Development of Variational Data Assimilation for Active Tectonics Studies

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Acknowledgements: Laura Ehret, Ed Zaron, Boon Chua, and scientists at JPL (Jet Propulsion Laboratory) and Indiana University

@ ~18 days ~55 days ~91 days ~146 days ~164 days 20 **Preview** strike₁₀ Fault slip Dimension [km] **Fault Plane** slip @ ~18 days ~55 days ~91 days ~146 days ~164 days 2.5 0.5 1.5 strike Displacement (cm) slip Fault Say, there is a hypothetical Dimension [km] regional fault (fault motion on the plane shown above) that is continuously creeping. We have CGPS 20 data every 20 km or so, dip irregularly. Our timeslip dependent inversion results for such fault motion are on the left. 3 2 6 Displacement (cm)

Preview

Observe

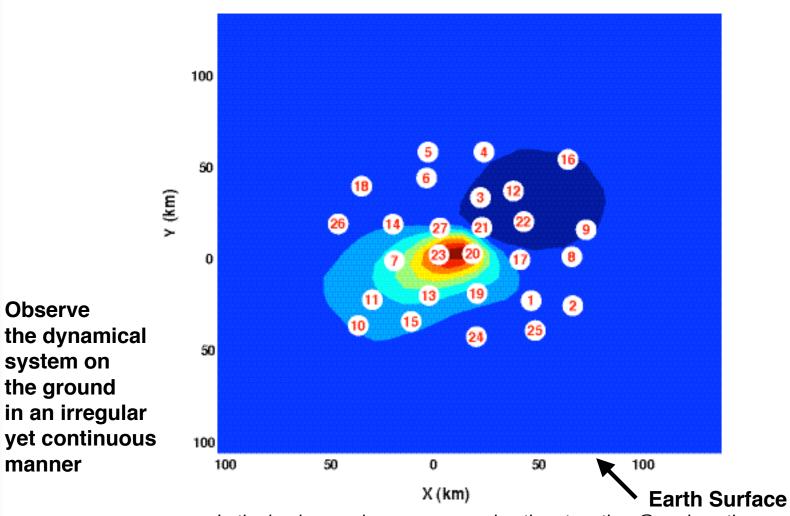
system on the ground

manner

the dynamical

in an irregular

Observatory Array Design

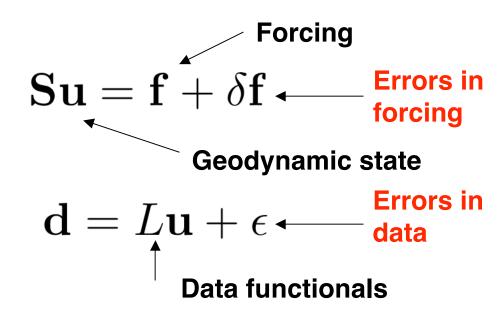


In the background, you see creeping thrust motion @ a given time

Preview

Dynamical Equations

Data



We have uncertainties and errors in physics, forcing, parameters, medium, nonlinearity, and data

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Outline

- a) General Concepts
- b) Equations
- c) Active Tectonics Motivation Slow Slip
- d) Experiments
- e) Midnight Oil

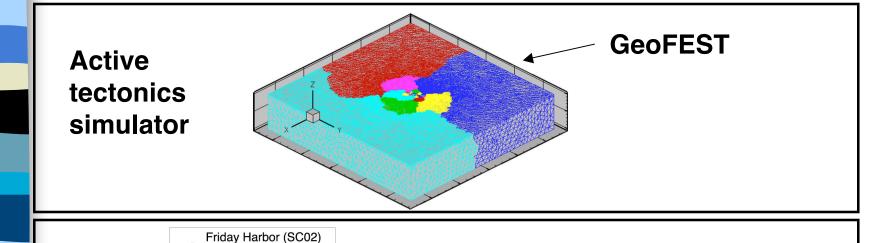
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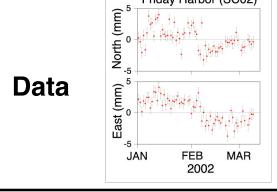
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Worked on

Inverse problems for time-dependent, non-linear processes in active tectonics using "data assimilation" methods

