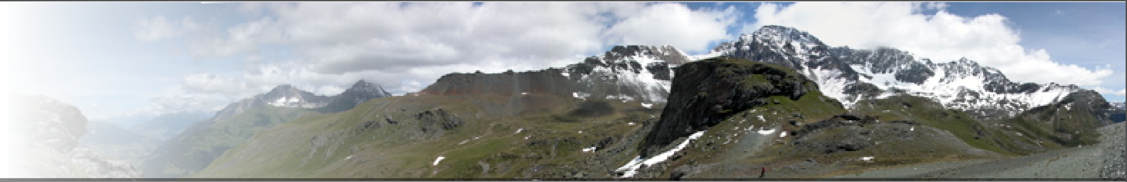


Mantle convection, plate boundaries and plasticity.

Dave May
Louis Moresi
Hans Mühlhaus
Mirko Velic
Wendy Sharples
Dave Stegman

Outline

- Observations about plate boundaries
- Isotropic formulations
- Anisotropic formulations
- Some numerical experiments

Rheological

Outline

- Observations about plate boundaries
- Isotropic formulations
- Anisotropic formulations
- Some numerical experiments

Rheological

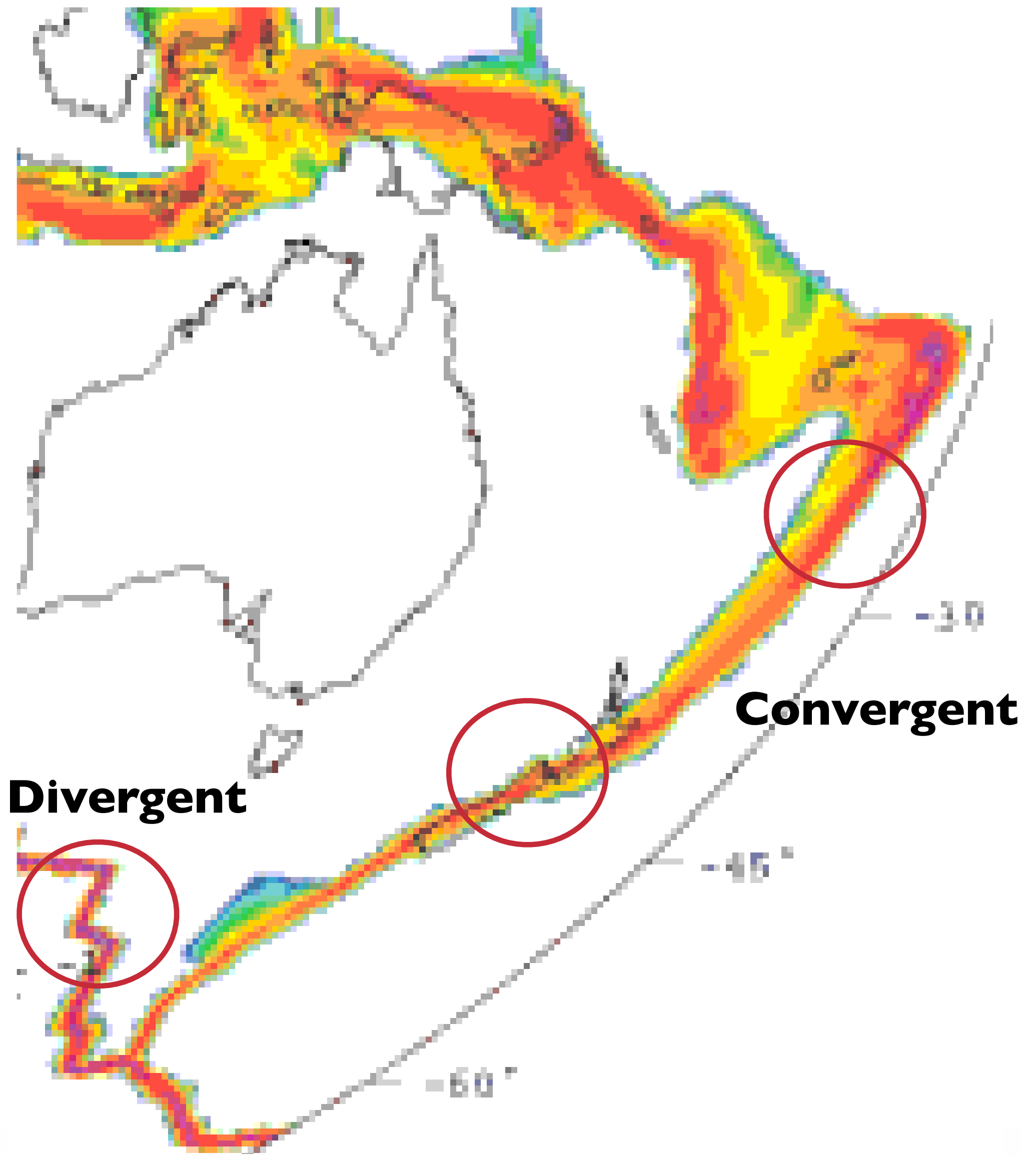
- The Material Point Method
- A Schur complement preconditioner for variable viscosity

Numerical

Plate boundaries deformations — up close ?

Different plate boundary environments have different geological structures ...

- On the global scale they are very narrow "linear" features
- But these lines have an internal structure (distinctive geology)
- The development of this structure probably influences the large-scale plate boundary geometry and properties.
- 2nd inv. of strain-rate does not show significant difference between different plate boundary types



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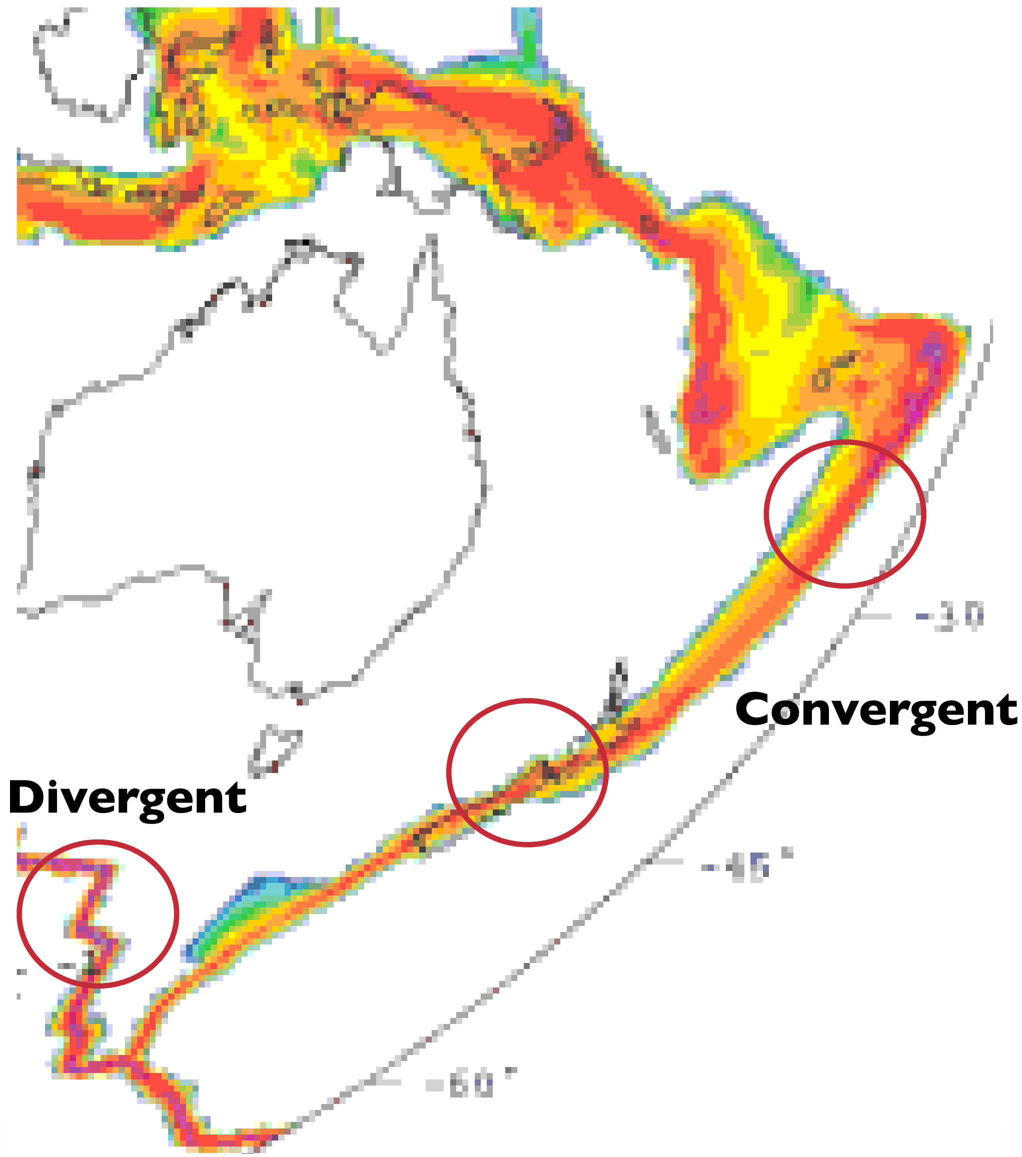

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Depends upon stress orientation as well as magnitude

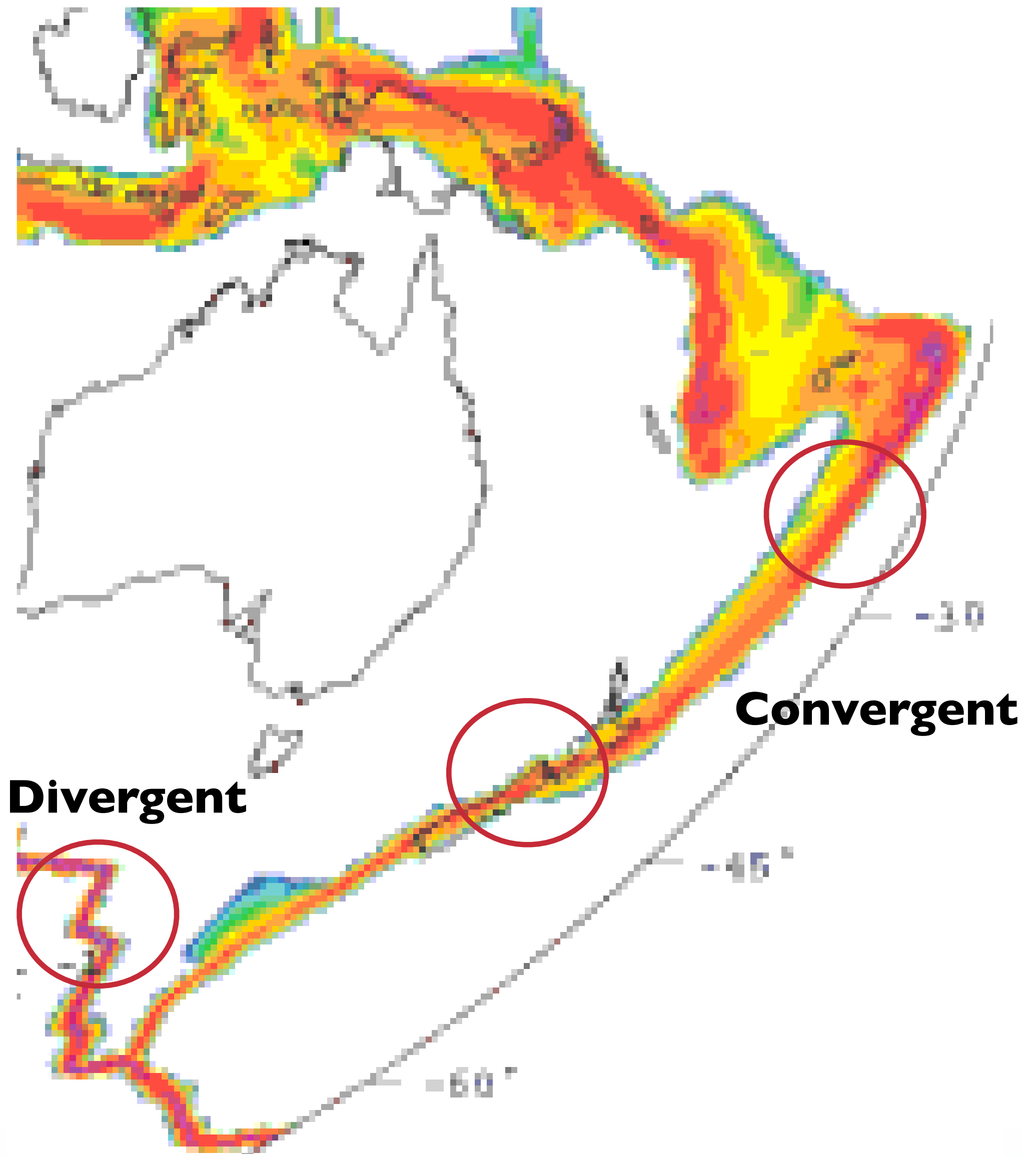


Plate boundaries kinematics ?

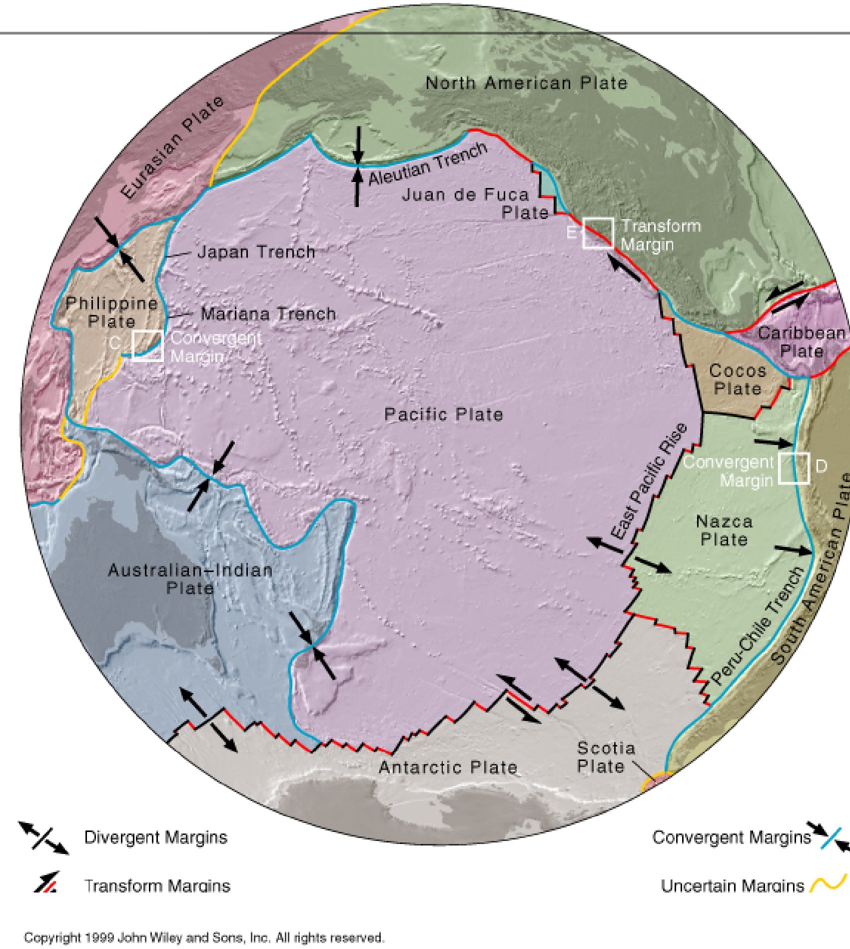


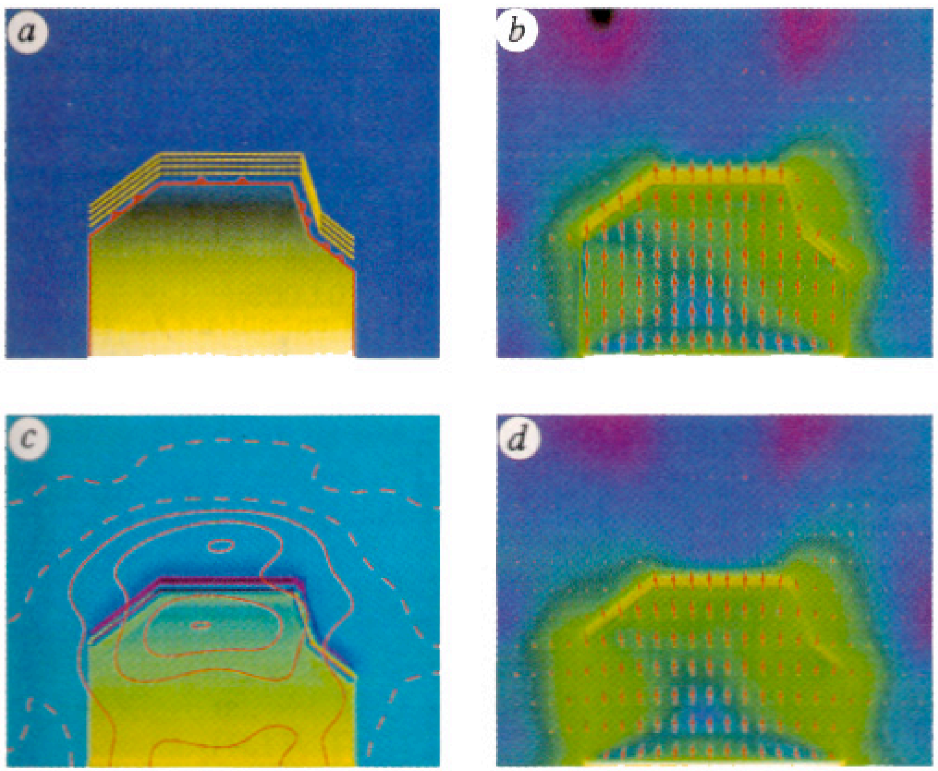
Plate boundaries are **systems** with plenty of small scale physics going on

