Including Gravitational Stresses in Finite Element Simulations

CIG
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INFRASTRUCTURE
for GEODYNAMICS

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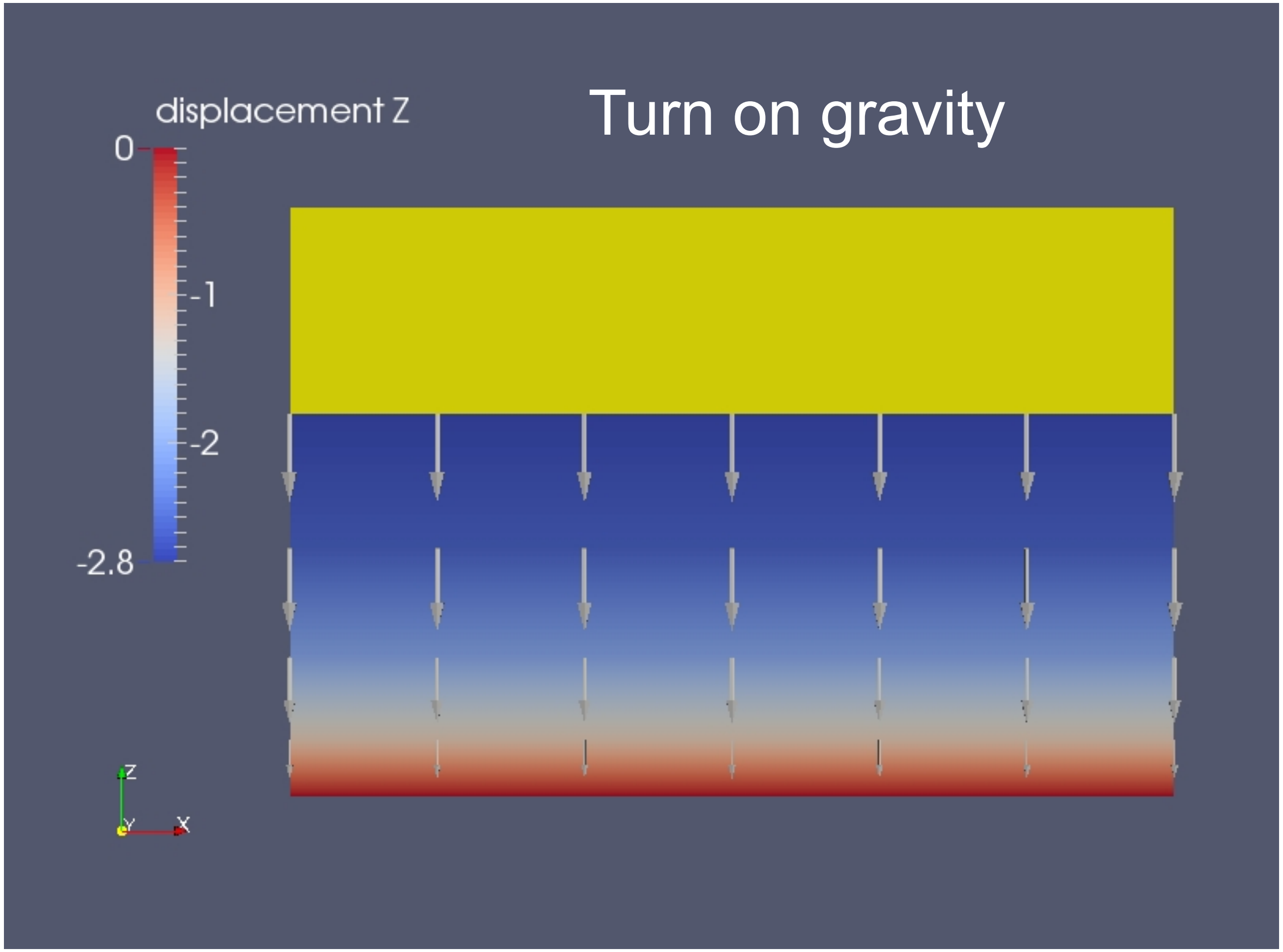
When do we need to include gravitational stresses?

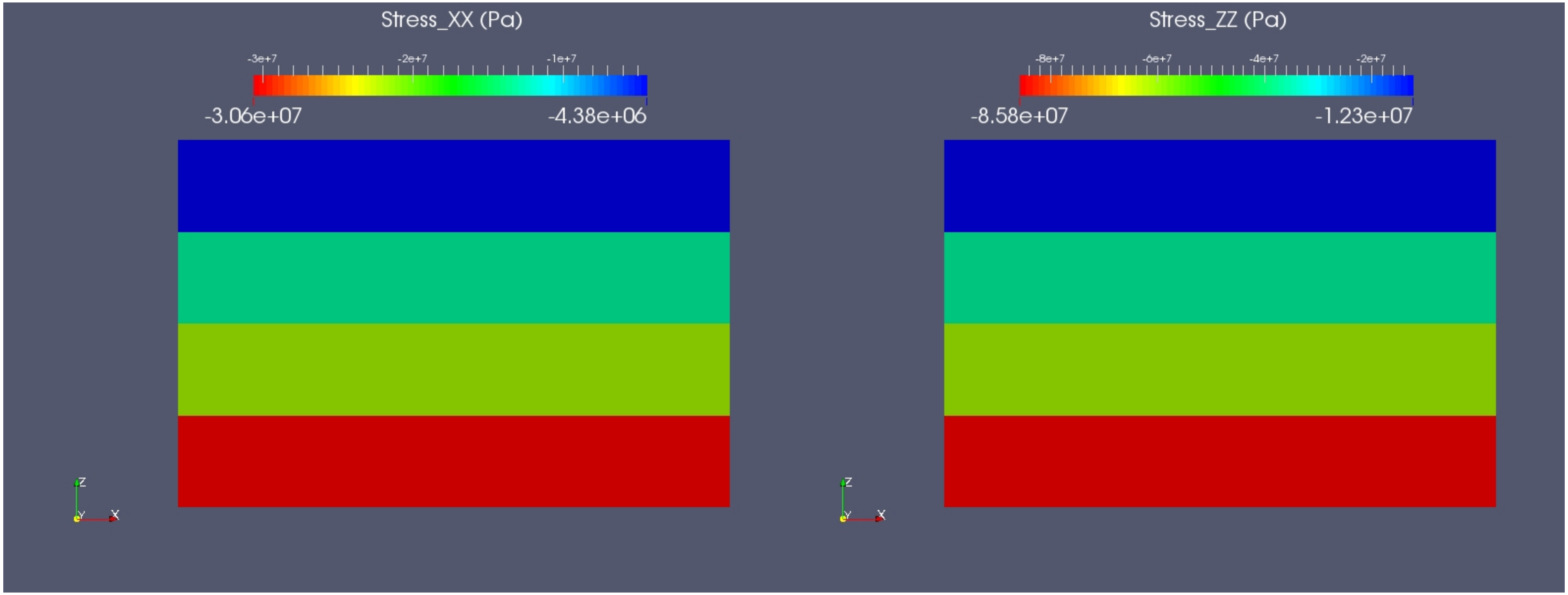
- Pressure/stress-dependent rheology.
  - Pressure-dependent bulk rheology (e.g., plasticity).
  - Stress-dependent fault rheology (e.g., friction).
- Viscoelastic simulations where we care about vertical deformation.
- Other simulations where we care about the absolute stress state.

