Flow Laws for the Lower Crust and Upper Mantle

David Kohlstedt
Department of Geology and Geophysics
University of Minnesota

Constitutive Equations / Flow Laws

$$\dot{\varepsilon} = \dot{\varepsilon}\left(\sigma, T, P, f_{\mathrm{H_2O}}, a_{\mathrm{ox}}, \ldots, S, d, \ldots, \Phi, \phi, \ldots\right)$$

$$\eta = \eta\left(\sigma, T, P, f_{\mathrm{H_2O}}, a_{\mathrm{ox}}, \ldots, S, d, \ldots, \Phi, \phi, \ldots\right)$$

Flow Laws for Steady-State Deformation – Diffusion Creep

Grain-matrix diffusion

$$\dot{\varepsilon}_{\mathrm{NH}} = \alpha_{\mathrm{NH}} \frac{\sigma V_{\mathrm{m}}}{\mathrm{R}T} \frac{D_{\mathrm{gm}}}{d^2}$$

$$D_{\mathrm{gm}} = D^{\mathrm{o}}_{\mathrm{gm}} \exp\left(-\frac{\Delta E_{\mathrm{gm}} + P\Delta V_{\mathrm{gm}}}{\mathrm{R}T}\right)$$

$$= D^{\mathrm{o}}_{\mathrm{gm}} \exp\left(-\frac{\Delta H_{\mathrm{gm}}}{\mathrm{R}T}\right)$$

Grain-boundary diffusion

$$\dot{\varepsilon}_{\mathrm{C}} = \alpha_{\mathrm{C}} \frac{\sigma V_{\mathrm{m}}}{\mathrm{R}T} \frac{\delta D_{\mathrm{gb}}}{d^3}$$

$$D_{\mathrm{gb}} = D^{\mathrm{o}}_{\mathrm{gb}} \exp\left(-\frac{\Delta E_{\mathrm{gb}} + P\Delta V_{\mathrm{gb}}}{\mathrm{R}T}\right)$$

$$= D^{\mathrm{o}}_{\mathrm{gb}} \exp\left(-\frac{\Delta H_{\mathrm{gb}}}{\mathrm{R}T}\right)$$

Flow Laws for Steady-State Deformation – Diffusion Creep

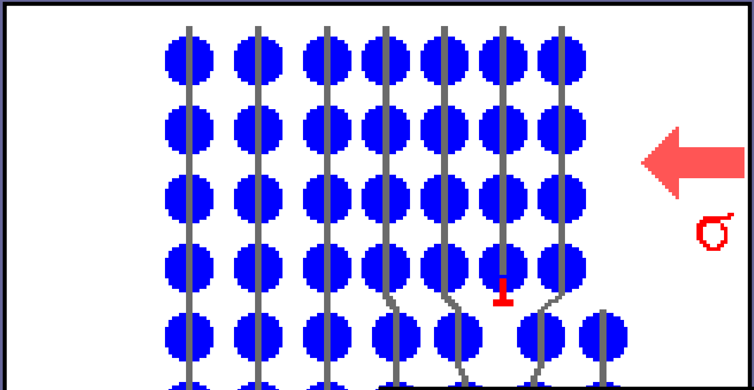
$$\dot{\varepsilon}_{\mathrm{diff}} = 14\left(\frac{\sigma V_{\mathrm{m}}}{\mathrm{R}T}\right)\left(D_{\mathrm{gm}} + \frac{\pi\delta D_{\mathrm{gb}}}{d}\right)\left(\frac{1}{d^{2}}\right)$$

$$D = D^{\mathrm{o}} \exp\left(-\frac{\Delta H}{\mathrm{R}T}\right)$$

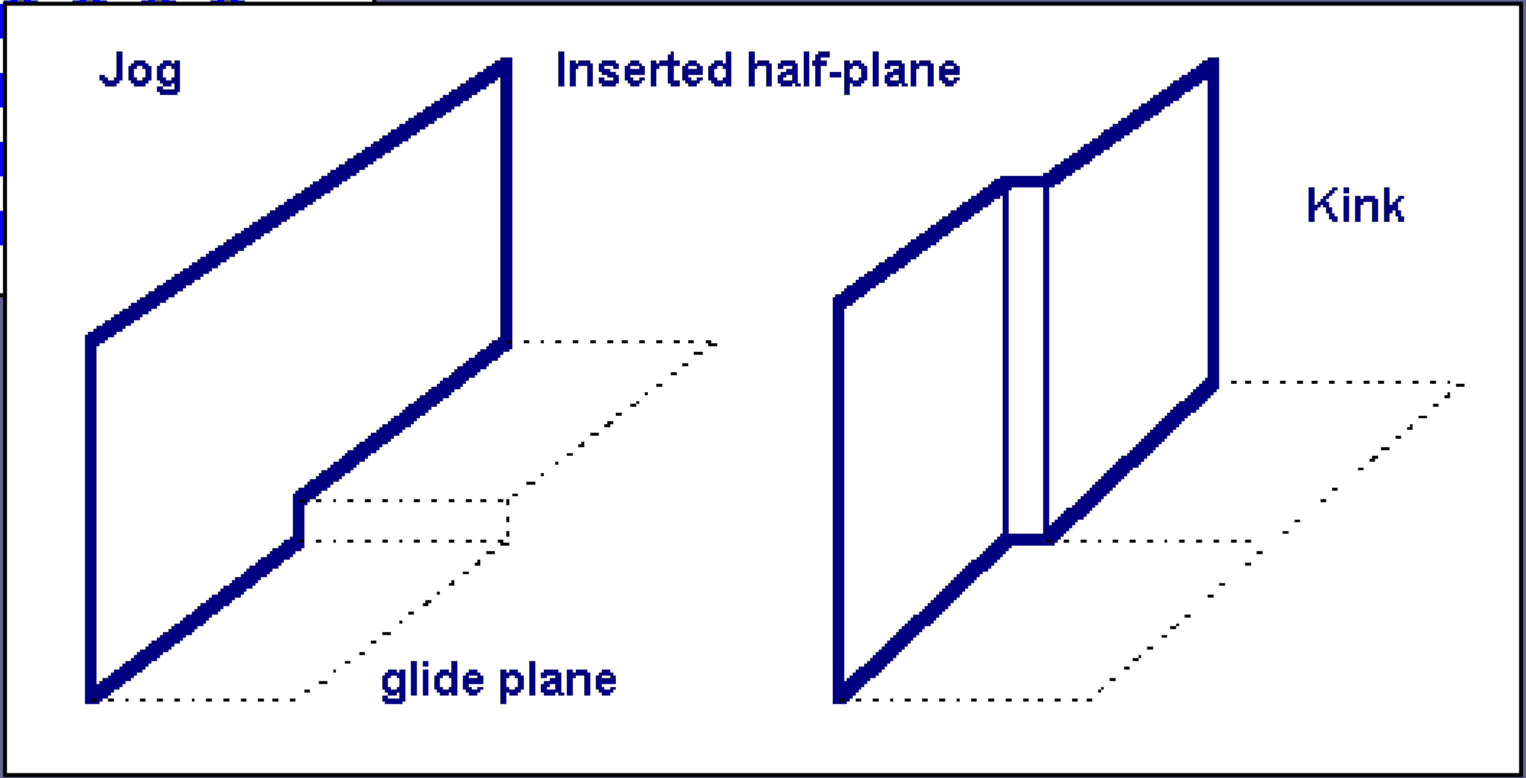
$$\Delta H_{\mathrm{gb}} < \Delta H_{\mathrm{gm}}$$

slowest ion along fastest path

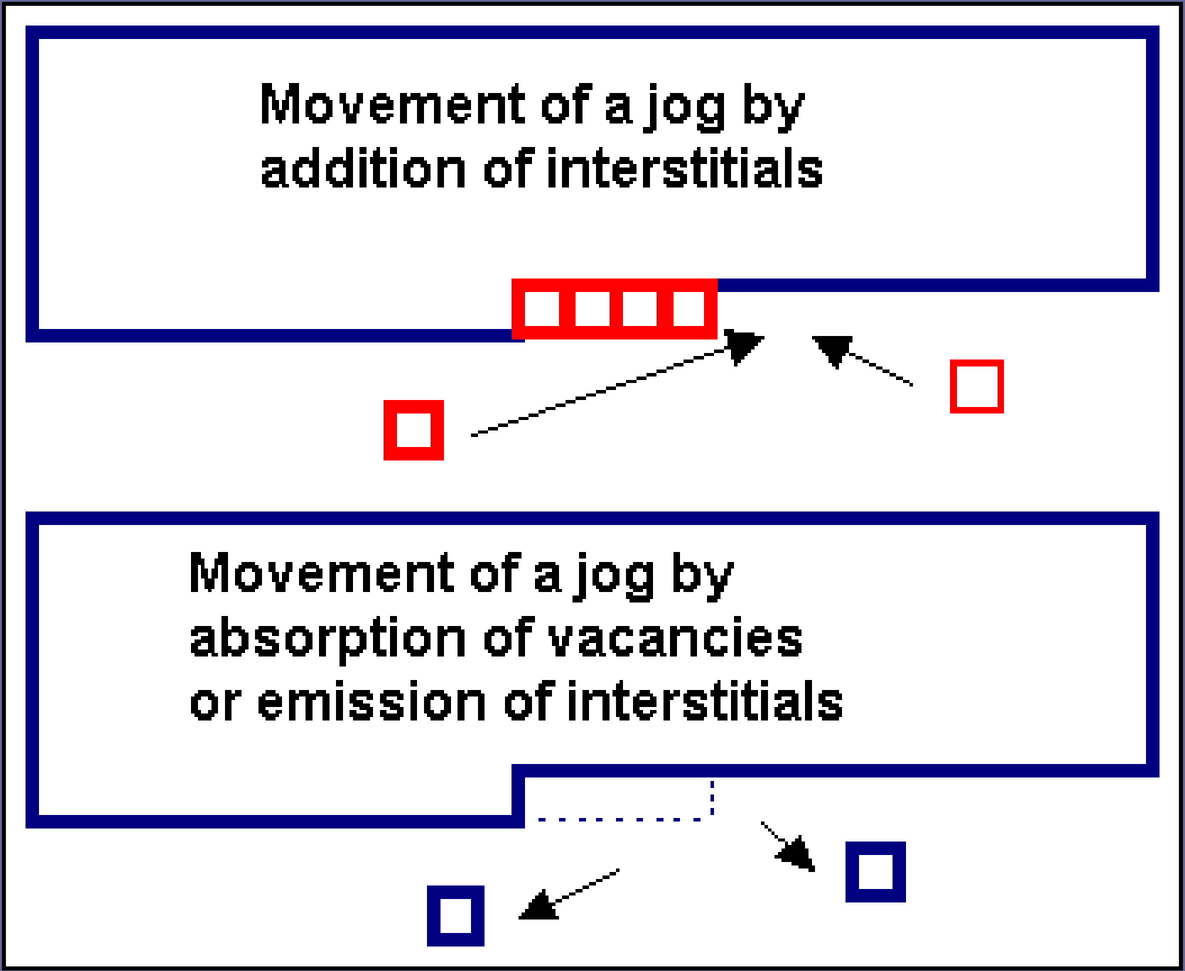
Flow Laws for Steady-State Deformation – Dislocation Creep



http://www.techfak.uni-kiel.de/matwis/amat/def_en/index.html



High-Temperature Deformation – Dislocation Climb



http://www.techfak.uni-kiel.de/matwis/amat/def_en/index.html

Flow Laws for Steady-State Deformation – High-Temperature Dislocation Creep

$$\dot{\varepsilon} = \mathrm{A}\frac{\sigma^n}{d^m} f_{\mathrm{O}_2}^{p} f_{\mathrm{H}_2\mathrm{O}}^{q} \exp\left(-\frac{Q_{\mathrm{cr}}}{\mathrm{R}T}\right)$$

$$\begin{tikzcd}
 & \dot{\varepsilon} = \rho b \bar{\boldsymbol{v}} \arrow[dl] \arrow[dr] & \\
\rho \approx \left(\frac{\sigma}{Gb}\right)^2 \arrow[ddr] & & \bar{\boldsymbol{v}} = \frac{\ell_{\mathrm{g}} + \ell_{\mathrm{c}}}{t_{\mathrm{g}} + t_{\mathrm{c}}} \approx \frac{\ell_{\mathrm{g}}}{\ell_{\mathrm{c}}} \boldsymbol{v}_{\mathrm{c}} \arrow[ddl] \\
 & & \boldsymbol{v}_{\mathrm{c}} = 2\pi \frac{\sigma V_{\mathrm{m}}}{RT} \frac{D}{b} \frac{1}{\ln(R_{\mathrm{o}} / r_{\mathrm{c}})} \\
 & \dot{\varepsilon} = 2\pi \frac{G V_{\mathrm{m}}}{\mathrm{R}T} \left(\frac{\sigma}{\mathrm{G}}\right)^3 \frac{D}{b^2} \frac{1}{\ln(G/\sigma)} \frac{\ell_{\mathrm{g}}}{\ell_{\mathrm{c}}} &
\end{tikzcd}$$

