling: Example



Obstacle #1: Model Construction

Faults (and topography) are complex, nonplanar surfaces

- Often described by triangular facets
- Intersections frequently involve small angles
- Usually can't be assembled to form airtight volumes



Fault surfaces in Los Angeles area from SCEC CFM

Obstacle #1: Model Construction

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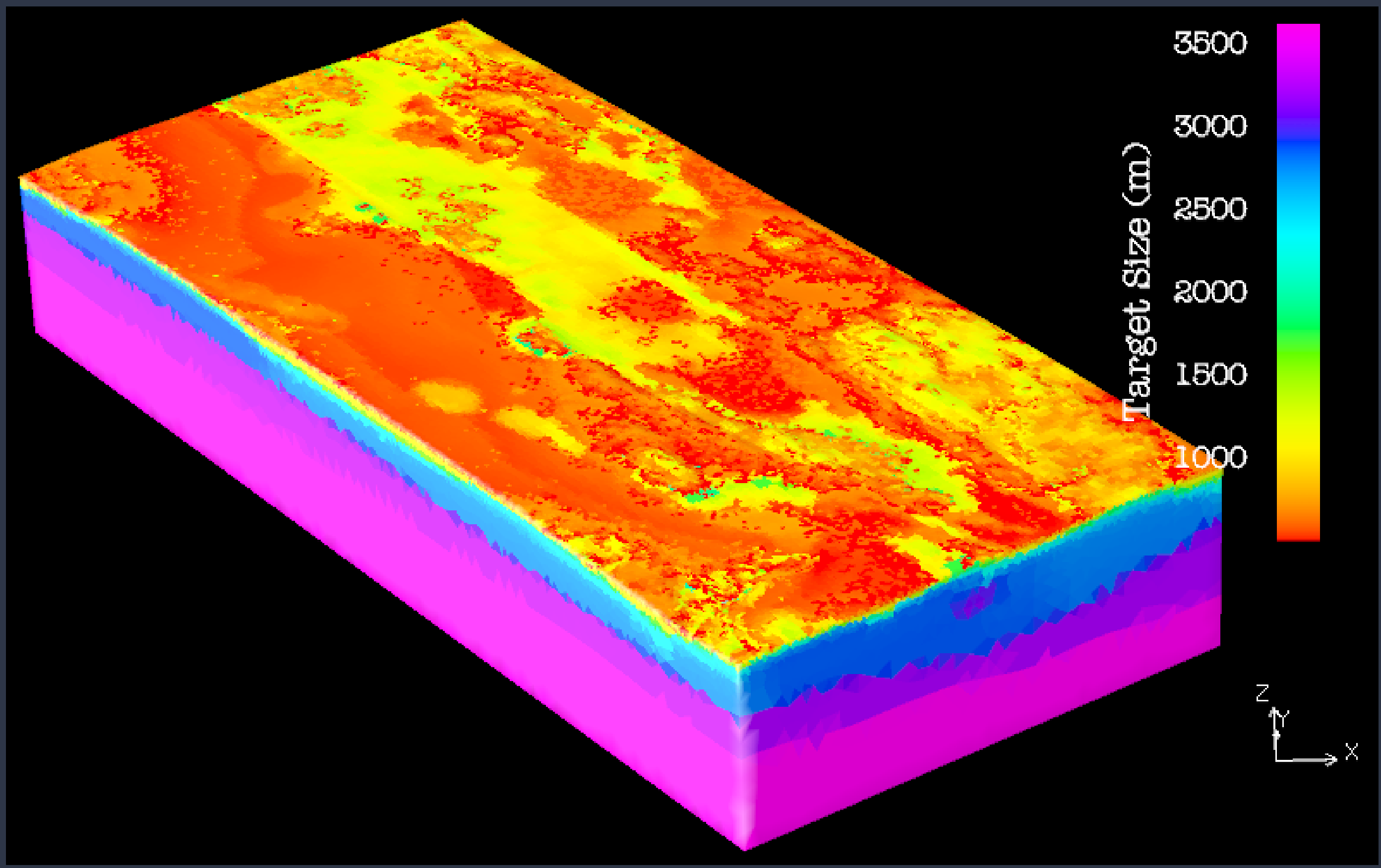

