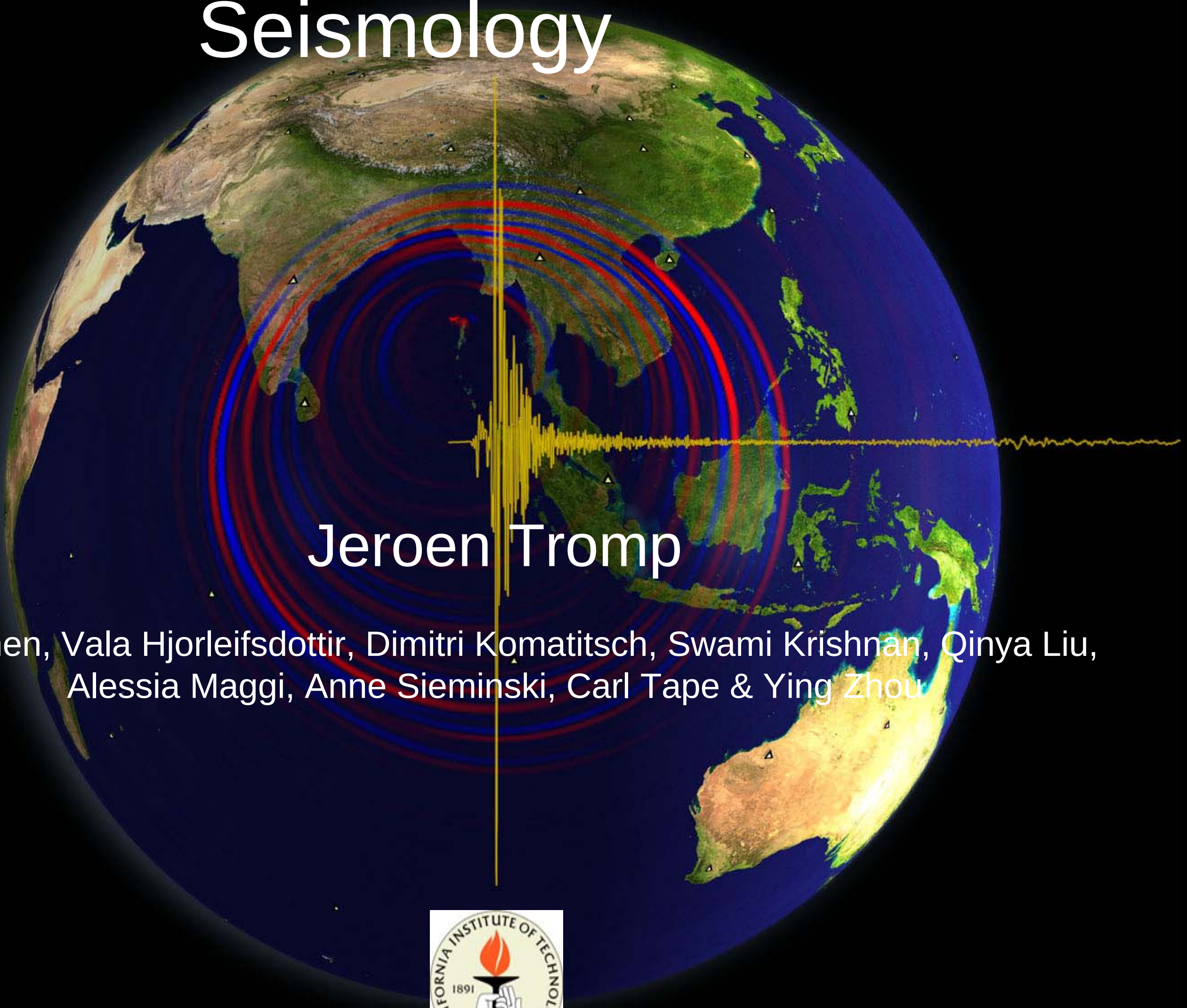
Spectral-Element and Adjoint Methods in Seismology



Jeroen Tromp

Min Chen, Vala Hjorleifsdottir, Dimitri Komatitsch, Swami Krishnan, Qinya Liu,
Alessia Maggi, Anne Sieminski, Carl Tape & Ying Zhou

Governing Equations

Equation of motion:

$$\rho\,\partial_t^2\mathbf{s} - \boldsymbol{\nabla}\cdot\mathbf{T} = \mathbf{f}$$

Boundary condition:

$$\hat{\mathbf{n}}\cdot\mathbf{T} = \mathbf{0}$$

Initial conditions:

$$\mathbf{s}(\mathbf{x},0) = \mathbf{0}, \qquad \partial_t\mathbf{s}(\mathbf{x},0) = \mathbf{0}$$

Earthquake source:

$$\mathbf{f} = -\mathbf{M}\cdot\boldsymbol{\nabla}\delta(\mathbf{x}-\mathbf{x}_{\mathrm{s}})S(t)$$

Weak Form

$$\int_{\Omega} \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{x} = -\int_{\Omega} \boldsymbol{\nabla}\mathbf{w}:\mathbf{T}\,\mathrm{d}^3\mathbf{x} + \mathbf{M}:\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_{\mathrm{s}})S(t)$$

- Weak form valid for any test vector
- Boundary conditions automatically included
- Source term explicitly integrated

Finite-fault (kinematic) rupture:

$$\mathbf{M}:\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_{\mathrm{S}})\,S(t)\rightarrow\int_{S_{\mathrm{s}}}\mathbf{m}(\mathbf{x}_{\mathrm{S}},\,t):\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_{\mathrm{S}})\,d^2\mathbf{x}_{\mathrm{S}}$$

The Diagonal Mass Matrix

Representation of the displacement:

$$\mathbf{s}(\mathbf{x}(\xi,\eta,\zeta),t) = \sum_{i=1}^{3}\hat{\mathbf{x}}_i \sum_{\sigma=0}^{n}\sum_{\tau=0}^{n}\sum_{\nu=0}^{n} s_i^{\sigma\tau\nu}(t) h_\sigma(\xi) h_\tau(\eta) h_\nu(\zeta)$$

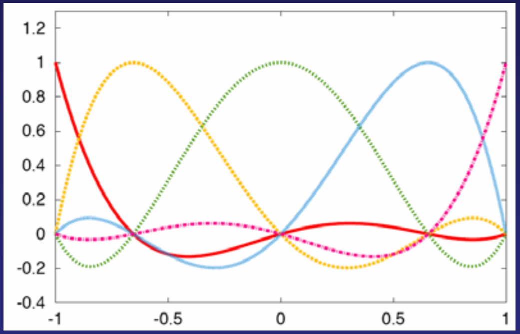
Representation of the test vector:

$$\mathbf{w}(\mathbf{x}(\xi,\eta,\zeta)) = \sum_{i=j}^{3}\hat{\mathbf{x}}_j \sum_{\sigma=0}^{n}\sum_{\tau=0}^{n}\sum_{\nu=0}^{n} w_i^{\alpha\beta\gamma} h_\alpha(\xi) h_\beta(\eta) h_\gamma(\zeta)$$