Origin of Dependence of Visicosity on Fugacity

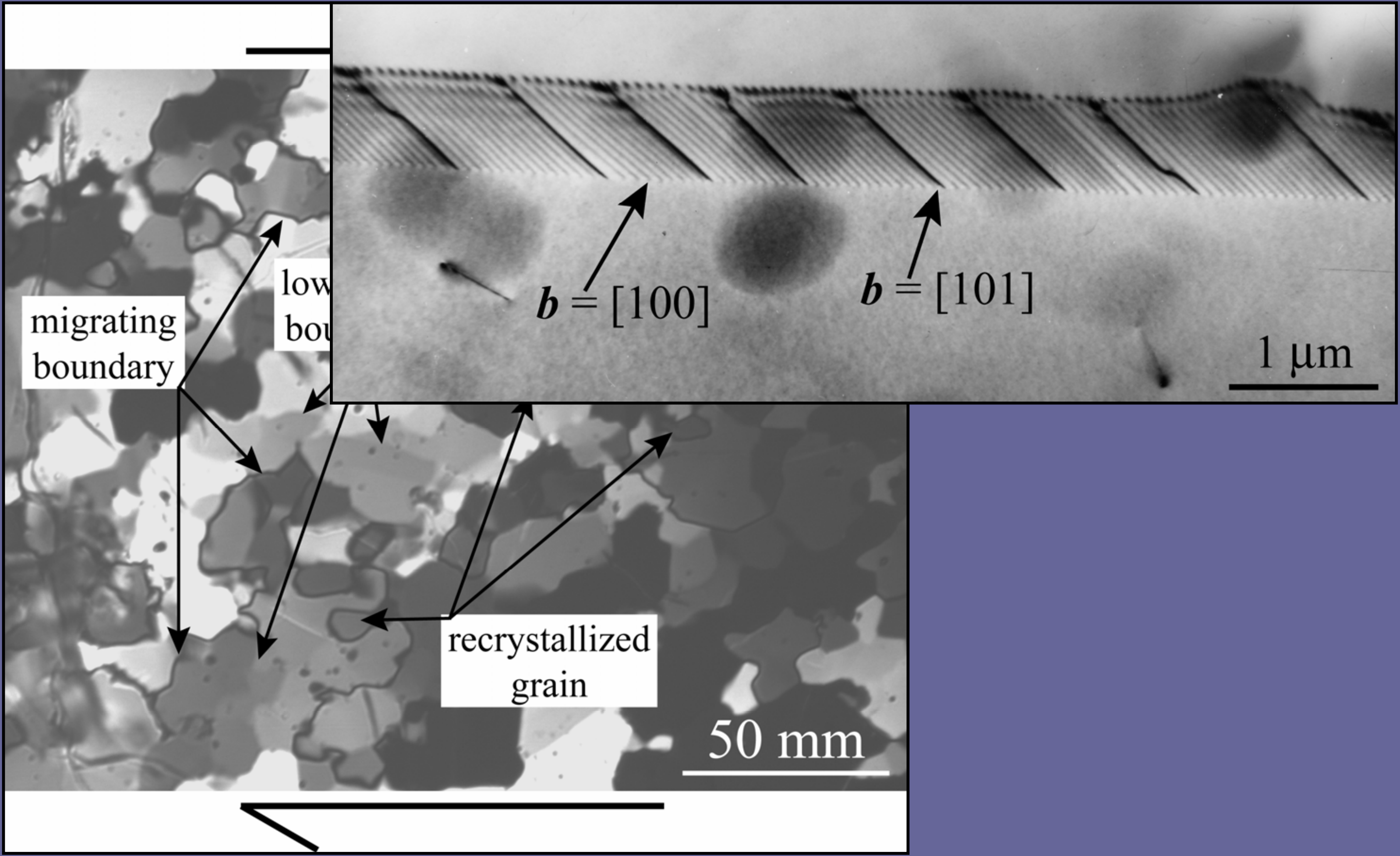
$$X_{\mathrm{ion}} D_{\mathrm{ion}} = X_{\mathrm{V}} D_{\mathrm{V}}$$

$$X_{\mathrm{ion}} = \left(1 - X_{\mathrm{V}}\right) \approx 1$$

$$D_{\mathrm{ion}} \approx X_{\mathrm{V}} D_{\mathrm{V}}$$

$$D_{\mathrm{ion}} \ll D_{\mathrm{V}} \qquad D_{\mathrm{ion}} \propto X_{\mathrm{V}} \propto f_{\mathrm{O_2}}^{p} f_{\mathrm{H_2O}}^{q}$$

Dislocation-Accommodated Grain Boundary Sliding

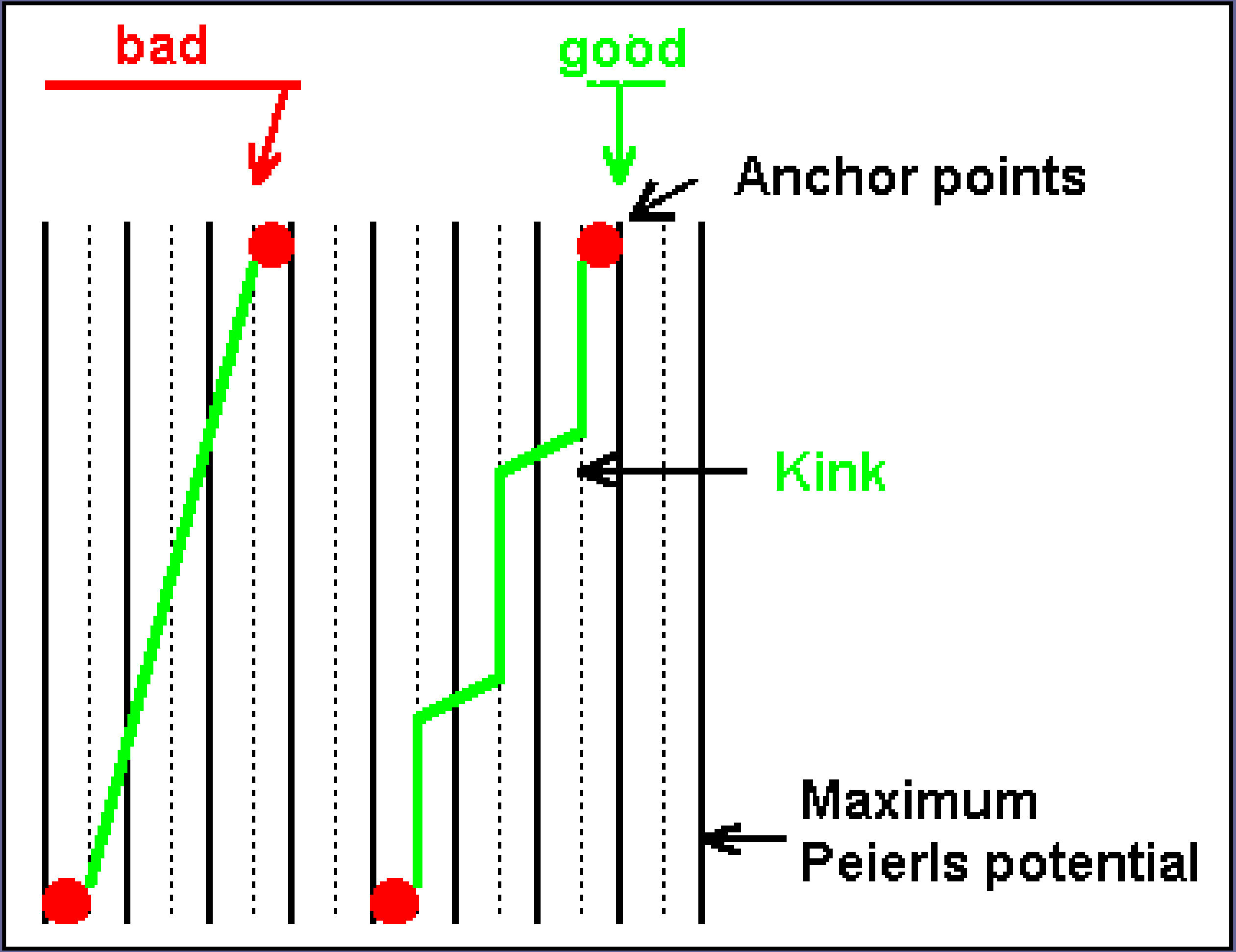


Flow Laws for Steady-State Deformation – Dislocation-Accommodated Grain Boundary Sliding

$$\dot{\varepsilon} = \mathrm{B}_{\mathrm{gbs}} \frac{D_{\mathrm{gm}}}{d^1} \frac{GV_{\mathrm{m}}}{\mathrm{R}T} \left(\frac{\sigma}{G} \right)^3 \qquad d_{sgs} < d$$

$$\dot{\varepsilon} = \mathrm{A}_{\mathrm{gbs}} \frac{D_{\mathrm{gb}}}{d^2} \frac{GV_{\mathrm{m}}}{\mathrm{R}T} \left(\frac{\sigma}{G} \right)^2 \qquad d_{sgs} > d$$

Low-Temperature Deformation – Dislocation Glide



http://www.techfak.uni-kiel.de/matwis/amat/def_en/index.html

Flow Laws for Steady-State Deformation – Low-Temperature Dislocation Creep

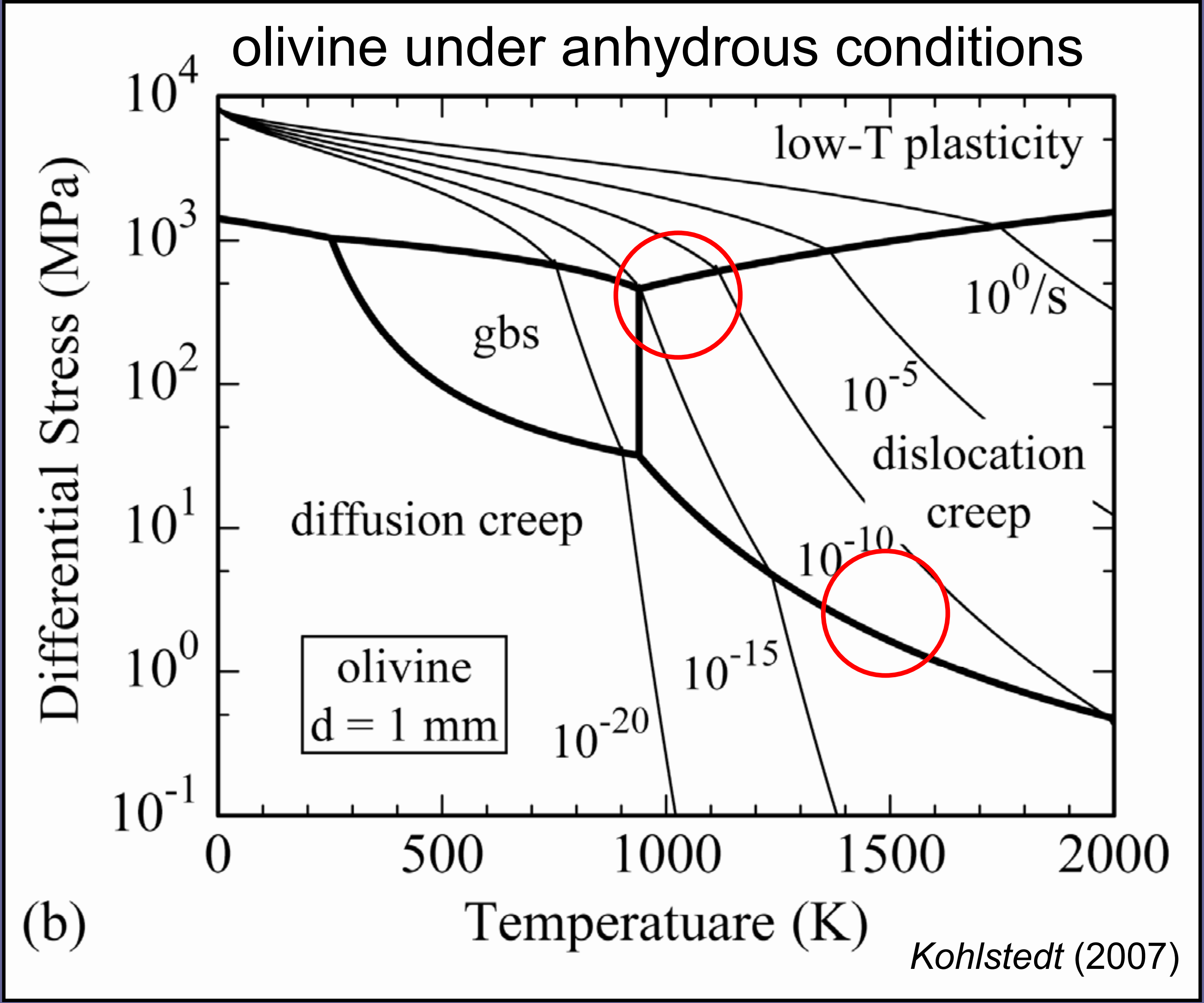
$$\bar{\boldsymbol{v}} \approx \boldsymbol{v}_{\mathrm{g}} = c_{\mathrm{k}} \boldsymbol{v}_{\mathrm{k}}$$

$$\Delta H_k(\sigma) = \Delta H_k^{\mathrm{o}} \left[1 - \left(\frac{\sigma}{\sigma_{\mathrm{P}}} \right)^r \right]^s$$

$$\dot{\varepsilon} = \dot{\varepsilon}_{\mathrm{P}} \left(\frac{\sigma}{G} \right)^2 \exp - \left\{ \frac{\Delta H_k^{\mathrm{o}}}{\mathrm{R}T} \left[1 - \left(\frac{\sigma}{\sigma_{\mathrm{P}}} \right)^r \right]^s \right\}$$

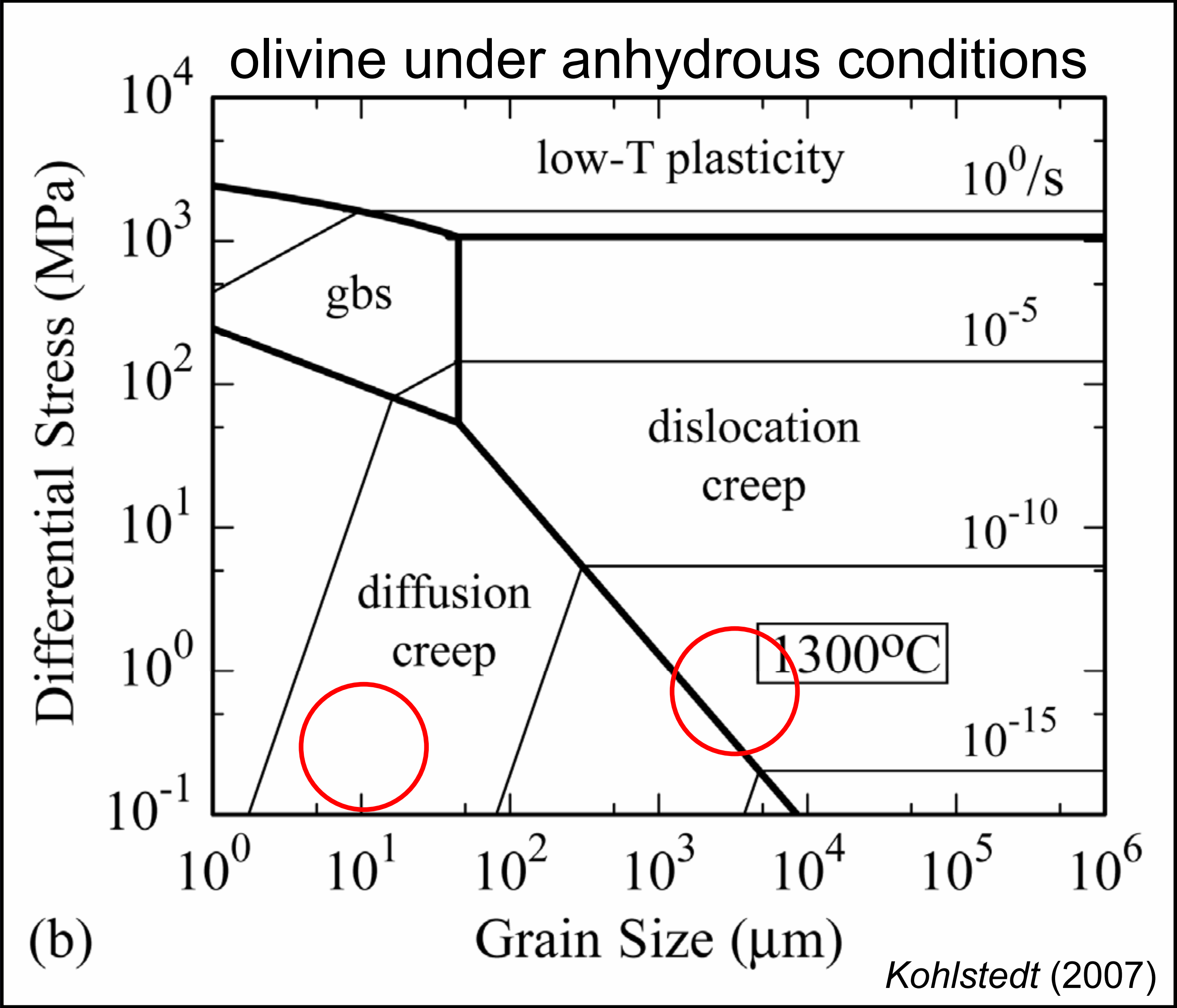
σ_{P} = Peierls stress, intrinsic lattice resistance to glide