Obvious (and wrong) approach to including gravitational stresses



examples/3d/hex8/step15.cfg

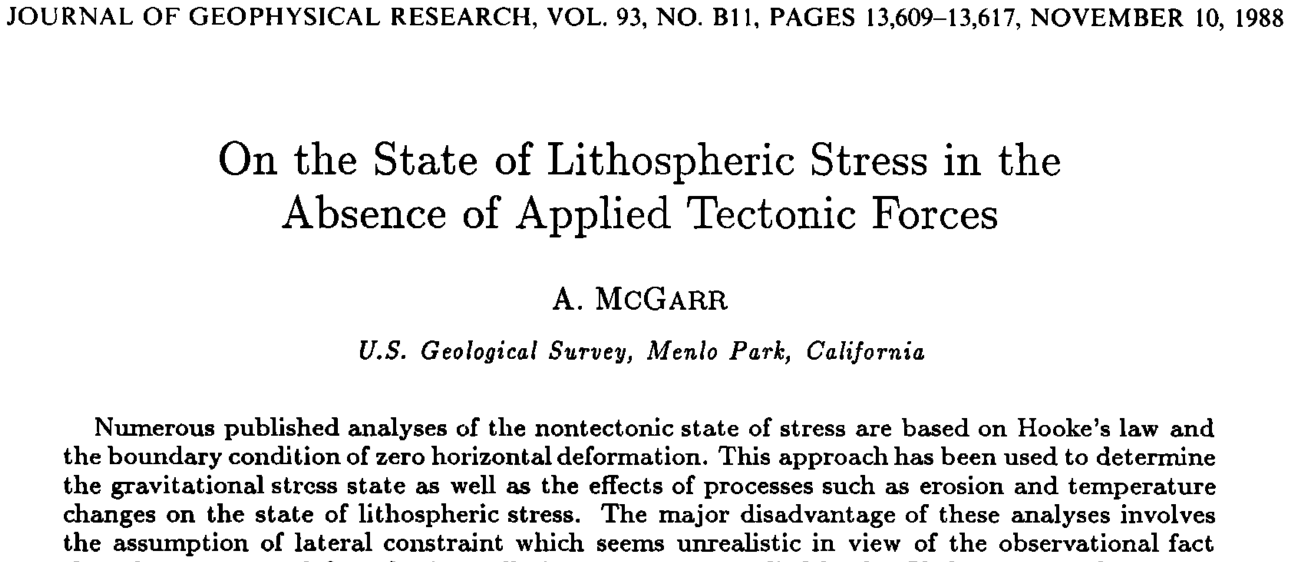
Obvious approach (cont.)



$$\sigma_{xx} = \sigma_{yy} = \frac{\nu}{1-\nu}\sigma_{zz}$$

examples/3d/hex8/step15.cfg

What is a realistic initial stress state?



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On the State of Lithospheric Stress in the Absence of Applied Tectonic Forces

A. McGarr

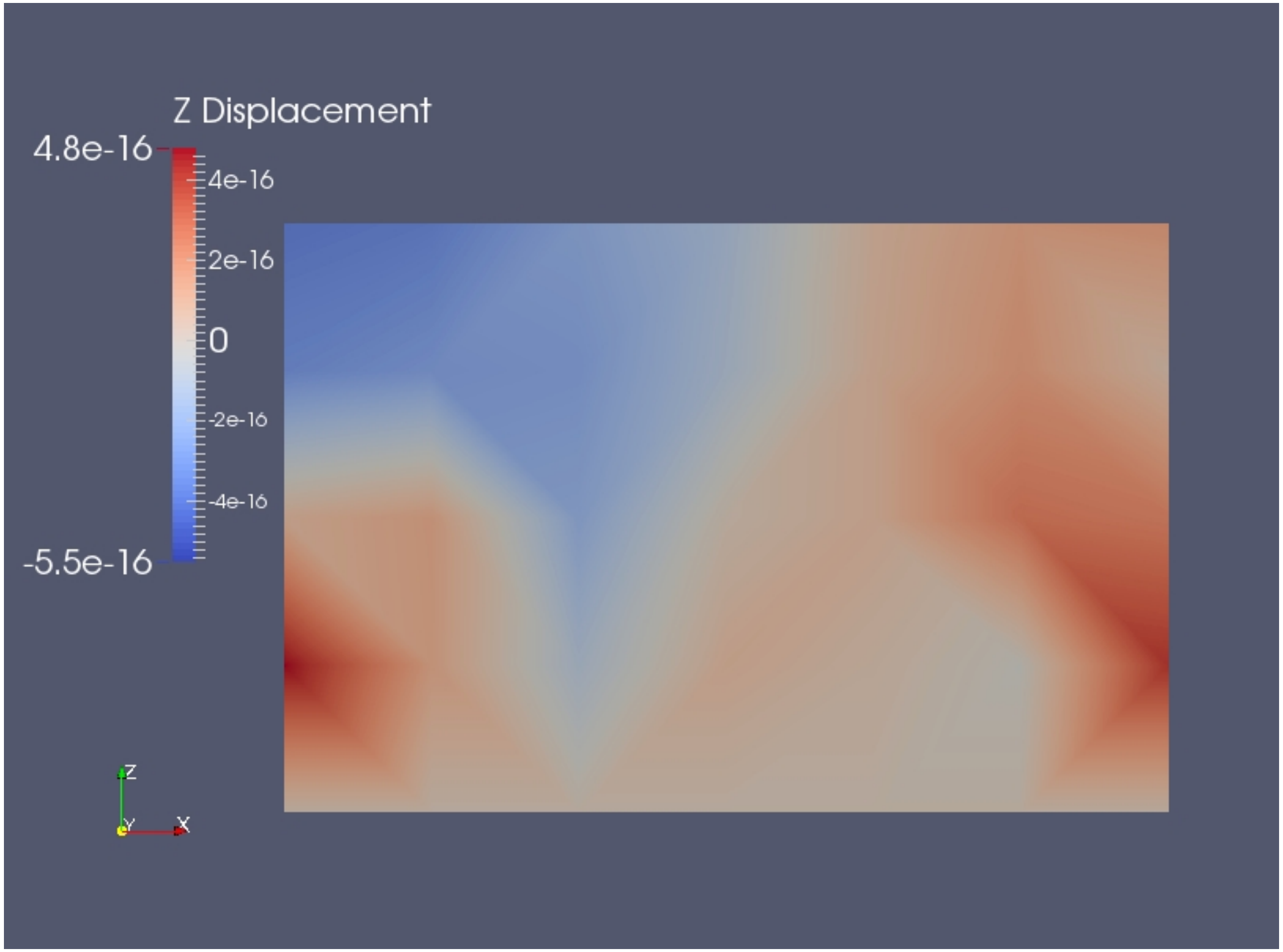
U.S. Geological Survey, Menlo Park, California

