# A Constitutive Model for Layer Development in Shear Zones near the Brittle-Ductile Transition

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## Abstract

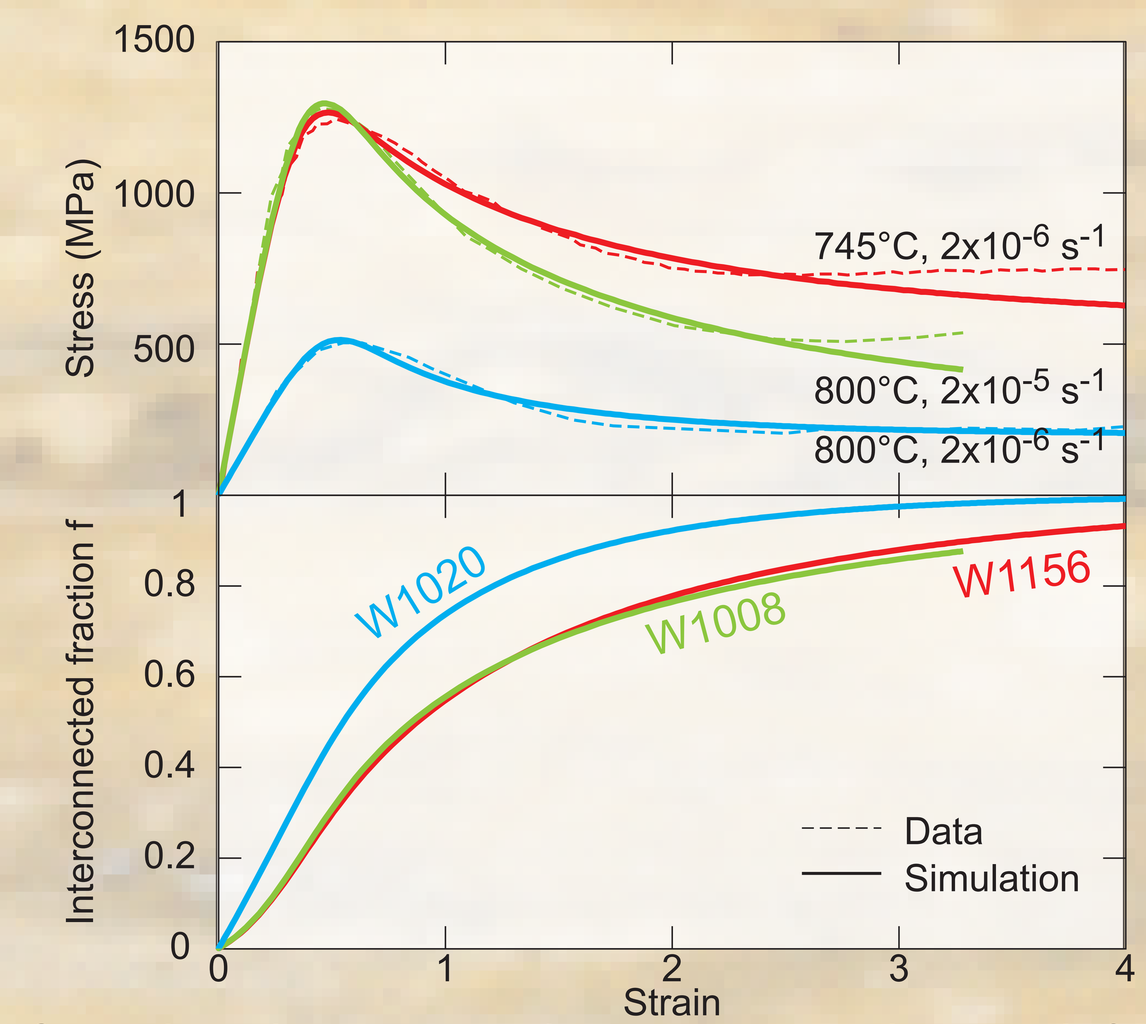
The microstructure of ductile shear zones differs from that of surrounding wallrocks. In particular, compositional layering is a hallmark of shear zones. As layered rocks are weaker than their isotropic protolith when loaded in simple shear, layering may hold the key to explain localization of ductile deformation onto ductile shear zones. I propose here a constitutive model for layer development. A two-level mixing theory allows the strength of the aggregate to be estimated at intermediate degrees of layering. A probabilistic failure model is introduced to control how layers develop in a deforming aggregate. This model captures one of the initial mechanism of phase interconnection identified experimentally by Holyoke and Tullis [2006a,b], fracturing of load bearing grains. This model reproduces the strength evolution of these experiments and can now be applied to tectonic modeling.