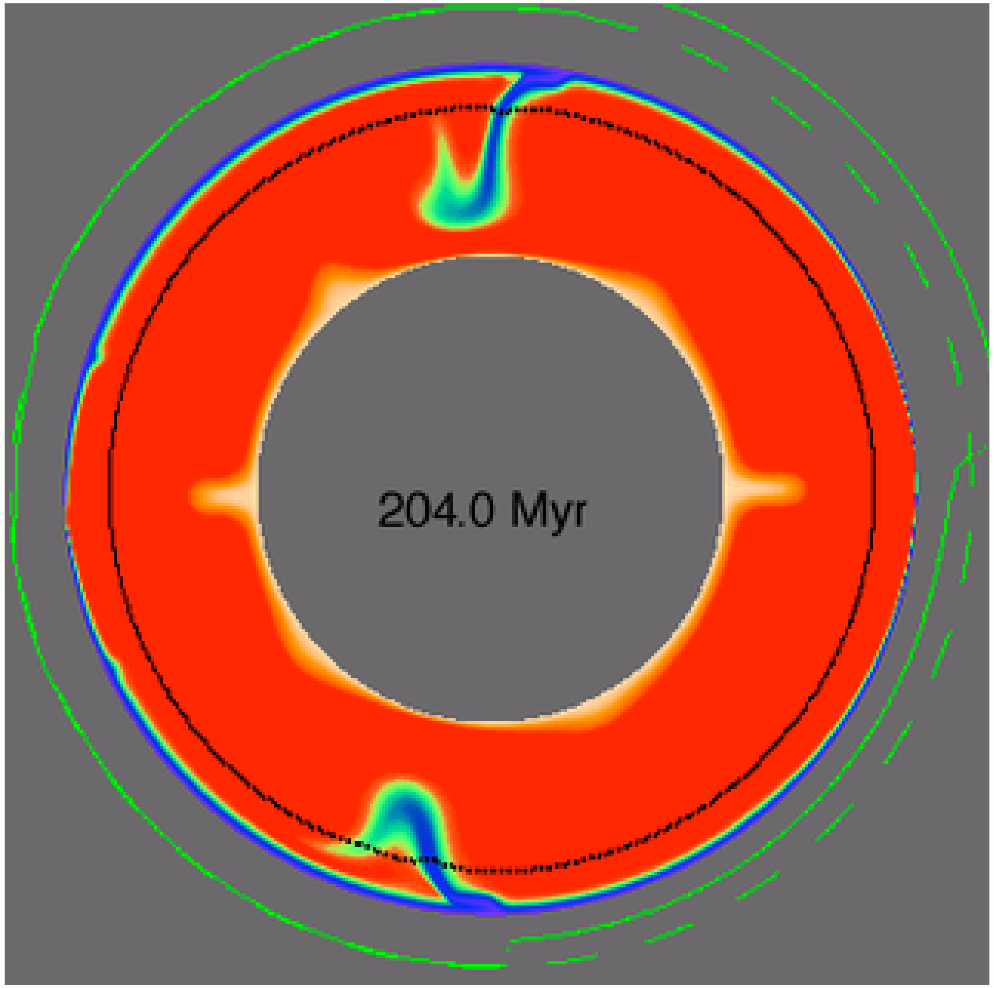
- continuum codes need to use an appropriately scaled representation of the physics at the large scale.
- kinematics of plate boundary evolution is very well documented

Lithospheric faults

Macro-scale faults are effective for accurate models of subduction zones

- Asymmetry
- Accurate trench topography
- More plate-like surface deformation

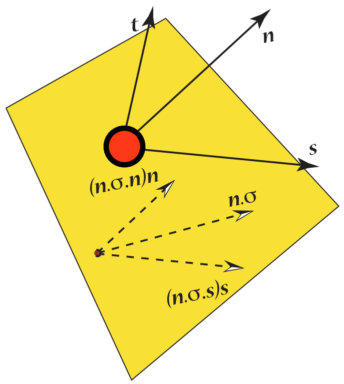


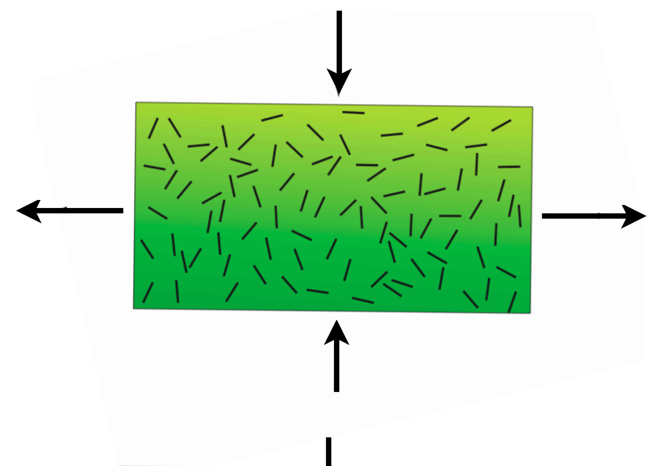


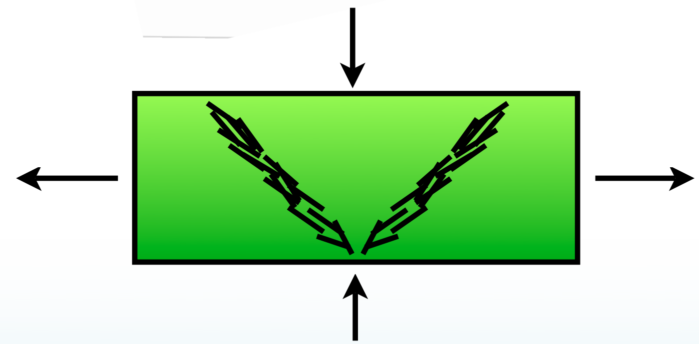
Zhong, Gurnis

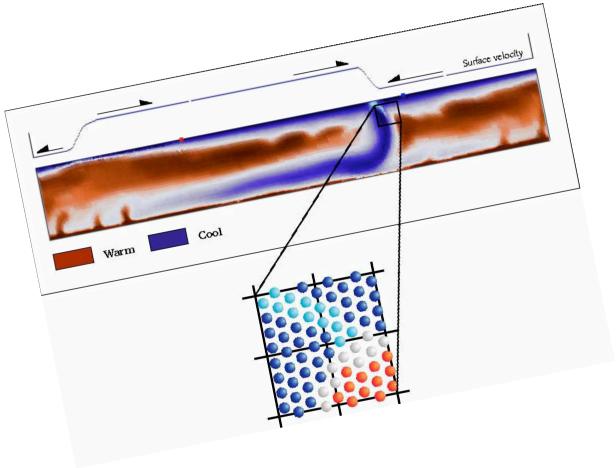
Anisotropic formulation

- Rather than constrain velocity directly, construct a rheology which produces a similar effect.
- Oriented slip surface on a particle (small scale). Strong in the normal dir., weak in the tangential dir.
- An ensemble of such particles can produce a macroscopic feature (fault).
- Connections to multiscale methods.
- + Include observed fabric and orientation into dynamic models where we know that information.
- + Easy to implement in a particle code.









A basic formulation for convection with various failure modes

Equation of motion for

- Viscoplastic behaviour

Constitutive laws for various yielding behaviours

- “Granular” flow
- Prescribed fault zone fabric
- Anderson faulting behaviour

Governing Equations

$$\tau_{ij,j} - p_{,i} = \rho(\phi, C, \dots)g_i$$
$$u_{i,i} = 0$$

Momentum and Mass conservation

$$\frac{\tau_{ij}}{\eta} + \alpha\Lambda_{ijkl}\tau_{kl} = \frac{1}{2}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right]$$

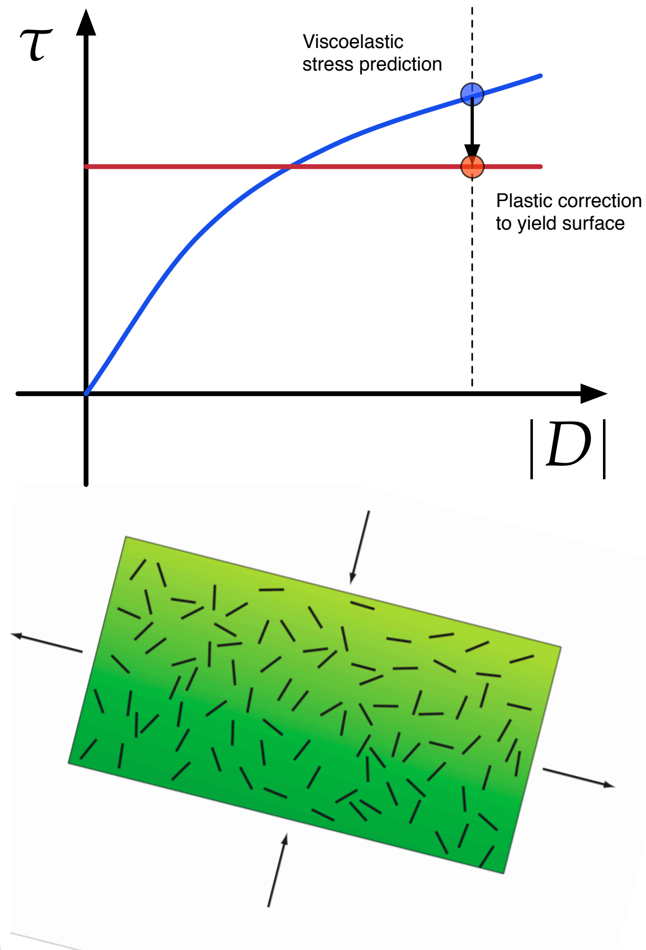
Flow law

$$\phi_t + u_i\phi_i = (\kappa\phi_i)_i + Q$$

Energy conservation

$$C_{,t} + u_i C_{,i} = 0$$

Material tracking



Failure models

Compliance & Constitutive behaviour

$$\bar{S}_{ijkl}\tau_{kl} = D_{ij}$$ [Compliance equation or flow law]

$$\left(S_{ijkl} + S'_{ijkl}\right)\tau_{kl} = D_{ij}$$

isotropic anisotropic

$$\bar{C}_{ijkl}\bar{S}_{ijkl}\tau_{kl} = \bar{C}_{ijkl}D_{kl}$$

$$\bar{C}_{ijkl}\bar{S}_{ijkl} = \delta_{ik}\delta_{jl}$$

$$\tau_{ij} = \bar{C}_{ijkl}D_{kl}$$

$$= \left(C_{ijkl} + C'_{ijkl}\right)D_{kl}$$

$$C_{ijkl} = 2\eta\,\delta_{ik}\delta_{jl}$$ isotropic

$$C'_{ijkl} = -\beta\Pi_{ijkl}$$ anisotropic

Compliance & Constitutive behaviour

$$\tau_{ij} = 2\eta D_{ij} - \beta \Pi_{ijkl} D_{kl}$$ [Constitutive equation]

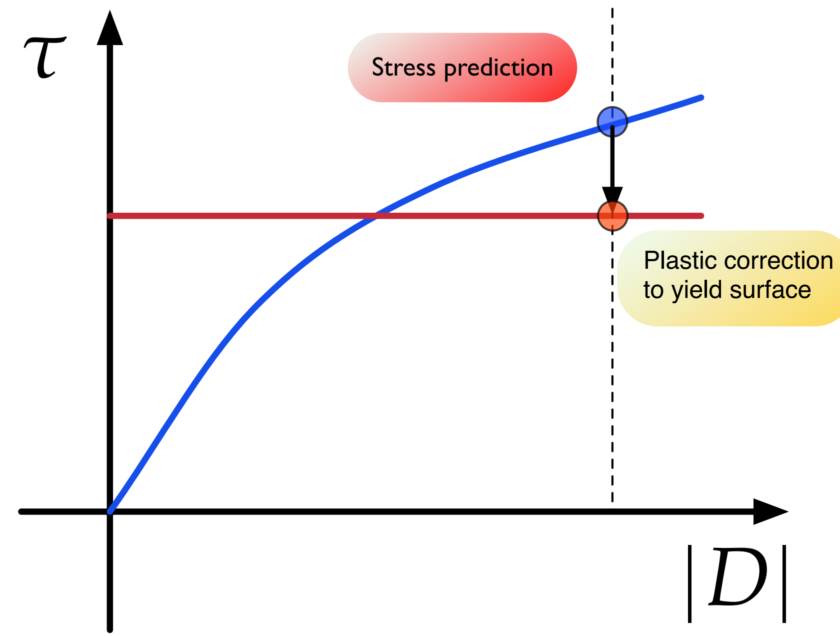
$$= T_{ij} - \beta \Pi_{ijkl} D_{kl}$$

How do we define the correction term $\beta \Pi_{ijkl}$??

Yielding

$$f(\boldsymbol{\tau}) < 0$$

$$\tau_{ij} = T_{ij} - \beta \Pi_{ijkl} D_{kl}$$



- Process is nonlinear

$$T_{ij}^{(I)} = 2\eta^{(I)} D_{ij}^{(I)}$$

- Start with a yield criterion and develop a constitutive model with the smallest number of free parameters which we can vary to ensure the yield criterion can be satisfied.

OR

- Try to find a flow rule which matches the observed deformation pattern & which can be used in conjunction with the rest of the convection formulation

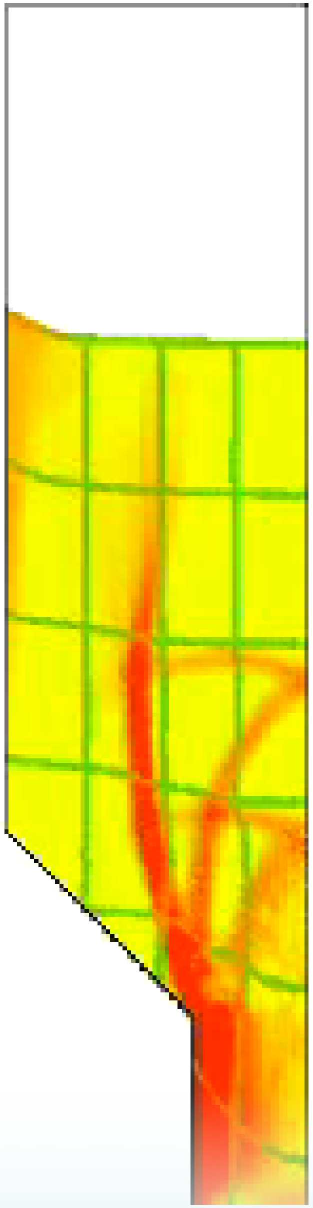
Drucker-Prager yield criterion

$$|\tau| - p\tan\varphi - C \leq 0$$

Standard viscous / viscoplastic behaviour

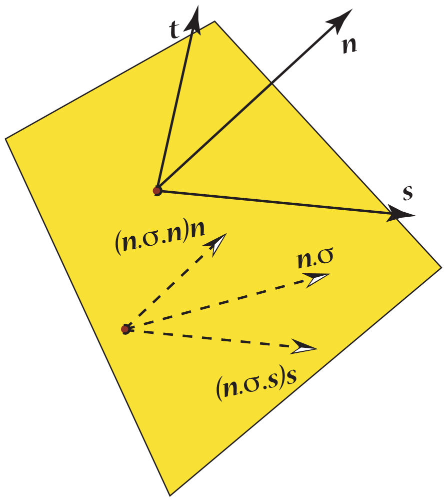
$$\tau_{ij} = \boxed{2\eta D_{ij}} + \boxed{2(\eta' - \eta)D_{ij}}$$

$$\boxed{\eta'^{(I)} = \frac{\tan\varphi p^{(I)} + C}{2|D^{(I)}|}}$$



Yield criterion for an individual fault fragment

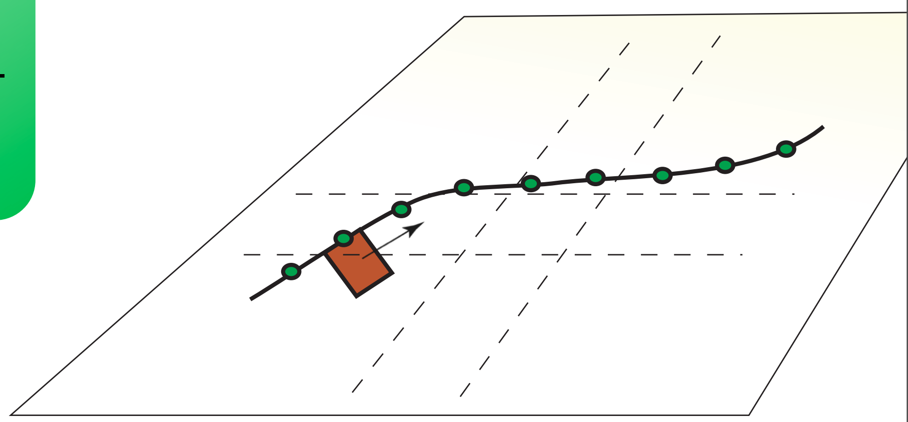
$$\tau_s - \sigma_n \tan\varphi - C \le 0 \qquad \sigma_n = n_i \tau_{ij} n_j + p \quad \tau_s = s_i \tau_{ij} n_j$$



Standard viscous / viscoplastic behaviour

$$\tau_{ij} = 2\eta D_{ij} + 2(\eta' - \eta)\Pi_{ijkl} D_{kl}$$

$$\eta'^{(I)} = \frac{\tan\varphi(2\eta D_{nn}^{(I)} + p^{(I)}) + C}{2D_{ns}^{(I)}}$$



$$\Pi_{ijkl} = -2 n_i n_j n_k n_l +$$

$$\left(n_i n_k \delta_{lj} + n_j n_k \delta_{il} + n_i n_l \delta_{kj} + n_j n_l \delta_{ik} +\right)/2$$