Numerous published analyses of the nontectonic state of stress are based on Hooke's law and the boundary condition of zero horizontal deformation. This approach has been used to determine the gravitational stress state as well as the effects of processes such as erosion and temperature changes on the state of lithospheric stress. The major disadvantage of these analyses involves the assumption of lateral constraint which seems unrealistic in view of the observational fact

Since gravity is not 'turned on', and since materials are either emplaced or relaxed by inelastic processes, an isotropic initial stress state is more realistic.

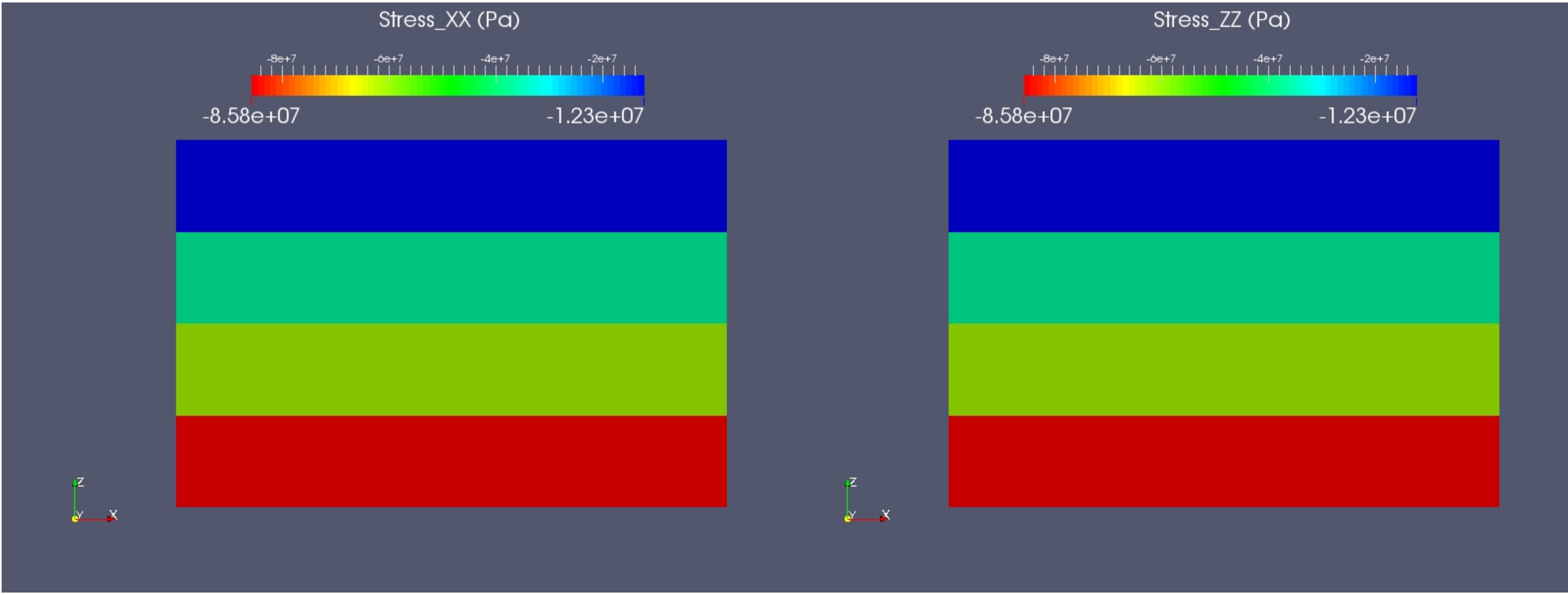
$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \rho g h$$