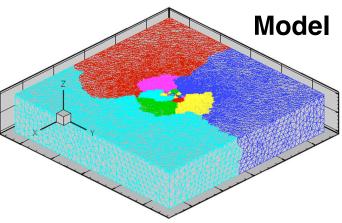




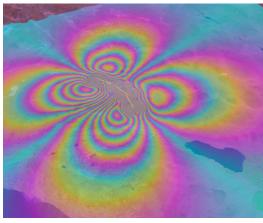
GPS data for active tectonics

From Meghan Miller's @ University of Central Washington webpage

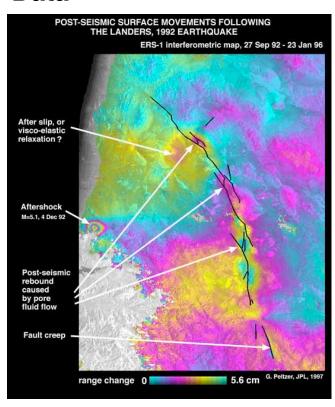
GeoFEST



Modeled response



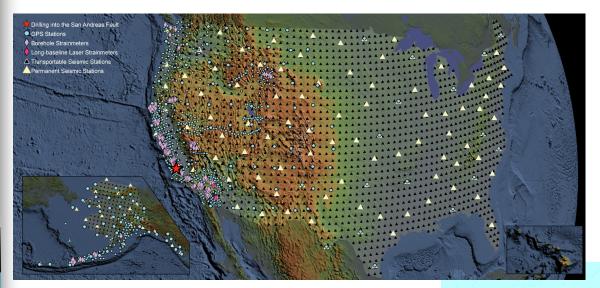
Data



Above: Figures from

Jet Propulsion Laboratory
California Institute of Technology

Motivation



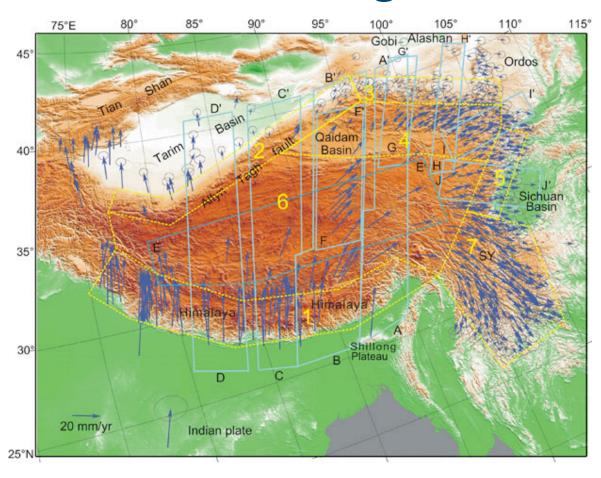
Left: Proposed Data Coverage by Earthscope

FUJISAN:

Right: Geographical Survey Institute (GSI), Japan - 1,224 GPS control points for real time crustal motions.

Start of dense coverage and observatory-style monitoring

Inversion for forcing



GPS (Global Positioning System) data in and around the Tibet. India "forcing" into Eurasia. Figure from Peter Molnar's @ UC Boulder webpage.

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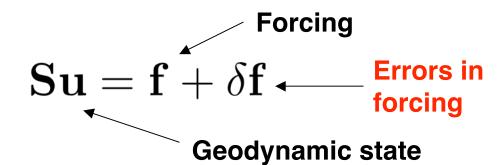
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- b) **Equations**
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Variational Data Assimilation

Dynamical Equations

Data



$$\mathbf{d} = L\mathbf{u} + \epsilon$$
 Errors in data

Data functionals

Minimize the penalty functional

$$\mathcal{J}[\mathbf{u}] = (\mathbf{d} - L\mathbf{u})^* W_d (\mathbf{d} - L\mathbf{u}) + (\mathbf{S}\mathbf{u} - \mathbf{f})^* W_f (\mathbf{S}\mathbf{u} - \mathbf{f})$$
 $W_d = C_d^{-1} \quad W_f = C_f^{-1} \quad \text{Inverses of error covariances}$

Maximum likelihood estimate of u for a Gaussian error processes

Variational Data Assimilation

Errors in data

Errors in forcing

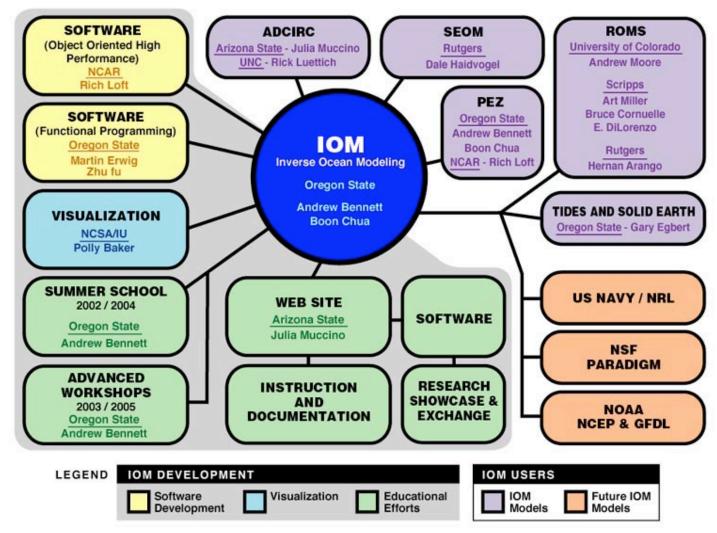
Minimize the penalty functional

$$\mathcal{J}[\mathbf{u}] = (\mathbf{d} - L\mathbf{u})^* W_d^* (\mathbf{d} - L\mathbf{u}) + (\mathbf{S}\mathbf{u} - \mathbf{f})^* W_f (\mathbf{S}\mathbf{u} - \mathbf{f})$$