## Comparison with experiments

The proposed model of fabric development and resultant strength decrease is compared with the laboratory experiments of Holyoke and Tullis' [2006]. The evolution equation and rheological formulation are coupled with a elastic loading equation that describe the experimental apparatus.

$$d\sigma/dt = KH(\varepsilon_p - \varepsilon)$$

where *KH* represents the elasticity of the machine and $\varepsilon_p$ is the externally imposed strain rate, constant in time. The system of ODE is integrated forward in time with initial conditions *f*=0 and σ=0. I assumed *C*=13%. The stiffness *KH* is determined from the initial linear portion of the stress/strain curves.




**Comparison between experimental stress-strain curved from Holyoke and Tullis [2006b] (dashed lines) and simulation (solid line) usign the parameters in the table below**

| | W1156 | W1008 | W1020 |
|---|---|---|---|
| Temperature,°C | 745 | 800 | 800 |
| Strain rate, $s^{-1}$ | $1.9 \times 10^{-6}$ | $1.3 \times 10^{-5}$ | $1.3 \times 10^{-6}$ |
| $n_a$ | 3 | 3 | 3 |
| $n_b$ | 18 | 18 | 18 |
| $s_y$, MPa | 2000 | 2000 | 2000 |
| c, MPa | 1500 | 1500 | 1500 |
| $B_a$, MPa | 1700 [a] | 1984 [a] | 904 [a] |
| $B_b$, MPa | 500 [b] | 203 [b] | 178 [b] |
| 1/l | 3.1 | 3.2 | 9.5 |
| K, MPa | 4027 | 4047 | 1297 |

[a] $A_a = 3.867 \times 10^{-6}\,\mathrm{MPa}^{-3}\mathrm{s}^{-1}$, $Q_a$ = 250 kJ/mol.
[b] $A_b = 4.981 \times 10^{-55}\,\mathrm{MPa}^{-3}\mathrm{s}^{-1}$, $Q_b$ = 3000 kJ/mol (apparent value reflecting reaction ).

Background image and hand samples: Ductile shear zone sample from South Armorican Shear Zone, courtesy of Frédéric Gueydan, Géosciences Rennes, France

## Aggregate Description

The aggregate is formed of several minerals with abundance $\phi_i$, each obeying a power law relation between strain and stress

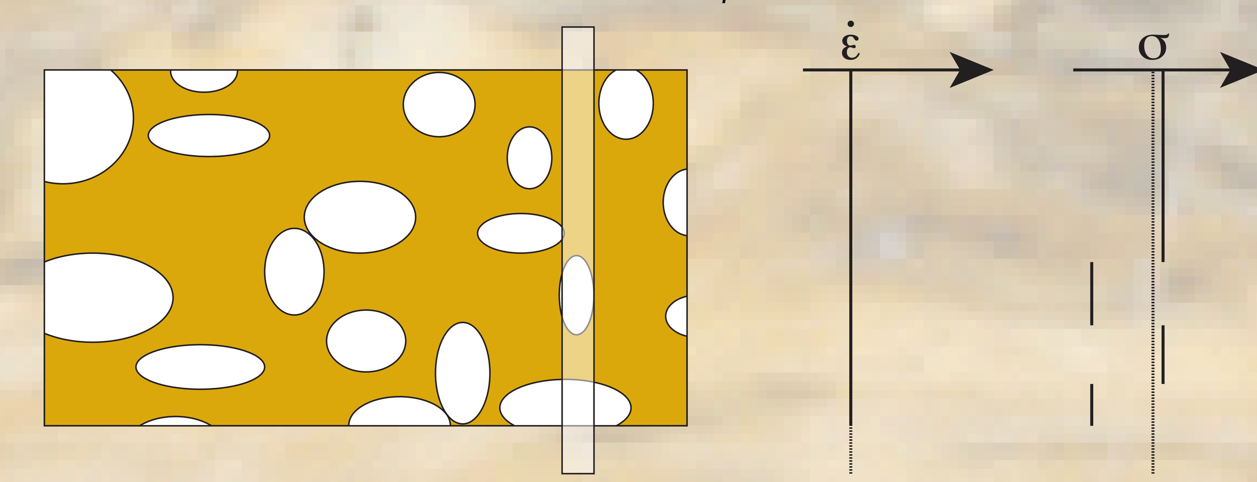
$$\varepsilon = A_i \sigma^{n_i}$$

The rheology of the aggregate is generally bounded by two endmember mixture relations