Using initial stresses to provide realistic stresses/displacements



examples/3d/hex8/step16.cfg

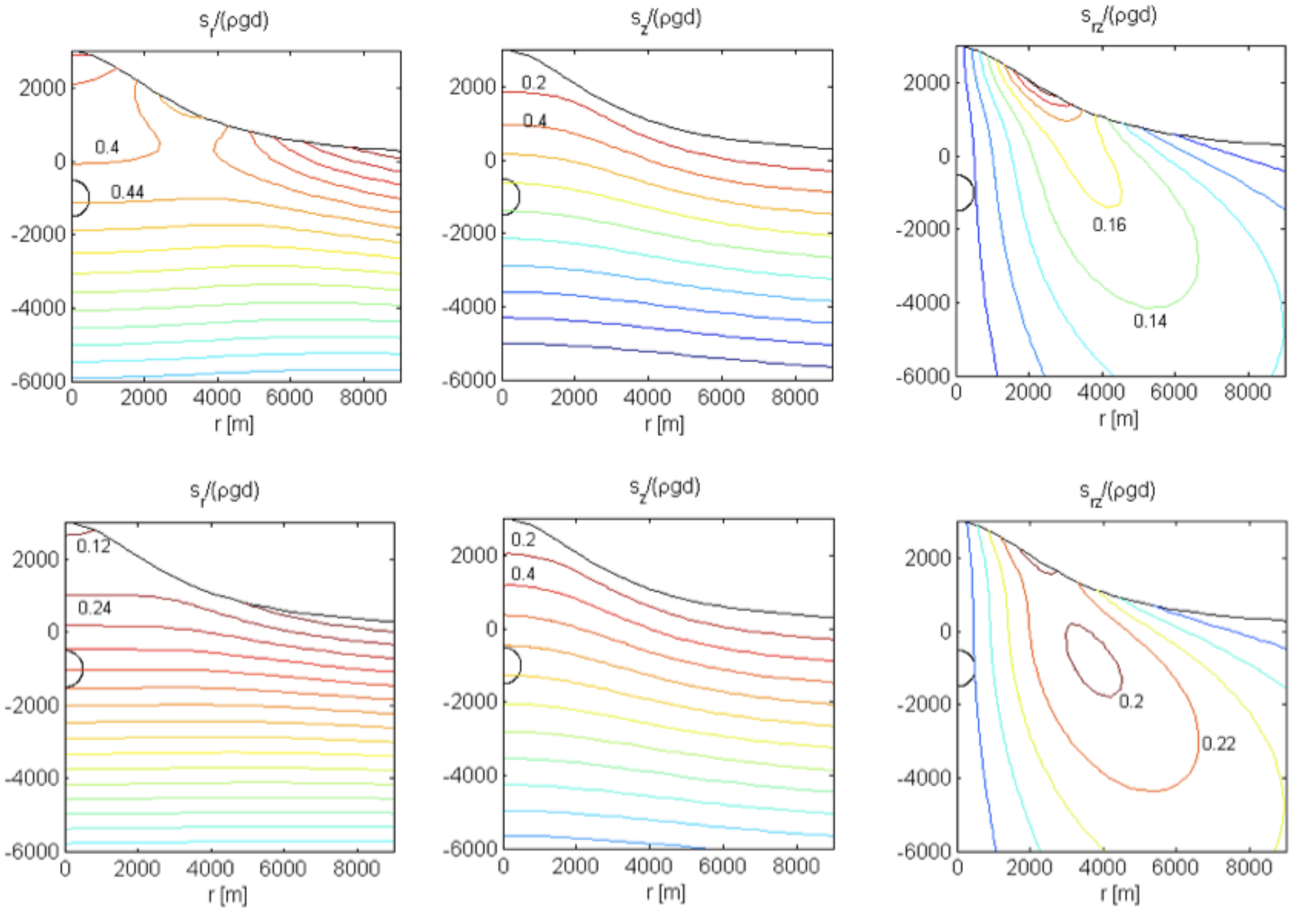
Using initial stresses (cont.)



$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \rho g h$$

examples/3d/hex8/step16.cfg

How to handle topography?



Uniaxial strain

Isotropic stress

From Currenti and Williams (2014).

Gravitational stresses with topography

① Compute an initial set of stresses by just ‘turning on’ gravity.

a) Use $\nu = 0.5$ to represent incompressible material (not yet possible in PyLith).

b) Apply far-field traction/displacement BC to counteract uniaxial strain effects.

② Use stresses computed in step 1 as initial stresses.

Viscoelastic problems with vertical deformation

- Will deformation be driven by gravity or by applied tractions/displacements/slip?
  - Do we want the initial stress state to induce deformation?
- The default infinitesimal strain formulation does not recompute body forces consistent with the deformed configuration.

Simple set of 'floating box' examples (fun with gravity)

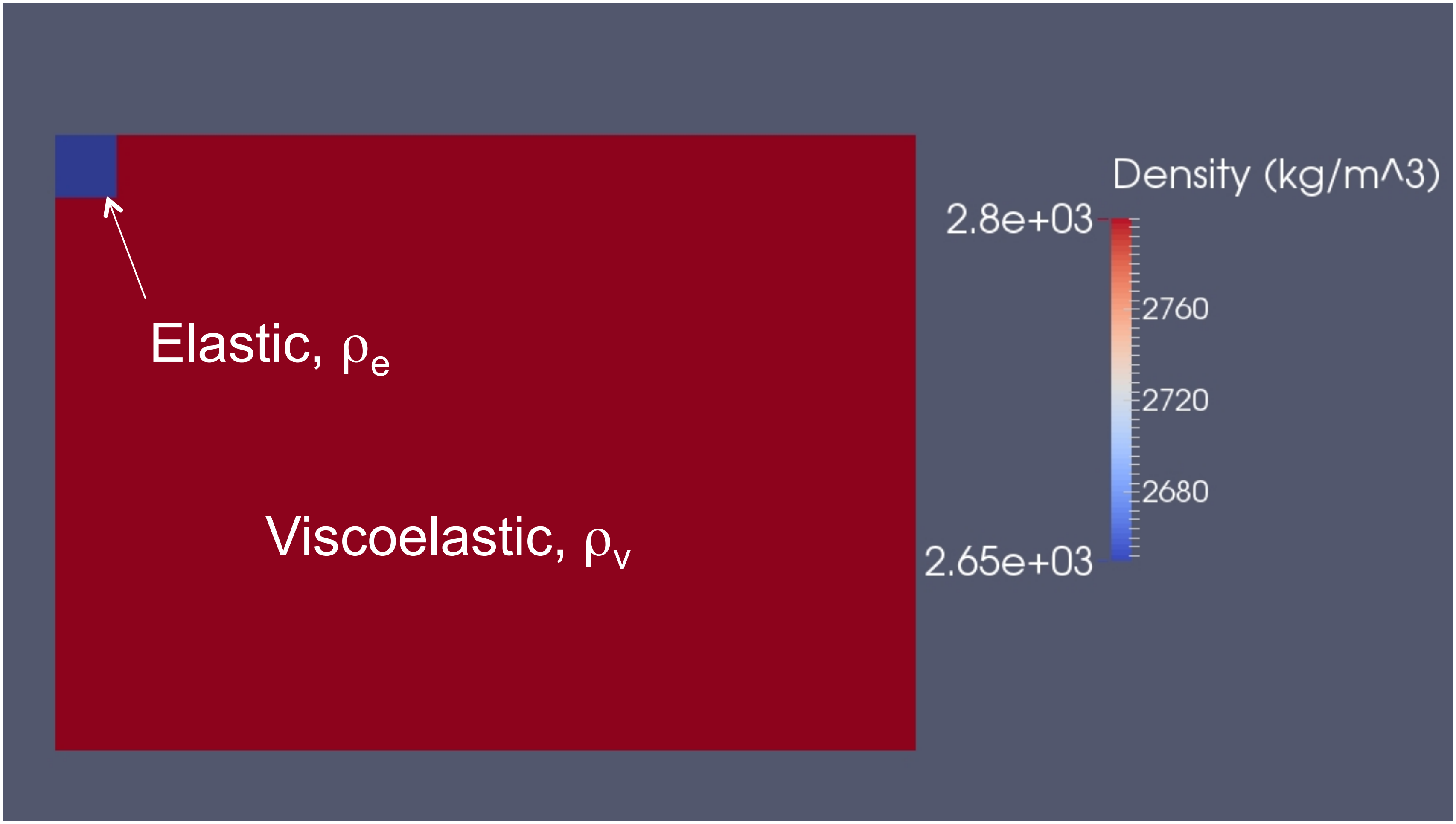
① Compute exact gravitational stresses and apply these as initial stresses for the viscoelastic problem.