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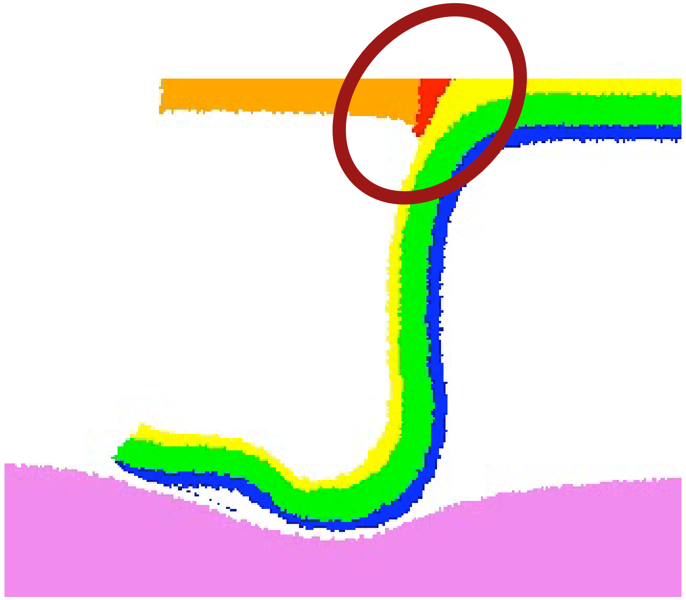

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Standard viscous constitutive law

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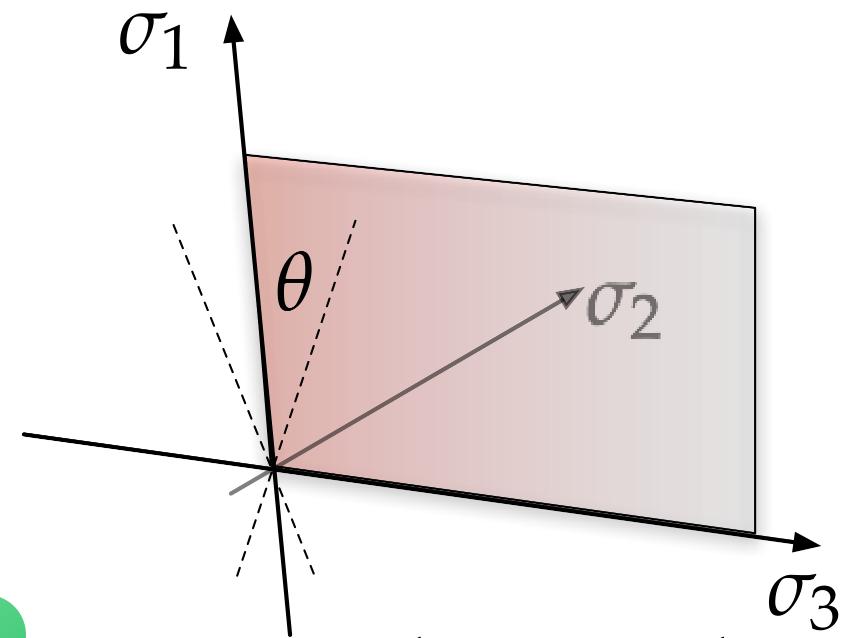
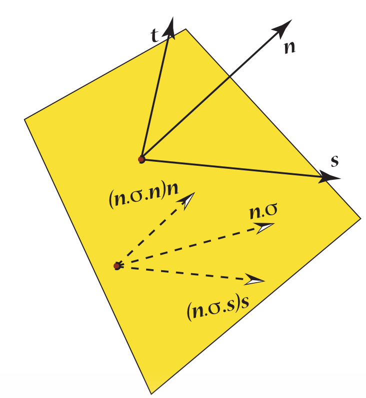


$$\theta = \pm\frac{1}{2}\arctan\left(\frac{1}{\tan\varphi}\right)$$

As before but this time the orientation is determined locally by the ambient stress tensor



Anderson faulting model

$$\tau_s - \sigma_n \tan\varphi - C \leq 0$$

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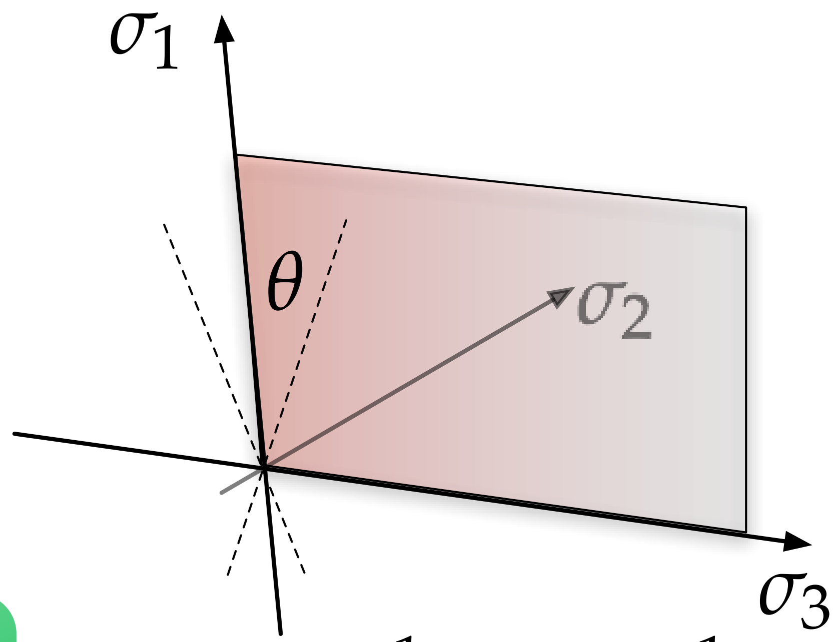

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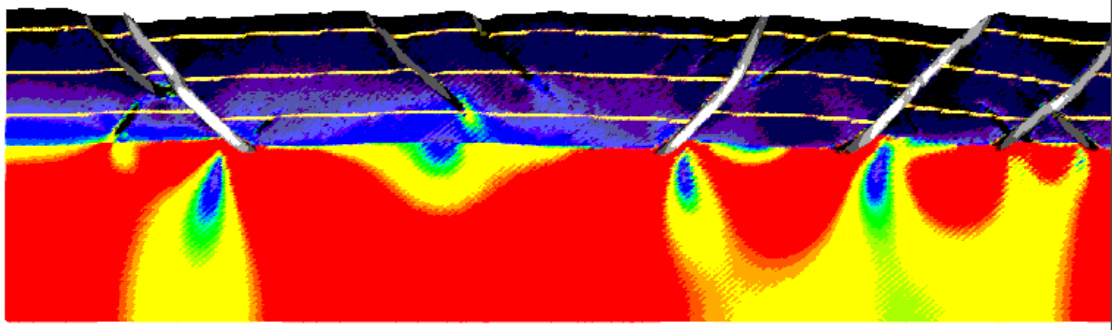
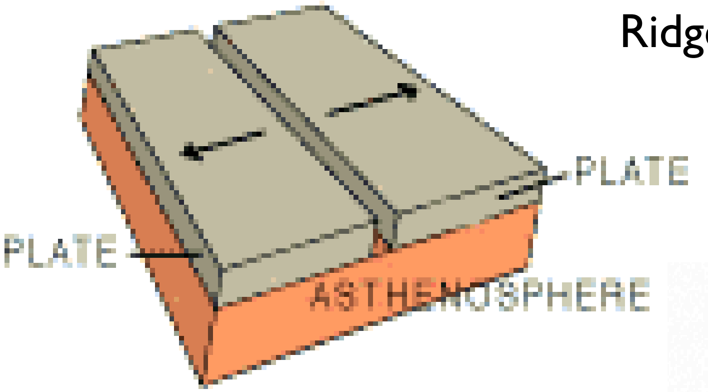
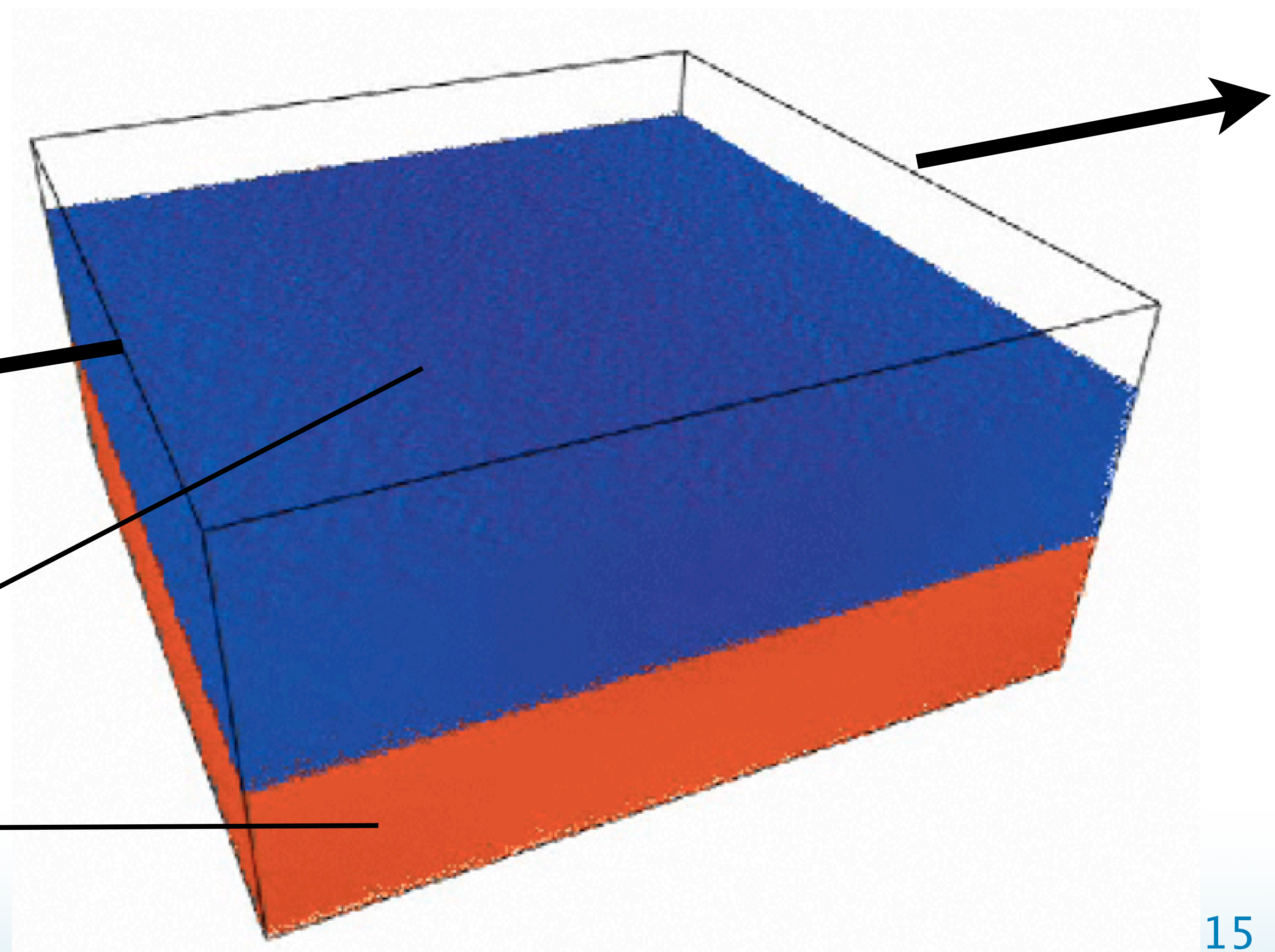