Earthquake Rupture Modeling: Current Obstacles

- Computational obstacles
  - Model construction (generating the mesh)
    - Faults (and topography) are complex, nonplanar surfaces
    - Discretization size varies with material properties
    - Poor mesh quality affects time step (aspect ratios $>$ 0.4–0.5)
    - Problem size limited by capacity of (serial) mesh generators
- Geoscience obstacles
  - Many parameters are poorly constrained
    - We only know a few basic features of fault constitutive behavior
    - Use rough approximations of strain accumulation and loading of faults

Modeling the Interseismic Process

For a given region, we want to know:

- How and where is strain accumulating?
- How is inelastic (viscoelastic) deformation redistributing strain?
- How does slip on fault A affect the stresses on fault B?
- How do gravitational forces affect tectonic deformation?

Modeling Pre- and Post-Earthquake Deformation

- Overview of approach
  - Prescribe driving forces or deformation
  - Allow constitutive model to control accumulation of strain
  - Quasi-static elasticity equation yields time-dependent deformation

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- Ingredients
  - Geologic structure
  - Constitutive models
    - Linear and nonlinear viscoelastic and viscoelastoplastic rheologies
    - Friction models for fault surfaces (often excluded)
  - Initial conditions/boundary conditions
    - Prescribe slip on fault surfaces, or
    - Set velocities at boundaries according to plate motions

Interseismic Modeling: Governing Equations

- Solve static elasticity equation at each time step

$$\nabla \cdot \underline{\sigma}(t, \overrightarrow{u}) + \overrightarrow{f} = 0$$

- Subject to tractions or displacements on faults and boundaries

$$\underline{\sigma}(t) \cdot \overrightarrow{n} = \overrightarrow{T}(t, \overrightarrow{x})$$
$$\overrightarrow{u}(t, \overrightarrow{x}) = \overrightarrow{u}_o(t, \overrightarrow{x})$$

Interseismic Modeling: Solution Scheme

- Discretize volume with finite elements
- Explicitly include faults as interior surfaces in solid model
- Use implicit time integration (quasi-static solution)
    - Time step is controlled by resolving time-dependent constitutive behavior