② Assume simple initial stresses consistent with uniform density.

   a) Perform simulation using default infinitesimal strain formulation.
   b) Perform simulation using finite strain formulation.
   c) Same as b) with smaller time step.

examples/2d/gravity

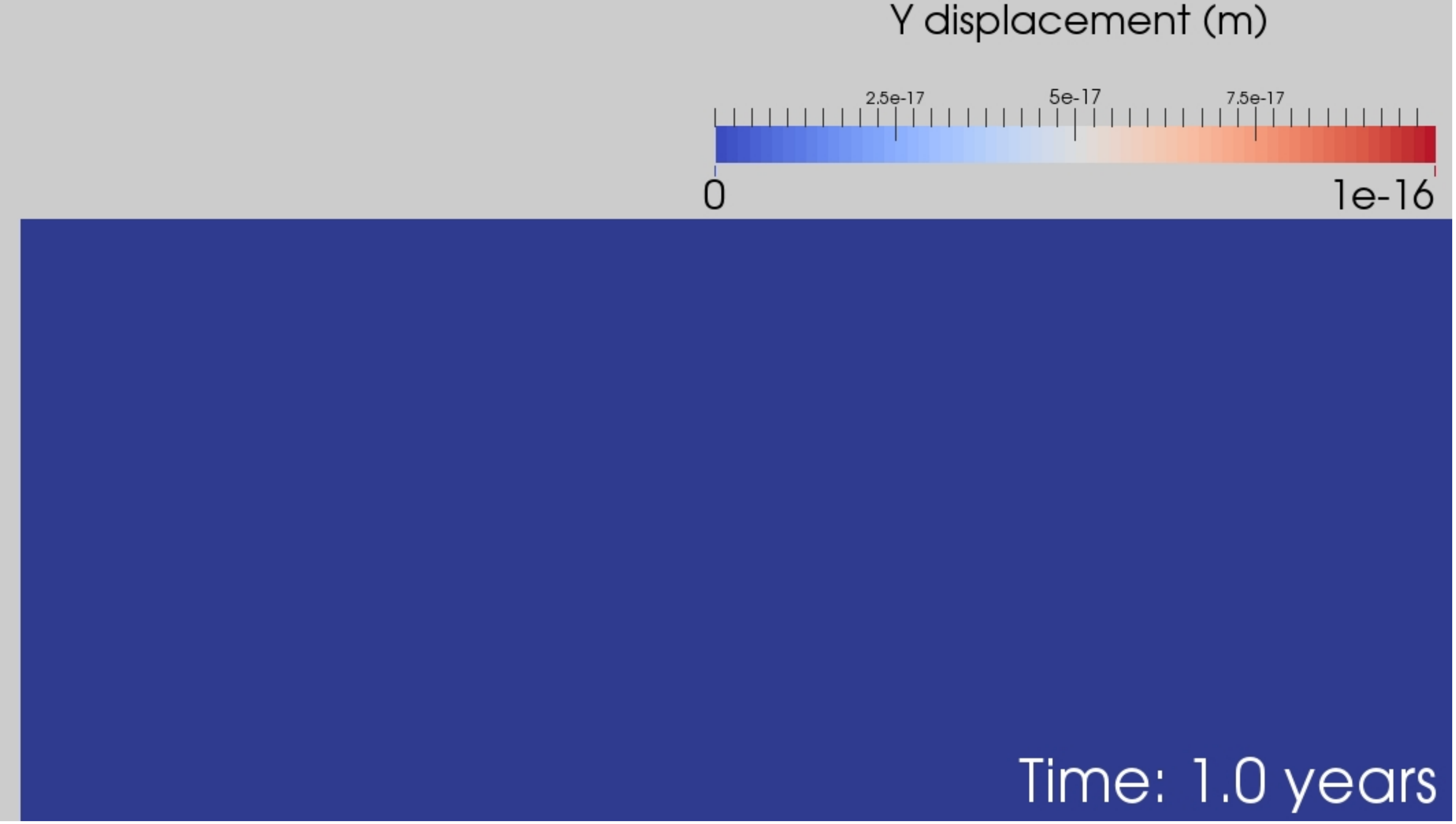
Problem setup



examples/2d/gravity

Case 1: Initial stress = exact gravitational stress

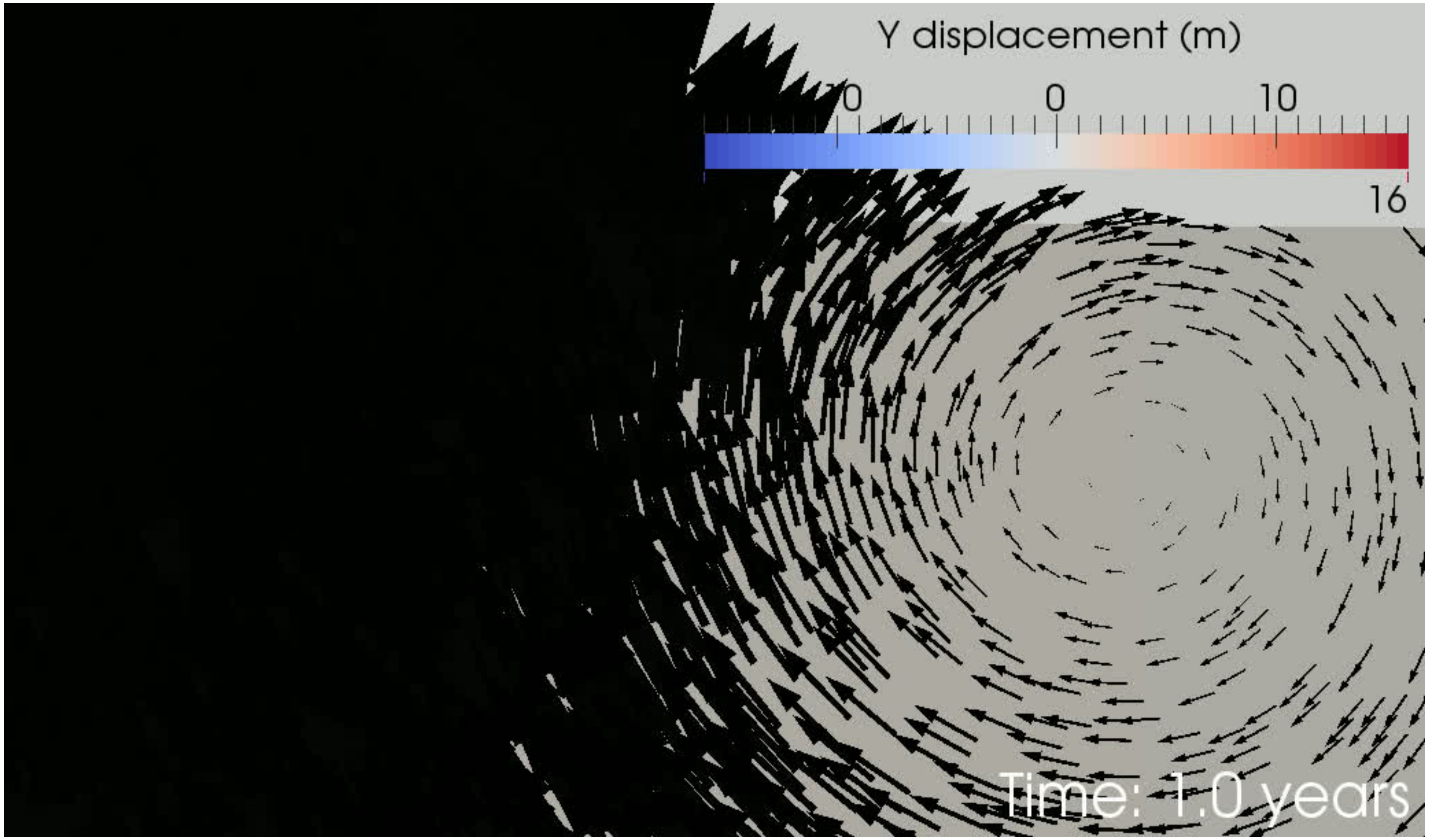
examples/2d/gravity/grav_stress_finite_is1.cfg