Example (1): Extension

Louis

Ridge



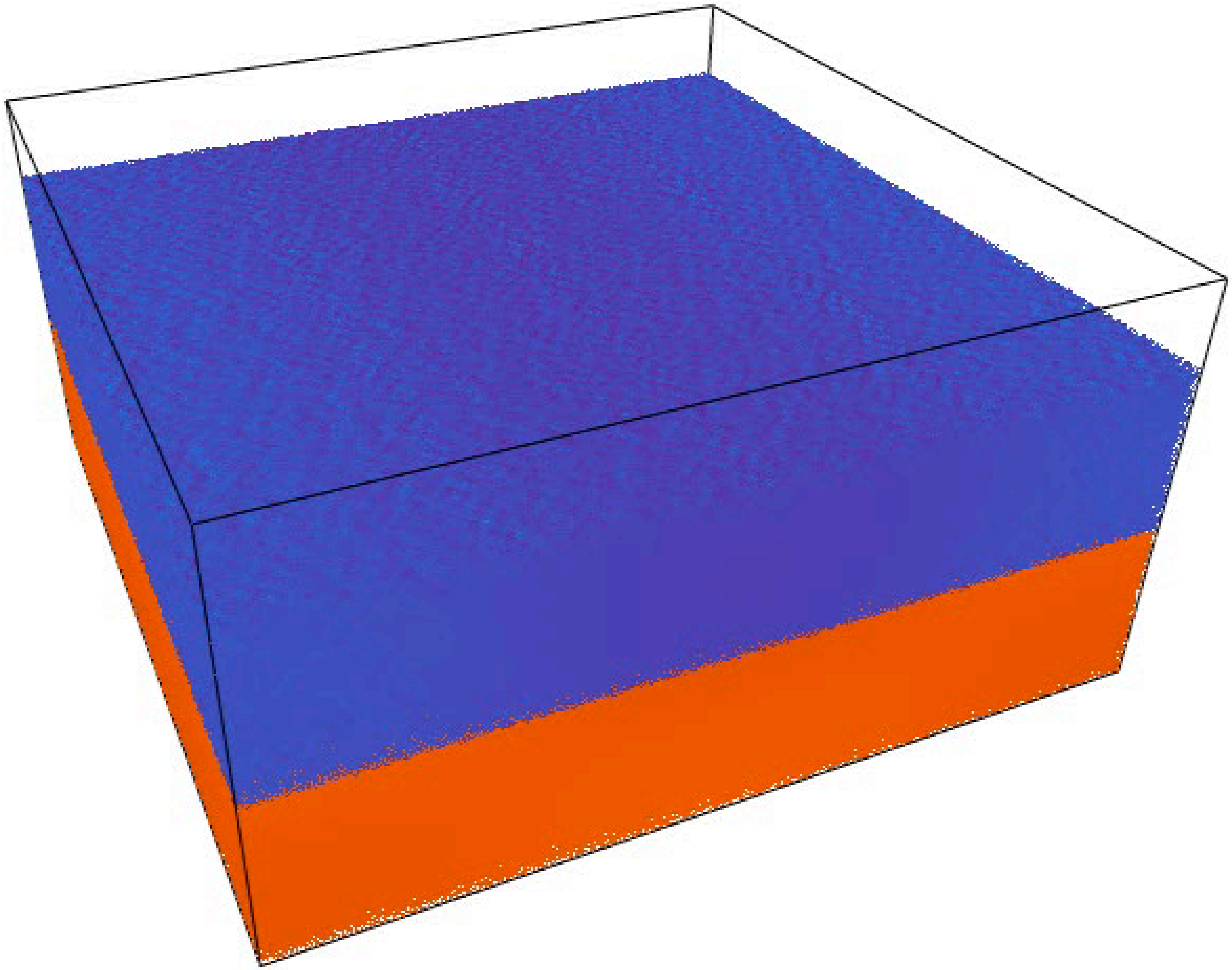


Anisotropic

Viscous

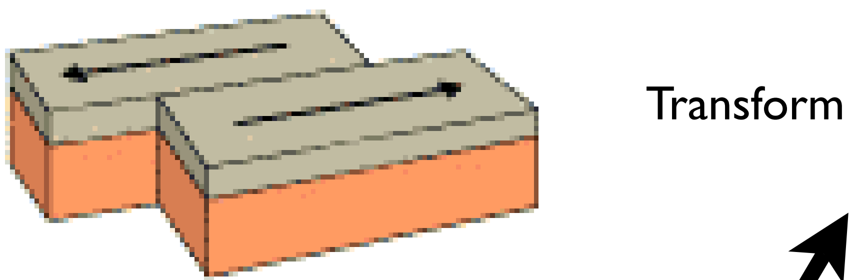
Example (1): Extension

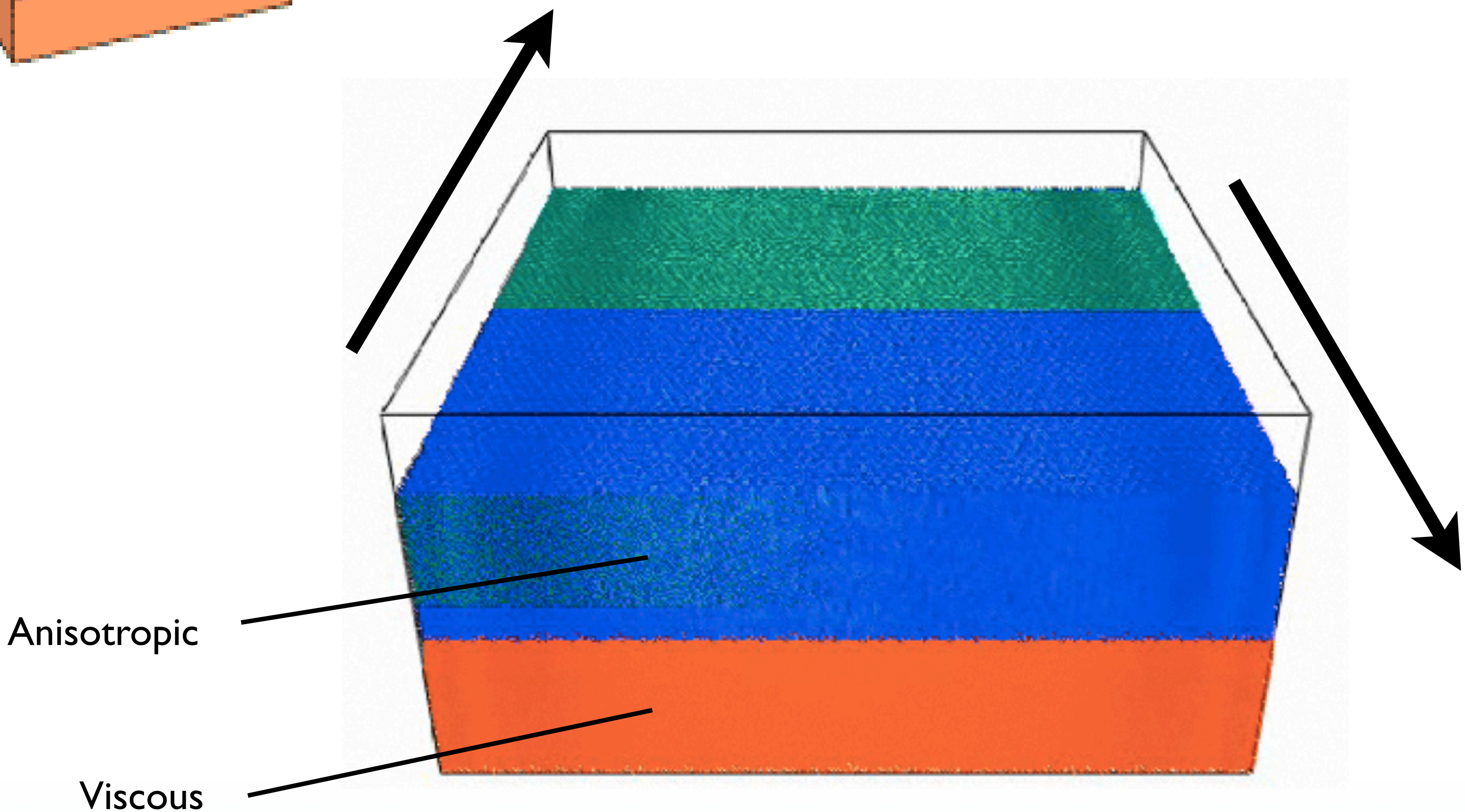
Louis



Example (2): Shear

Louis





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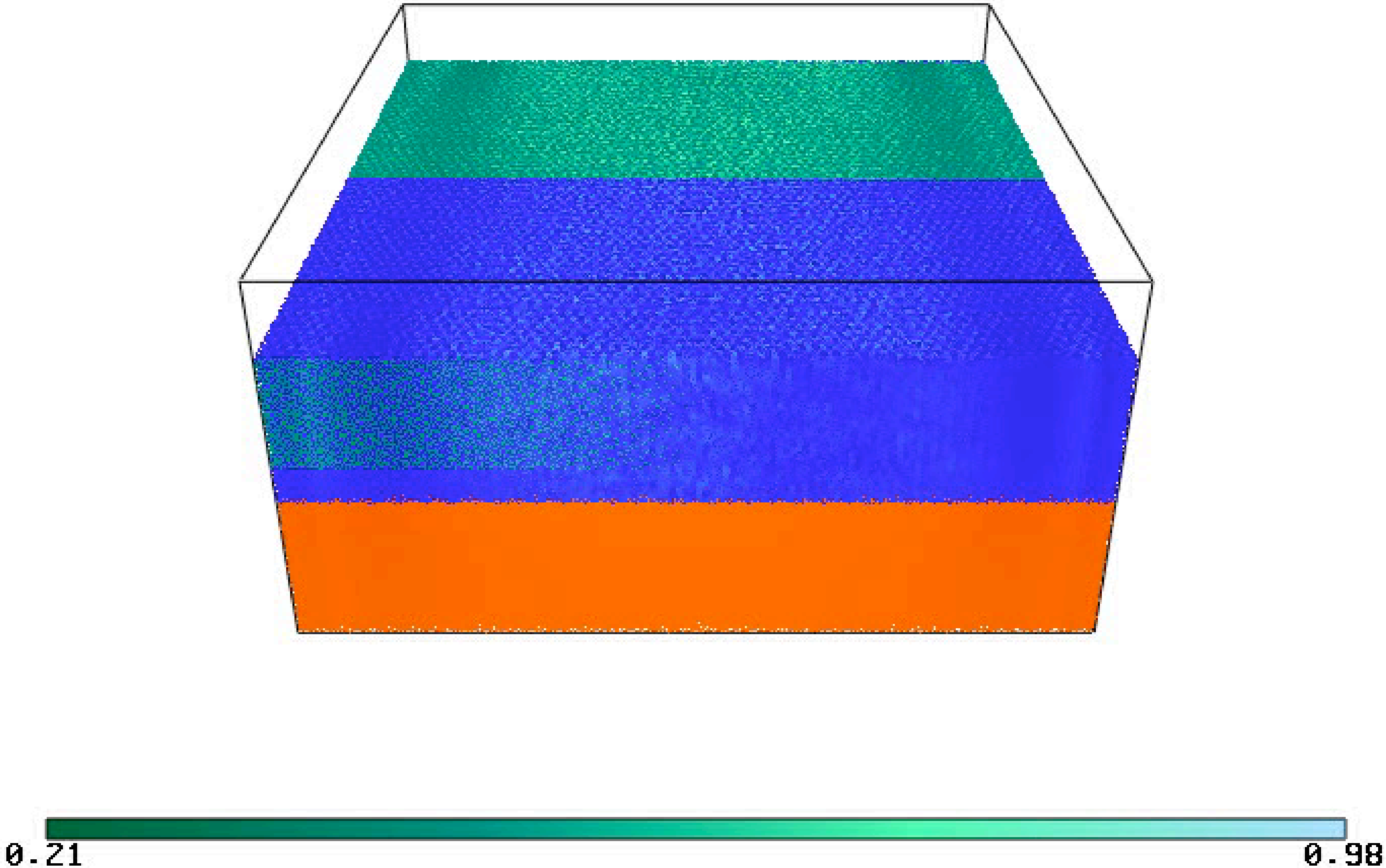
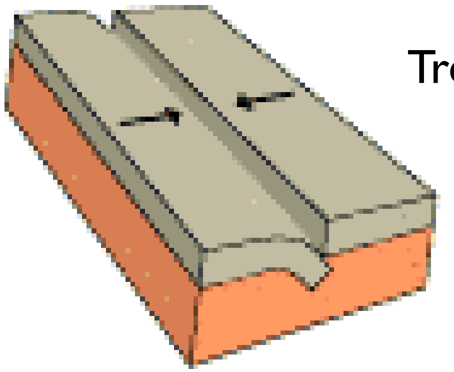


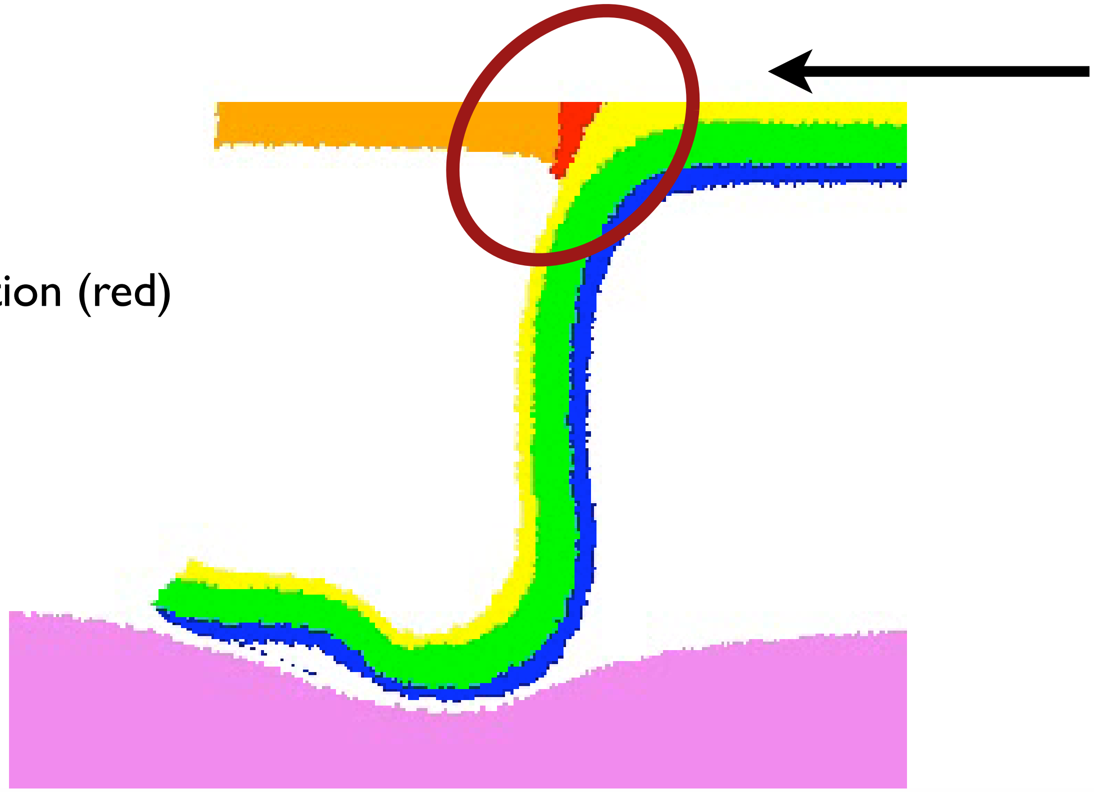
Example (3): Subduction

Wendy Sharples



Trench

Buoyant arc:	Anisotropic, fixed orientation (red)
Overriding plate:	Strong viscous (brown)
Upper layer:	Visco-plastic (yellow)
Core:	Viscous (green)
Lower layer:	Visco-plastic (blue)



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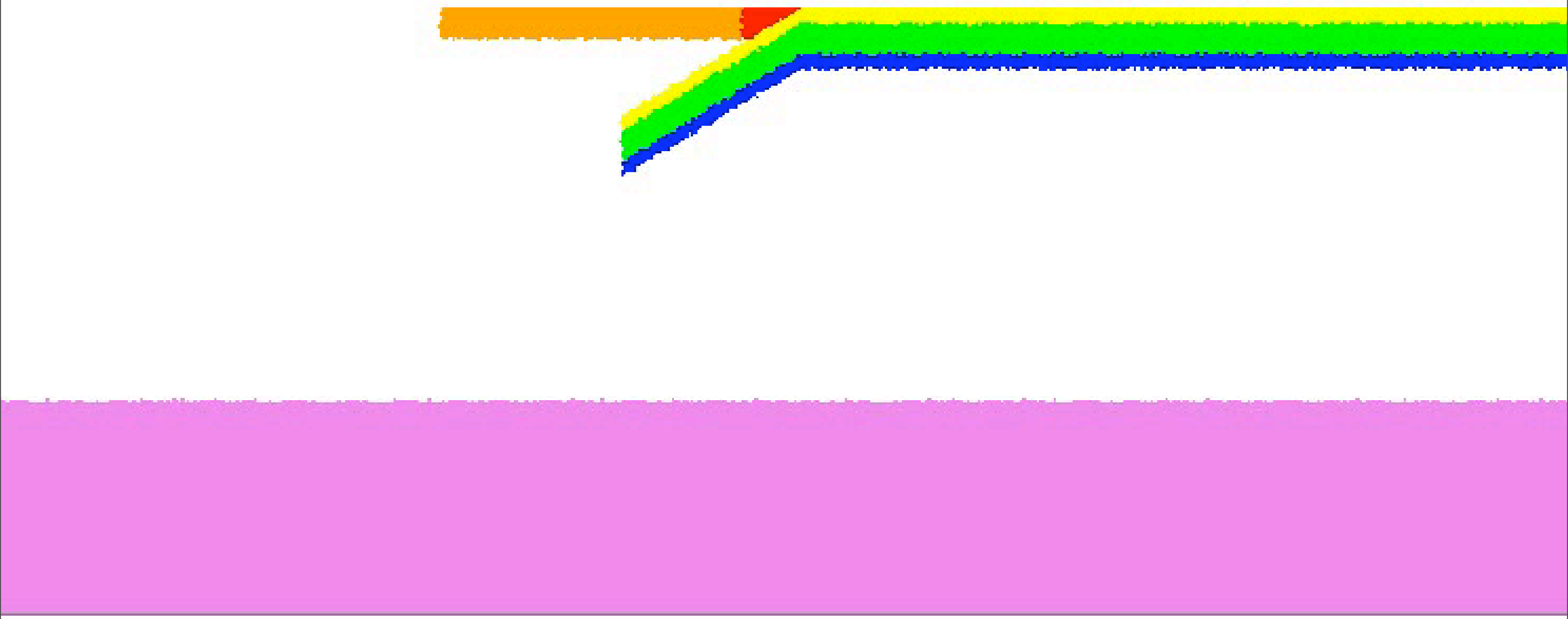
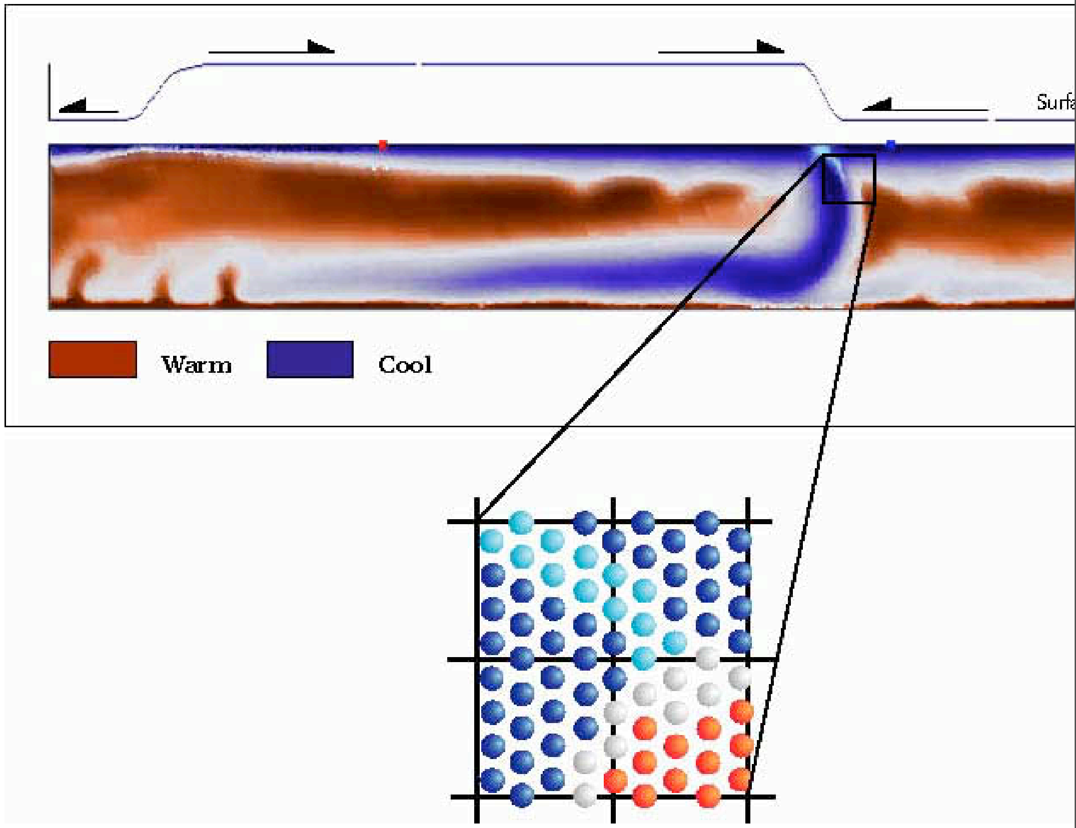


Numerical Realisation

In the style of the Material Point Method
"Fixed mesh with moving particles"

- Standard mixed Finite Elements used to obtain the velocity and pressure solution.
  - Inherits robustness, versatility of FEM
  - Admits general constitutive relations



- Lagrangian reference frame for:
  - Compositional tracking
  - *Stress- history tensor*
  - *Plastic strain history (scalar / tensorial)*
  - *Material orientation (anisotropy)*

This is the approach adopted in Underworld
www.mcc.monash.edu.au

Material Point Method

The connection between the FE formulation and the Lagrangian points is via evaluation of the weak form.

$$\mathbf{K}^E = \int_{\Omega_E} \mathbf{B}^T(\mathbf{x})\mathbf{C}(\mathbf{x})\mathbf{B}(\mathbf{x})d\Omega$$

$$\mathbf{K}^E = \sum_p w_p \mathbf{B}_p^T(\mathbf{x}_p)\mathbf{C}_p(\mathbf{x}_p)\mathbf{B}_p(\mathbf{x}_p)$$

Lagrangian points coincide with the quadrature points used to evaluate the weak form
- quadrature weights are defined locally over each element.
- weights are given by an approximate voronoi diagram.

The constitutive behaviour is associated with each particle "p" and is thus naturally incorporated into the quadrature.

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The discrete form of the momentum and continuity generated by the FEM may be expressed as

$$\begin{pmatrix} K & G \\ G^T & 0 \end{pmatrix} \begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} f \\ h \end{pmatrix}$$

The MPM formulation provided the means to discretize the Stokes flow.
But the issue of obtaining the flow field (u,p) is now a linear algebra problem...