Weak form:

$$\int_\Omega \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{x} = -\int_\Omega \boldsymbol{\nabla}\mathbf{w}:\mathbf{T}\,\mathrm{d}^3\mathbf{x} + \mathbf{M}:\boldsymbol{\nabla}\mathbf{w}(\mathbf{x}_s)S(t)$$

Degree 4 Lagrange polynomials:

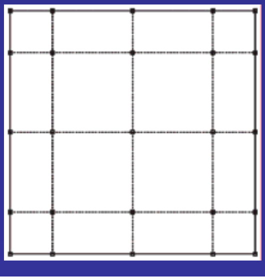


Diagonal mass matrix:

$$\int_{\Omega_e} \rho\,\mathbf{w}\cdot\partial_t^2\mathbf{s}\,\mathrm{d}^3\mathbf{x} = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho(\mathbf{x}(\boldsymbol{\xi}))\,\mathbf{w}(\mathbf{x}(\boldsymbol{\xi}))\cdot\partial_t^2\mathbf{s}(\mathbf{x}(\boldsymbol{\xi}),t)\,J(\boldsymbol{\xi})\,\mathrm{d}^3\boldsymbol{\xi} = \sum_{\alpha=0}^{n}\sum_{\beta=0}^{n}\sum_{\gamma=0}^{n}\omega_\alpha\omega_\beta\omega_\gamma J^{\alpha\beta\gamma}\rho^{\alpha\beta\gamma}\sum_{i=1}^{3} w_i^{\alpha\beta\gamma}\ddot{s}_i^{\alpha\beta\gamma}$$

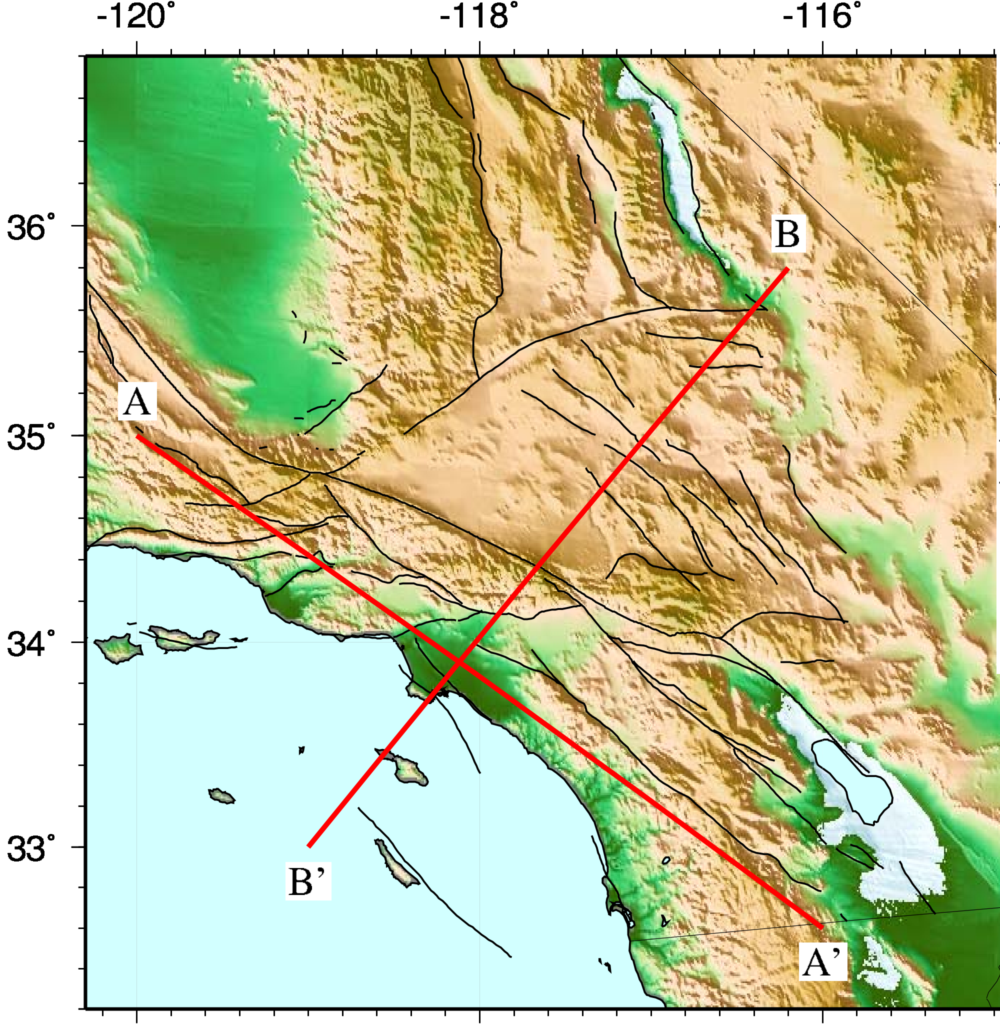
- Integrations are pulled back to the reference cube
- In the SEM one uses:
    - interpolation on GLL points
    - GLL quadrature

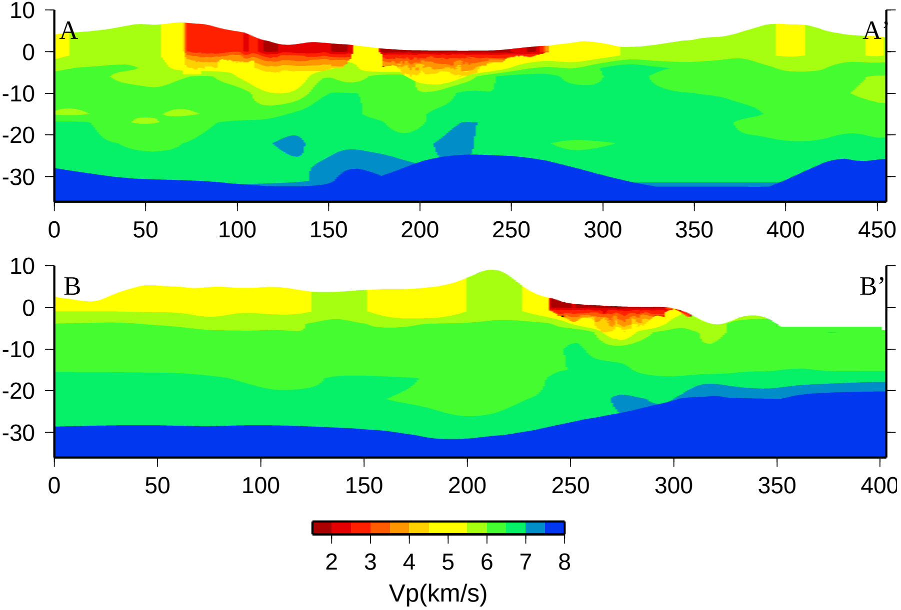
Degree 4 GLL points:

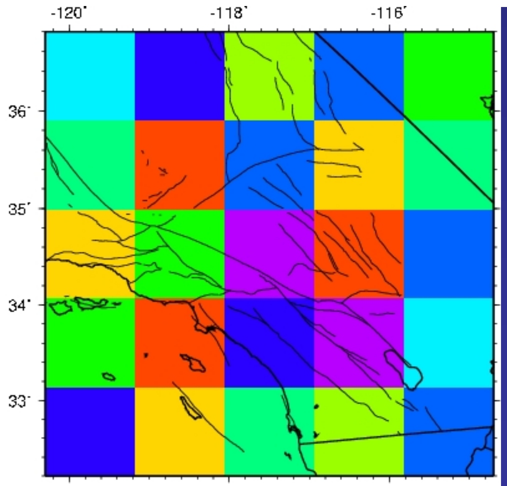


Southern California Simulations

SEISMOLOGICAL LABORATORY
California Institute of Technology







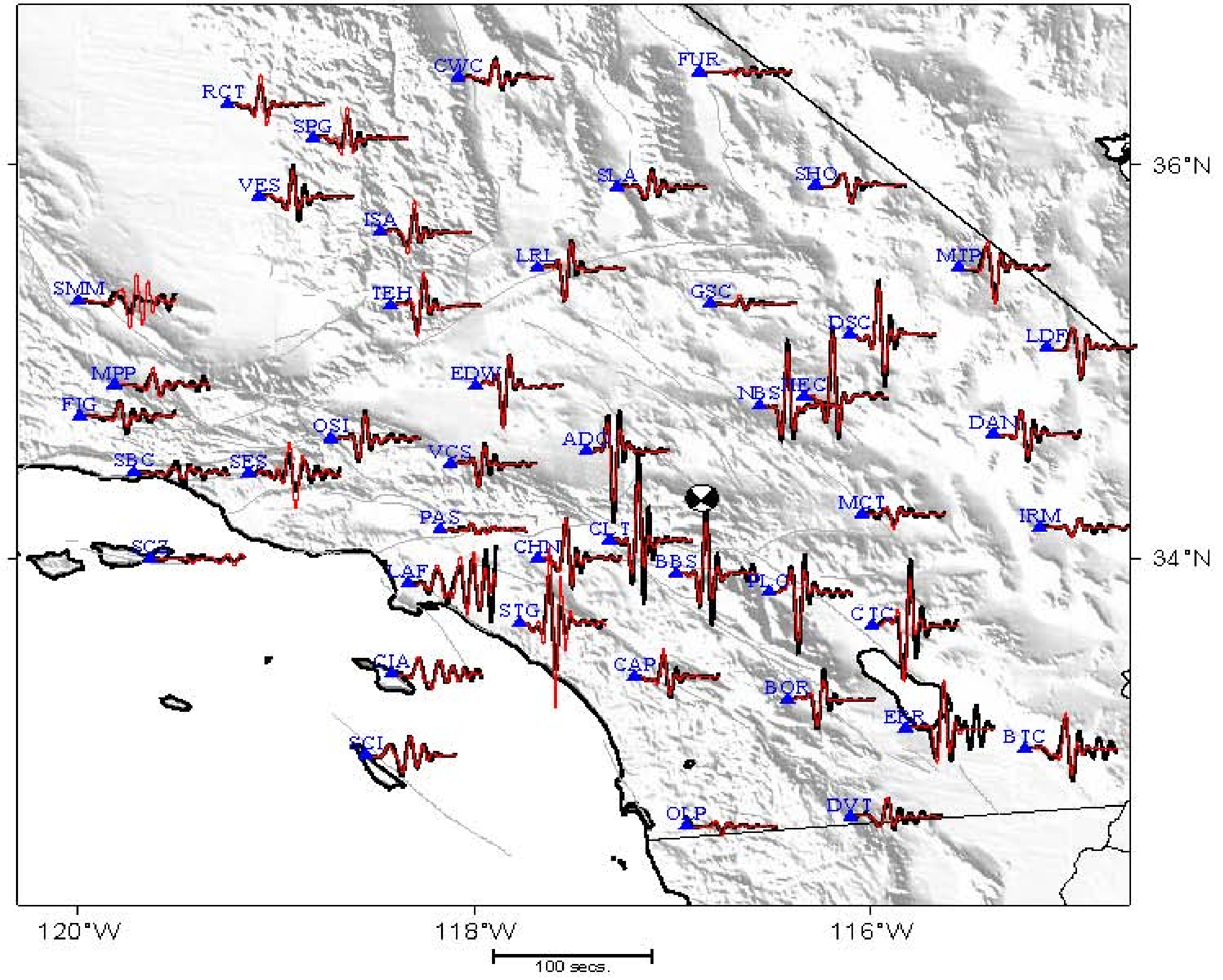
n x *m* mesh slices

SPECFEM3D_BASIN: geodynamics.org

June 12, 2005, M=5.1 Big Bear

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

3D Regional Forward Simulations



Periods > 6 s

June 12, 2005, M=5.1 Big Bear

Near Real-Time Applications

- Automated near real-time simulations of all M>3.5 events
- ShakeMovies at http://www.shakemovie.caltech.edu/
- Soon:
  - CMT source solutions
  - Synthetic seismograms

[SHAKEMOVIE] CALTECH'S SOUTHERN CALIFORNIA SEISMIC EVENT PORTAL

southern california ShakeMovie

:: CALTECH'S NEAR REAL TIME SIMULATION OF SOUTHERN CALIFORNIA SEISMIC EVENTS PORTAL :: STATUS [ALIVE] :: Sunday, September 24, 2006 :: Sunday, September 24, 2006 ::

MOST RECENT EARTHQUAKE

3.8 10207681
5 miles WSW of Ocotillo, CA5 miles WSW of Ocotillo, CA
Thu Sep 14 00:11:06 2006 utc
(32.71 , -116.06)

OTHER RECENT EVENTS

3.8 14239764
3 miles NE of Coso Junction, CA3 miles NE of Coso Junction, CA
Wed Jul.12.22:20:49.2006 utc
(36.07 , -117.91)

3.7 14239184
7 miles NNE of Perris, CA7 miles NNE of Perris, CA
Mon Jul.10.02:54:43.2006 utc
(33.87 , -117.09)

4.2 14236768
6 miles WSW of Salton City, CA6 miles WSW of Salton City, CA
Fri Jun.30.00:28:05.2006 utc
(33.24 , -116.04)

3.7 10186185
10 miles ESE of Ocotillo, CA10 miles ESE of Ocotillo, CA
Fri Jun.2.00:56:14.2006 utc
(32.68 , -115.85)

3.7 10185473
33 miles SE of Calexico, CA33 miles SE of Calexico, CA
Sun May.28.12:06:35.2006 utc
(32.30 , -115.20)

4.4 10185465
27 miles SE of Calexico, CA27 miles SE of Calexico, CA
Sun May.28.11:55:23.2006 utc
(32.38 , -115.23)

MOST RECENT :: **MOST RECENT** :: event: **5 miles WSW of Ocotillo, CA5 miles WSW of Ocotillo, CA** :: Thu Sep 14 00:11:06 2006

Event Id:
10207681
UTC:
Thu Sep 14 00:11:06 2006
MW:
3.8
5 miles WSW of Ocotillo, CA5 miles WSW of Ocotillo, CA
Latitude: Longitude:
32.7085 **-116.0607**

[download movies]

Southern California
i: 320x240 I mpeg

[view maps]
Peak Ground Acceleration
i: 500x488 I jpg

LINKS +
FAQ +

Welcome to **ShakeMovie**: Caltech's Near Real Time Simulation of Southern California Seismic Events Portal. This portal has been designed to present the public with *near real time* visualizations of recent significant seismic events in the Southern California Region. These movies are the results of simulations carried out on a large computer cluster. *Earthquake movies will be available for download approximately* ***45 mins*** *after the occurrence of a quake of magnitude 3.5 or greater.*

FACTS

When an earthquake occurs, seismic waves are generated which propagate away from the fault rupture.