Deformation Mechanism Map – σ vs *T*



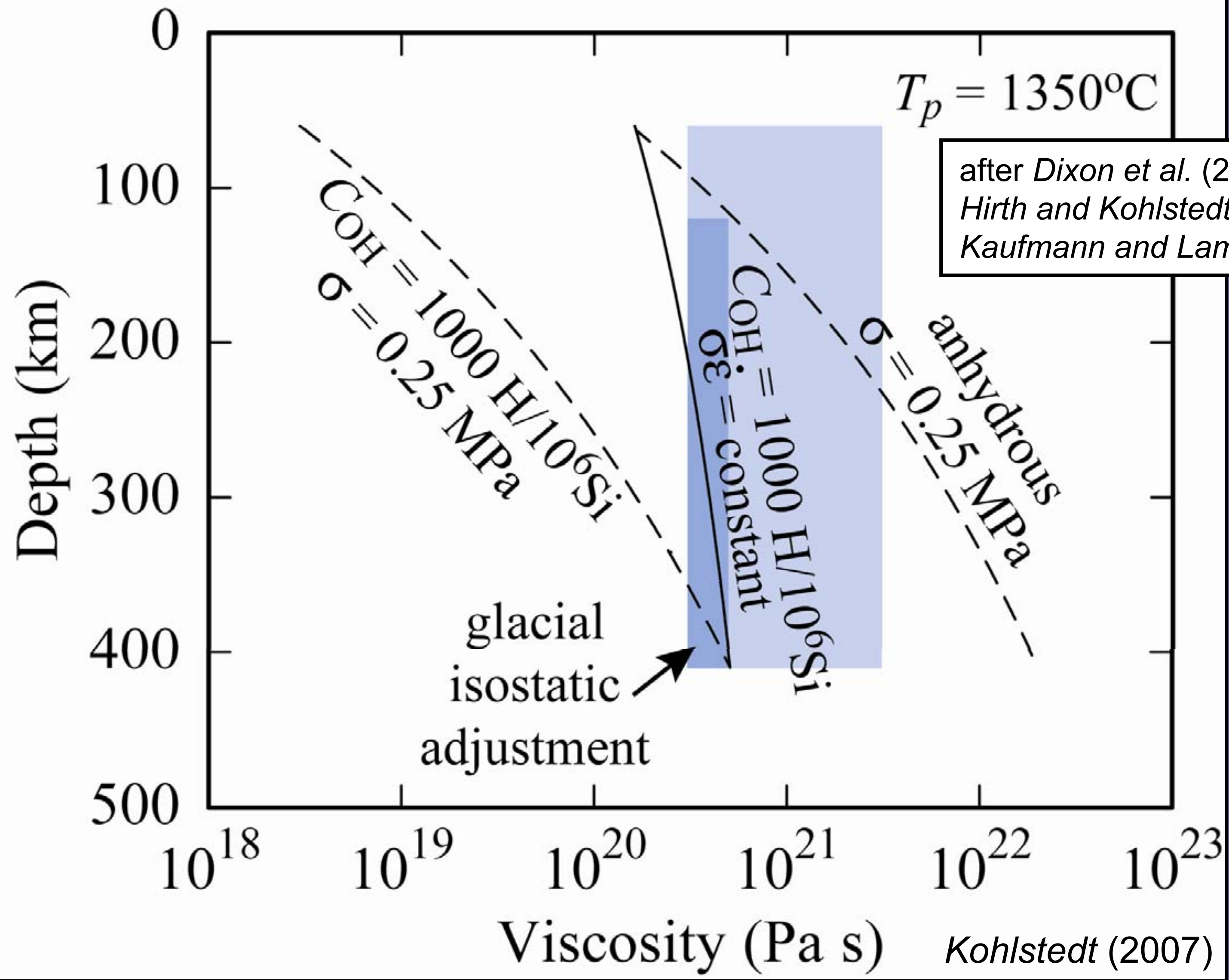
Kohlstedt (2007)

Deformation Mechanism Map – σ vs *d*



Kohlstedt (2007)

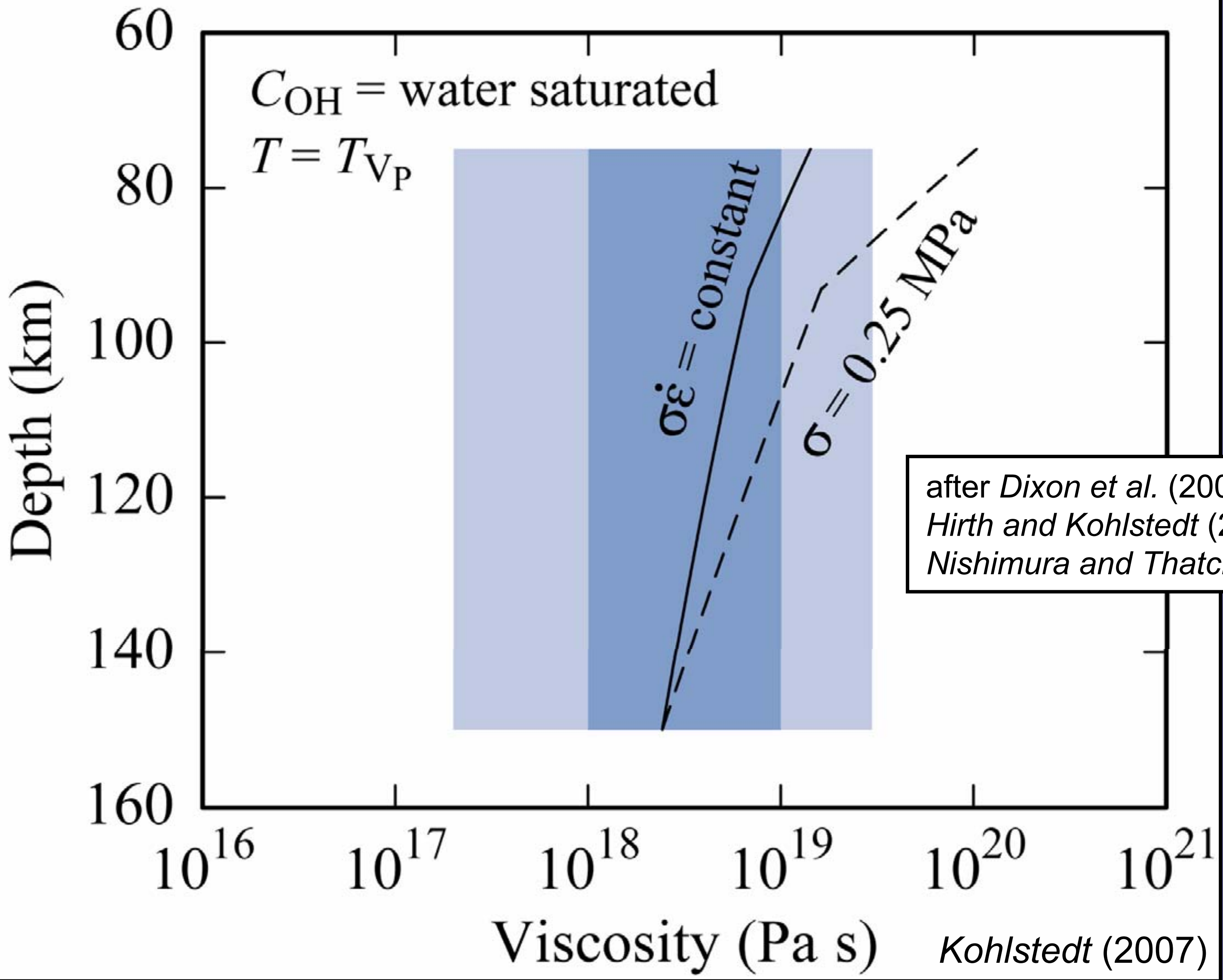
Viscosity Profiles vs Glacial Isostatic Adjustment
Global Average



after *Dixon et al.* (2004)
Hirth and Kohlstedt (2003)
Kaufmann and Lambeck (2002)

Kohlstedt (2007)

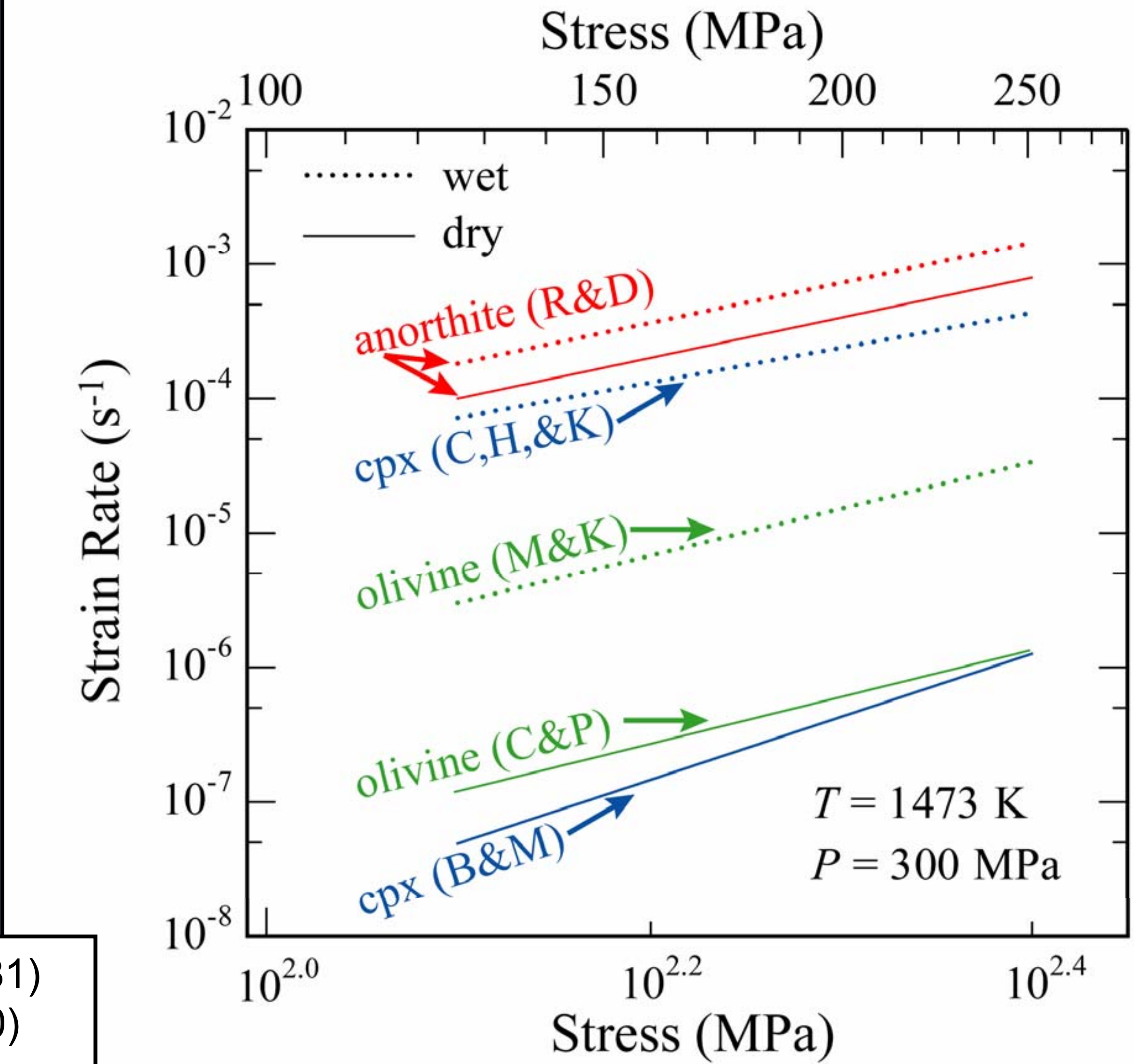
Viscosity Profiles vs Glacial Isostatic Adjustment Western U.S.



after *Dixon et al.* (2004)
Hirth and Kohlstedt (2003)
Nishimura and Thatcher (2003)

Kohlstedt (2007)

Comparison of Flow Behavior of Several Single-Phase Rocks Deformed Under Wet and Dry Conditions



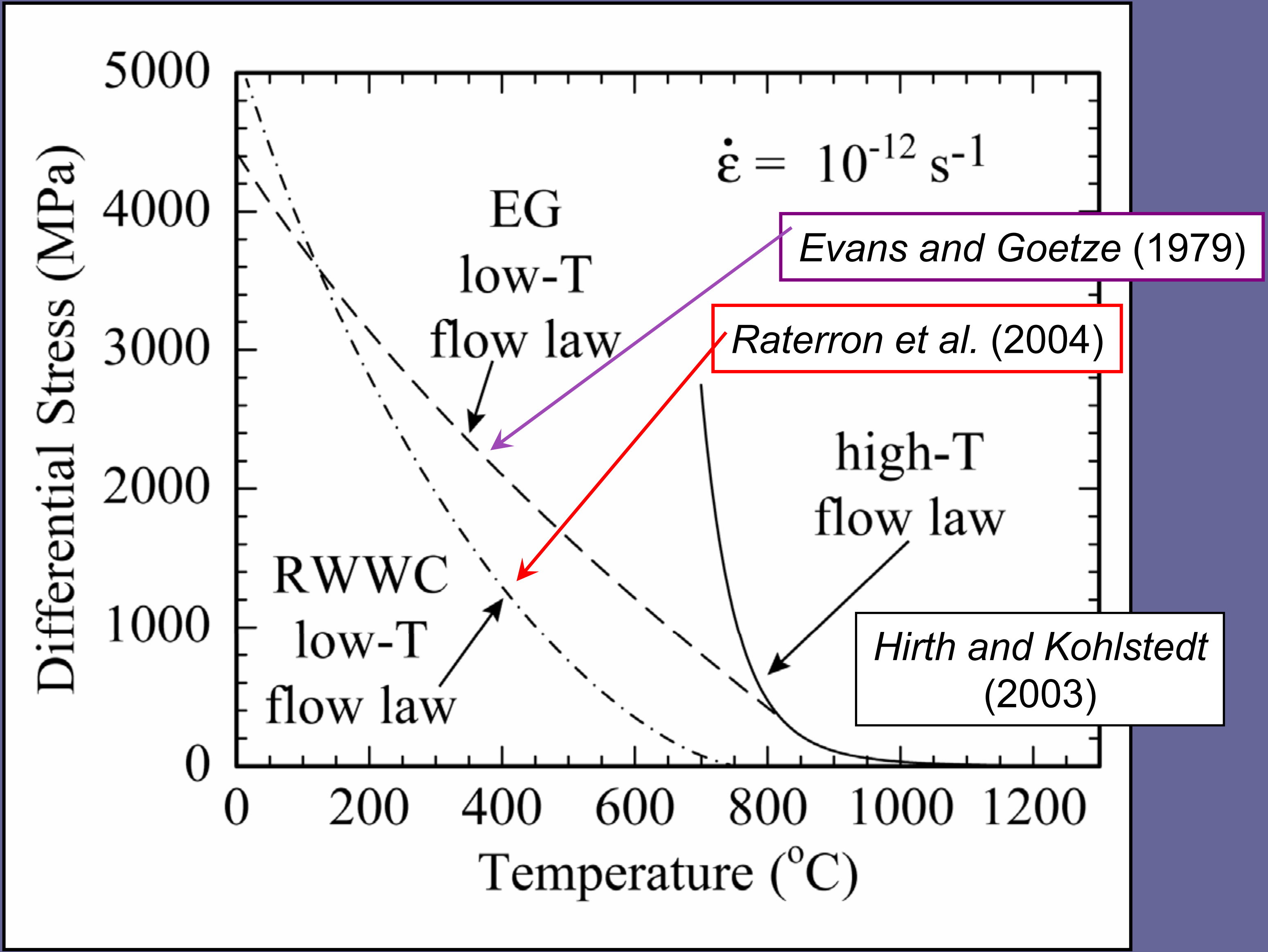
Kohlstedt (2007)