Constant strain rate: $\sigma_d = \sum_i \varphi_i (\varepsilon / A_i)^{1/n_i}$

Constant stress: $\varepsilon = \sum_i \varphi_i A_i \sigma_c^{n_i}$

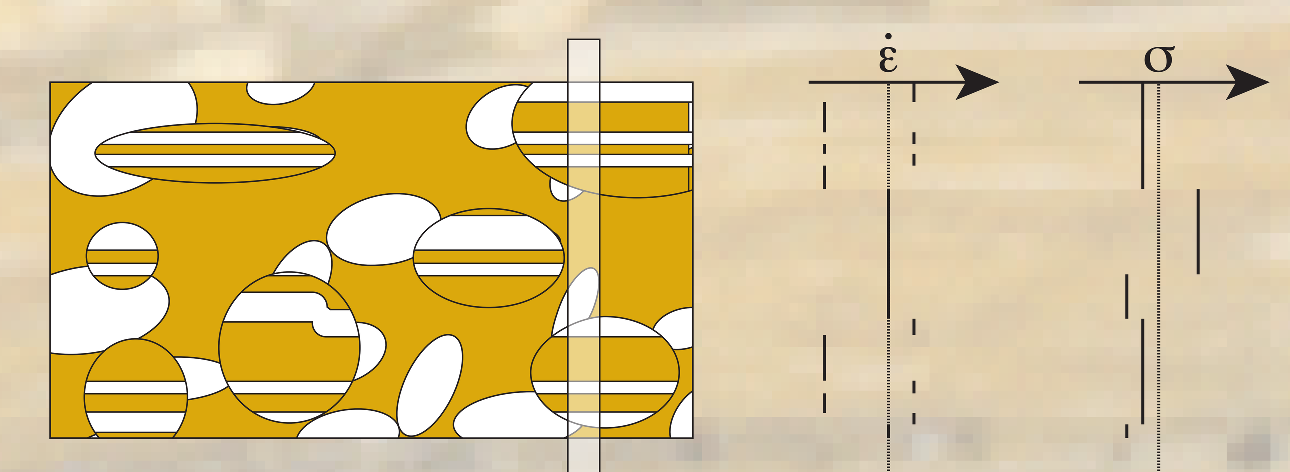
## Initial State: Isotropic




In the protolith, the stronger minerals form a load-bearing framework, forcing a uniform strain rate throughout the aggregate. If only two phases *a* and *b* are present, with the abundance of phase *b* (conceptually weaker) being *C*, the rheology of the starting material is approximately