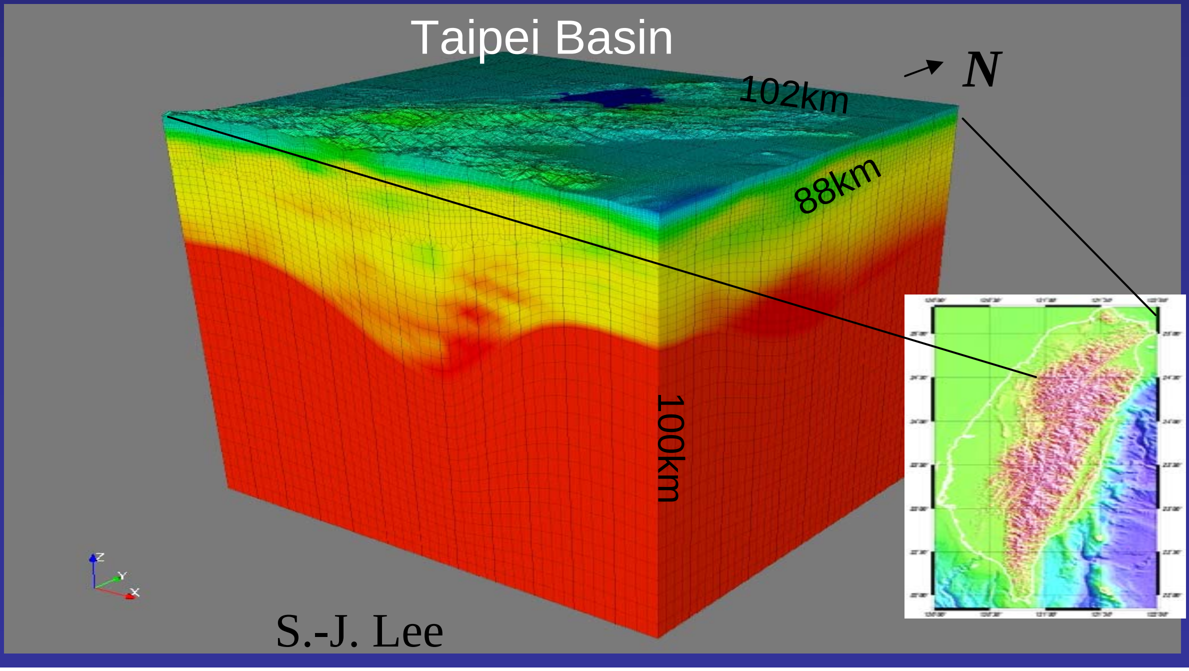
Here we see the up-and-down velocity of the Earth's surface. Strong blue waves indicate the surface is moving rapidly downward. Strong red waves indicate rapid upward motion.

When the waves pass through soft soils (sediments) they slow down and amplify. Waves speed up when they pass through hard rock.

The color of the waves oscillates between

SPECFEM3D_BASIN: Future Plans

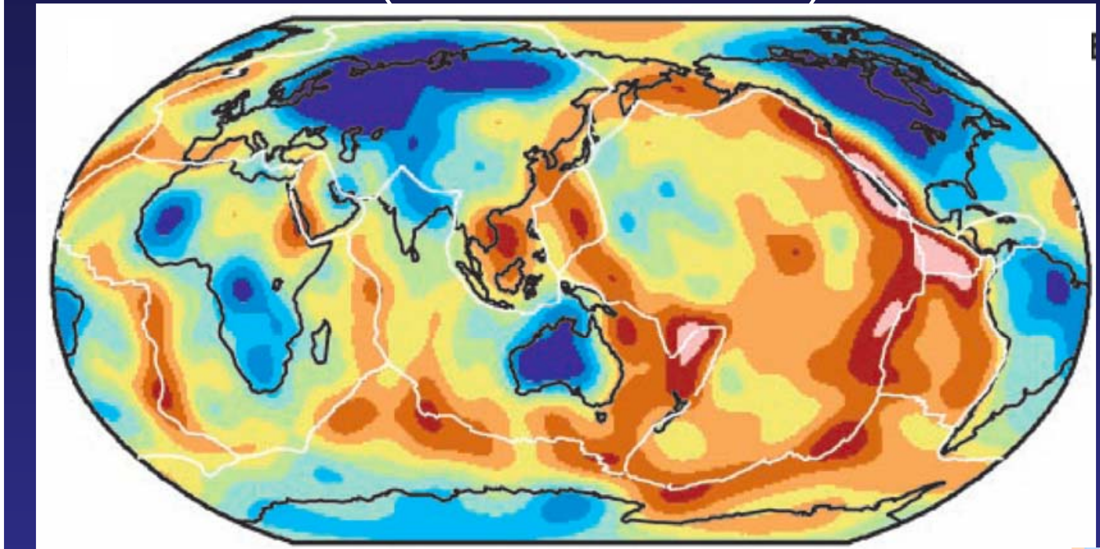
- Switch to a (parallel) CUBIT hexahedral finite-element mesher (Casarotti, Lee)
  - Topography & bathymetry
  - Major geological interfaces
  - Basins
  - Fault surfaces
- Use ParMETIS or SCOTCH for mesh partitioning & load-balancing
- Retain the SPECFEM3D_BASIN solver (takes ParMETIS meshes; Komatitsch)
- Add dynamic rupture capabilities (Ampuero, Lapusta, Kaneko)