Chopra and Paterson (1981)
Rybacki and Dresen (2000)
Mei and Kohlstedt (2000)
Bystricky and Mackwell (2001)
Chen et al. (2006)

Why the Emphasis on Single-Phase Rather than Multi-Phase Rocks?

eutectic melting

Low-Temperature Plasticity

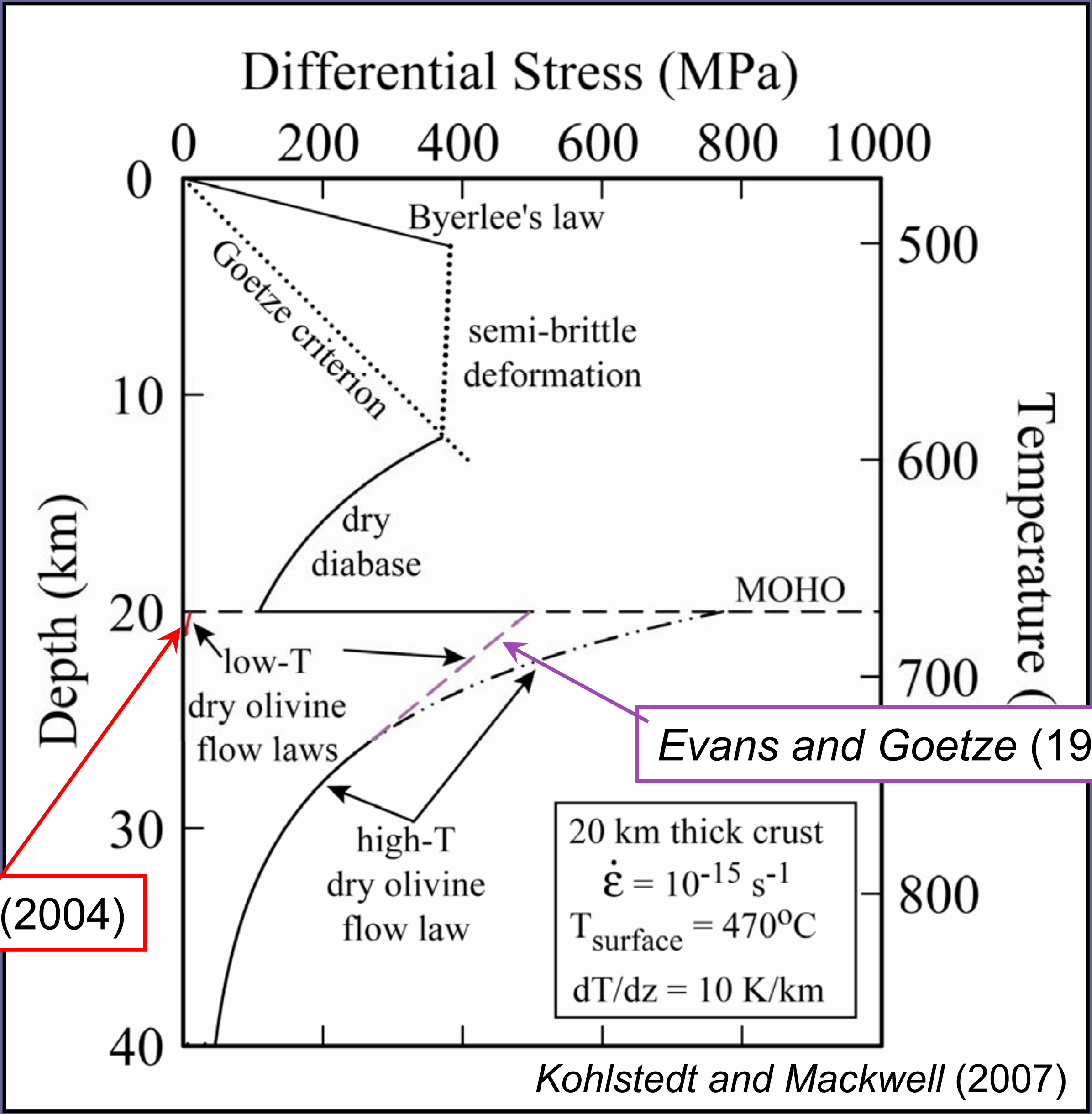


Evans and Goetze (1979)

Raterron et al. (2004)

Hirth and Kohlstedt (2003)

Low-Temperature Plasticity

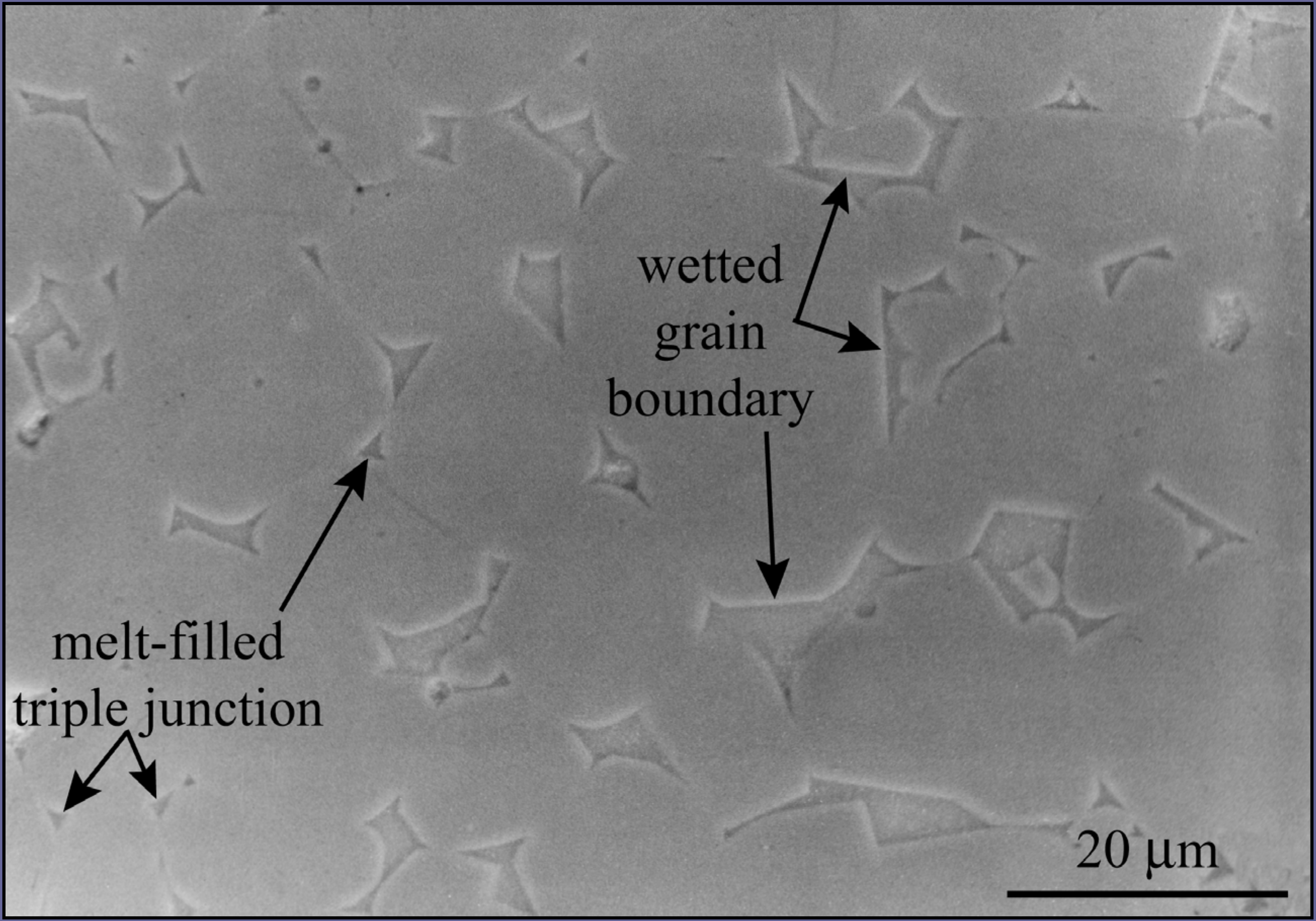


Raterron et al. (2004)

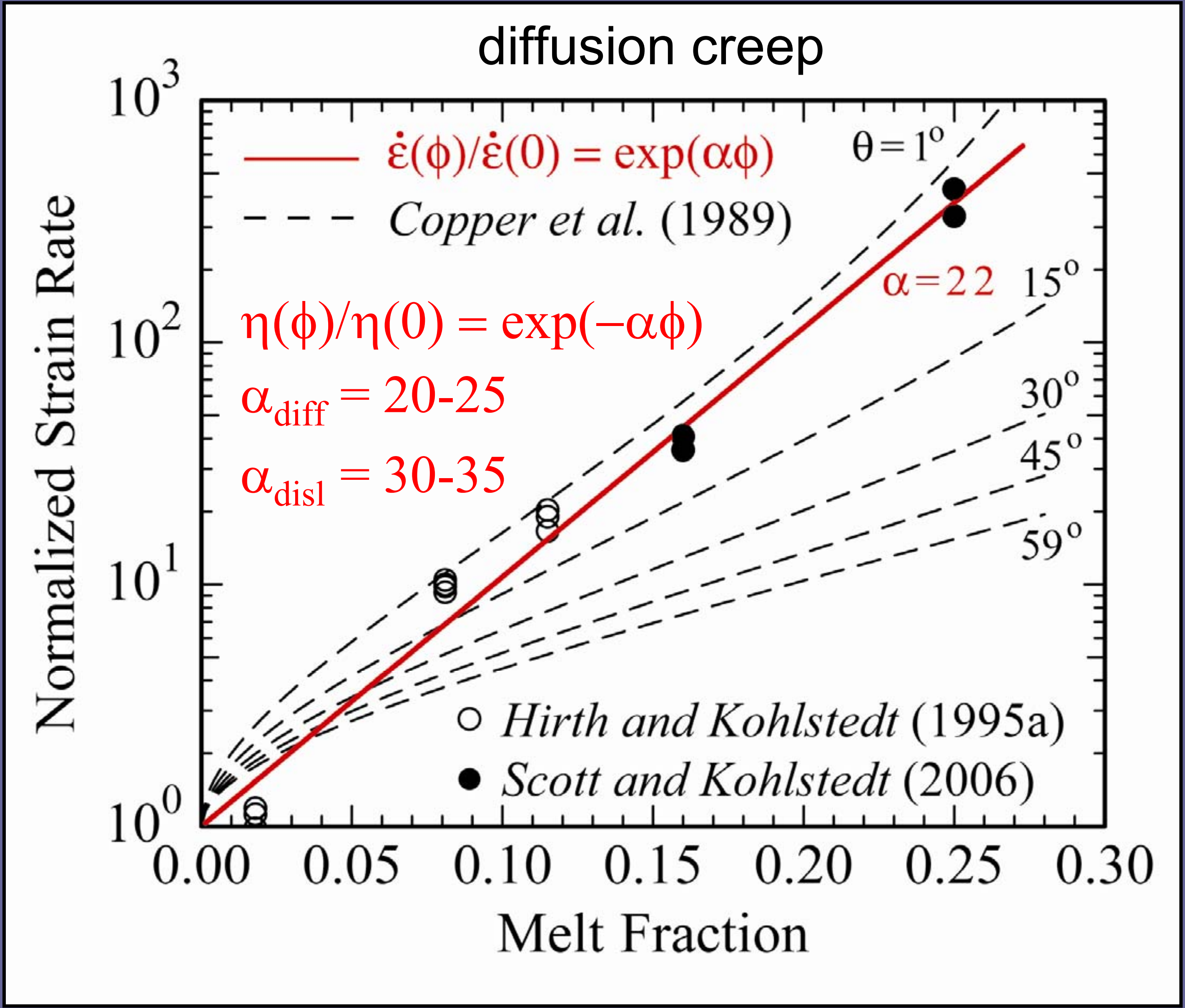
Evans and Goetze (1979)

Kohlstedt and Mackwell (2007)