No displacement!

Case 2a: $\sigma_{xx} = \sigma_{yy} = \rho_v gh$
Infinitesimal strain

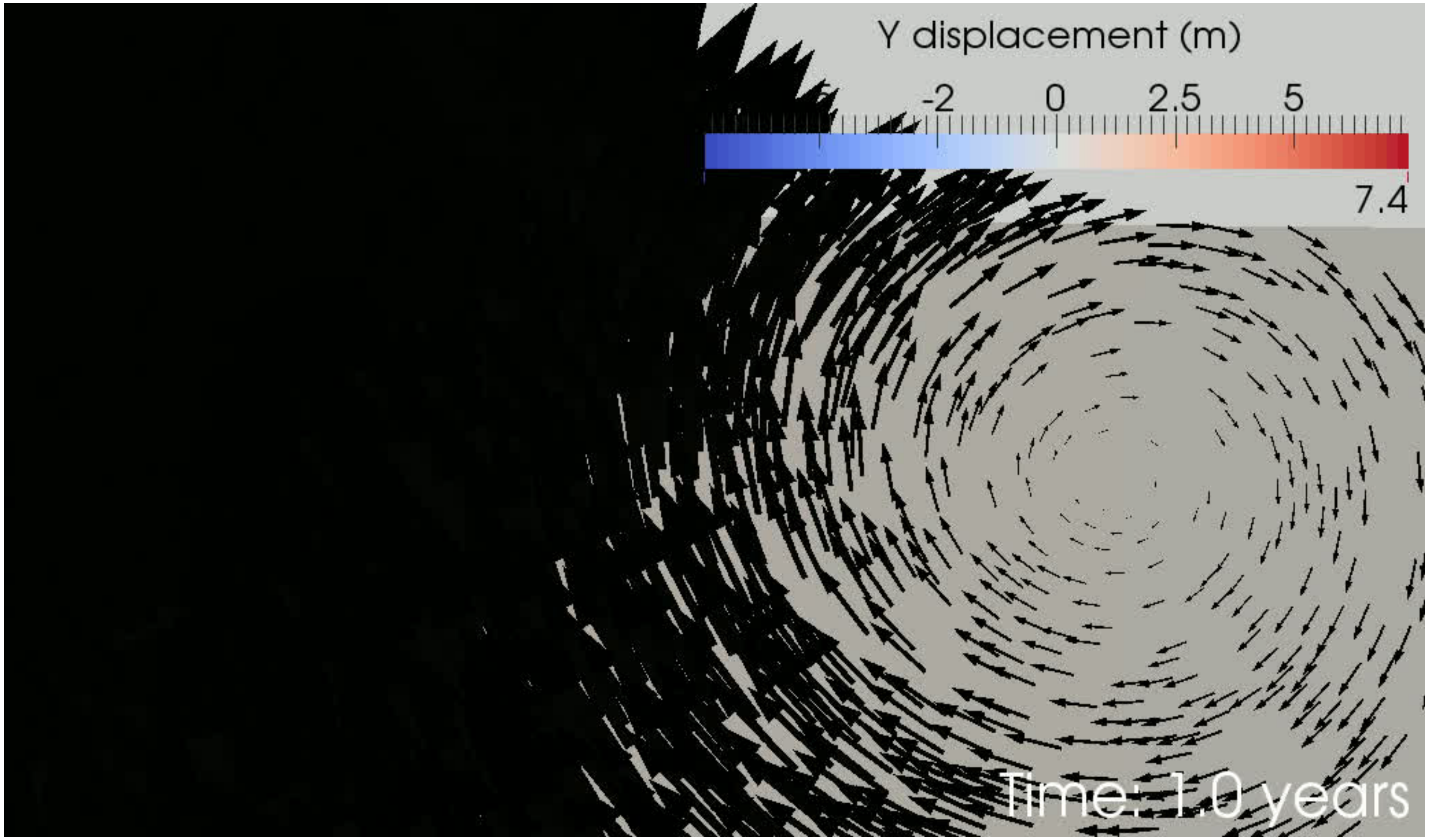
examples/2d/gravity/grav_stress_infin_is2.cfg



Too much displacement!