$$\sigma_d = (1-C)(\varepsilon/A_a)^{1/n_a} + C(\varepsilon/A_b)^{1/n_b}$$

## Intermediate state

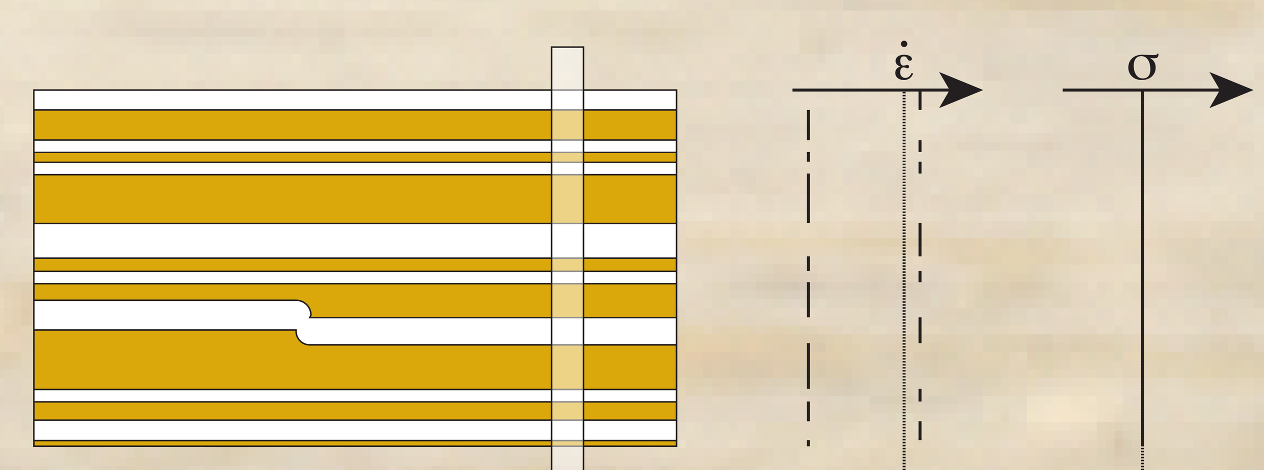



If a fraction *f* of the aggregate is layered, I consider that two pseudophases are present in the aggregate, one with isotropic structure, the other with layered structure. Their rheology is given by the constant strain rate and constant stress approximation, respectively. As the pseudophases are randomly distributed, I assume that the strain rate is the same between each pseudophawse. The aggregate behavior is then given by:

$$\varepsilon = \sum_i \varphi_i A_i \sigma_c^{n_i} \qquad \sigma = (1-f)\,\sigma_d + f\,\sigma_c$$

$$\sigma = \sigma_c \quad f + (1-f)(1-C)\left[(1-C) + C\frac{A_b\sigma_c^{n_b}}{A_a\sigma_c^{n_a}}\right]^{1/n_a} + (1-f)C\left[C + (1-C)\frac{A_a\sigma_c^{n_a}}{A_b\sigma_c^{n_b}}\right]^{1/n_b}$$

## Final state: Layered

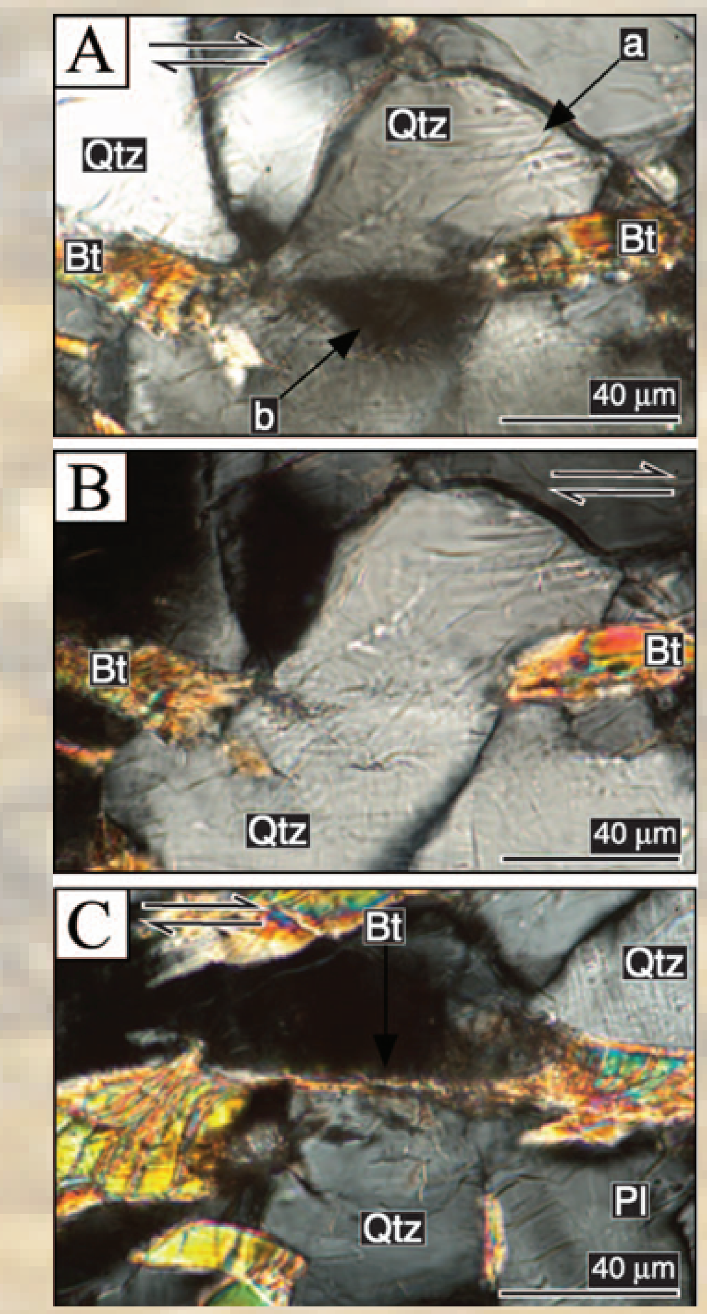



In the shear zone, each phase forms an inconnected layer so that the response to layer-parallel shear is given by the constant stress approximation