 $W_d = C_d^{-1}$ $W_f = C_f^{-1}$ Inverses of error covariances

Note: The physics is embedded in the penalty functional

b) Variational Data Assimilation/Inverse Framework



We provide the model, its tangent linear, and the adjoint IOM provides the framework for the inverse/ optimatization problem

Euler-Lagrange (E-L) Equations

Necessary conditions for a minimum

$$\mathcal{J}$$

Backward:
$$\mathbf{S_{TL}}^*\lambda = L^*W_d(\mathbf{d} - L\mathbf{u})$$

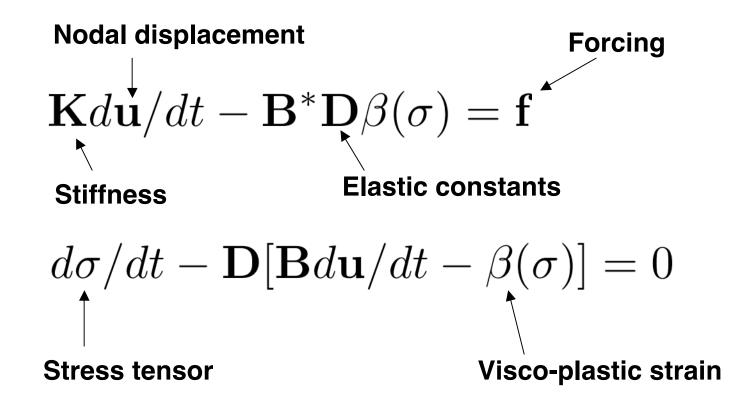
Forward :
$$\mathbf{S}\mathbf{u} = \mathbf{f} + W_f \lambda$$

$$S_{TL}$$
 = tangent linear S_{TL}^* = adjoint of the tangent linear

Uses representer approach

Forward Equations

Forward Model Equations (S)



Tangent Linear Equations

Tangent Linear Equations ($\mathbf{S_{TL}}$)

Linearize viscous rheology (if needed)

$$\mathbf{K}d\mathbf{u}/dt - \mathbf{B}^*\mathbf{D}\beta'\overset{\mathbf{f}}{\sigma} = \mathbf{f}$$

$$d\sigma/dt - \mathbf{D}[\mathbf{B}d\mathbf{u}/dt - \beta'\sigma] = \epsilon$$

Errors in stress equation

Adjoint Equations ($\mathbf{S_{TL}}^*$)

Mapping:
$$\lambda, \epsilon \longleftarrow \mathbf{u}, \sigma$$

$$\mathbf{K}d\lambda/dt - \mathbf{B}^*\mathbf{D} = d\mathbf{u}/dt$$

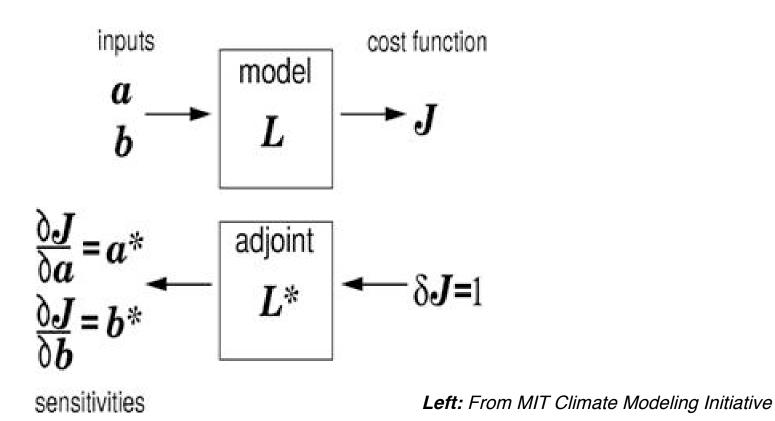
$$d\epsilon/dt - \beta' \mathbf{D}[\mathbf{B}d\lambda/dt - \epsilon] = \sigma$$