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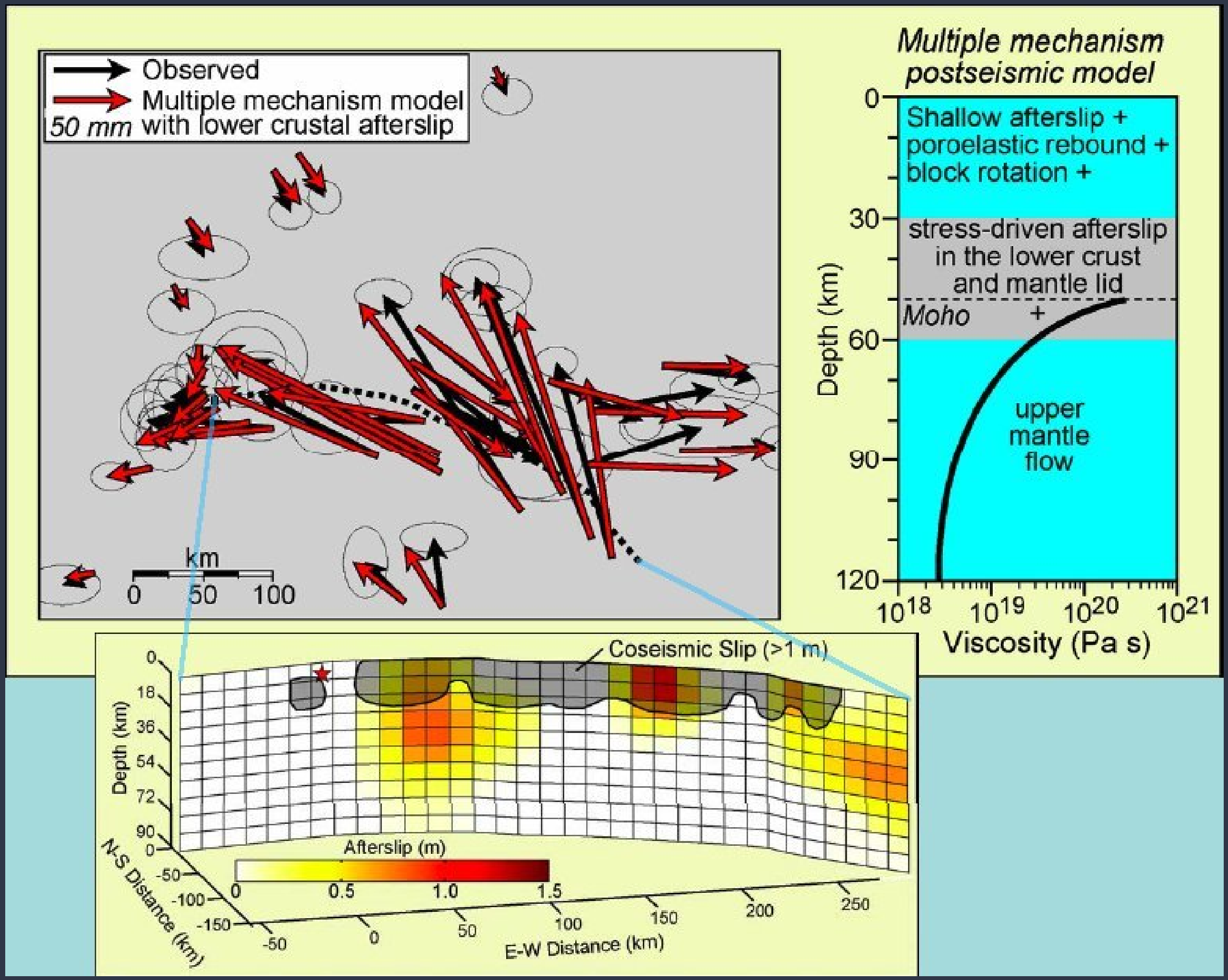
Interseismic Modeling: Example

Post-earthquake deformation for 2002 M7.9 Denali earthquake

Multiple mechanisms required to explain GPS displacements (Freed *et al.*, 2005)



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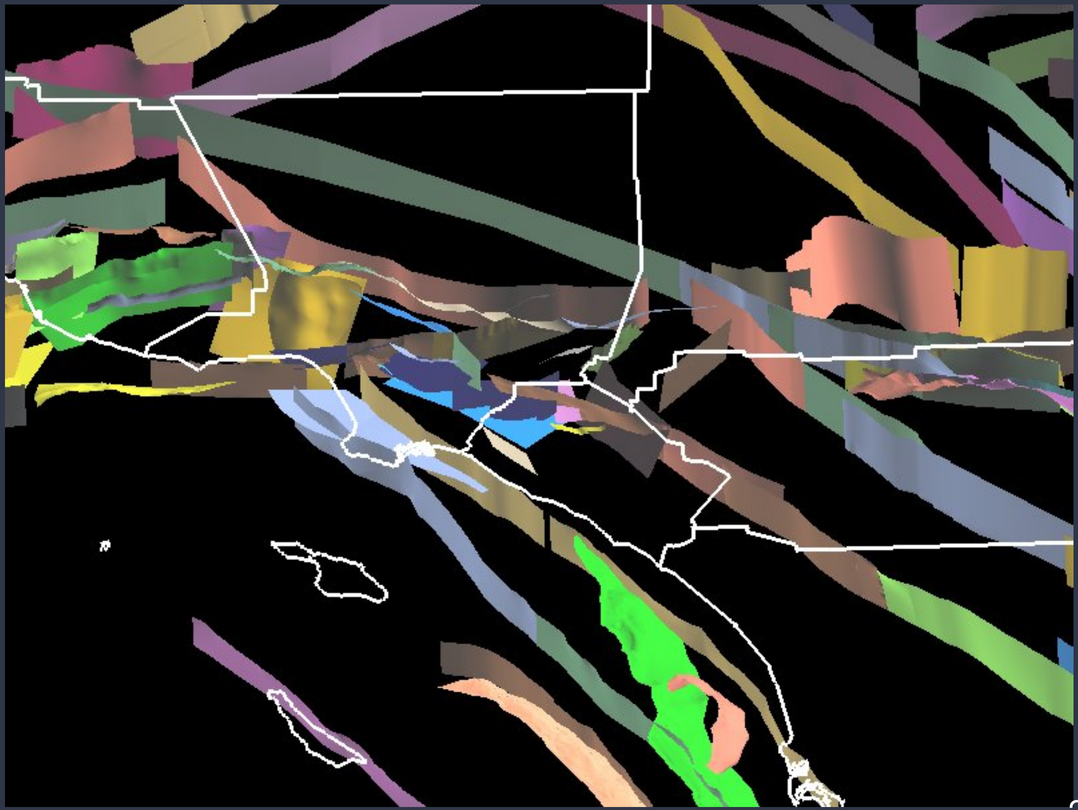
Same issues as eq rupture modeling