Taipei Basin

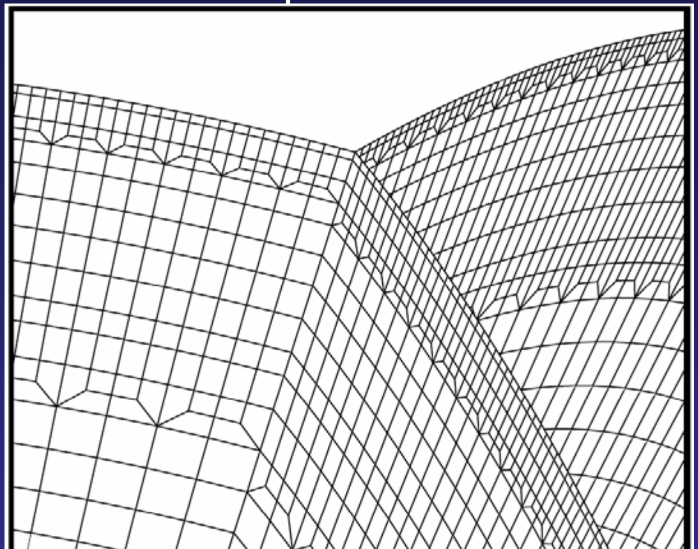
S.-J. Lee

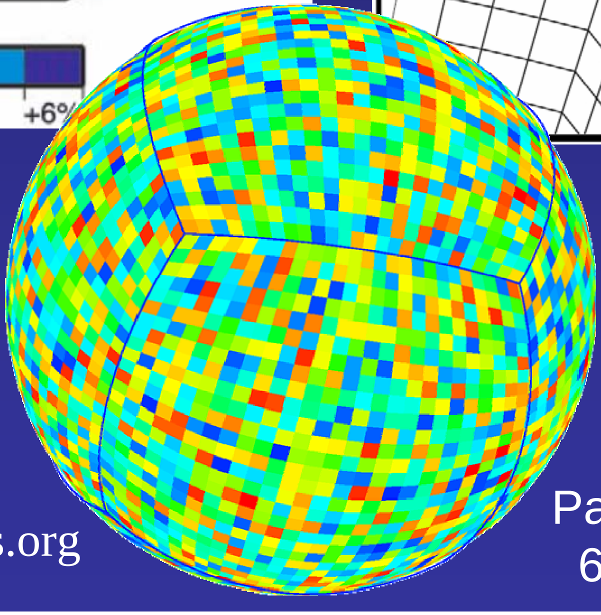
Global Simulations

S20RTS (Ritsema et al. 1999)



Cubed sphere mesh



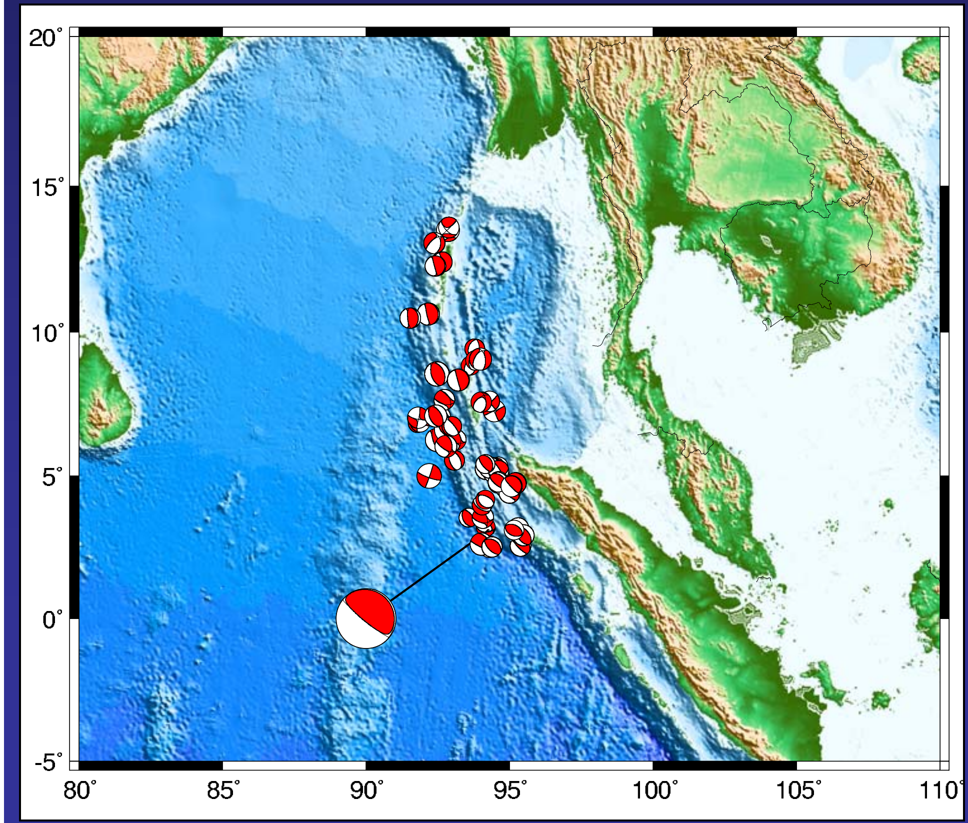


SPECFEM3D_GLOBE: geodynamics.org

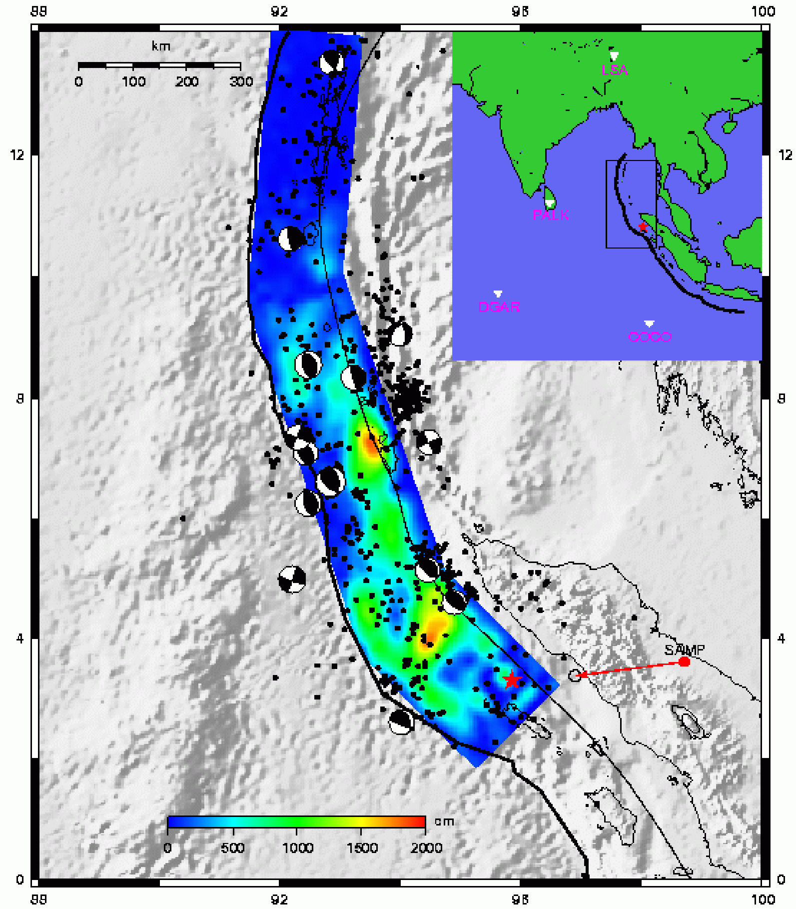
Parallel implementation:
6 n^2 mesh slices

Great 2004 Sumatra-Andaman Earthquake

Main shock & aftershocks (Harvard)