Adjoint is required for gradient of penalty functional and for representer calculation u sed to minimize \mathcal{J}

$$\mathcal{J}[\mathbf{u}] = (\mathbf{d} - L\mathbf{u})^* W_d (\mathbf{d} - L\mathbf{u}) + (\mathbf{S}\mathbf{u} - \mathbf{f})^* W_f (\mathbf{S}\mathbf{u} - \mathbf{f})$$

Broad class of geological problems: Same formulation and software

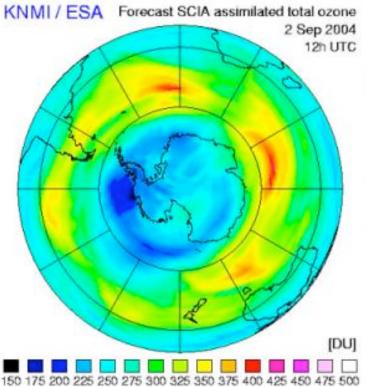
Adjoint Based Modeling - Rational



Sensitivity of small number of output parameters with respect large number of input variables

Errors, sensitivity, variational - extremely powerful tool

Adjoint Based Modeling - Rational



Left: Ozone layer thickness is predicted using adjoint models. From KNMI/ ESA

Right: Paths of hurricanes are predicted using adjoint models, Sensitivity to errors tweaks the initial conditions, and are used for forecasting.



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c) Active Tectonics Motivation

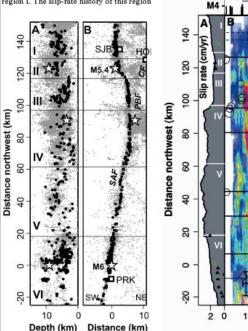
Initial application: Aseismic slip/silent earthquakes

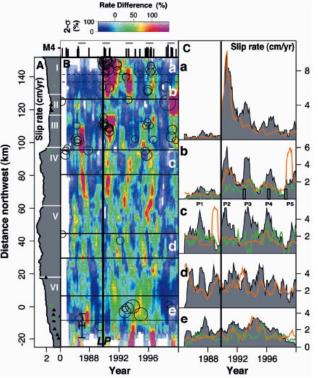
Left: Analysis of earthquake and creepmeter data along San Andreas Fault. From Nadeau and McEvilly, Science, 9 January 2004.

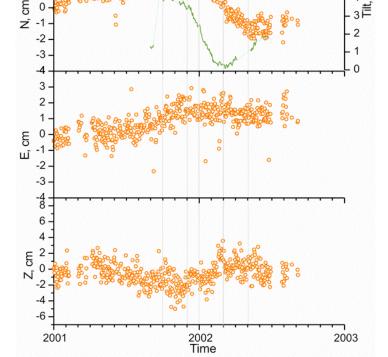
ACAPULCO

shielding effects associated with the Mo locked zone are substantially modifying a quasiperiodic signal.

The 1989 M7.1 Loma Prieta earthquake (2) occurred to the immediate northwest of region I. The slip-rate history of this region







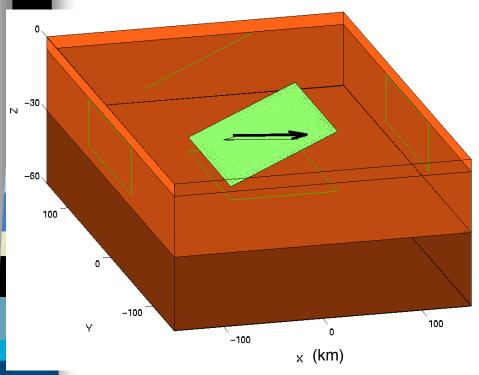
Right: GPS data from Guerrero, Mexico. From Vladimir Kostoglodov's @UNAM, Mexico webpage

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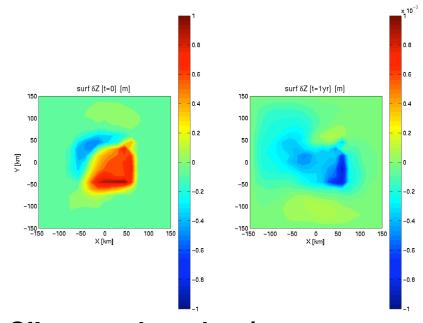
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- e) Midnight Oil

d) Experiments Experiment setup



Right: Typical GeoFEST (similar to GPS data) viscoelastic response to an impulse forcing, say a single earthquake. Vertical motion on surface in meters