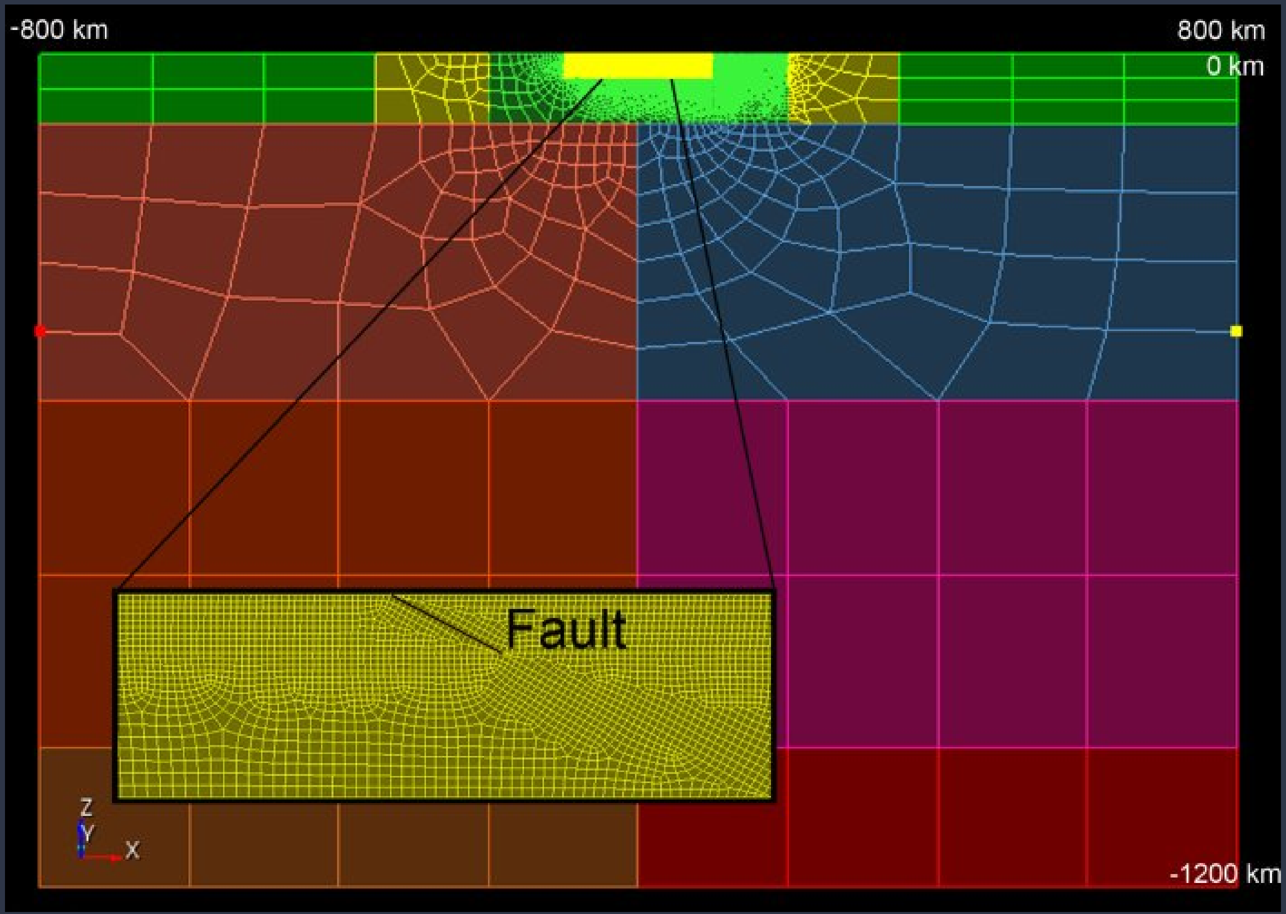


Interseismic Modeling Obstacle #2: Discretization

Need adaptive mesh refinement and coarsening

- Discretization size depends on constitutive behavior and fault BC
- Only known qualitatively *a priori* (usually use rules of thumb)