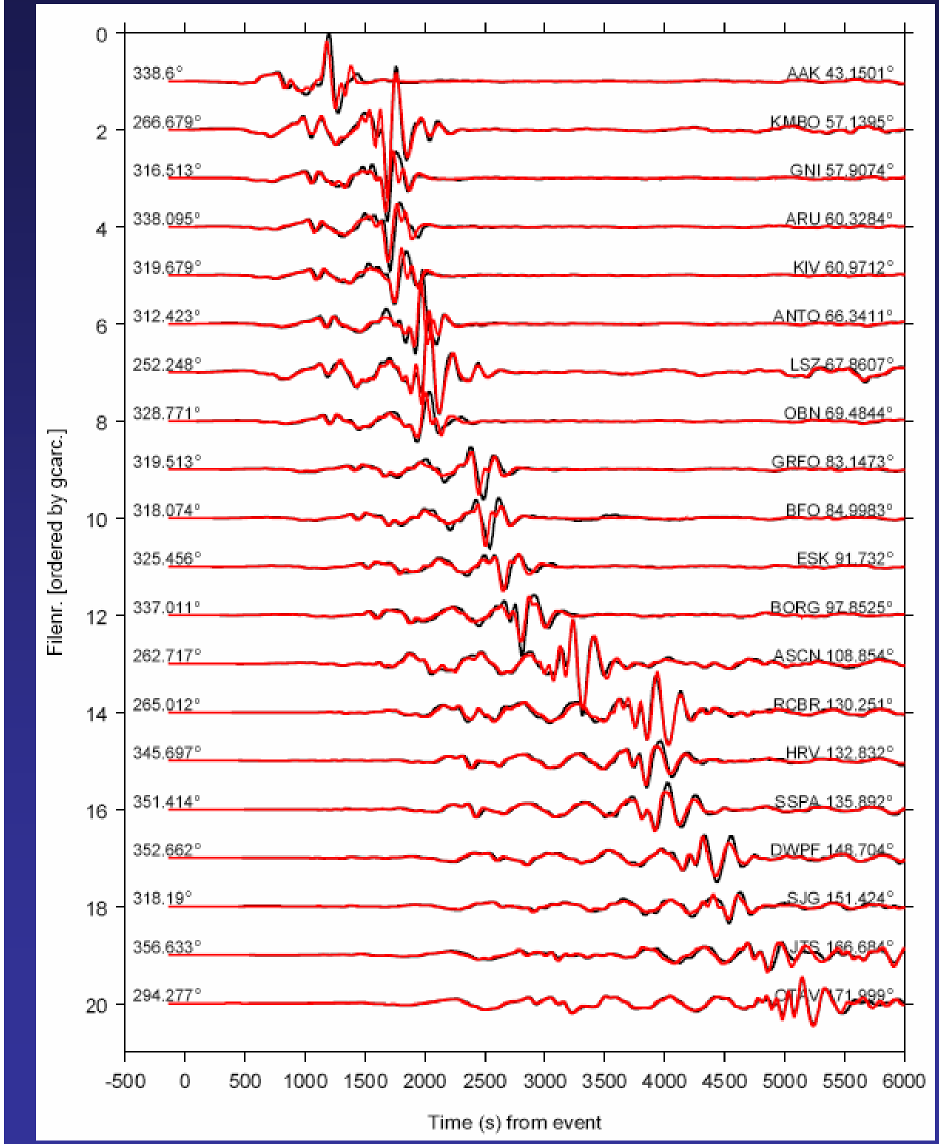
Finite slip model (Chen et al., 2005)

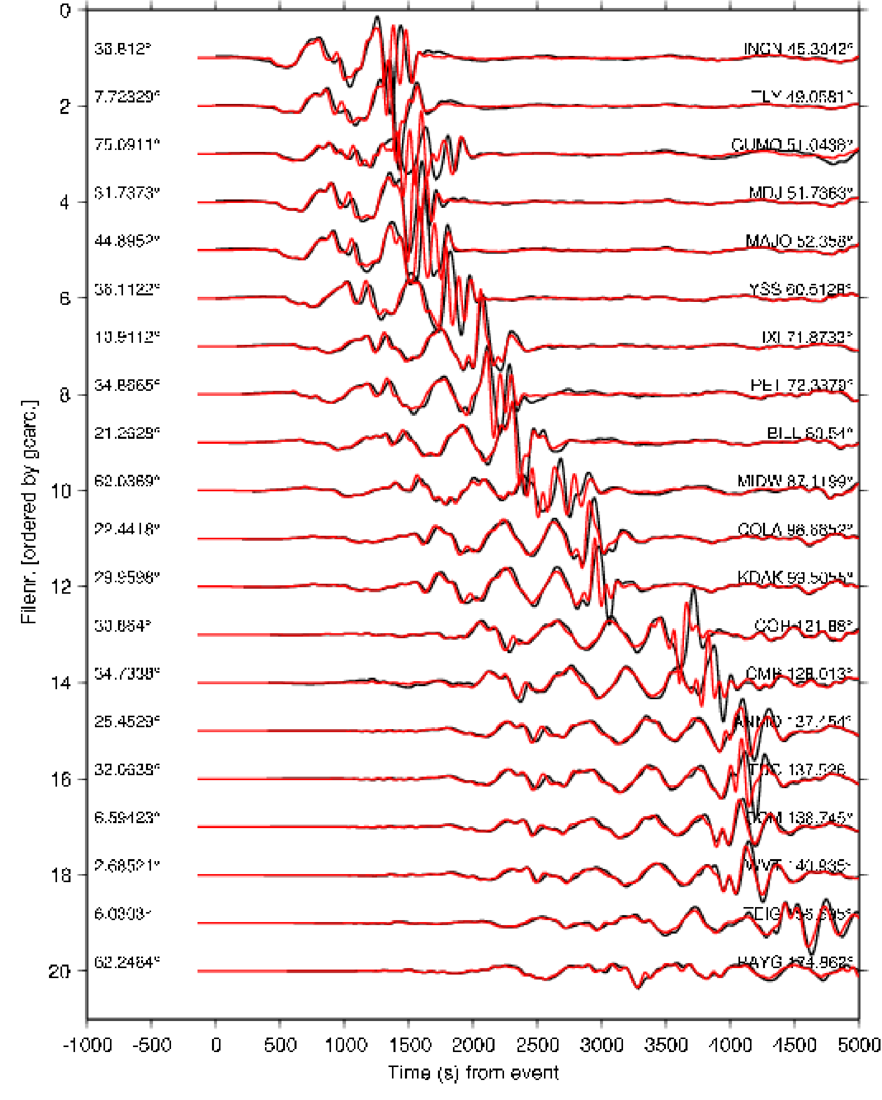


Sumatra Surface Waves

QuickTime™ and a
YUV420 codec decompressor
are needed to see this picture.