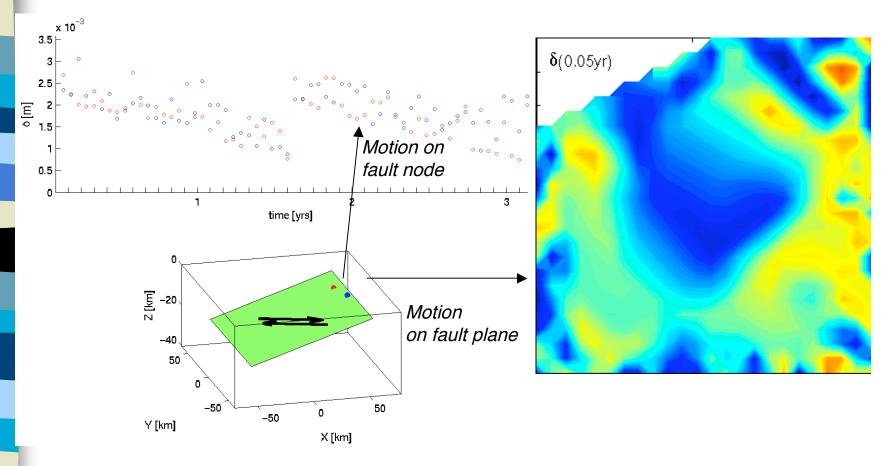
Left: GeoFEST Model - Three layer Earth, viscoelastic, 100-20 km long blind fault. Finite element with 1000s of nodes and elements



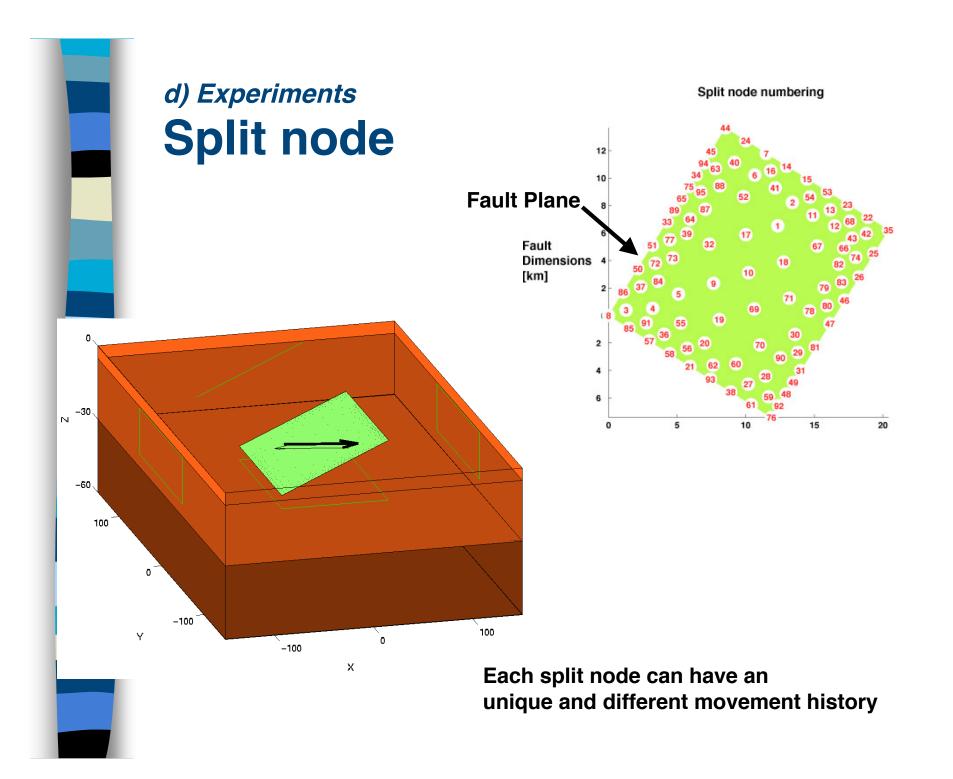
Silent earthquakes/ aseismic slips are forced differently......

d) Experiments

Forcing an aseismic fault



Estimate aseismic fault slip spatially and temporarily from GPS/ creepmeter/ earthquake data



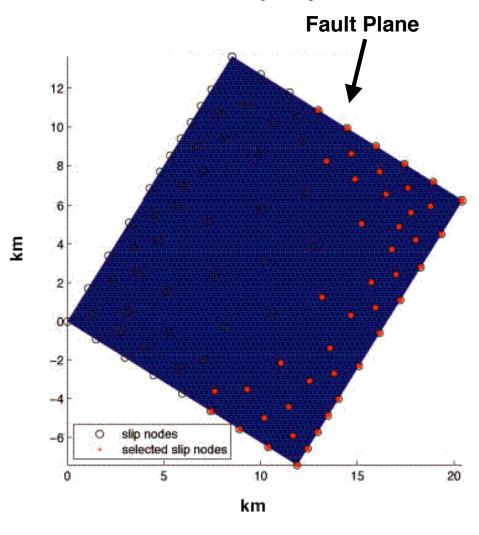
d) Experiments **Creating forcing** Selecting split nodes for motion spatially and temporally **Fault Plane** 12 10 -60、 100 100 _100 Х slip nodes selected slip nodes 20 15 10 km