Iterative solution techniques for Stokes' flow

The ideal approach should be *optimal* in the sense that the convergence rate of method will be bounded independently of

- any discretisation parameters *(Example; grid resolution)*
- the constitutive parameters *(Example; smoothly varying viscosity vs. discontinuous viscosity)*
- the constitutive behaviour *(Example; isotropic vs. anisotropic)*

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Our approach decouples the velocity and pressure unknowns. We solve

$$\boxed{Sp = G^T K^{-1} f - h,} \quad \text{where } S = G^T K^{-1} G$$

for pressure using an (outer) iterative method. This requires the operation,

$$y = Sx$$

S is defined in a matrix-free sense (i.e. not explicitly formed). The action of the inverse of K applied to vector necessitates another (inner) iterative method to define;

$$u^* \approx K^{-1} f^*$$

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EFFECTIVE PRECONDITIONERS FOR S ARE REQUIRED

Schur complement preconditioners

Simple diagonal approximation to S

Given $S = G^T K^{-1} G$, we let

$$K_d = \mathrm{diag}\,(K)\,,$$

then an approximate Schur complement is

$$\hat{S} = G^T K_d^{-1} G. \qquad \text{(explicit)}$$

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Preconditioning requires us to define the operation

$$y = \hat{S}^{-1} x.$$

This can be given via a factorisation such as Incomplete Cholesky, ICC(k)

The choice

$$\hat{S}_d = \mathrm{diag}\left(G^T K_d^{-1} G\right)$$

is often made. The mantle convection code CITCOM is one example.

We will use $\hat{S}_d^{-1}$ as a reference preconditioner in our comparisons.

Schur complement preconditioners

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Observation: We don't anticipate the diagonal preconditioner to be spectacular since it ignores ALL the coupling in the K block.

This can be given via a factorisation such as Incomplete Cholesky, ICC(k)

The choice

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Approximate Commutator

Consider using commutators to construct an approximation for S,
as is done in: Kay, et. al, *SIAM J. Sci. Comput.*, **24** (2002)

For isoviscous Stokes, the commutator relation used is;

$$\bar{Z} = \left(\eta \nabla^2\right) \nabla - \nabla \left(\eta \nabla^2\right) = 0$$

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$$Z = KG - GK_p \approx 0$$

Elman, et. al, *SIAM J. Sci. Comput.*, **27** (2006)

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Elman, et. al, *SIAM J. Sci. Comput.*, **27** (2006)

Minimise via the normal equation to give

- $K_p = \left(G^T G\right)^{-1} G^T K G$

Assuming $Z = 0$, we can define

- $\hat{S}_b = \left(G^T G\right) K_p$

Approximate Commutator

This leads to the preconditioner

$$
\begin{aligned}
\hat{S}_b^{-1} &= K_p^{-1}\left(G^T G\right)^{-1} \\
&= \left(G^T G\right)^{-1} G^T K G \left(G^T G\right)^{-1}
\end{aligned}
$$

which is the BFBt preconditioner from; Elman, *SIAM J. Sci. Comput.*, **17** (1996)

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This leads to the preconditioner

$$\hat{S}_b^{-1} = K_p^{-1} \left(G^T G\right)^{-1}$$
$$= \left(G^T G\right)^{-1} G^T K G \left(G^T G\right)^{-1}$$

+ No inversion of K required

+ Coupling in K preserved

- Two Poisson solves required

+ Most problems require low precision Poisson solves

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Approximate Commutator

This leads to the preconditioner

$$\hat{S}_b^{-1} = K_p^{-1}\left(G^T G\right)^{-1} = \left(G^T G\right)^{-1} G^T K G \left(G^T G\right)^{-1}$$

+ No inversion of K required

+ Coupling in K preserved

- Two Poisson solves required

+ Most problems require low precision Poisson solves

which is the BFBt preconditioner from; Elman, *SIAM J. Sci. Comput.*, **17** (1996)

For finite elements, the assumption $Z \sim 0$, is not automatically enforced. Scaling of the operators K and G is essential to reduce |Z|.

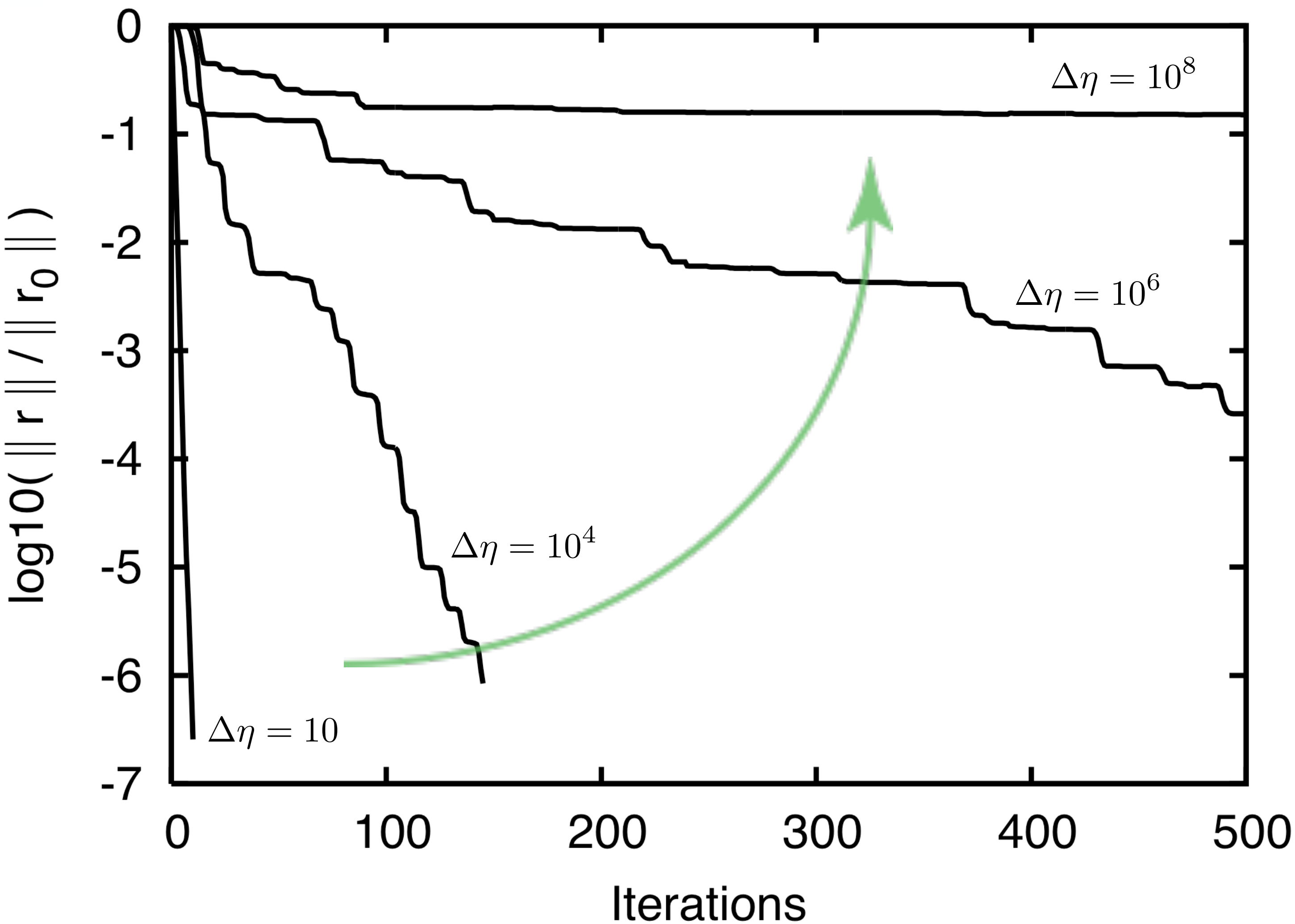
$$\begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}^{-1} \begin{pmatrix} K & G \\ G^T & 0 \end{pmatrix} \begin{pmatrix} X_1 & 0 \\ 0 & X_2 \end{pmatrix}^{-T} = \begin{pmatrix} K_s & G_s \\ G_s^T & 0 \end{pmatrix}$$

We let both X_1, X_2 be diagonal matrices and aim to enforce;

$$K_s \sim I \implies X_1^{-1} K X_1^{-T} \sim I$$

$$G_s^T G_s \sim I \implies G^T \left(X_1^{-T} X_1^{-1}\right) G \sim X_2 X_2^T$$

Effect of the scaling, X

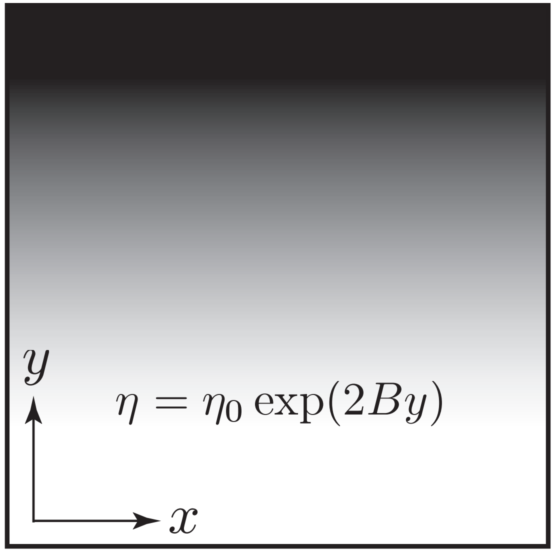


Scaling becomes more important as the viscosity contrast increases.

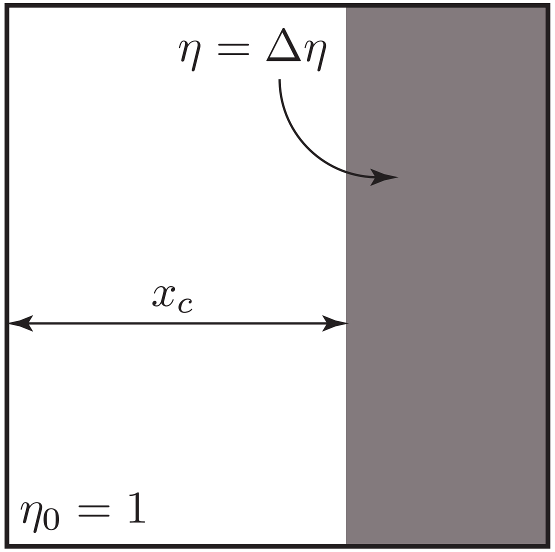
Prototypical Geodynamic Simulations

- As a starting point we only consider isotropic constitutive laws.
- Defined by single parameter, fluid viscosity eta.
- Only consider requirements 1 and 2

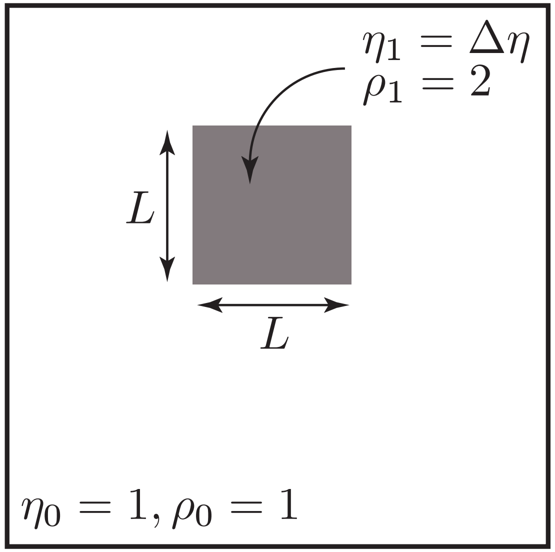
Exponentially varying with depth
"Exp(y)"



Step function in x
"Step(x)"



Step function in x & y
"Viscous sinker"



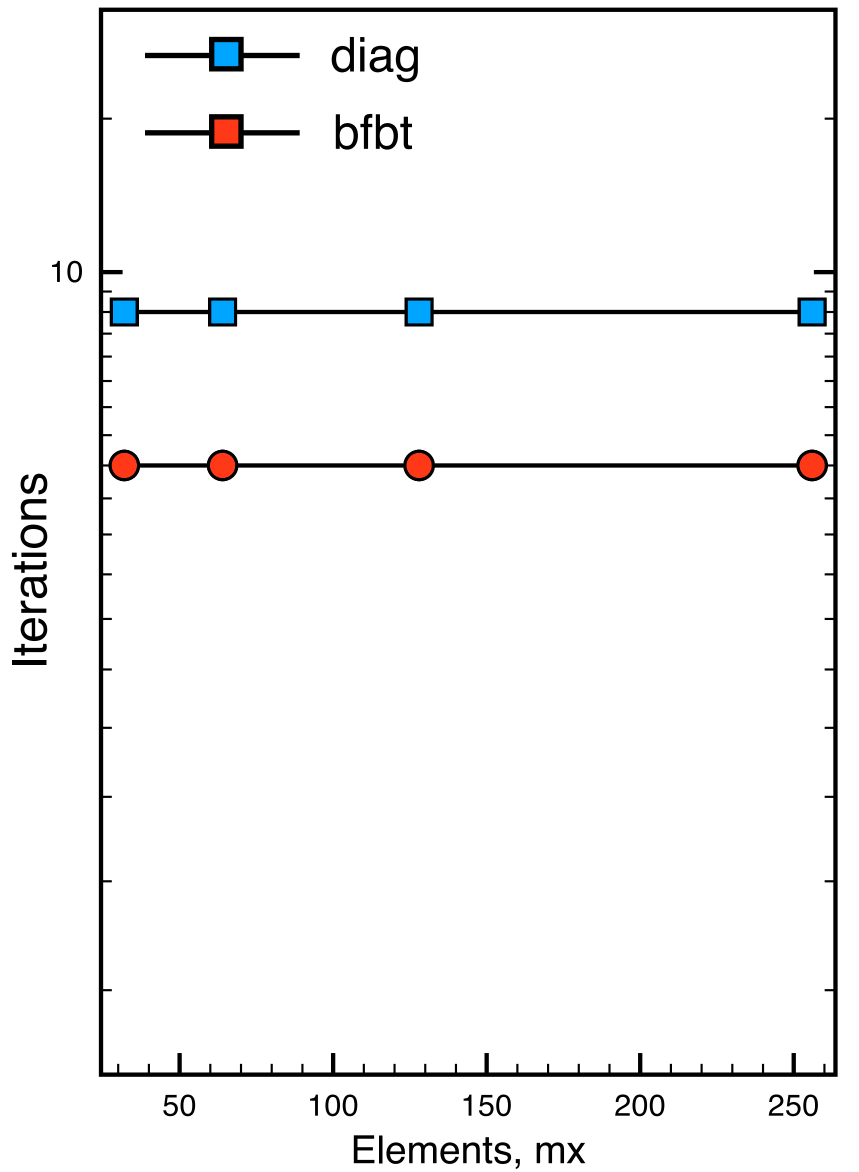
$$\text{Element resolutions} = \left\{\frac{1}{32}, \frac{1}{64}, \frac{1}{128}, \frac{1}{256}\right\}$$

$$\text{Viscosity contrasts} = \left\{10, 10^3, 10^6, 10^8\right\}$$

“Exp(y)”

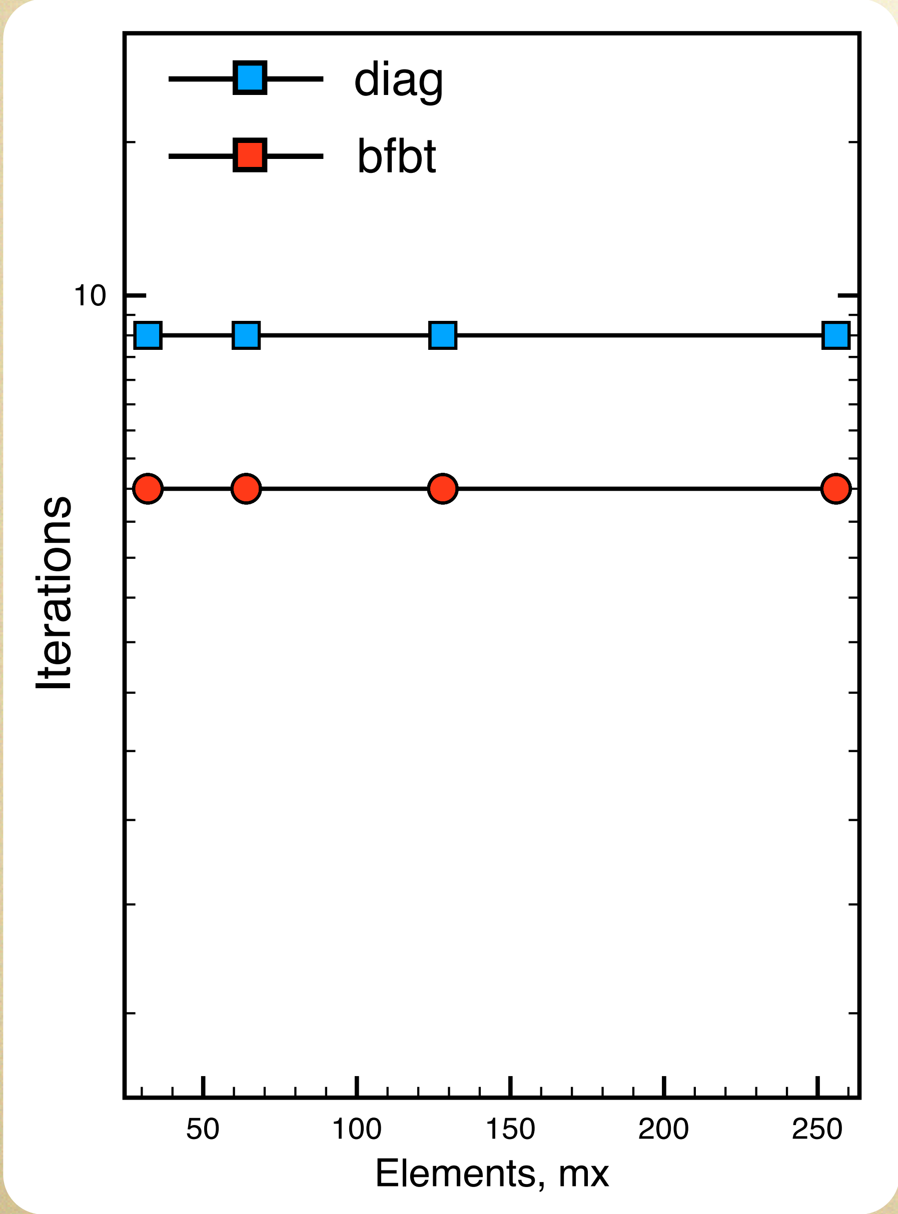
“Exp(y)”