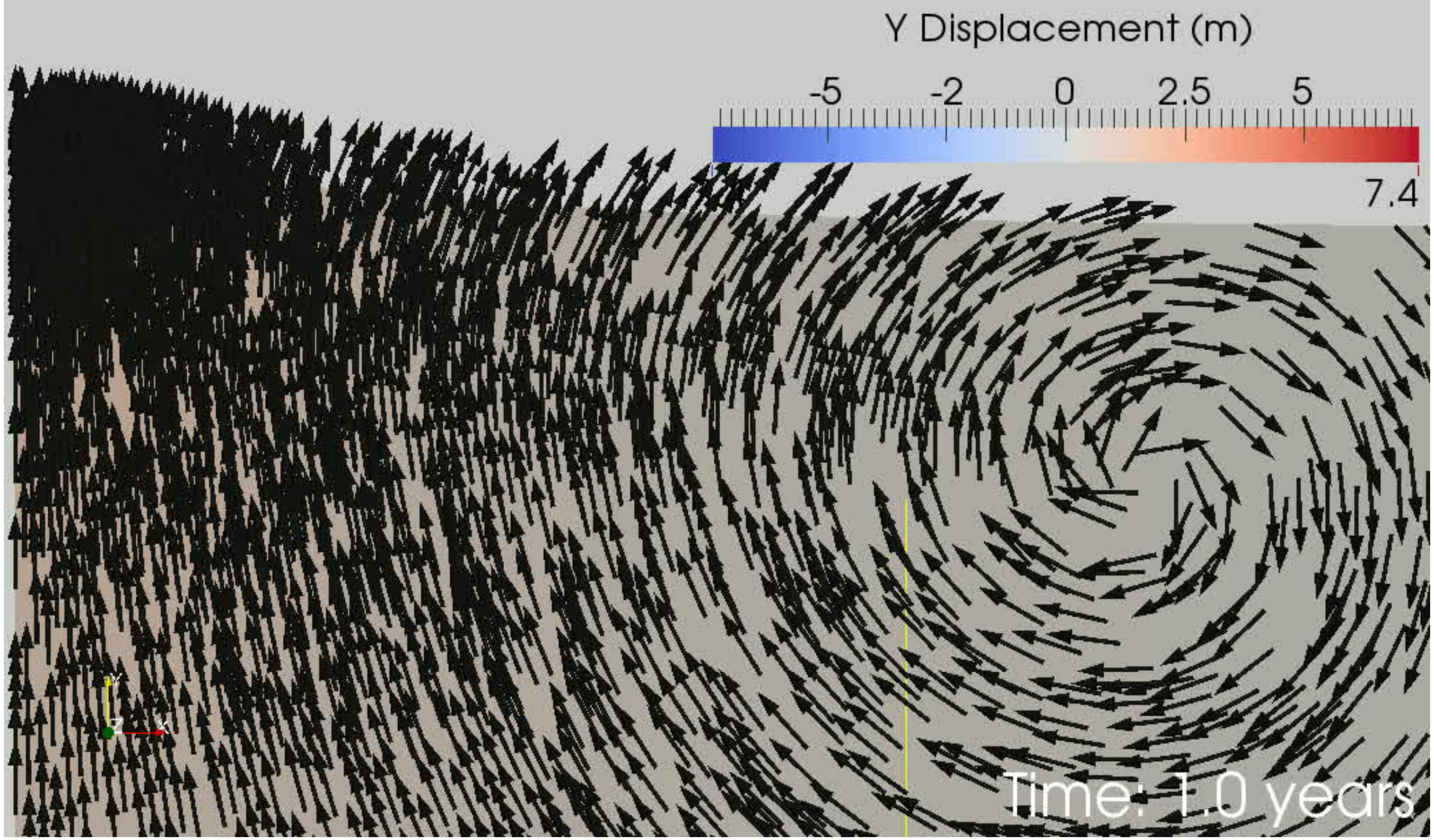
Case 2b: $\sigma_{xx} = \sigma_{yy} = \rho_v gh$
Finite strain

examples/2d/gravity/grav_stress_finite_is2.cfg



What's going on?

Case 2b: $\sigma_{xx} = \sigma_{yy} = \rho_v gh$
Finite strain (cont.)



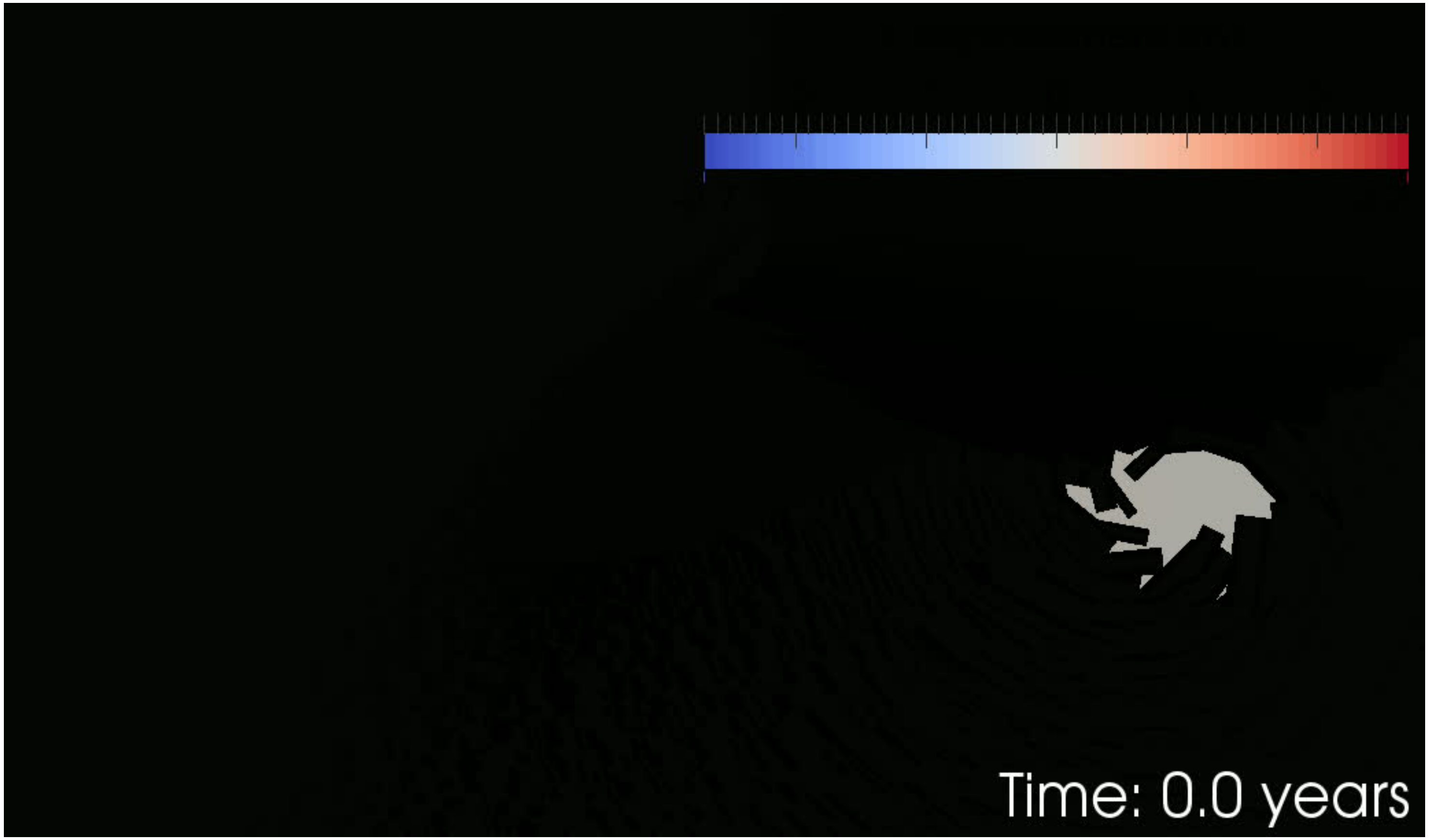
‘Drunken sailor’ instability!