Deformation from slip on fault in Taiwan (Hsu *et al.*, 2005)



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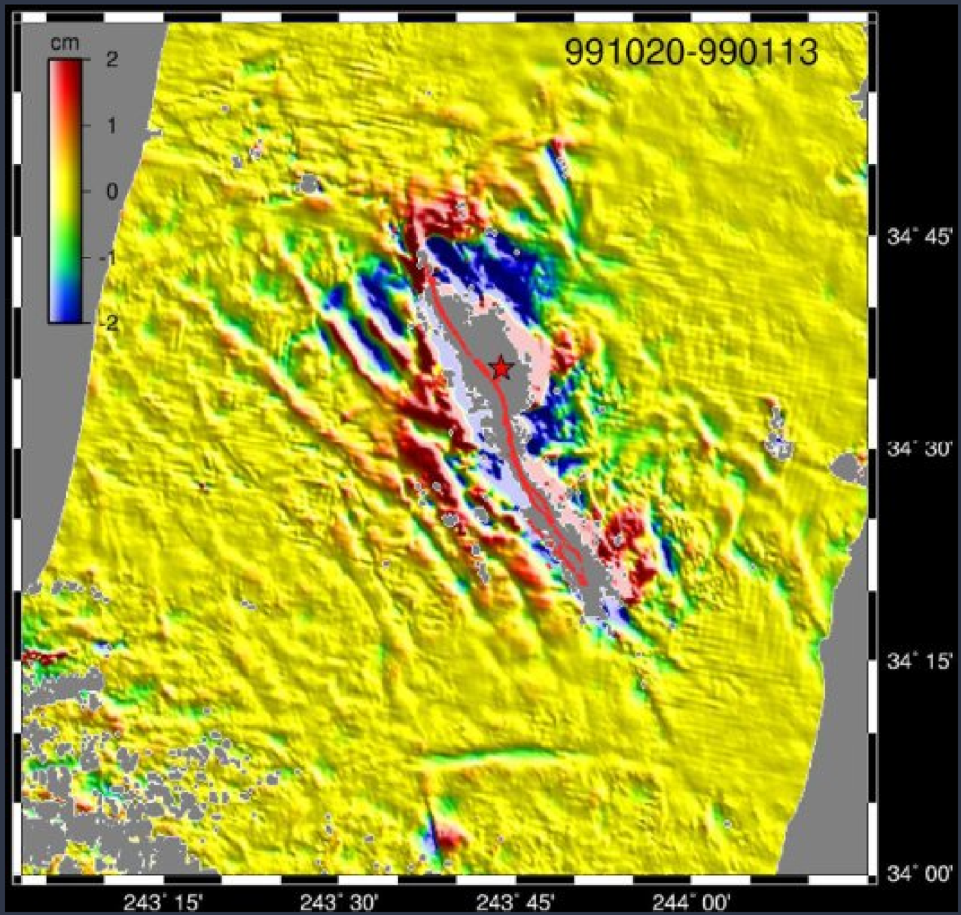
Localized deformation along compliant fault zones near Hector Mine earthquake from high-pass filtered InSAR image (Fialko *et al.*, 2002)