Surface-Wave Fits





Vala Hjorleifsdottir

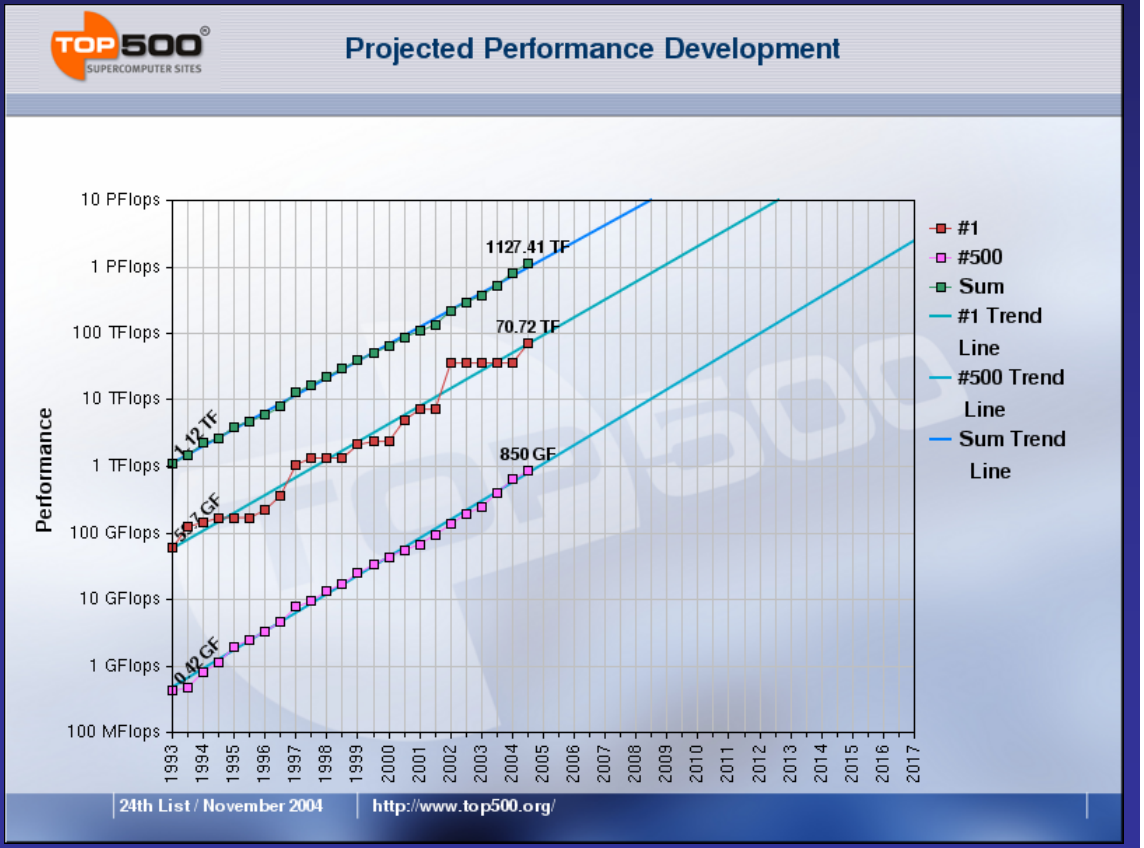
SPECFEM3D_GLOBE: Future Plans

On-demand TeraGrid applications:

- Automated, near real-time simulations of all M>6 earthquakes
- Analysis of past events (more than 20,000 events)
- Seismology Web Portal

Petascale simulations:

- Global simulations at 1-2 Hz
- New doubling brick
  (perfect load-balancing)



Adjoint Spectral-Element Simulations

Adjoint Tomography

PDE-constrained waveform tomography:

$$\chi = \tfrac{1}{2}\sum_r \int_0^T [\mathbf{s}(\mathbf{x}_r,t) - \mathbf{d}(\mathbf{x}_r,t)]^2\,\mathrm{d}t - \int_0^T\int_\Omega \boldsymbol{\lambda}\cdot(\rho\,\partial_t^2\mathbf{s} - \boldsymbol{\nabla}\cdot\mathbf{T} - \mathbf{f})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t$$

Change in the waveform misfit function:

$$\begin{aligned}
\delta\chi = {} & \int_0^T\int_\Omega \sum_r [\mathbf{s}(\mathbf{x}_r,t) - \mathbf{d}(\mathbf{x}_r,t)]\,\delta(\mathbf{x}-\mathbf{x}_r)\cdot\delta\mathbf{s}(\mathbf{x},t)\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t \\
& - \int_0^T\int_\Omega (\delta\rho\boldsymbol{\lambda}\cdot\partial_t^2\mathbf{s} + \boldsymbol{\nabla}\boldsymbol{\lambda}{:}\delta\mathbf{c}{:}\boldsymbol{\nabla}\mathbf{s} - \boldsymbol{\lambda}\cdot\delta\mathbf{f})\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t - \int_0^T\int_\Omega [\rho\partial_t^2\boldsymbol{\lambda} - \boldsymbol{\nabla}\cdot(\mathbf{c}{:}\boldsymbol{\nabla}\boldsymbol{\lambda})]\cdot\delta\mathbf{s}\,\mathrm{d}^3\mathbf{x}\,\mathrm{d}t \\
& - \int_\Omega [\rho(\boldsymbol{\lambda}\cdot\partial_t\delta\mathbf{s} - \partial_t\boldsymbol{\lambda}\cdot\delta\mathbf{s})]_T\,\mathrm{d}^3\mathbf{x} - \int_0^T\int_{\partial\Omega} \hat{\mathbf{n}}\cdot(\mathbf{c}{:}\boldsymbol{\nabla}\boldsymbol{\lambda})\cdot\delta\mathbf{s}\,\mathrm{d}^2\mathbf{x}\,\mathrm{d}t,
\end{aligned}$$

Adjoint Equations

Adjoint wavefield: $\mathbf{s}^{\dagger}(\mathbf{x},t) \equiv \boldsymbol{\lambda}(\mathbf{x},T-t)$

Adjoint equation of motion: $\rho\,\partial_t^2\mathbf{s}^{\dagger} = \boldsymbol{\nabla}\cdot\mathbf{T}^{\dagger} + \mathbf{f}^{\dagger}$

Adjoint boundary conditions: $\hat{\mathbf{n}}\cdot\mathbf{T}^{\dagger} = \mathbf{0}$

Adjoint initial conditions: $\mathbf{s}^{\dagger}(\mathbf{x},0) = \mathbf{0}, \qquad \partial_t\mathbf{s}^{\dagger}(\mathbf{x},0) = \mathbf{0}$

Adjoint source: $\mathbf{f}^{\dagger}(\mathbf{x},t) = \sum_{r=1}^{N}[\mathbf{s}(\mathbf{x}_r,T-t)-\mathbf{d}(\mathbf{x}_r,T-t)]\,\delta(\mathbf{x}-\mathbf{x}_r)$

Frechet derivative

The Frechet derivative may be expressed as:

$$\delta\chi = \int_\Omega (\delta\rho K_\rho + \delta\mathbf{c} :: \mathbf{K_c})\, \mathrm{d}^3\mathbf{x} + \int_0^T \int_\Omega \mathbf{s}^\dagger \cdot \delta\mathbf{f}\, \mathrm{d}^3\mathbf{x}\, \mathrm{d}t$$