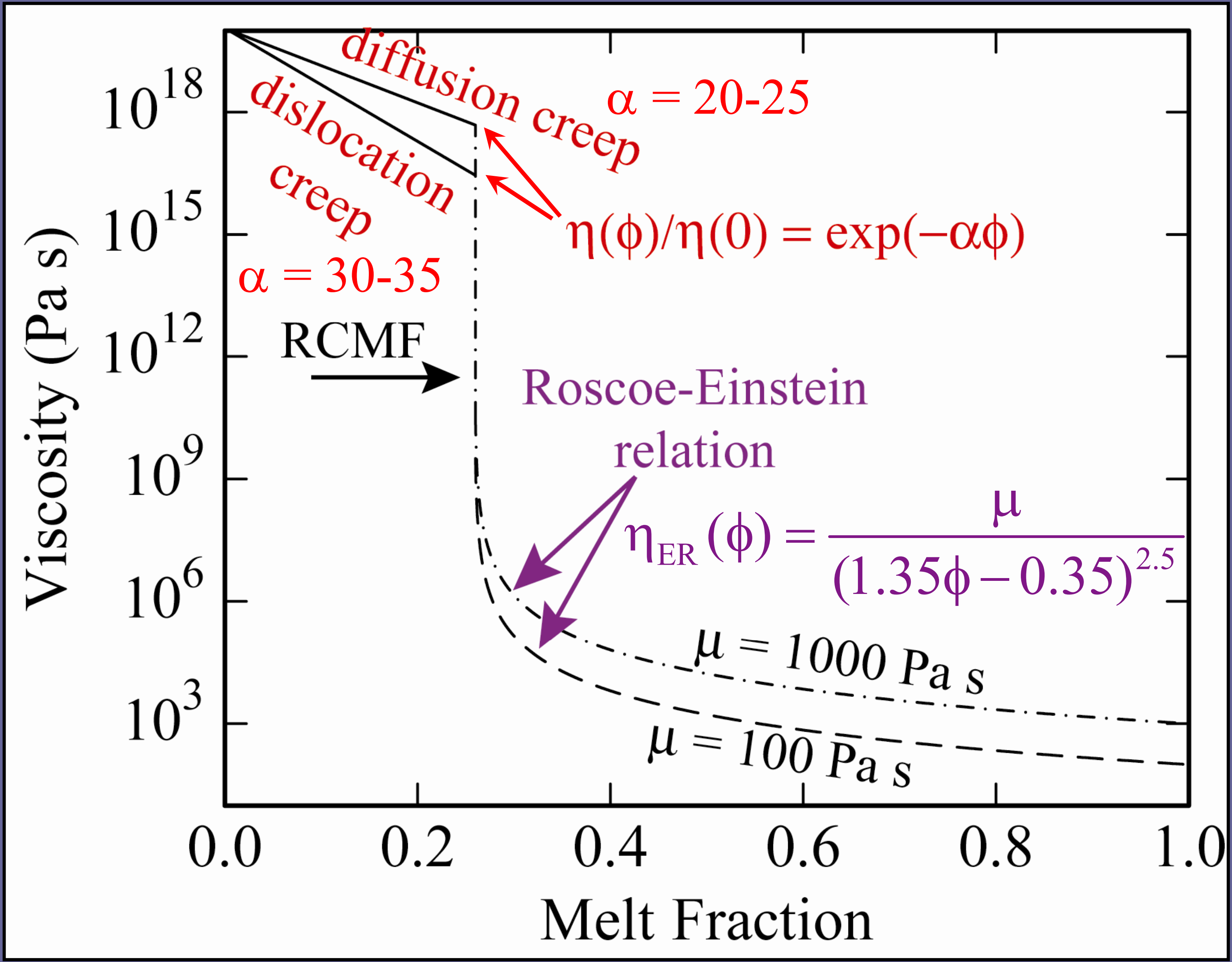
Melt Distribution in Partially Molten Rock



Flow Behavior of Partially Molten Rock

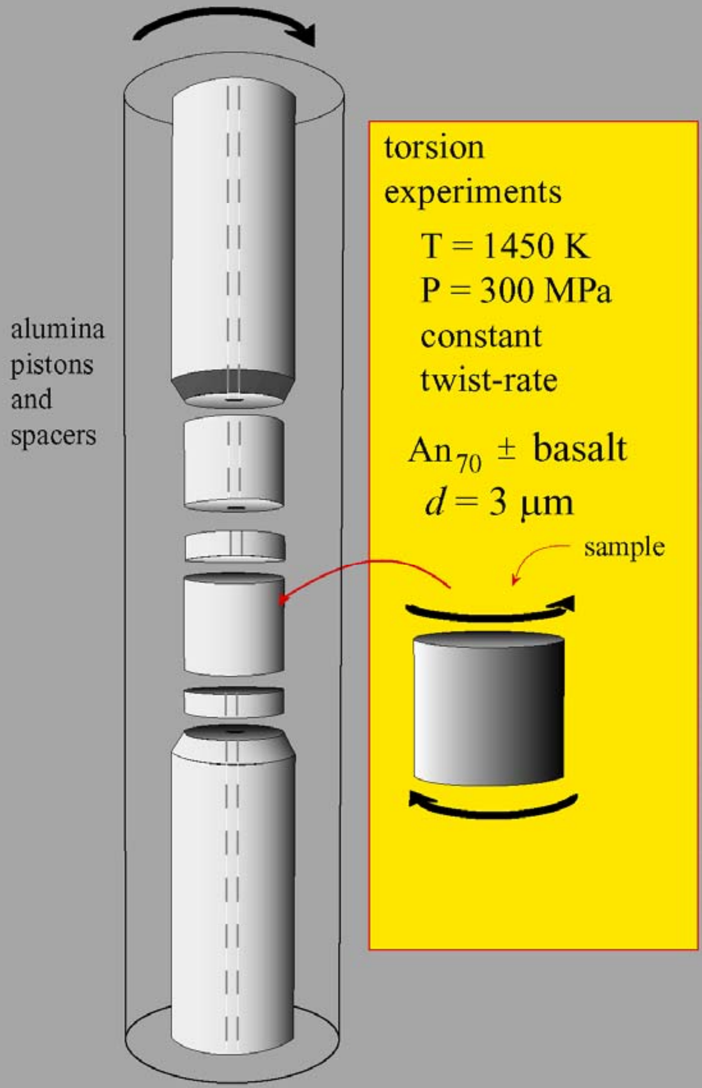


Flow Behavior of Partially Molten Rock



High-Strain Torsion Experiments

constant twist-rate or torque
applied from above



alumina
pistons
and
spacers

torsion
experiments

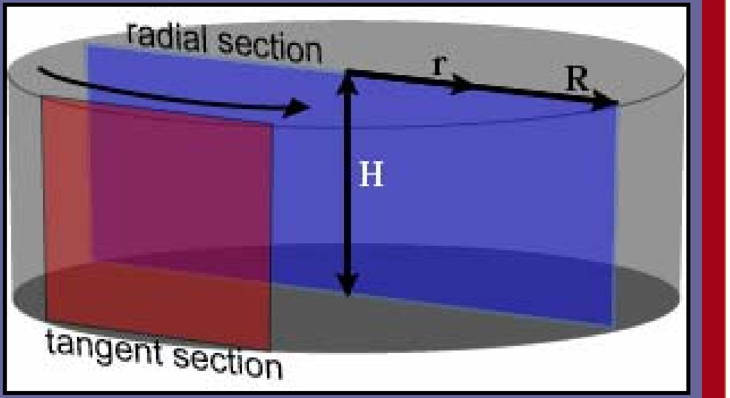
T = 1450 K
P = 300 MPa
constant
twist-rate

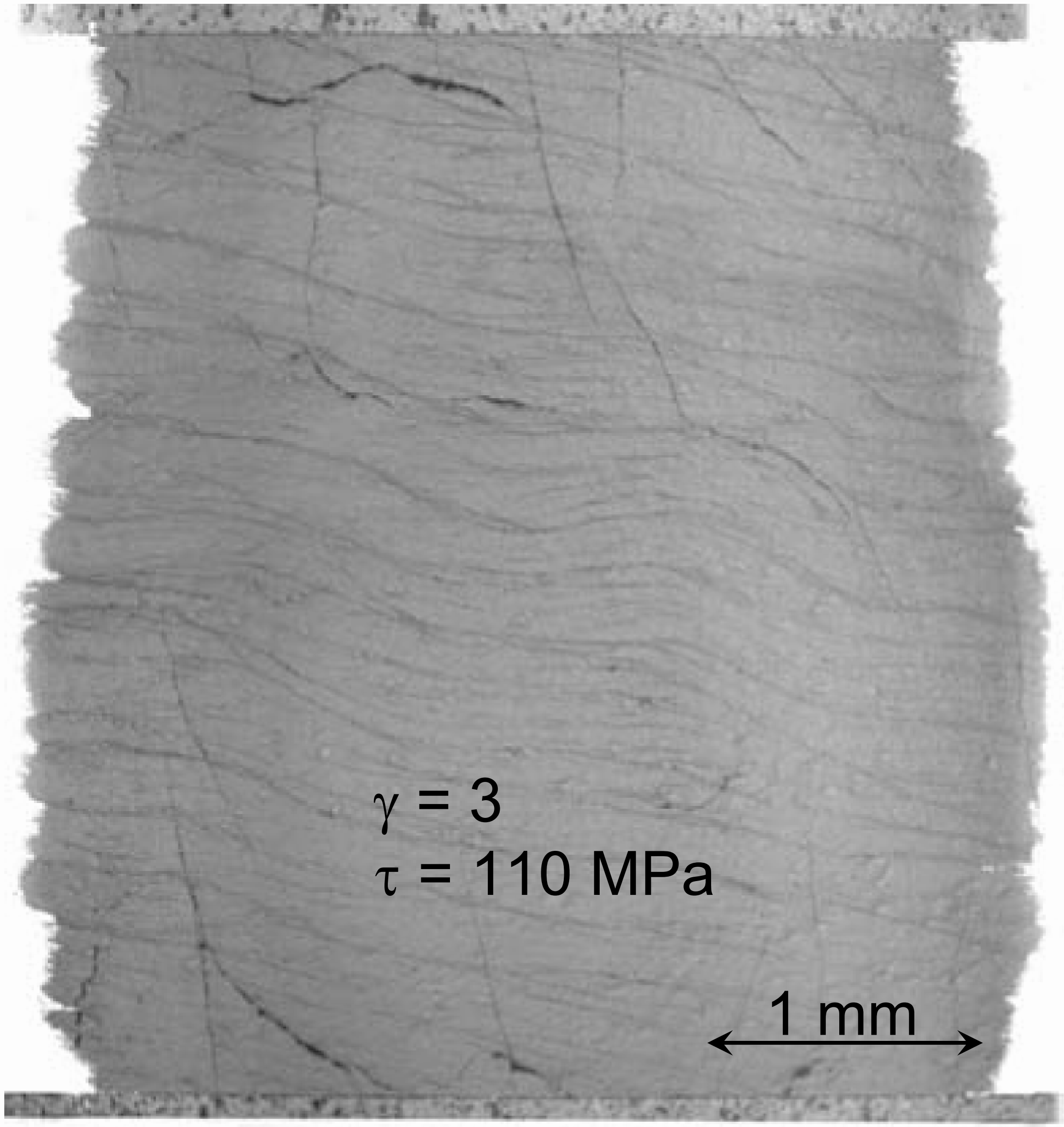
An_{70} ± basalt
$d = 3\ \mu m$

sample

gas-
medium
pressure
vessel

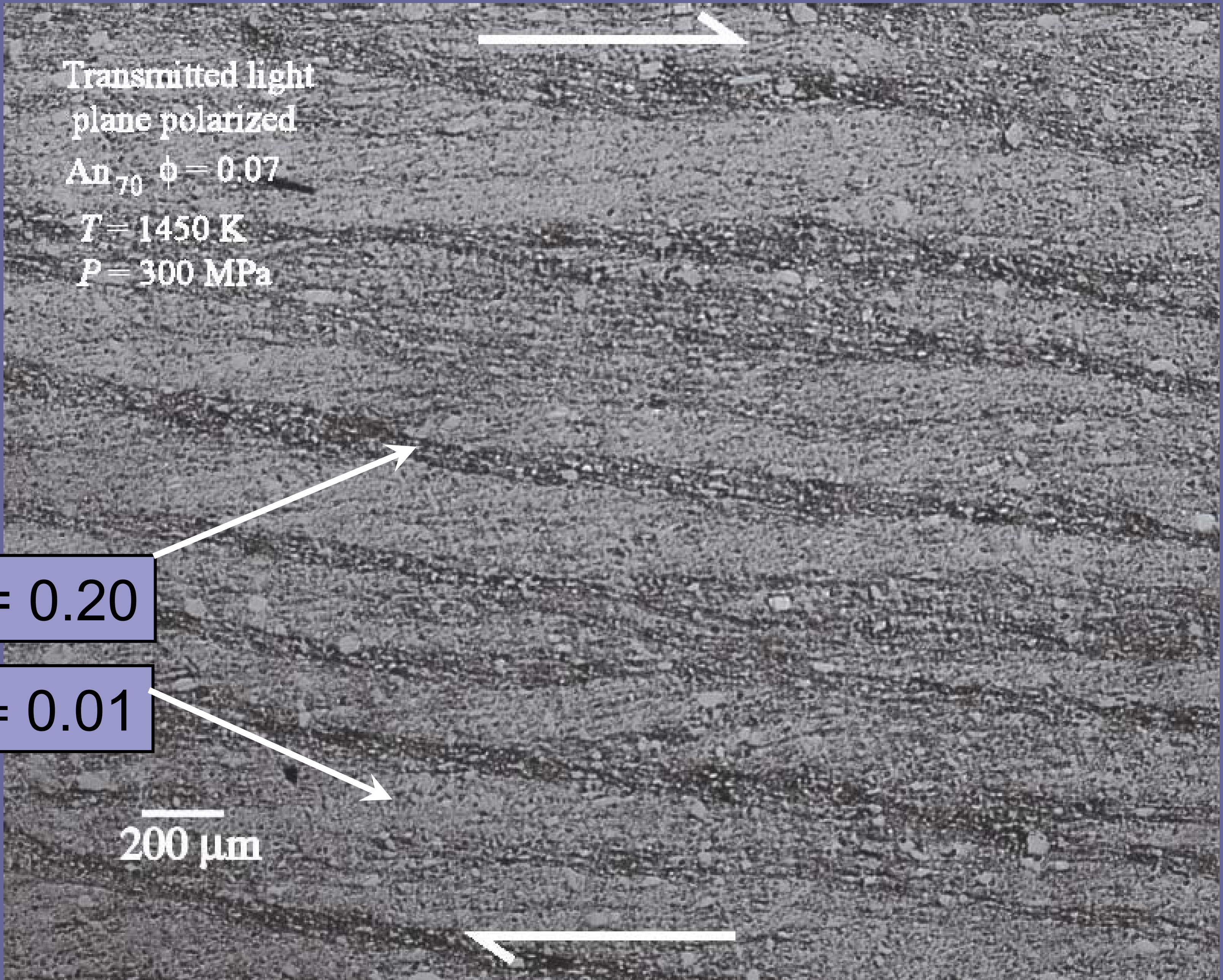
Torsional Deformation of Partially Molten Rock