$$\varepsilon = (1-C)\,A_a\sigma_c^{n_a} + C\,A_b\sigma_c^{n_b}$$

## Microstructural Evolution

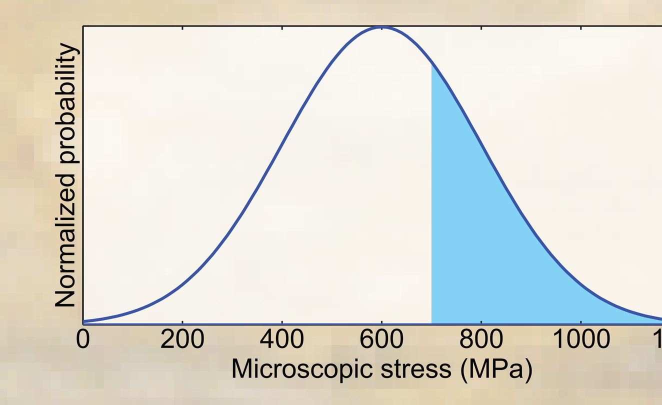
Layering develops through a combination of grain rotation, shearing, and breaking. Near the brittle-ductile condition, as in Holyoke and Tullis [2006] experiments, fracturing of load-bearing grains dominates.




**Optical micrographs from Holyoke and Tullis [2006a] near peak strength. The starting material is a gneiss minuti in which quartz and feldspar are strong load-bearing phases and the weak phase is mica (13%) A: Quartz grain between two biotite grains showing zone of localized strain (area b) B: Same as A, but polarizers rotated 45°. C: Biotite lining grain-scale shear zone in quartz formed in response to local stress concentrations between two biotite grains.**

To describe this phenomenon, I introduce a probabilistic failure model (as in Zhu et al., 2006). The local stress *s* is assumed to be normally distributed around a mean value $s_a = H\sigma$, where *H* is the stress enhancement factor, and $\xi = H\chi$ is the variance

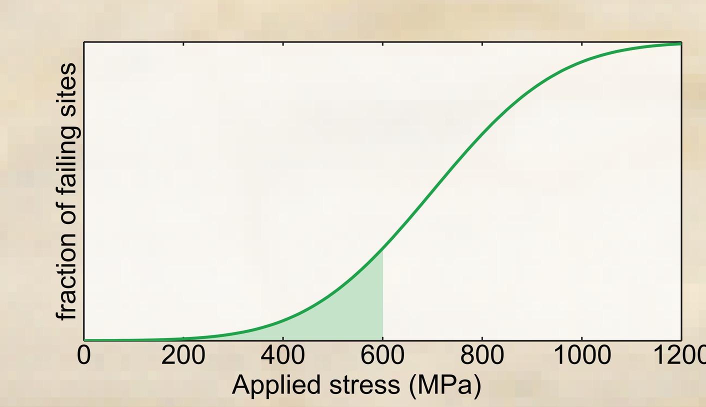
$$p(s \mid \sigma) = \frac{1}{\xi\sqrt{2\pi}} \exp\left(-\frac{(s-s_a)^2}{2\xi^2}\right)$$




**Distribution of local stresses in a sample assuming a Gaussian distribution with *s*=600 MPa and ξ=200 MPa. Shaded region represents states currently at failure if the yield strength is 700 MPa.**

Failure occurs at all the sites where $s > s_y = H\sigma_y$. The fraction of sites currently at failure is therefore.

$$N(\sigma) = \int (s > s_y)\, p(s \mid \sigma)\, ds = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\sigma - \sigma_y}{\chi\sqrt{2}}\right)\right]$$




**Fraction of damaged sites as a function of applied stress assuming again $s_y$=700 MPa and ξ=200 MPa**

The fraction of sites currently gaining a layered microstructure have to be at failure but be isotropic. There are *N*(1-*f*). such sites. Therefore, the fraction of the aggregate that has foliated microstructure evolves according to

$$\frac{df}{d\varepsilon} = \frac{1}{\lambda}(1-f)\frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\sigma - \sigma_y}{\chi\sqrt{2}}\right)\right]$$