Density and elastic tensor kernels:

$$K_\rho(\mathbf{x}) = -\int_0^T \mathbf{s}^\dagger(\mathbf{x}, T-t) \cdot \partial_t^2 \mathbf{s}(\mathbf{x}, t)\, \mathrm{d}t$$

$$\mathbf{K_c}(\mathbf{x}) = -\int_0^T \boldsymbol{\nabla}\mathbf{s}^\dagger(\mathbf{x}, T-t)\, \boldsymbol{\nabla}\mathbf{s}(\mathbf{x}, t)\, \mathrm{d}t$$

Numerical Implementation

$$K_\rho(\mathbf{x}) = -\int_0^T \mathbf{s}^\dagger(\mathbf{x}, T-t)\cdot\partial_t^2\mathbf{s}(\mathbf{x},t)\,\mathrm{d}t$$

Need simultaneous access to $\mathbf{s}^\dagger(\mathbf{x}, T-t)$ and $\mathbf{s}(\mathbf{x},t)$

- During calculation of adjoint field $\mathbf{s}^\dagger$, reconstruct $\mathbf{s}$ by solving the \`backward’ wave equation

  Need to store from a previous forward simulation:

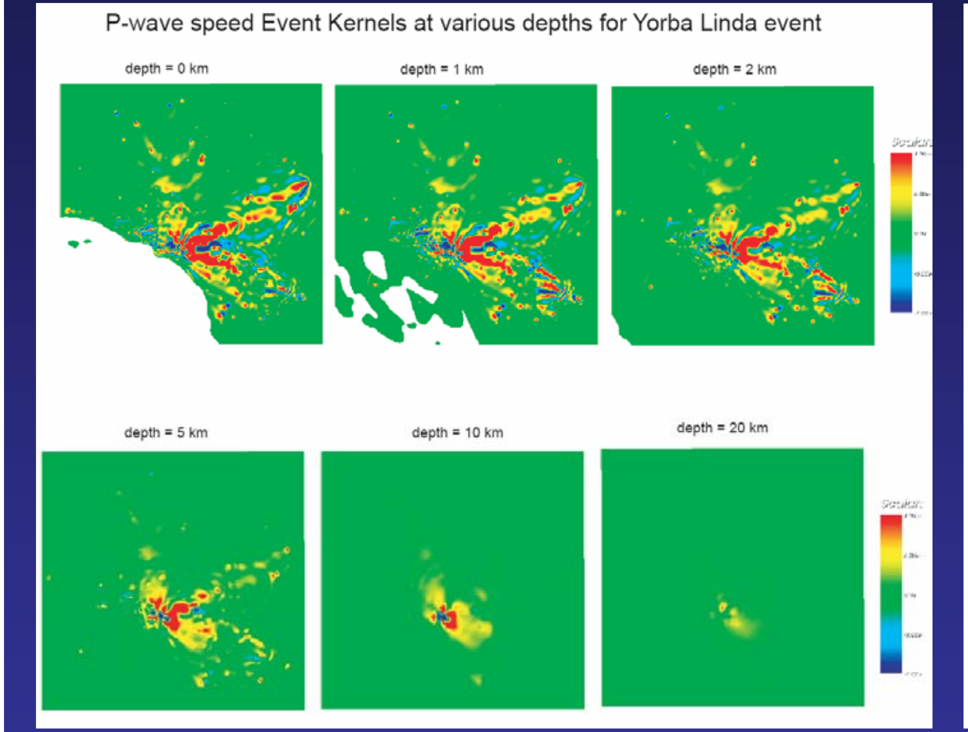
  – Last snapshot $\mathbf{s}(\mathbf{x},T)$

  – Wavefield absorb on artificial boundaries

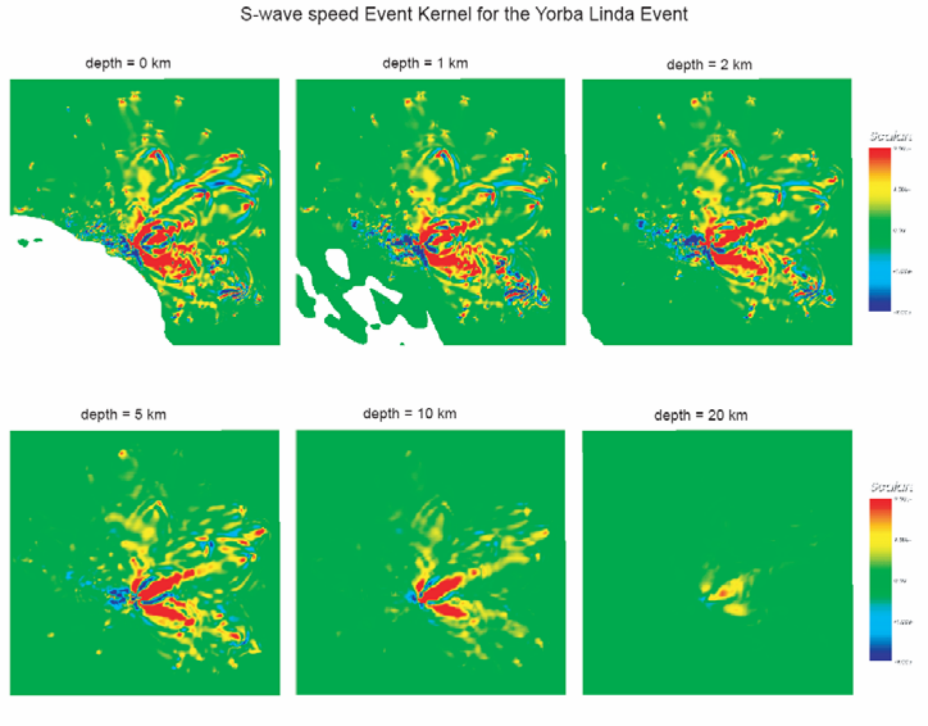
  Challenge:

  – \`Undoing’ attenuation

Toward 3D Tomography: SPECFEM3D Adjoint Capabilities



P-wave speed Event Kernels at various depths for Yorba Linda event



S-wave speed Event Kernel for the Yorba Linda Event

Conclusions

Adjoint methods:

- Choose an observable, e.g., waveforms or cross-correlation traveltimes
- Choose a measure of misfit, e.g., least-squares
- Determine the appropriate adjoint source for this observable & measurement
- Use fully 3D reference models
- Any arrival suitable for measurement
- No dependence on the number of stations, components, or measurements
- 3D sensitivity kernels may be calculated based upon two forward simulations for each earthquake
- Number of simulations: 3 * (# earthquakes) * (# iterations)
- Full anisotropy for the same cost
- Attenuation remains a challenge

Regional simulations:

- One 3 minute forward simulation accurate to 1.5 seconds takes 45 minutes on a 75 node cluster
- 150 events and 3 iterations would require 1800 simulations, i.e., three weeks of dedicated CPU time on 75 nodes
- Near real-time simulations

Global simulations:

- One 1 hour forward simulation accurate to 20 seconds takes 4 hours on a 75 node cluster
- 500 events and 3 iterations would require 6,000 simulations, i.e., 100 days on a 750 node cluster
- Near real-time simulations
- On-demand global seismology
- Petascale application