d) Experiments

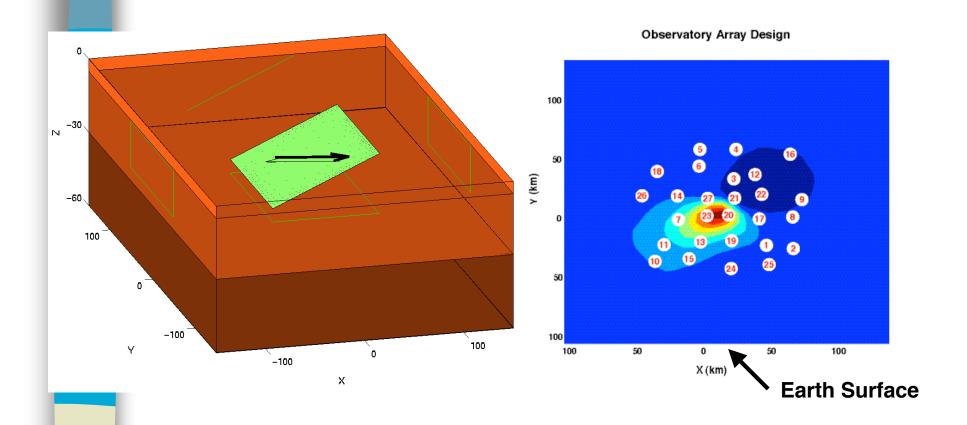
Creating "truth" models

- 1. Uniform, complete ramp-like forcing (simplest experiment)
- 2. Forcing in a limited region
- 3. Migratory slip
- 4. Forcing in a nonlinear medium
- 5. Ultra fast forcingforeshocks, after slip
- **6. Forcing from Different Plane**

Selecting split nodes for motion spatially and temporally



d) Experiments Listening

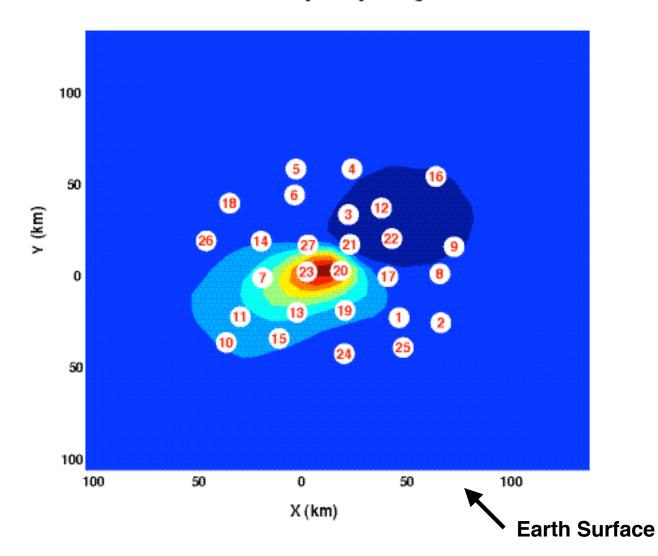


In the background, you see creeping thrust motion @ a given time

d) Experiments

Creating arrays

Observatory Array Design



Numbers = Continuous GPS Coverage

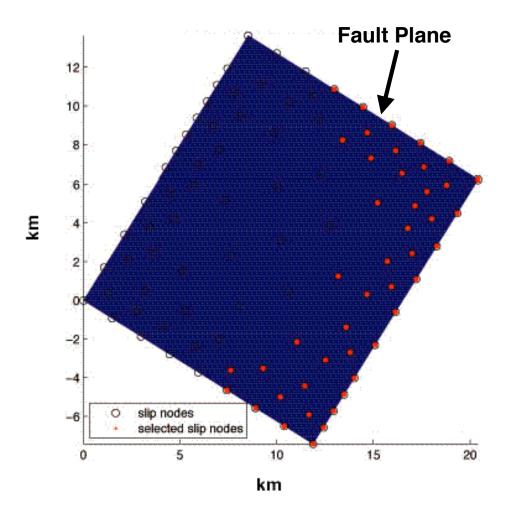
d) Experiments

Presenting results

1. Uniform, complete ramp-like forcing (simplest experiment)

2. Forcing in a limited region

Selecting split nodes for motion spatially and temporally

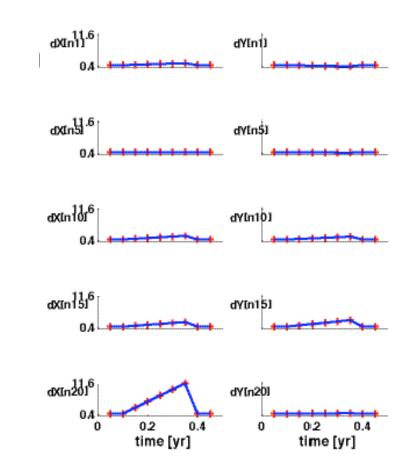


d) Experiments Perfect Data

Displacement (mm)