h - dependence

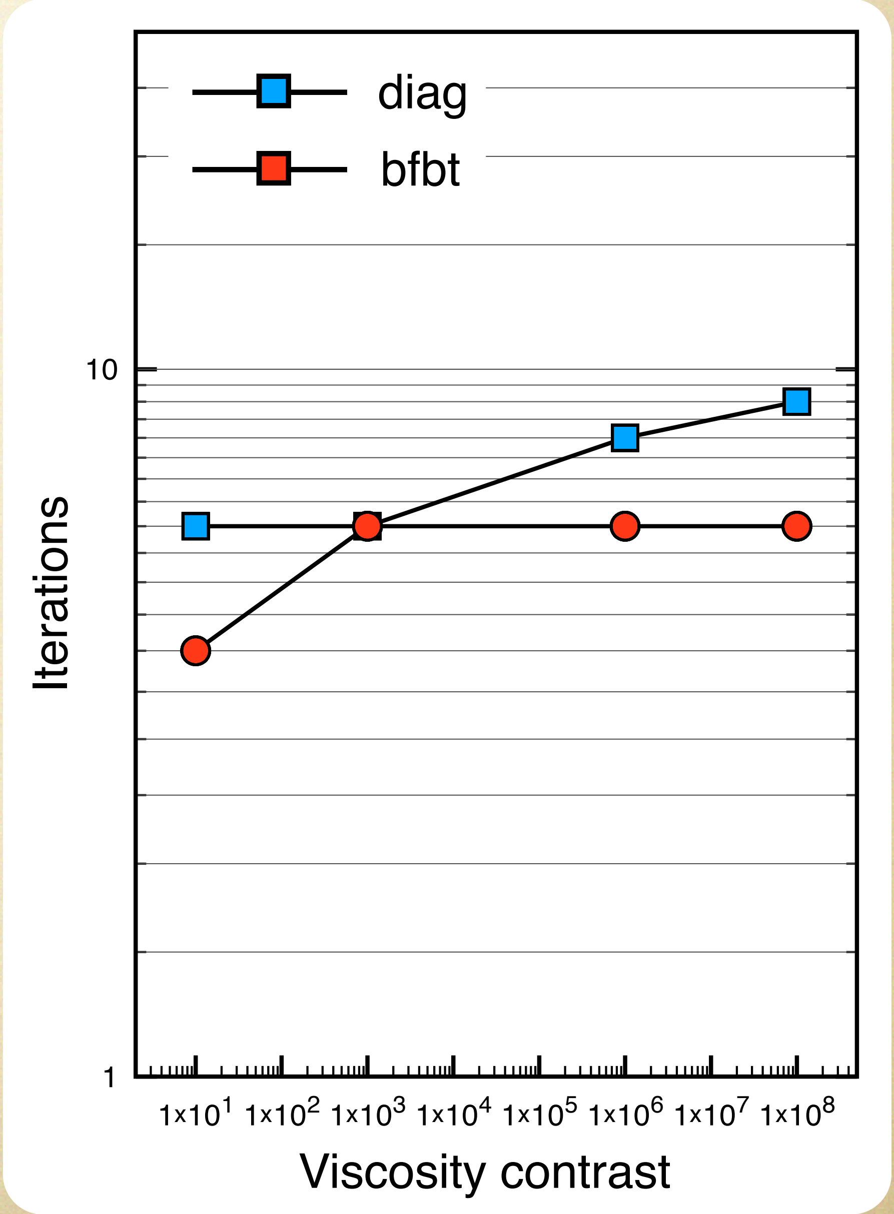


"Exp(y)"

h - dependence



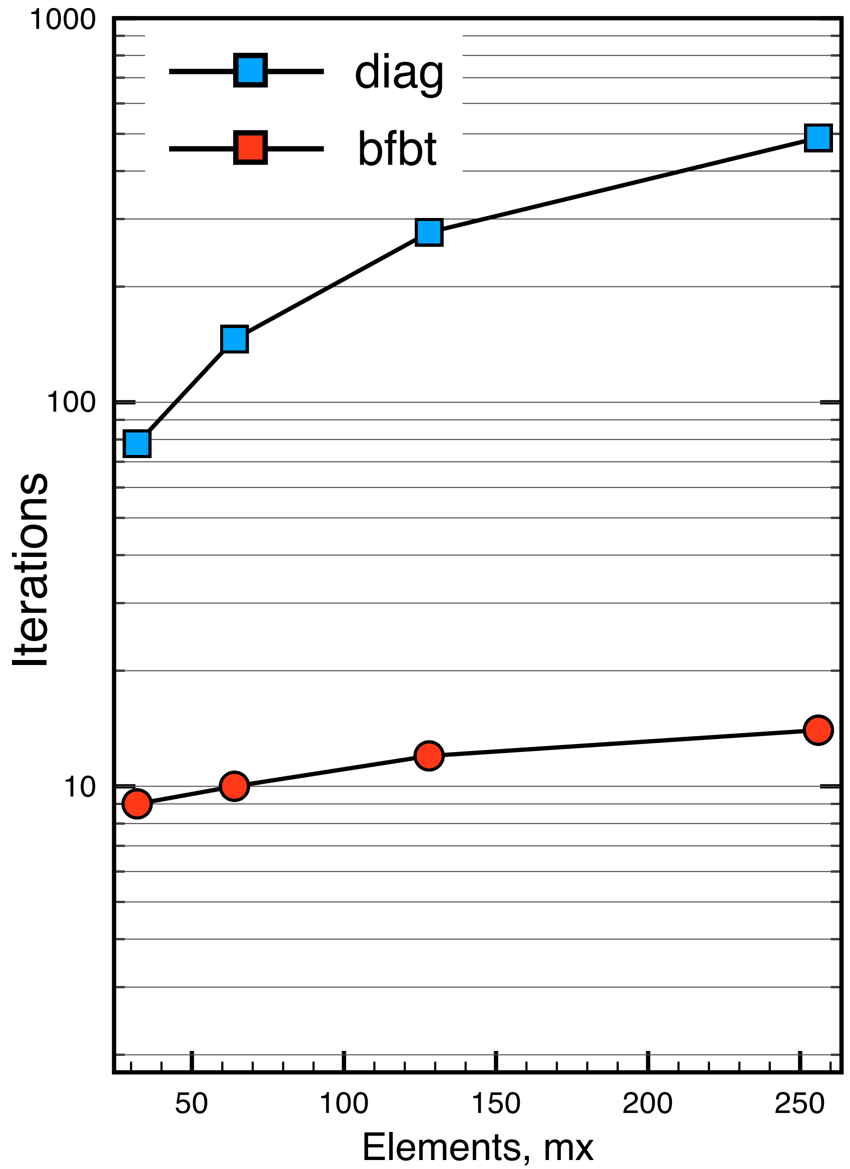
η - dependence



“Step(x)”

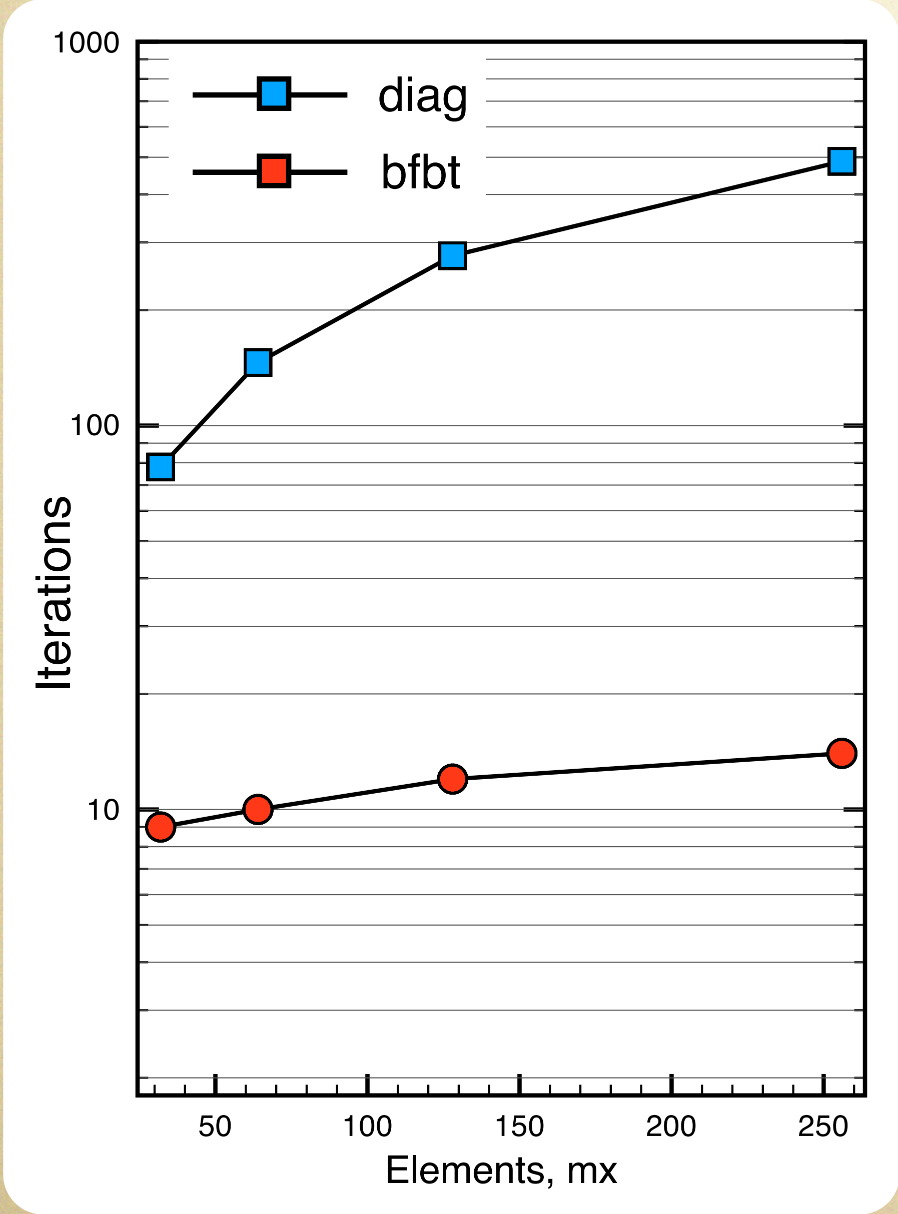
“Step(x)”

h - dependence

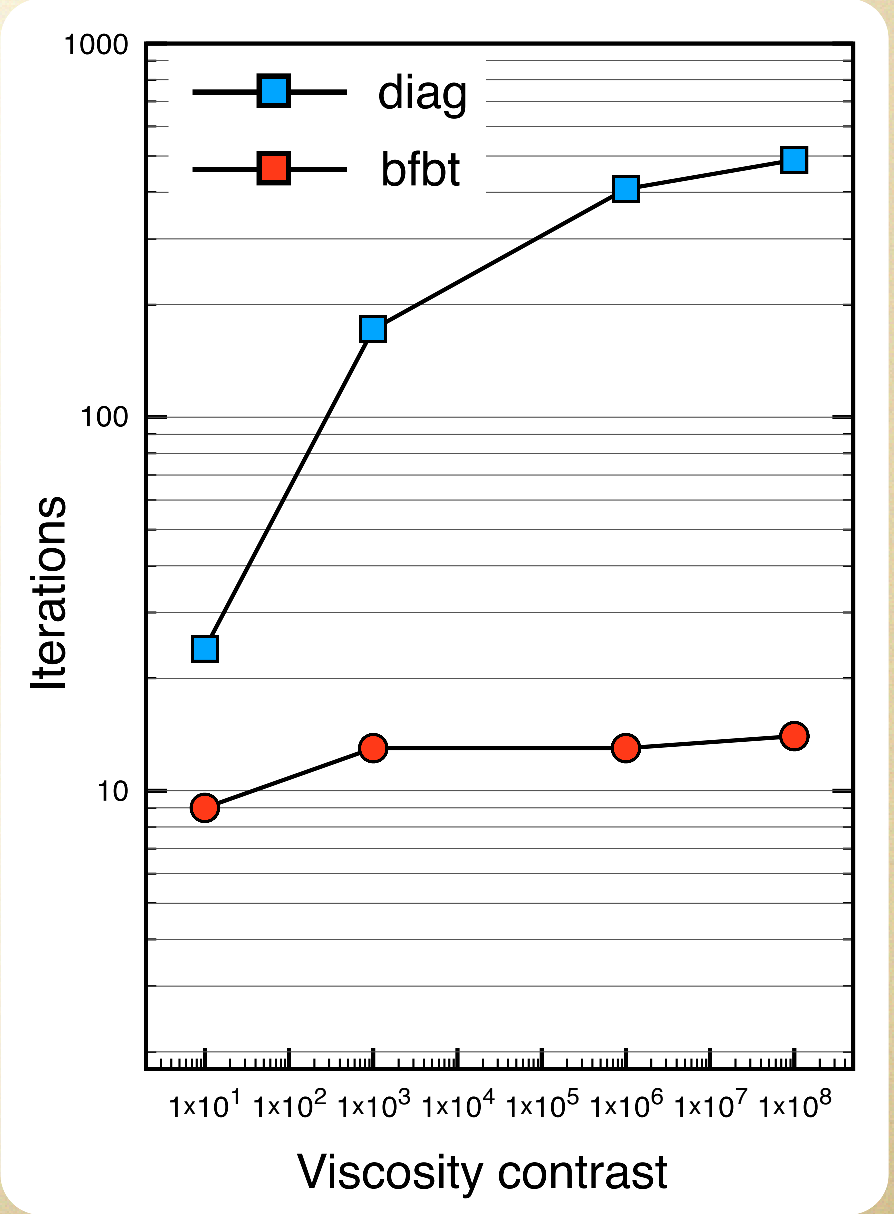


“Step(x)”

h - dependence



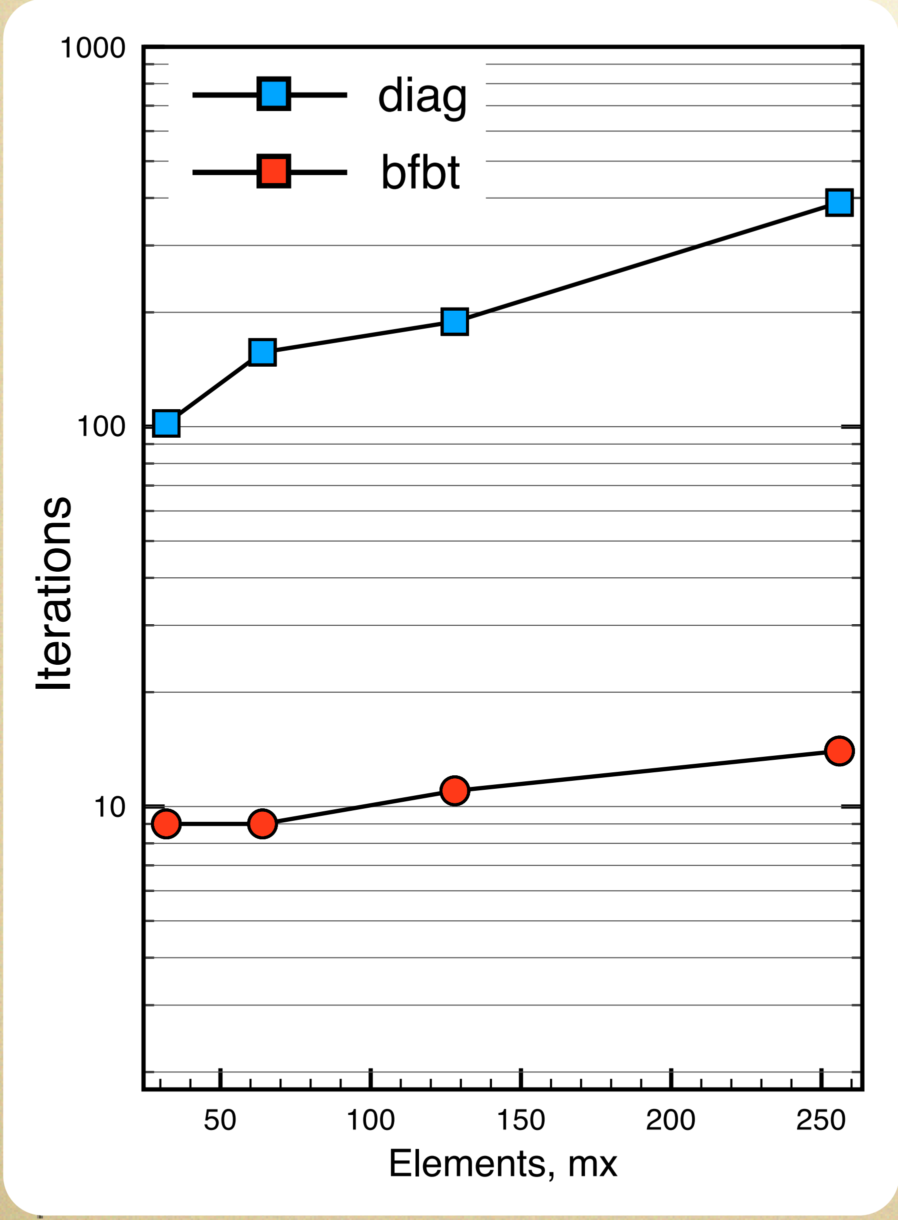
η - dependence



“Viscous sinker”