tangential section

Melt Segregation in Shear Partially Molten Rock



$\phi = 0.20$

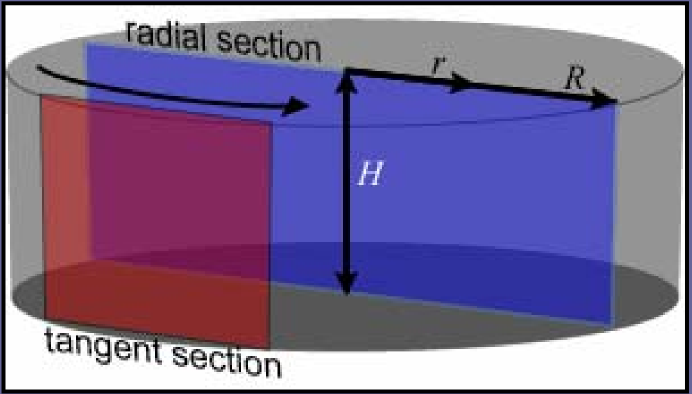
$\phi = 0.01$

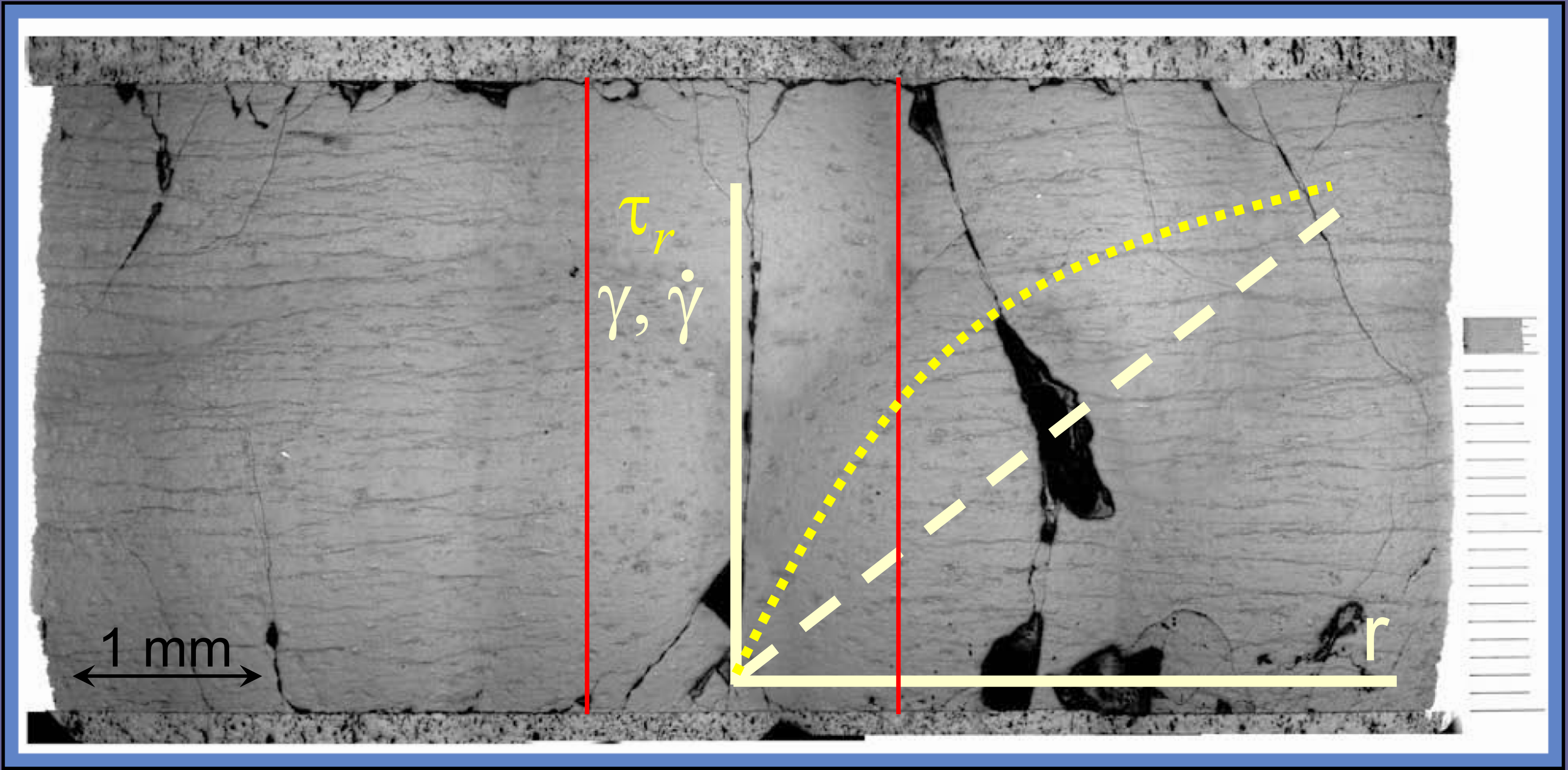
Torsional Deformation of Partially Molten Rock

$\gamma_r = \theta r/H$

$\tau_r \propto r^{1/n}$

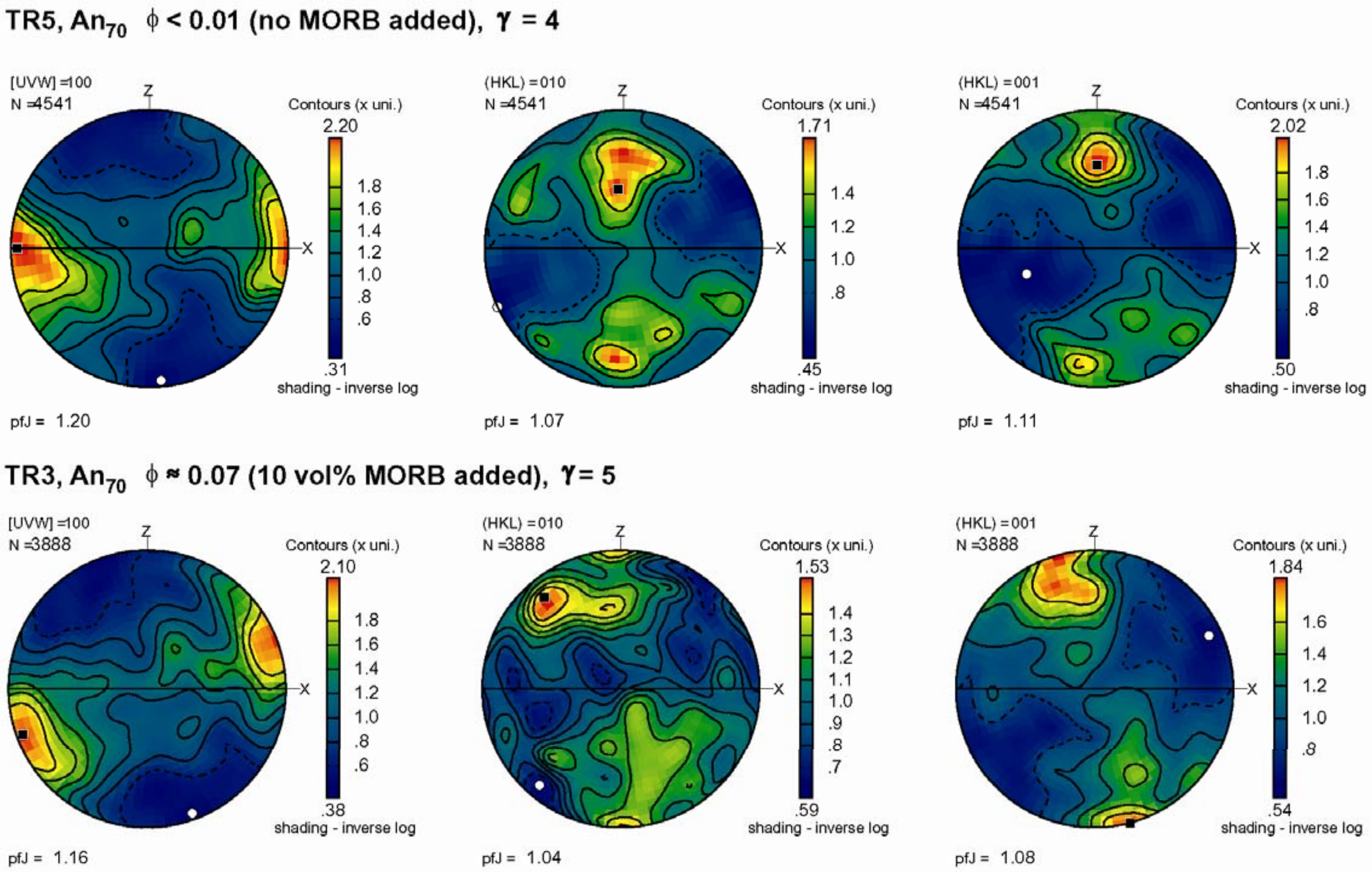
$\dot{\gamma} \propto \tau^n$





radial section

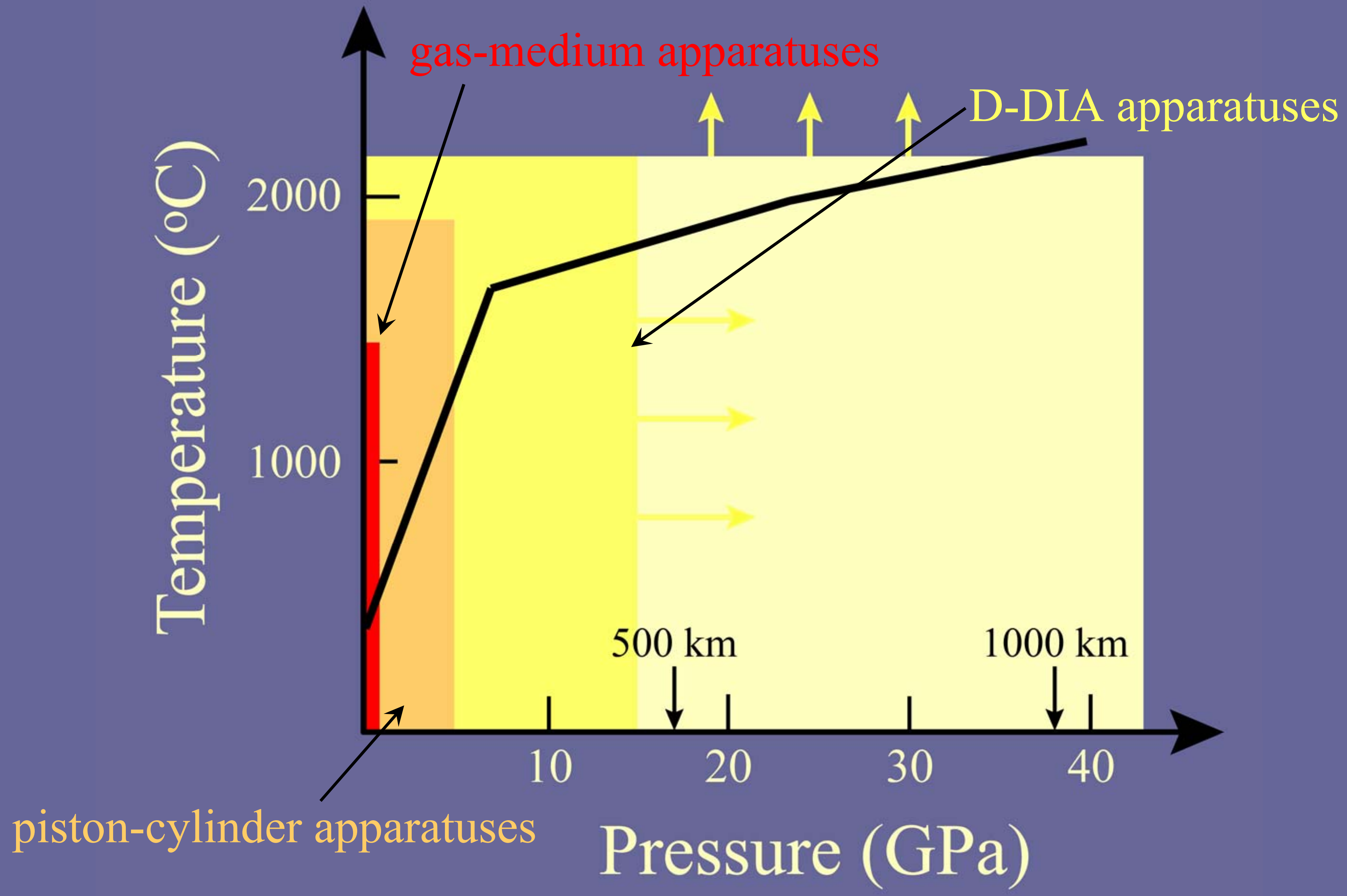
LPO from Shear Partially Molten Rock



TR5, An_{70} $\phi < 0.01$ (no MORB added), $\gamma = 4$

TR3, An_{70} $\phi \approx 0.07$ (10 vol% MORB added), $\gamma = 5$

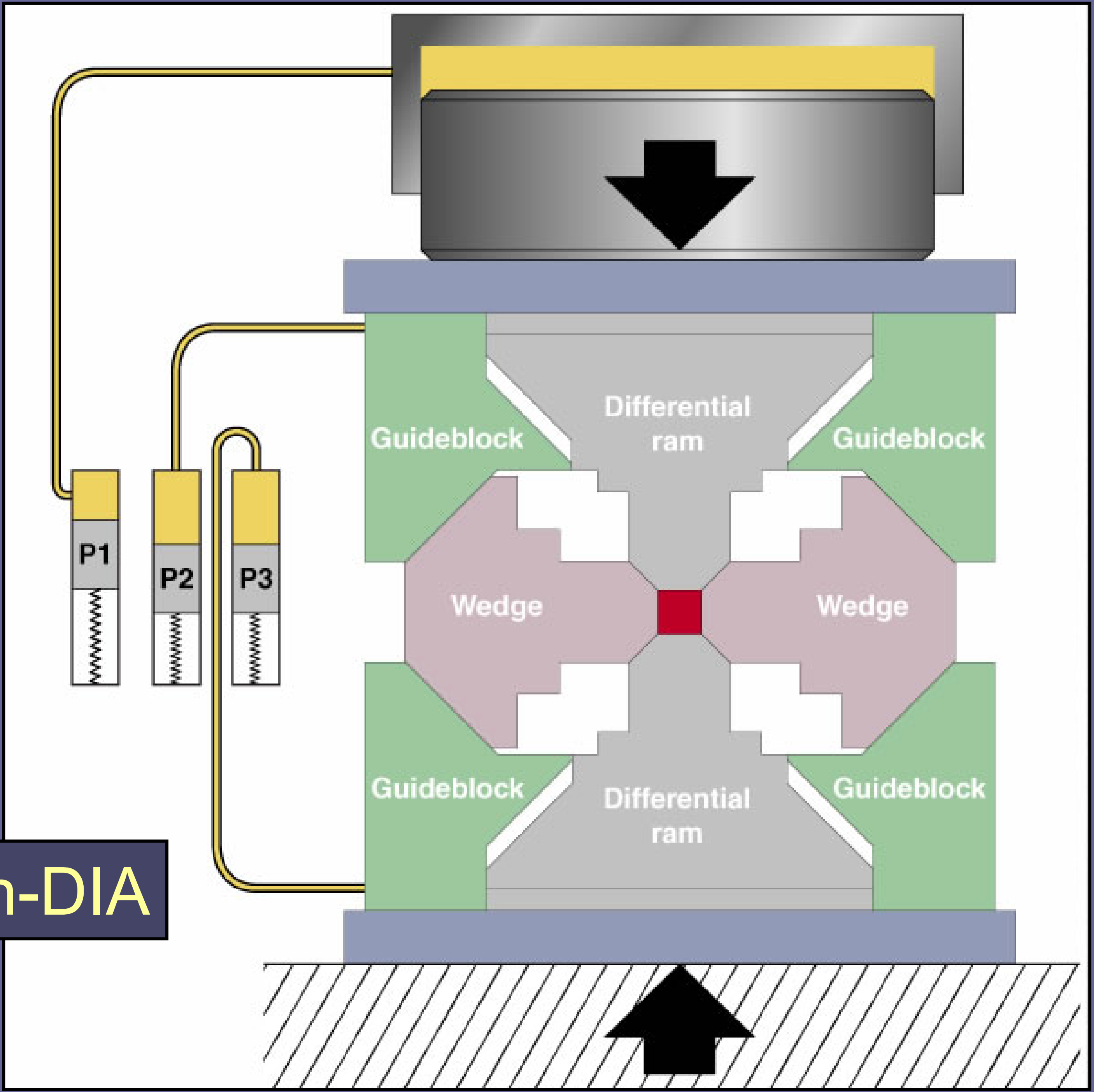
High-Pressure, High-Temperature Apparatuses