Simplest experiment: No noise

Comparison of ground response - data and computed from IOM at selected CGPS stations



Add Noise Later: Factor of standard deviation More noise in Z-component

d) Experiments Results at CGPS stations - Data Match

dXIn51 dYIn51

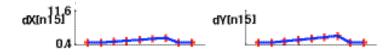
dXIn11.6

dxin1016 dyin101

Comparison of ground response - data and

dYIn11

computed from IOM at selected CGPS stations



n)

dXin2016 dYin201 dYin201 0.2 0.4 time [yr] time [yr]

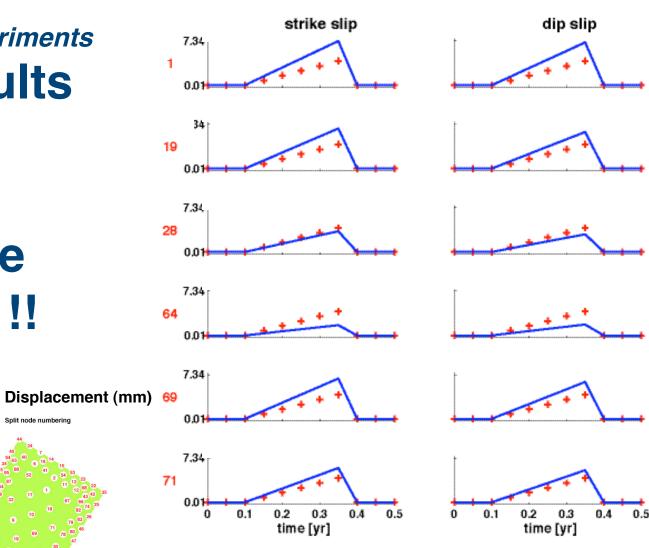
Displacement (mm)

Solid Line = Computed, + = Inversion through IOM

0.4

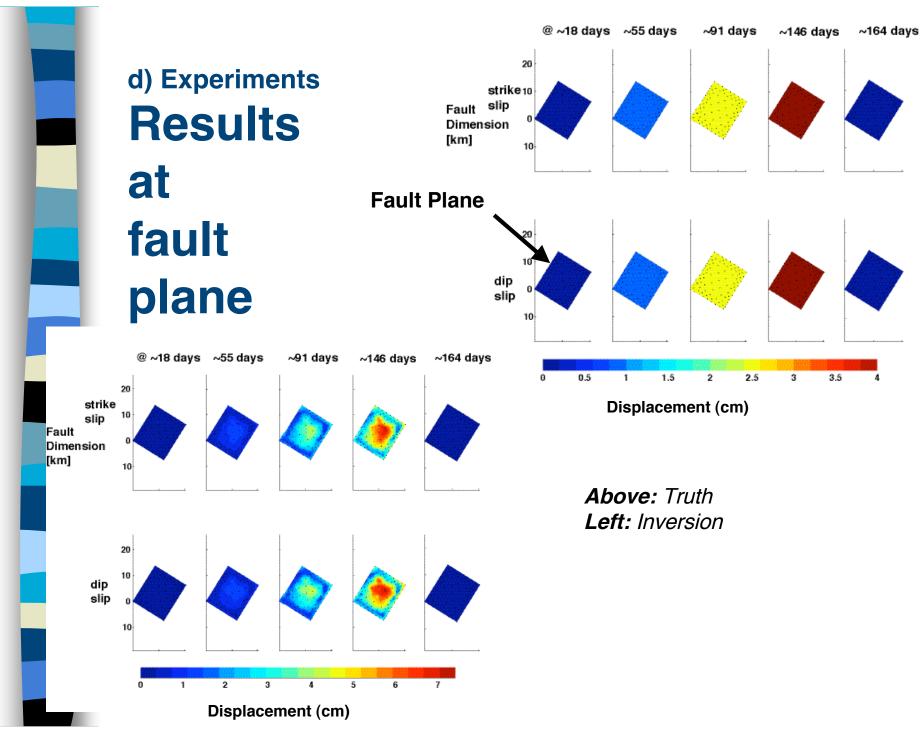
Comparison of true forcing and computed from IOM at slip/split nodes

d) Experiments Results at fault plane - Ah !!



......