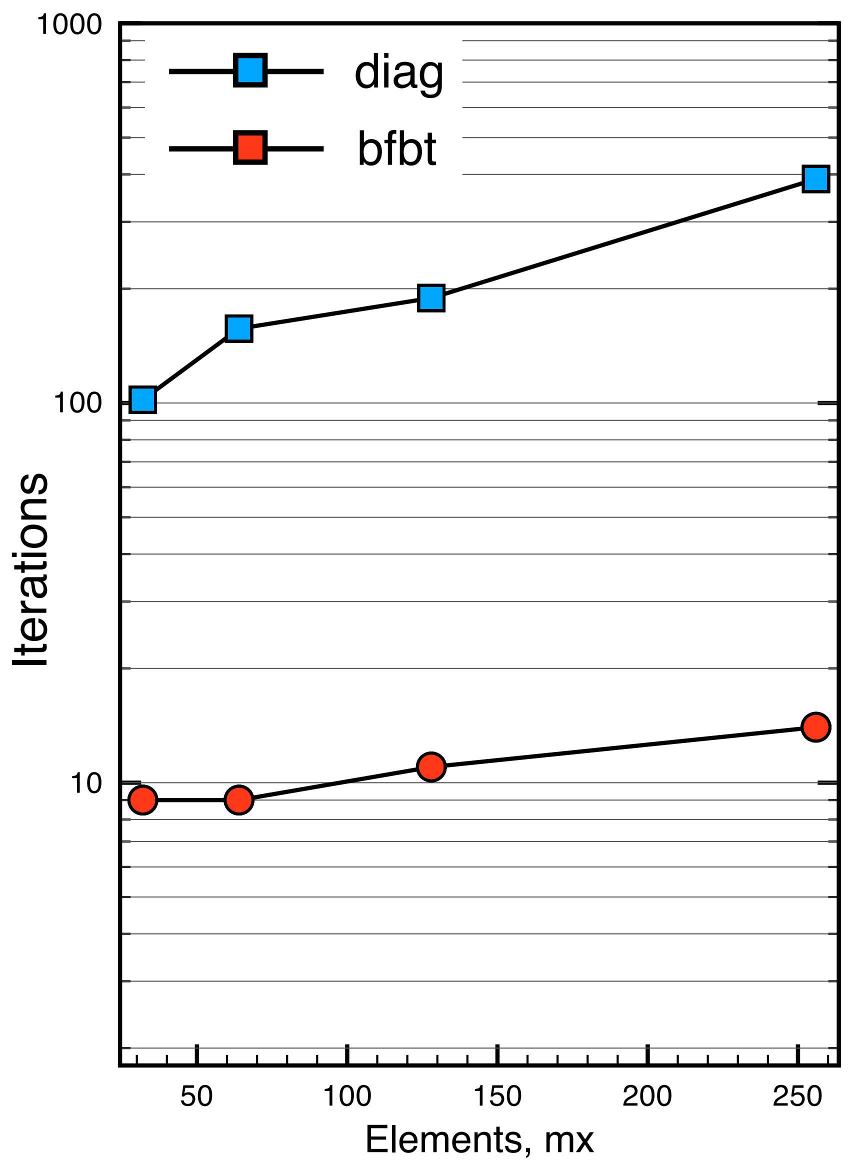
“Viscous sinker”

h - dependence

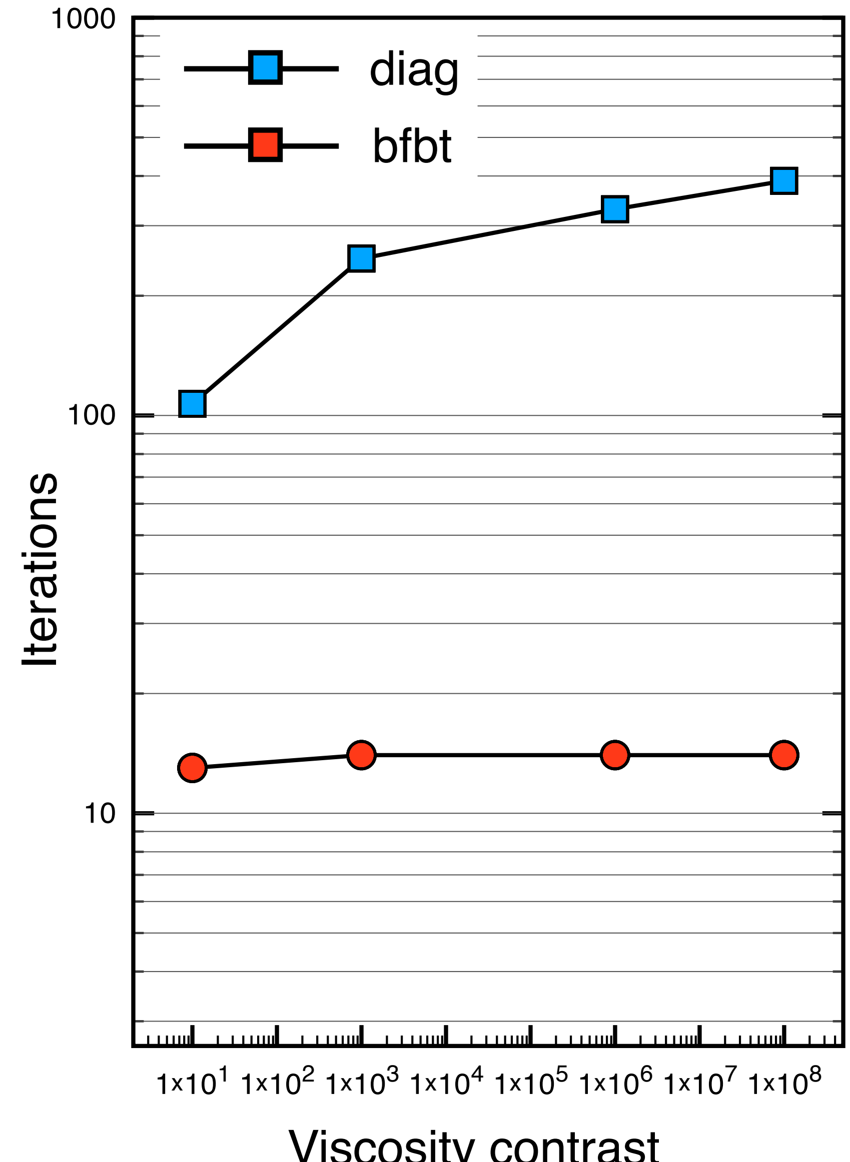


"Viscous sinker"

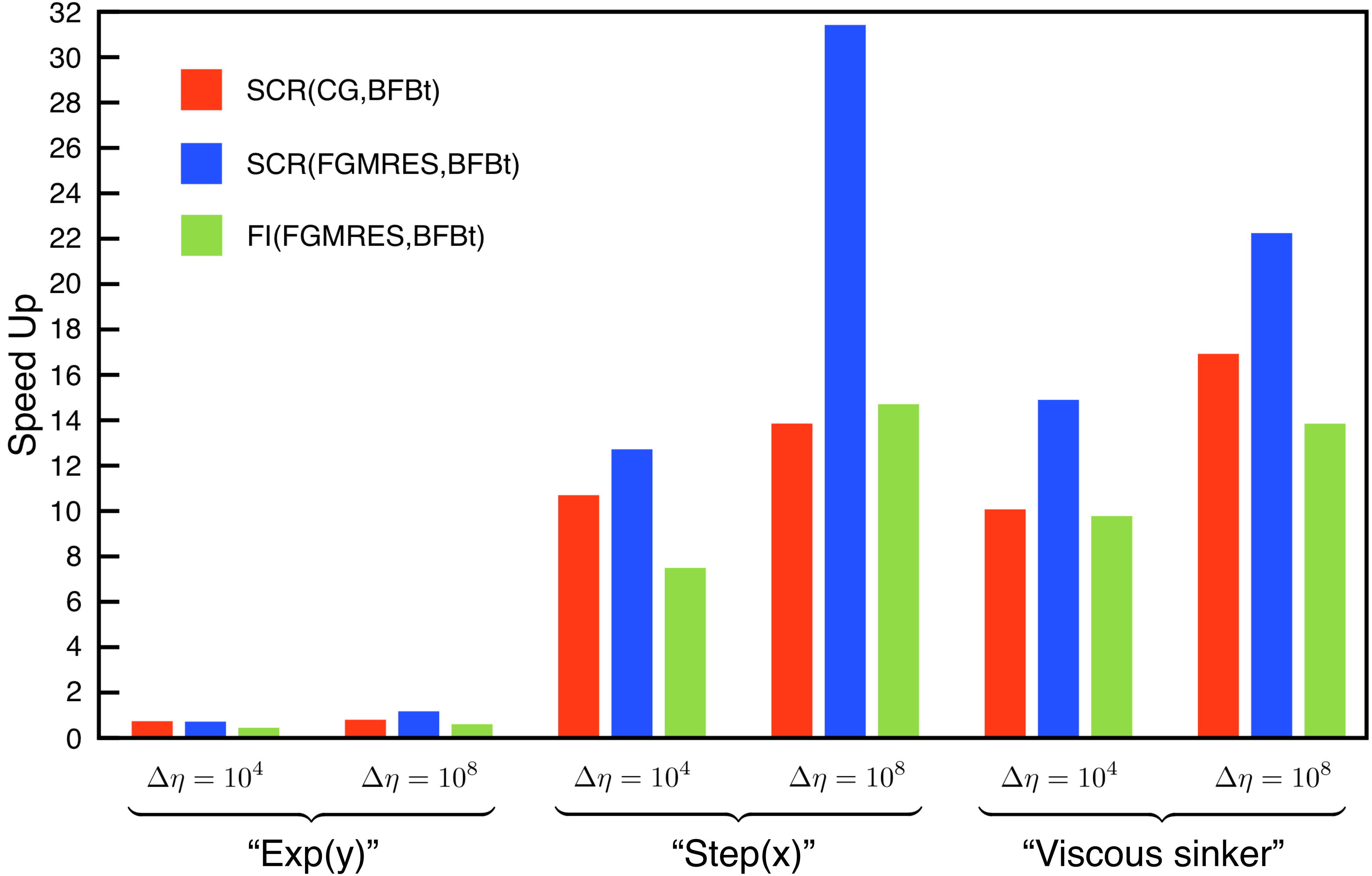
h - dependence



η - dependence



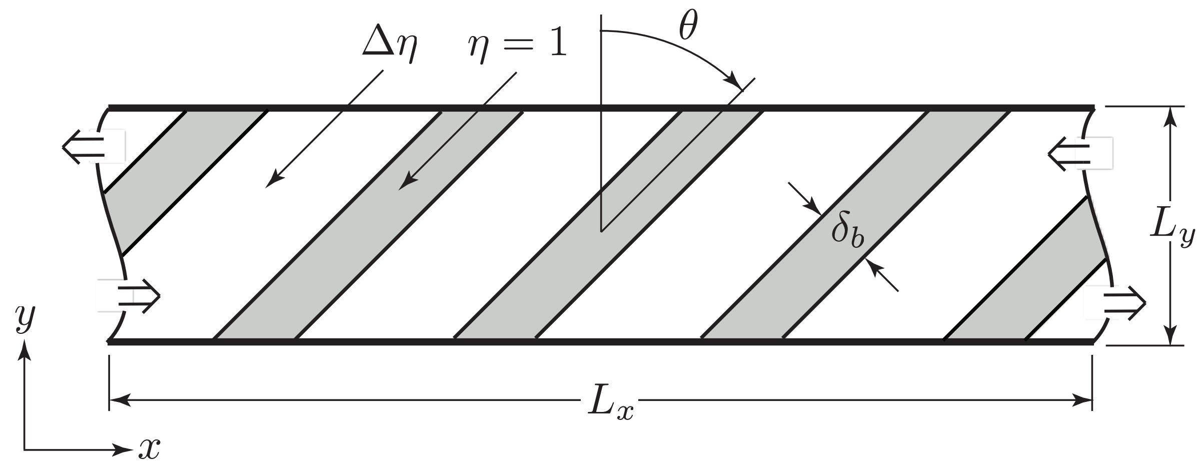
CPU Speed Up



Speed Up
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
32
SCR(CG,BFBt)
SCR(FGMRES,BFBt)
FI(FGMRES,BFBt)
$\Delta\eta = 10^4$
$\Delta\eta = 10^8$
$\Delta\eta = 10^4$
$\Delta\eta = 10^8$
$\Delta\eta = 10^4$
$\Delta\eta = 10^8$
"Exp(y)"
"Step(x)"
"Viscous sinker"

Applications: Localisation

Toy problem
Simple shear boundary conditions
Lx = Ly = 1
Band thickness: 1/64
Element resolution: 256 x 256



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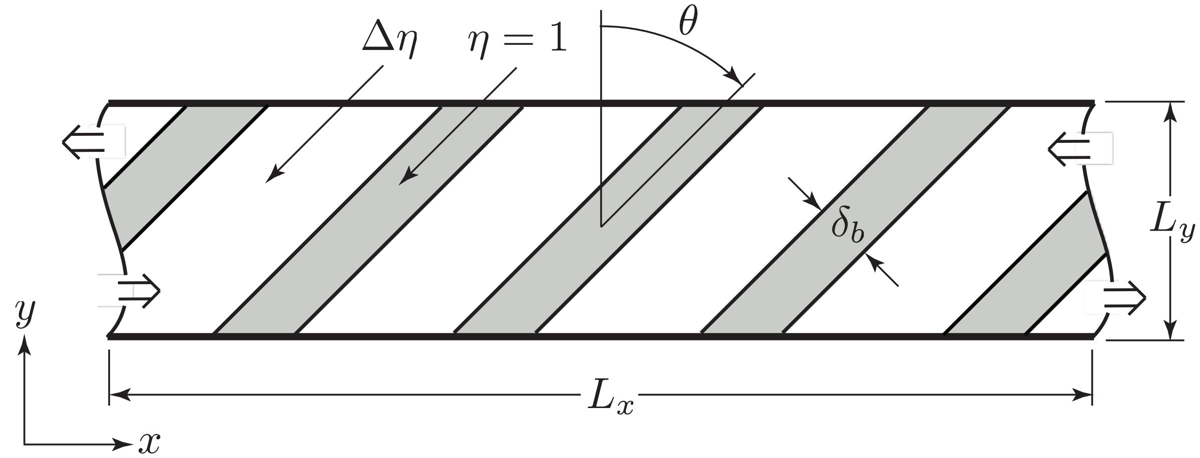

diag(S)

θ	$\Delta\eta = 10$	10^4	10^8
67.5°	321	3193	> 5000
45°	317	1490	> 5000
22.5°	310	1457	4402
0°	352	1010	2896
-22.5°	388	1495	4912
-45°	316	1324	> 5000
-67.5°	339	3083	> 5000

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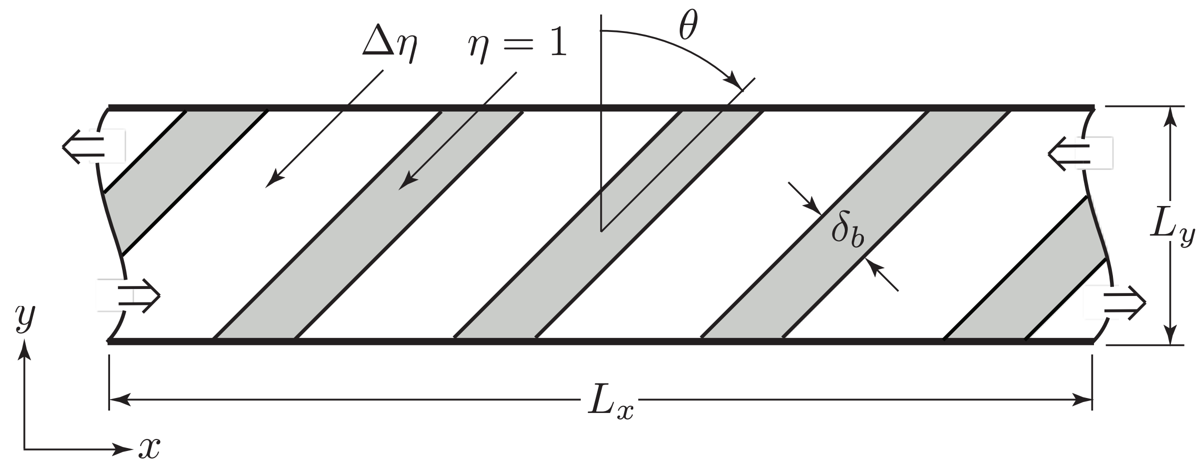
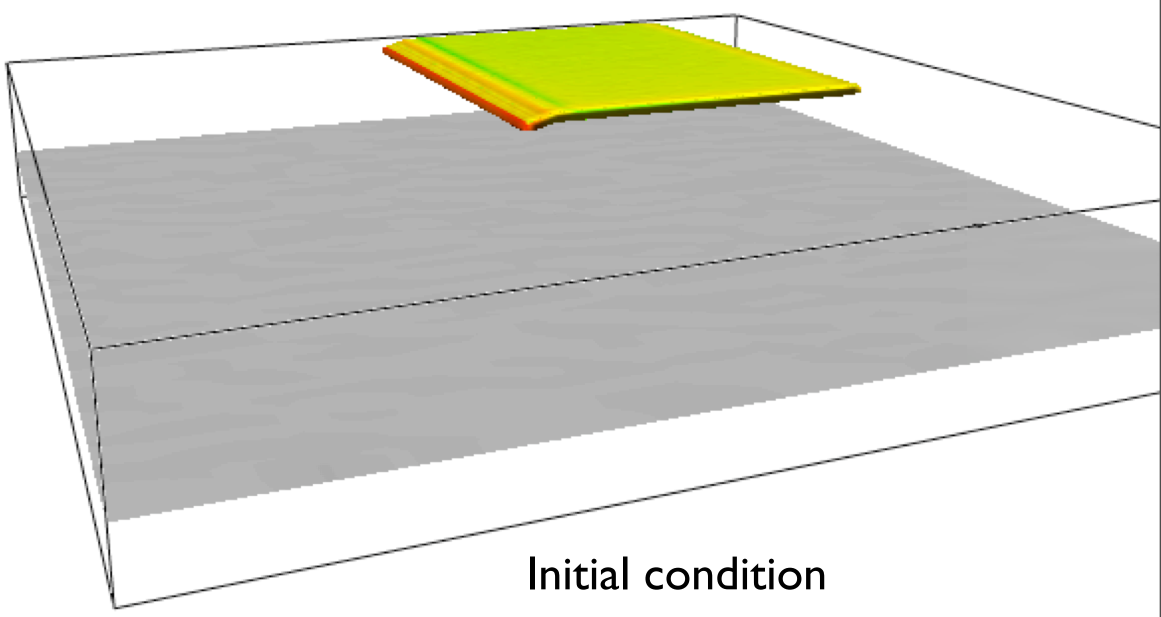


	diag(S)			BFBt		
	$\Delta\eta$			$\Delta\eta$		
θ	10	10^4	10^8	10	10^4	10^8
$67.5°$	321	3193	> 5000	42	59	19
$45°$	317	1490	> 5000	29	24	19
$22.5°$	310	1457	4402	28	21	18
$0°$	352	1010	2896	27	19	17
$-22.5°$	388	1495	4912	28	22	19
$-45°$	316	1324	> 5000	29	24	18
$-67.5°$	339	3083	> 5000	43	60	19