Problems with finite strain solution and standard time step size

- When we have gravity + viscoelasticity + free surface there is a known instability in the solution (‘drunken sailor’ or ‘sloshing’ instability).
- To overcome this instability, a Courant condition (Kaus et al., 2010) prescribes a time step size much smaller than the ‘standard’ stable time step size.

Case 2b: $\sigma_{xx} = \sigma_{yy} = \rho_v gh$
Finite strain + small dt

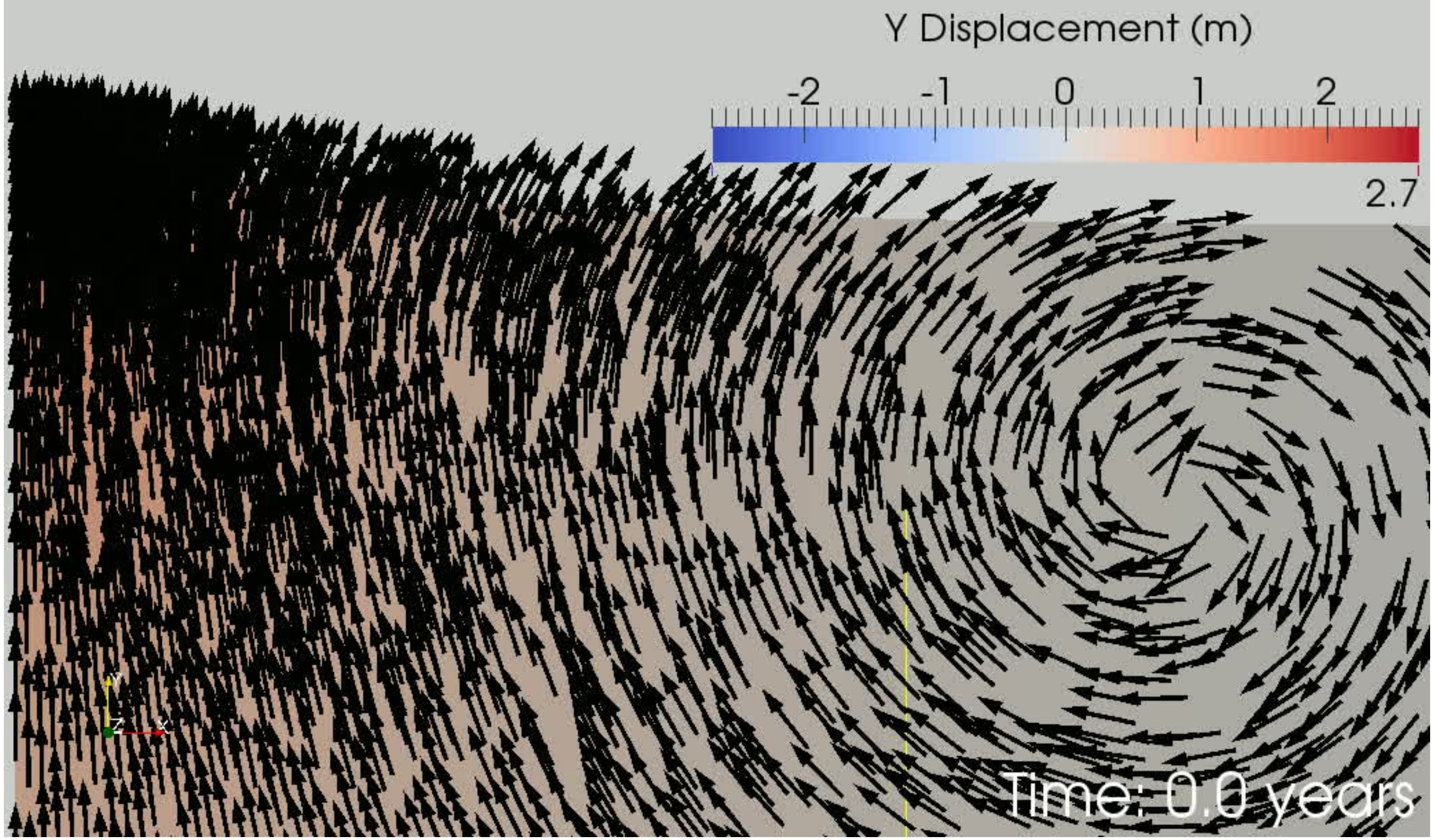
examples/2d/gravity/grav_stress_finite_is3.cfg



Much smaller displacements!

Case 2b: $\sigma_{xx} = \sigma_{yy} = \rho_v gh$
Finite strain + small dt (cont.)

examples/2d/gravity/grav_stress_finite_is3.cfg



Stable solution with smaller dt!

Summary

- Problems with no topography and simple density distributions:
  - Compensate gravitational stresses with a simple depth-varying initial stress state.
- Problems with topography and/or complex density distributions:
  - Run an initial simulation to compute equilibrium stresses and then use these as initial stresses in the actual simulation.

Summary (cont.)

- Problems with time-varying vertical displacement:
  - If we want subsequent deformation to be driven by a non-gravitational source:
    - Apply initial stresses equivalent to the gravitational stress.
  - If we want gravity to drive the deformation:
    - Apply initial stresses that do not equilibrate with density contrasts.
  - Need to use finite strain formulation, but this generally requires small time steps.

PyLith examples of interest

- examples/3d/hex8/step15.cfg
  - Gravity with no initial stresses.
- examples/3d/hex8/step16.cfg
  - Gravity with isotropic initial stresses.
- examples/3d/hex8/step17.cfg
  - Gravity with no initial stresses and finite strain.
- examples/2d/gravity
  - 2D gravity examples shown previously.