gas-medium apparatuses

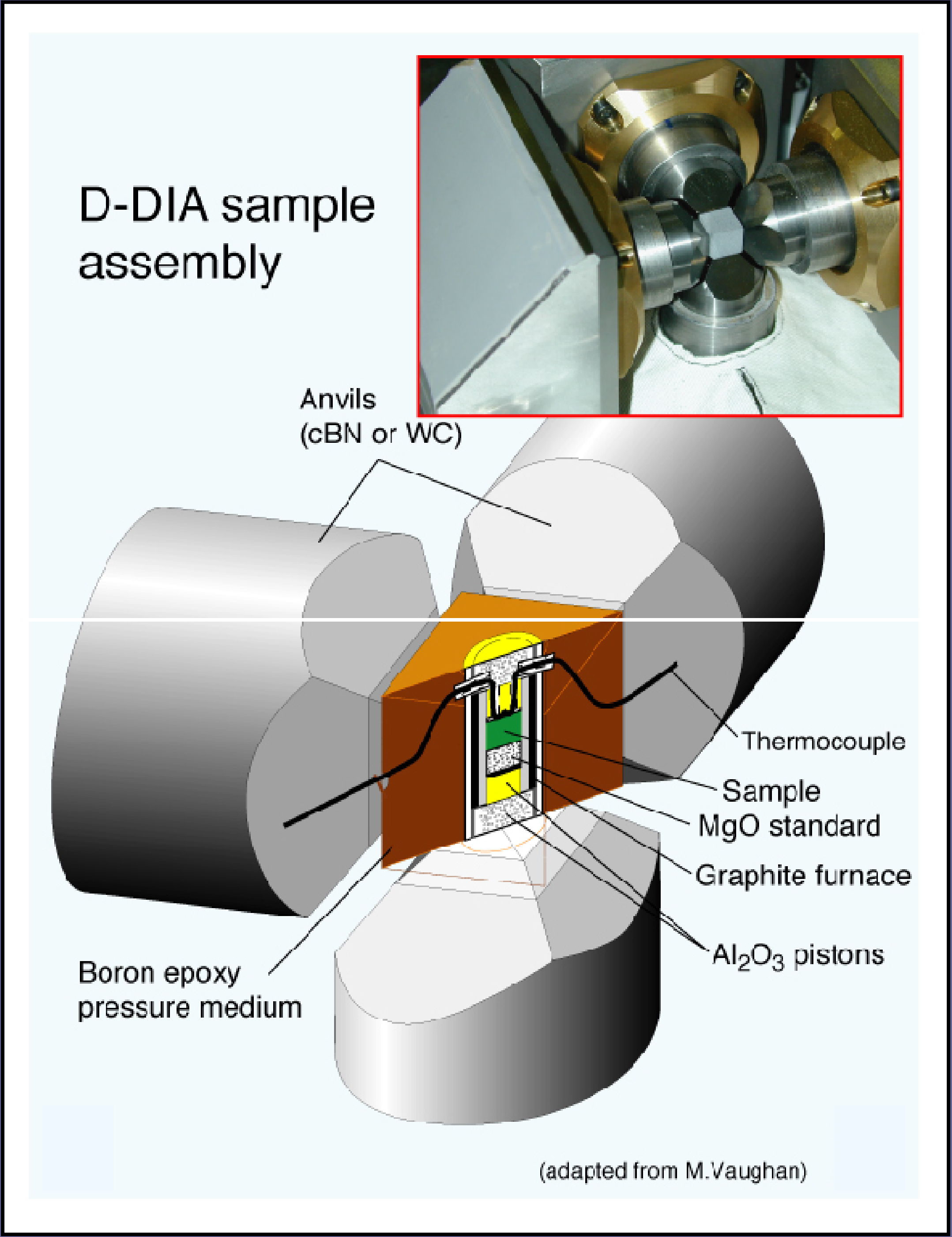
D-DIA apparatuses

piston-cylinder apparatuses



Deformation-DIA

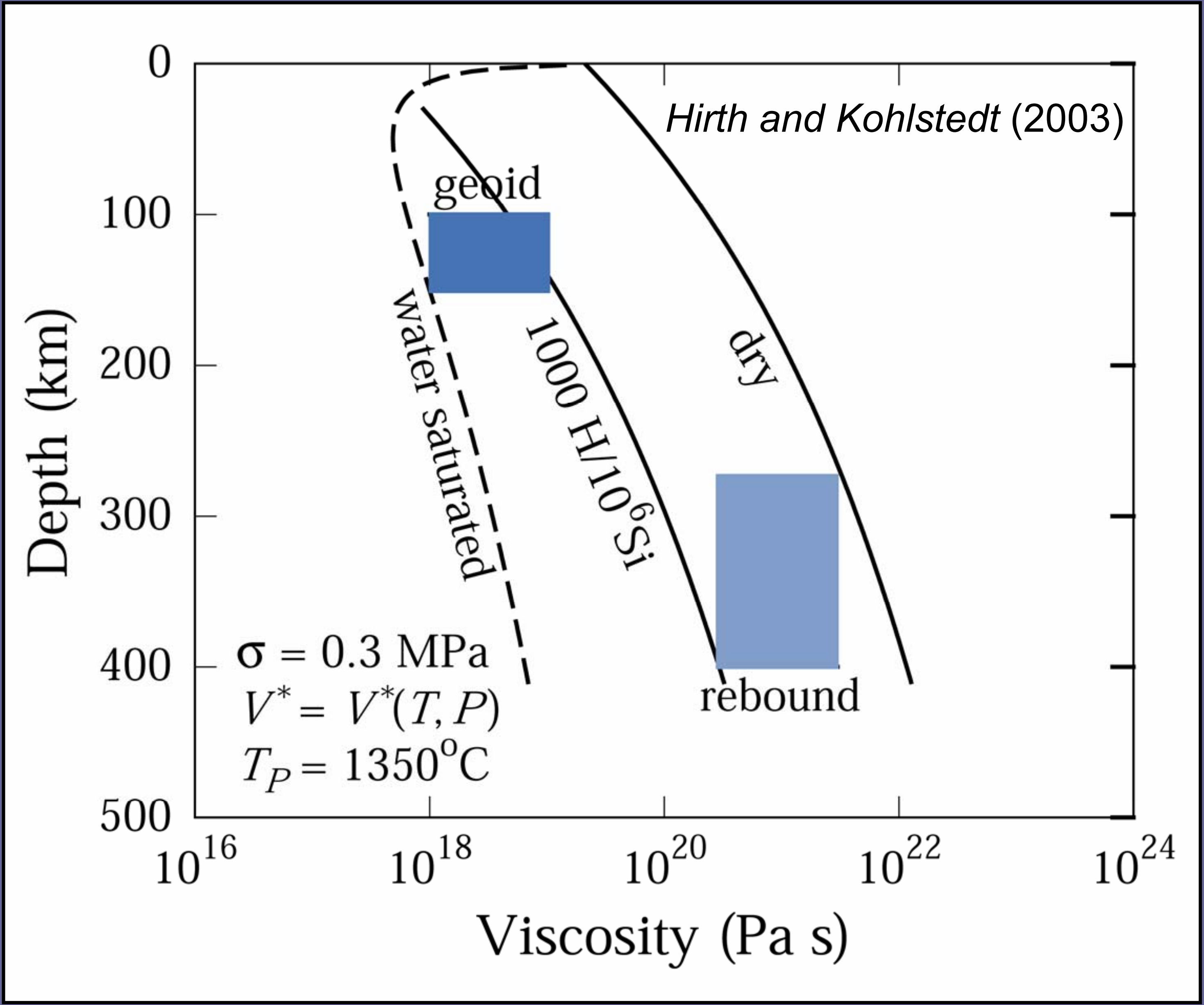
Deformation-DIA Sample Assembly



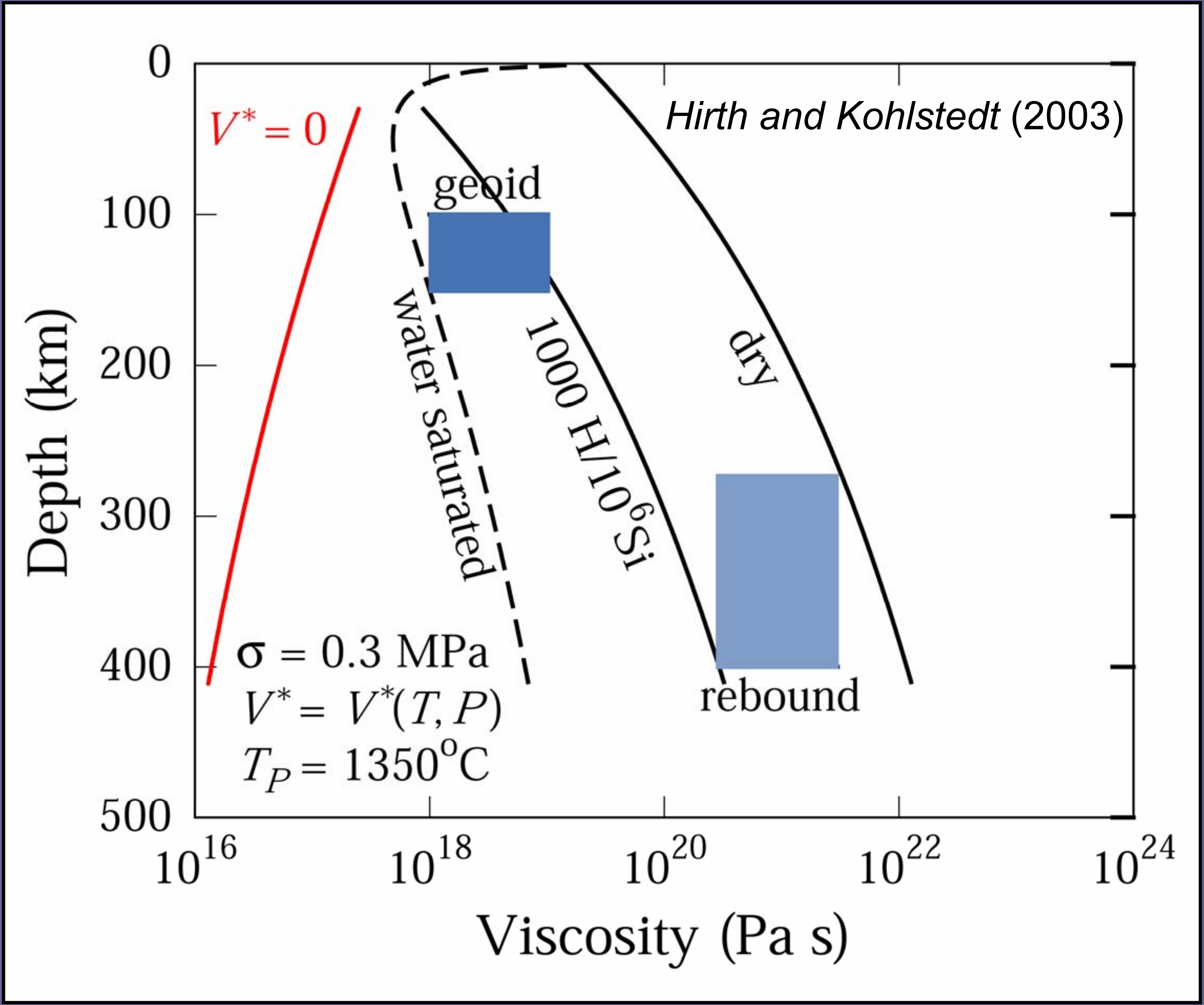
(adapted from M.Vaughan)

samples 1 mm in diameter

Viscosity Profiles vs Geoid and Rebound



Viscosity Profiles vs Geoid and Rebound



Constitutive Equations: Non-Steady State

$$\dot{\varepsilon} = \dot{\varepsilon}\left(\sigma, T, P, f_{\mathrm{H_2O}}, a_{\mathrm{ox}}, \ldots, S, d, \ldots, \Phi, \phi, \ldots\right)$$