Applications: Subduction

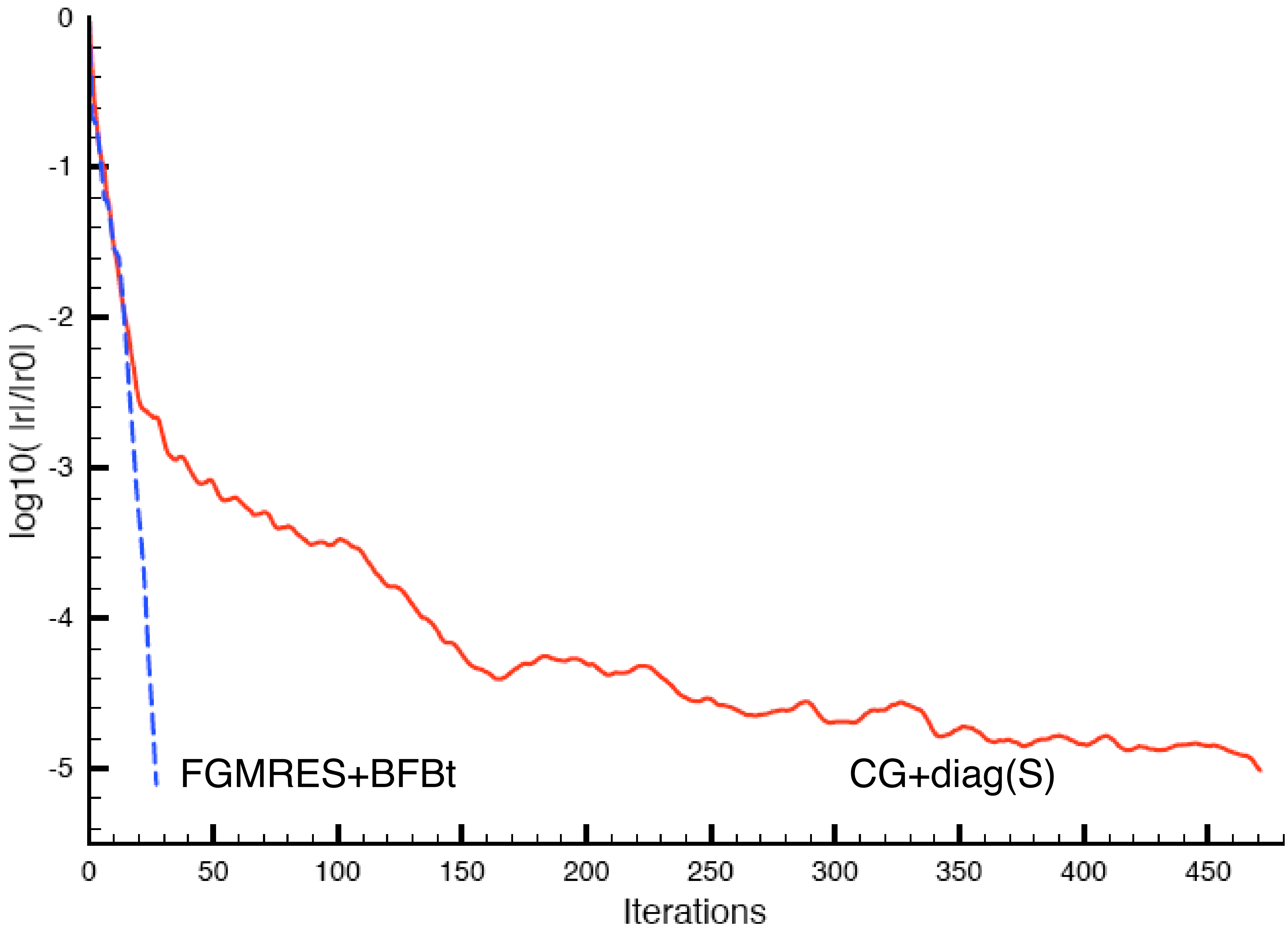
Take one of Dave Stegmans slabs... [Stegman etal. PEPI, In review (2008)]
  Rheological layering within the plate
  3 layers: viscoplastic - viscous - viscoplastic sandwich
  Maximum viscosity contrast is ~ 100

K solve: FGMRES/GMG
Lp solve: CG/ML
Fully parallel

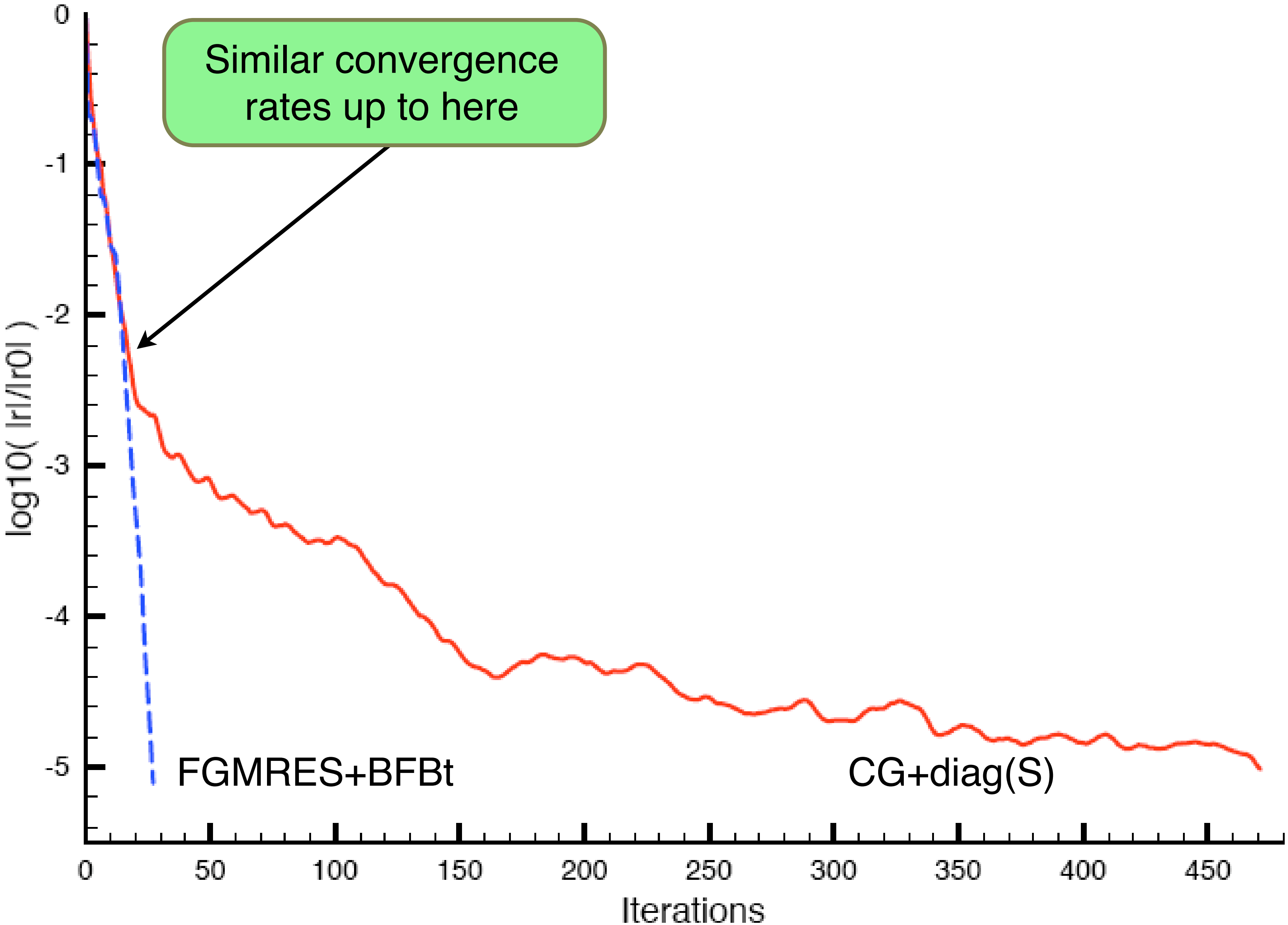


Initial condition

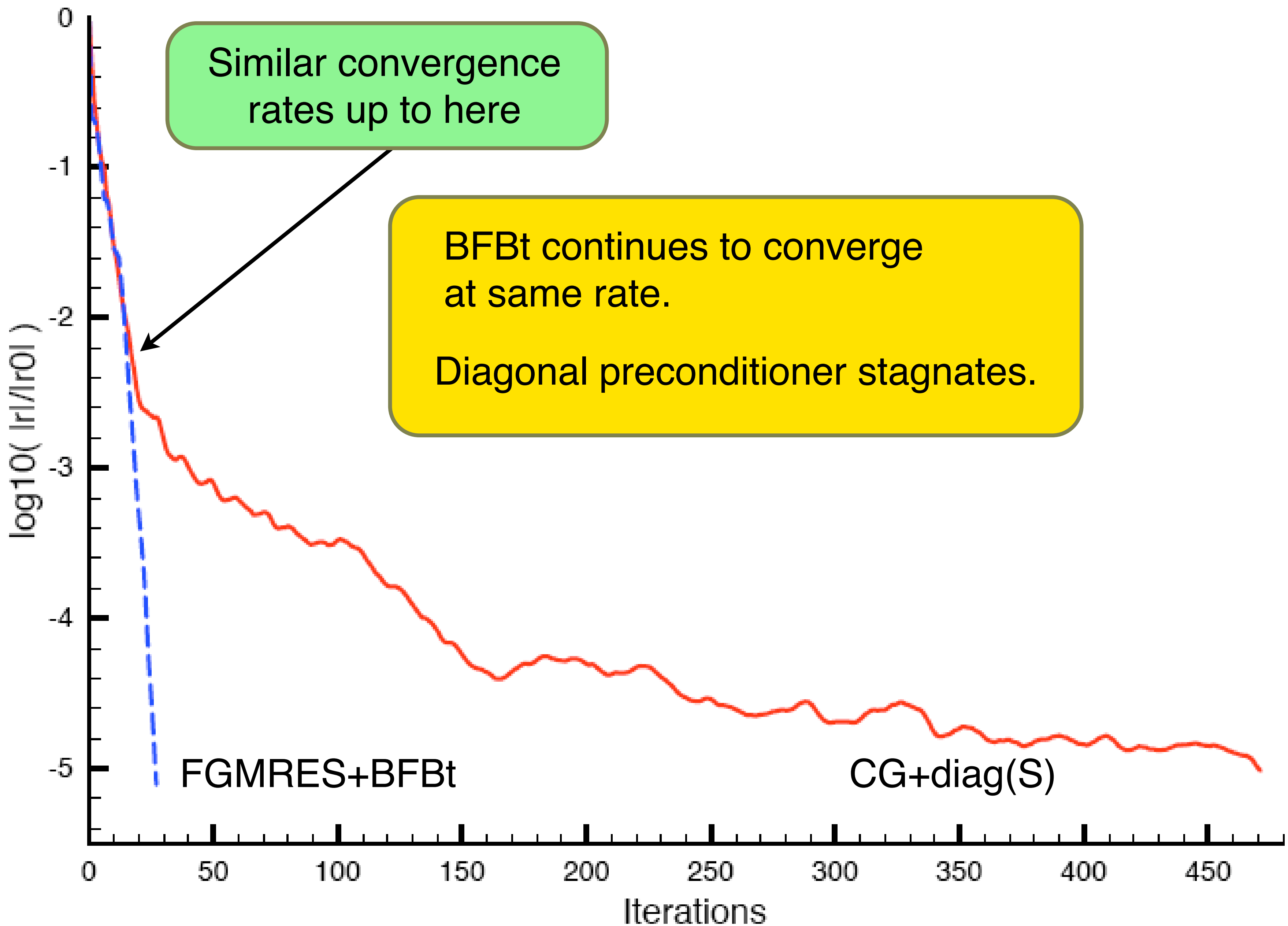
Applications: Subduction



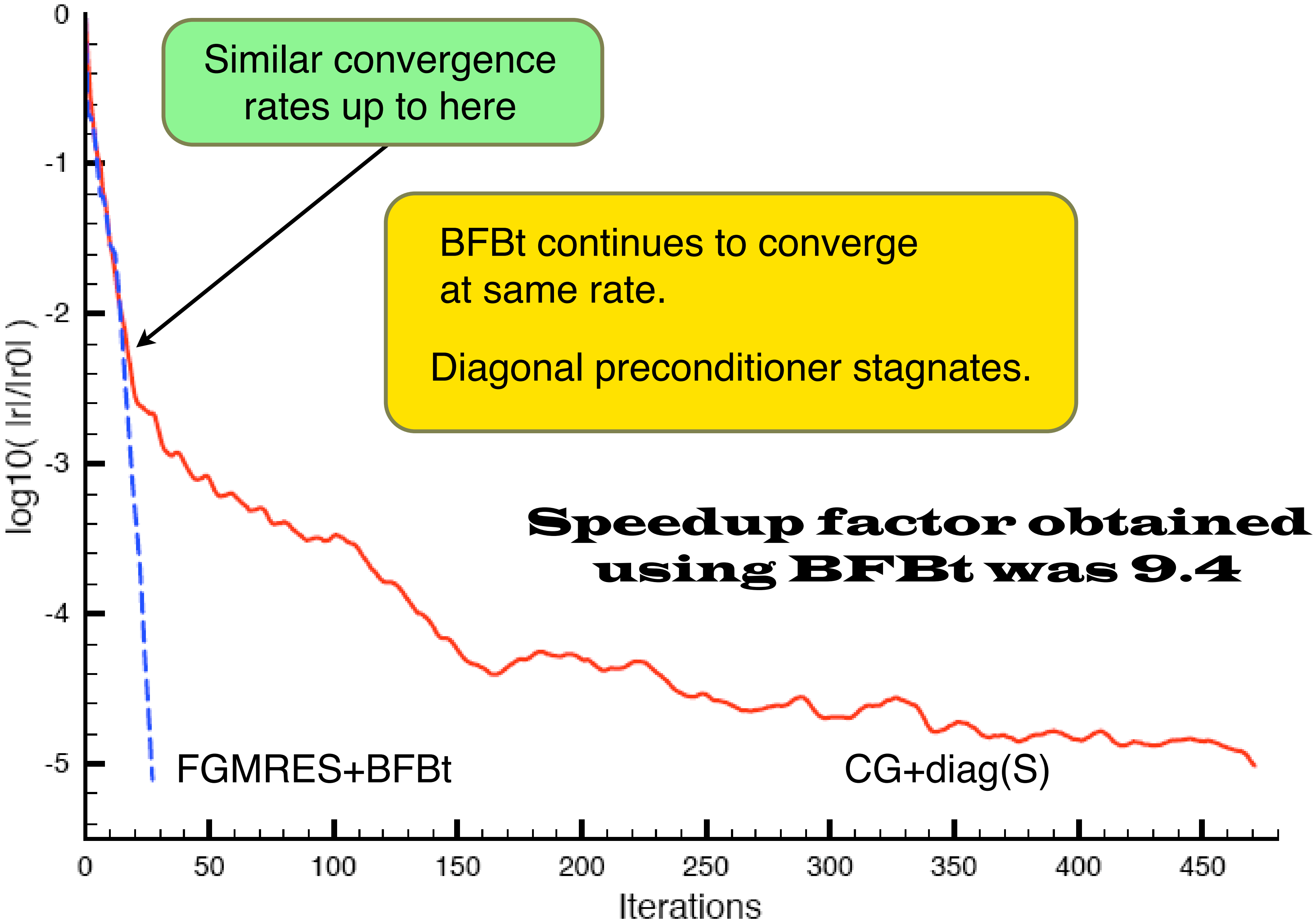
Applications: Subduction



Applications: Subduction



Applications: Subduction



Summary

- We considered two avenues to advance the numerical modelling of mantle convection/plate boundary
- 1) Rheology

  The anisotropic rheologies being investigated are starting to produce promising looking fault structures. More research is required to fully understand their behaviour and their use in representation plate boundaries.

- 2) Preconditioners

  The scaled BFBt preconditioner has proven to be an effective technique to solve discrete Stokes' problems relevant to geodynamics.

  Whilst it is not optimal for all experiments, the new approach is significantly faster and more robust than the commonly used diagonal approximation.

Questions?