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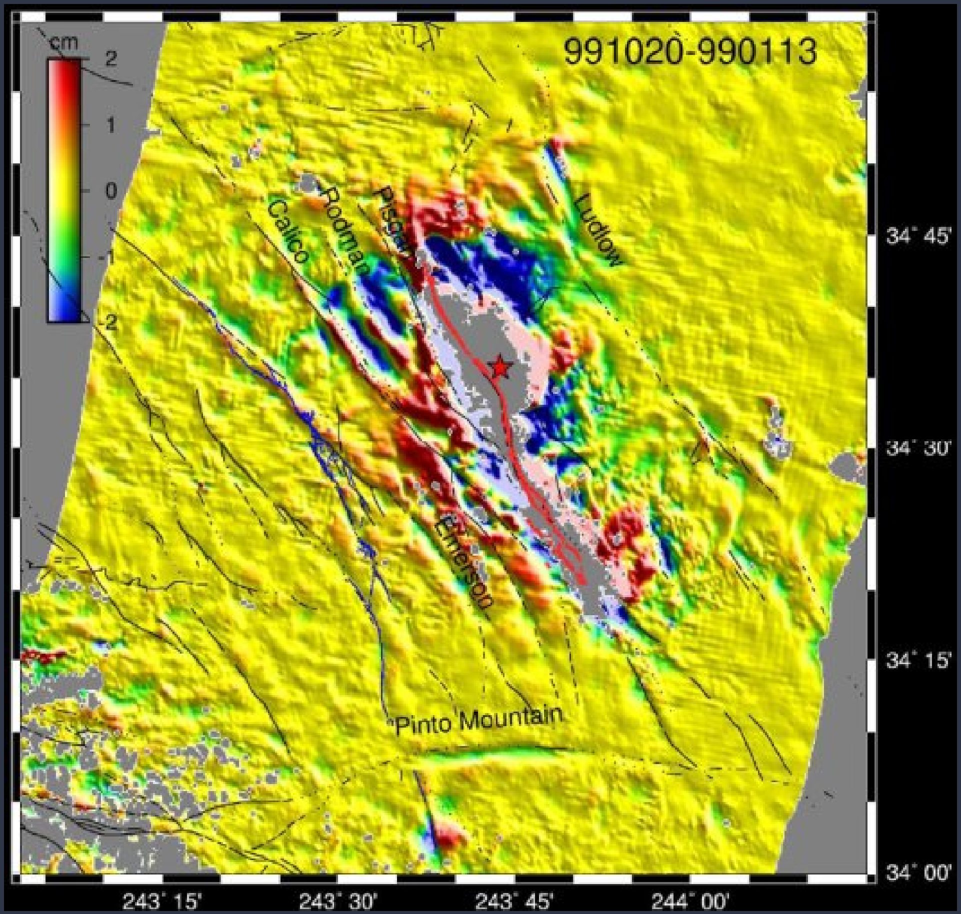

Interseismic Modeling: Current Obstacles

- Computational obstacles
  - Discretization
    - Faults (and topography) are complex, nonplanar surfaces
    - Need adaptive mesh refinement / coarsening
    - Poor mesh quality degrades solution (want aspect ratios $> \approx 0.4$)
    - Problem size limited by capacity of (serial) mesh generators
  - Solver issues
    - Deformation may be localized
    - Need robust solvers for nonlinear, time dependent rheologies
- Geoscience obstacles
  - Some parameters are poorly constrained
    - Many different viscoelastic rheologies used to explain same data
    - Rough approximations to earthquake dynamics

Overcoming Geoscience Obstacles

Couple/integrate seismic and interseismic modeling

- Tighter constraints on physics if we don't restrict models to isolated time frame and single process
- Can explore evolution of behavior over many earthquake cycles