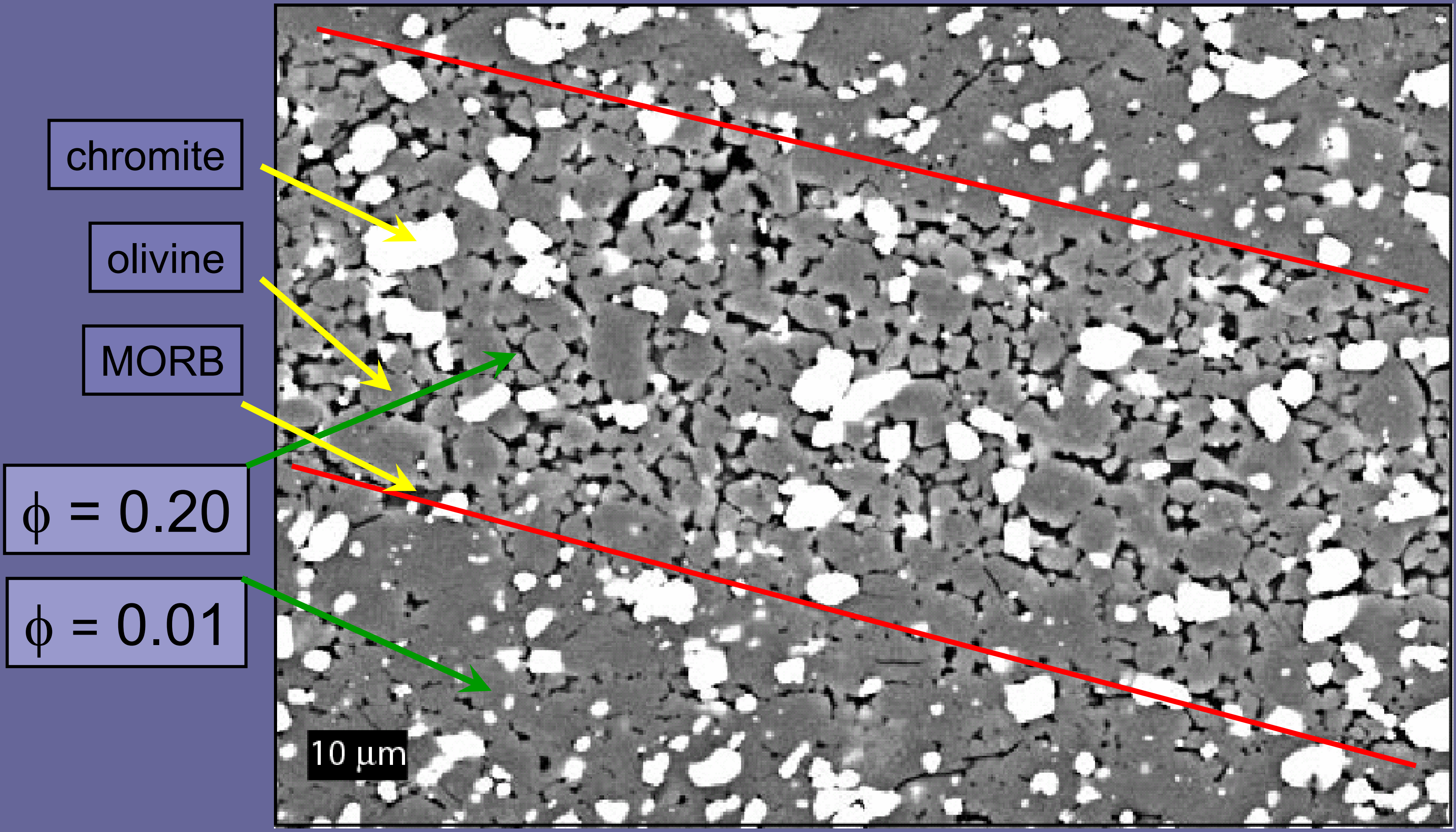
$$\cancel{\dot{\varepsilon} = \dot{\varepsilon}\left(\sigma, T, P, f_{\mathrm{H_2O}}, a_{\mathrm{ox}}, \ldots, S, d, \ldots, \Phi, \phi, \ldots, \varepsilon, t\right)}$$

$$\dot{\varepsilon} = \dot{\varepsilon}\left(\sigma, T, P, f_{\mathrm{H_2O}}, a_{\mathrm{ox}}, \ldots, S, d, \ldots, \Phi, \phi, \ldots, \sigma^*, d\sigma^*/dt\right)$$

σ^* = hardness parameter, *Hart* (1970)
measure of resistance of grains to dislocation movement
possibly correlates with sub-grain size, *Stone et al.* (2004)

Detailed View of a Melt-Rich Band

olivine + 25 vol % chromite + 6% MORB, $\gamma = 3$, $\sigma = 100$ MPa



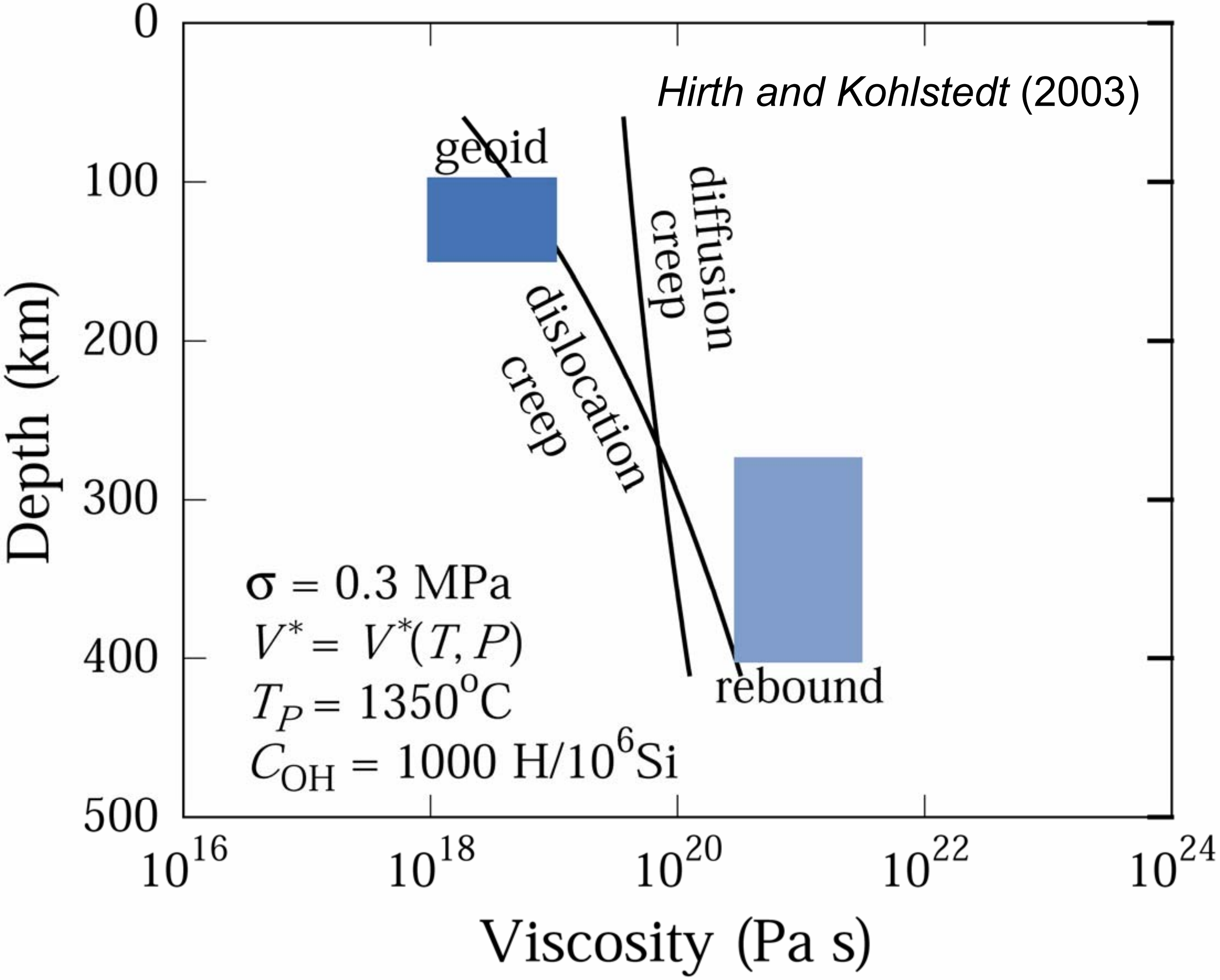
chromite

olivine

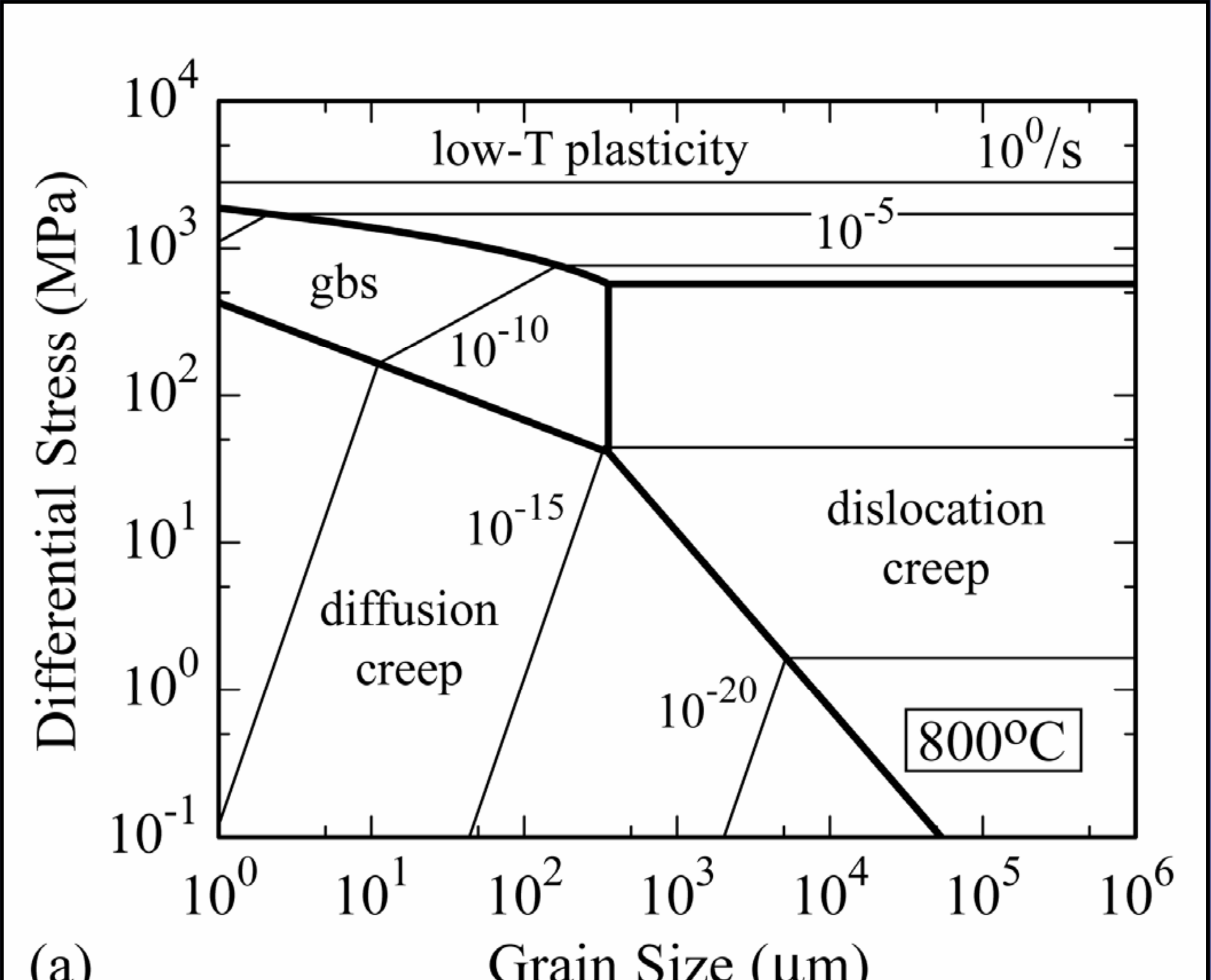
MORB

$\phi = 0.20$

$\phi = 0.01$

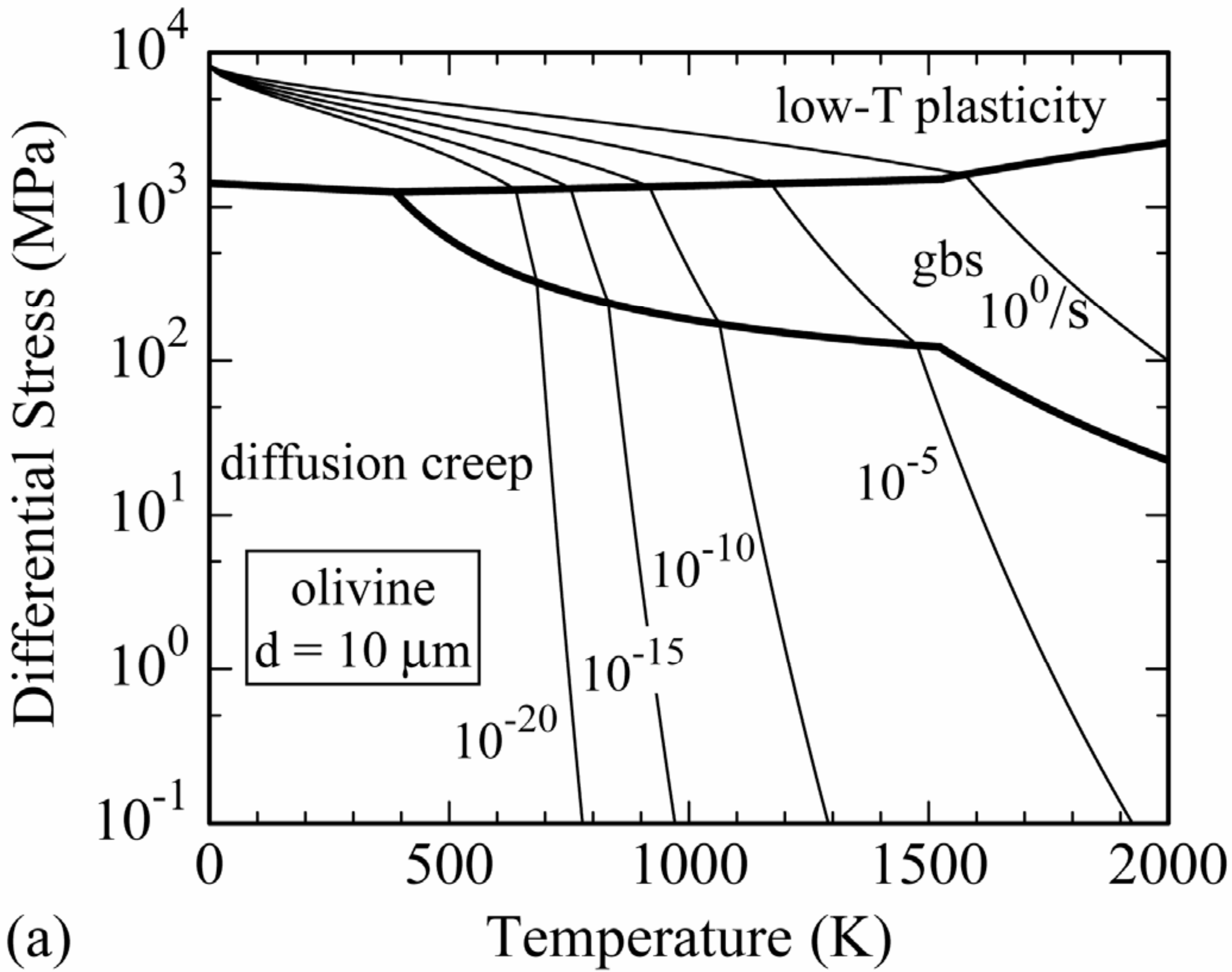

Hirth and Kohlstedt (2003)
geoid
diffusion creep
dislocation creep
rebound
σ = 0.3 MPa
$V^* = V^*(T, P)$
$T_P = 1350^{o}$C
$C_{OH} = 1000$ H/10^6Si
Depth (km)
0
100
200
300
400
500
Viscosity (Pa s)
10^{16}
10^{18}
10^{20}
10^{22}
10^{24}

Deformation Mechanism Map – σ vs *d*



Kohlstedt (2007)

Deformation Mechanism Map – σ vs *T*



Kohlstedt (2007)