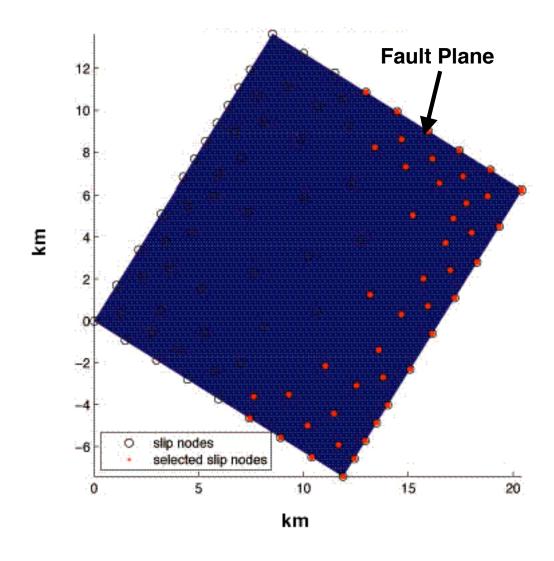
Solid Line = "Truth", + = Inversion through IOM

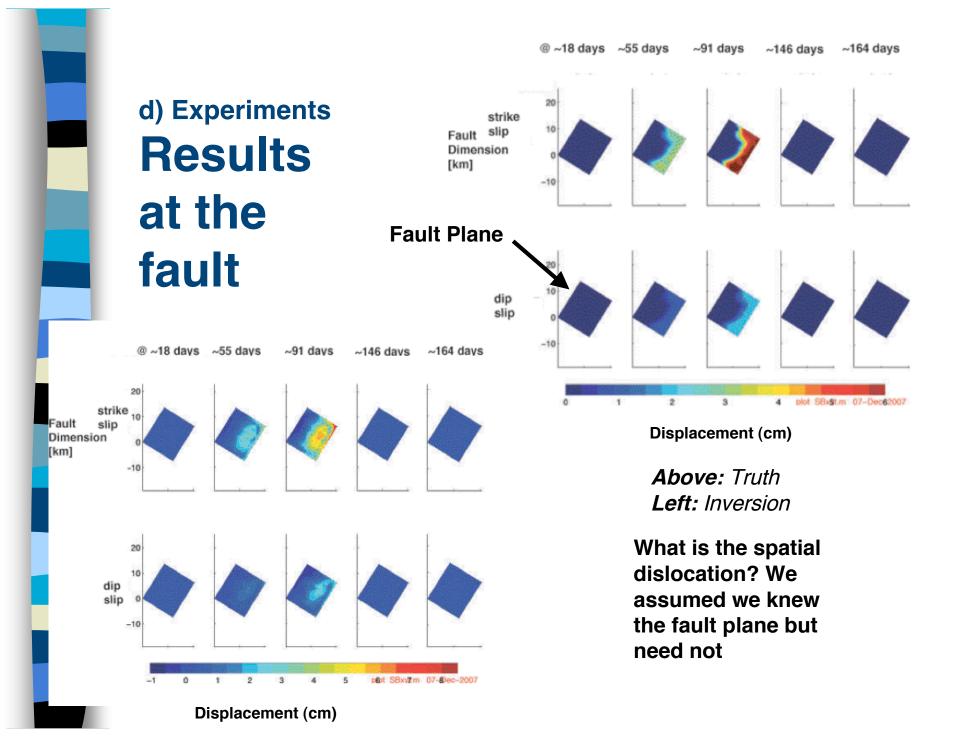


d) Experiments Forcing in a limited region

Depth controversy

Selecting split nodes for motion spatially and temporally





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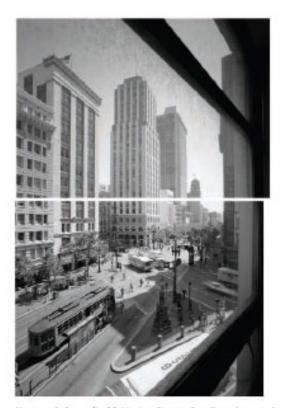
Midnight oil

 Request to CIG (Computational Infrastructure for Geodynamics) to adopt.

Shake the technique really hard - noise, more sparse data, errors, etc

Full mojo of adjoint

Publish





Now and then. (Left) Market Street, San Francisco, today, in a split composite photograph and (right) on 18 April 1906 as fire began to consume the Call Building (in center of photo). The building's steel frame was apparently undamaged by the earthquake, and it remains in use today. Note the debris from collapsed masonry buildings further up Market Street. [1906 photo by W.]. Street, courtesy Bancroft Library, Univ. of California, Berkeley. Rephotography by M. Klett with M. Lungren (8).]

Above: From William Ellsworth, Science, 14 April, 2006.