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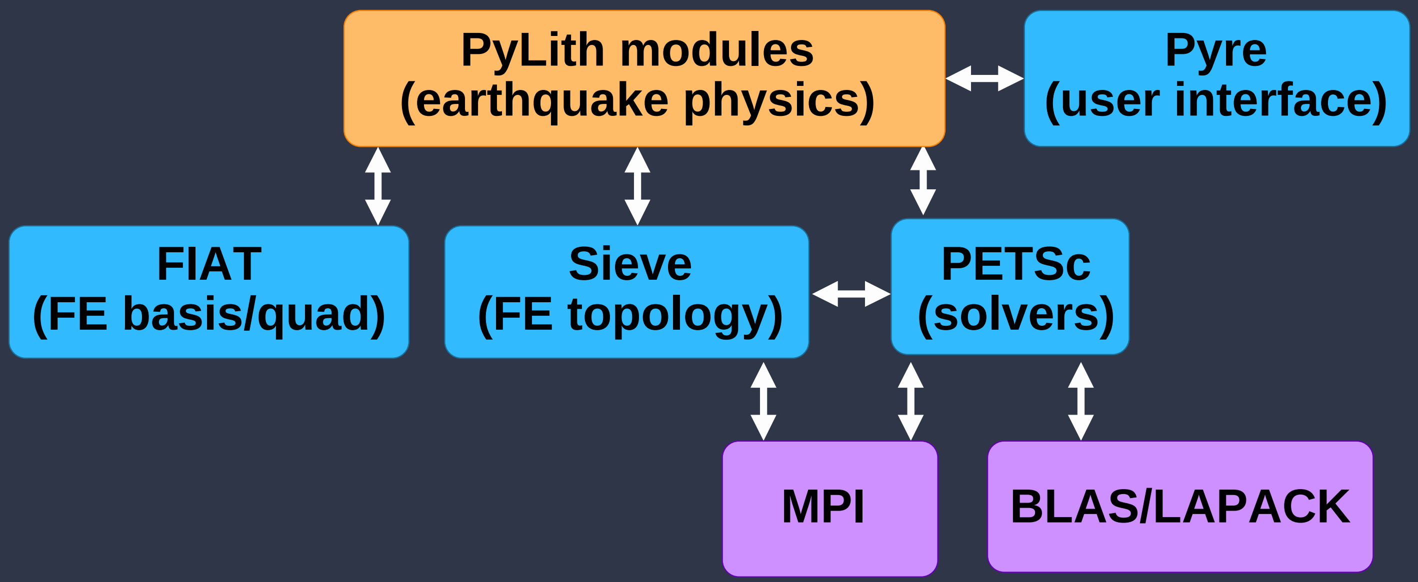
- Tighter constraints on physics if we don't restrict models to isolated time frame and single process
- Can explore evolution of behavior over many earthquake cycles
- Current cutting edge integrated models
  - Implementations restricted to very simple 2-D and 3-D problems
  - Neglect 3-D effects (material properties, complex geometry)
- CIG's PyLith: Redesigning two codes to form one suite of modules
  - EqSim: Earthquake rupture modeling code
  - Lithomop (formerly Tecton): Interseismic modeling code

PyLith

State-of-the-art code for crustal dynamics modeling

- Efficient, parallel, modular implementation
  - Linear, viscoelastic, and viscoelastoplastic elasticity
  - Traction and dislocation fault boundary conditions
  - Seismic (dynamic) and/or interseismic deformation (quasi-static)



- Target date of March 2007 for release of version 1.0

Developers: Brad Aagaard, Matt Knepley, Charles Williams

Modeling of Multiple Earthquake Cycles

New computational challenges emerge

- What is the best solution strategy?
  - Couple interseismic (quasi-static) and seismic (dynamic) solutions
  - New formulation integrating quasi-static and dynamic solutions
- Resolution of multi-scale problem
  - Multiple earthquake cycles with realistic behavior
    - Many small eq $\rightarrow$ resolve many dynamic eq rupture soln's

Summary of Computational Challenges

- Requirements for mesh generators
  - Complex, nonplanar surfaces (e.g., faults, topography)
  - Nontrivial variations in discretization size
  - Good mesh quality (want aspect ratios > 0.4–0.5)
  - Generate meshes with 10^6–10^8 elements
- Adaptive mesh refinement w/time-dependent refinement & coarsening
- Implementing dislocations/tractions on interior surfaces
  - Alternative implementations for propagating shear cracks?
    - Can we eliminate fault surfaces from our volume discretizations?
    - Are discontinuous Galerkin implementations a viable alternative?
- Solving quasi-static and dynamic elasticity equations
  - Are there more efficient solution schemes, solvers, or preconditioners?