Two and Three Dimensional Modeling Approaches to Lithosphere Deformation

Coupling with Surface Processes, Magmatism, and Fluid Flow

- Ritske Huismans
- Extended Dalhousie Geodynamics Group
- Chris Beaumont, Jean Braun,
- Philippe Fullsack, Ritske Huismans,
- Christina Morency, Karen Simon,
- Susanne Buiter, Lykke Gemmer, …..

Department of Oceanography, Dalhousie University, Halifax

Topics

- Fluid dynamics approach to Geodynamical/Tectonic problems
  – Viscous creeping flows (Stokes flows)
  – Solving plastic problems using a viscous approach
- Large displacement / Large Deformation Flows
  – Arbitrary Lagrangian Eulerian formulation
  – Importance of the upper free surface
  – Surface Processes - Erosion and Sedimentation
- Typical ALE model thermo-mechanical calculation
- Some examples of model results
- Future directions for two dimensional modeling
- Progress on state of the art three dimensional modeling tools

Viscous Creeping Flows

- Equilibrium Equation:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + \rho g_i = 0$$

- Viscous Incompressible Flow:

$$\sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\varepsilon}_{ij} \text{ and using } \dot{\varepsilon}_{ij} = \tfrac{1}{2}\left(\partial v_i / \partial x_j + \partial v_j / \partial x_i\right)$$

- Stokes Equation:

$$-\frac{\partial p}{\partial x_j} + \eta \frac{\partial}{\partial x_i}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) + \rho g_j = 0 \qquad j = 1,2$$

Rheologies

- Non-Linear Viscous Rheology:

$$\eta_{eff}^{v} = A^{-1/n} \cdot (\dot{I}_2^{/})^{(1-n)/2n} \cdot \exp\left[\frac{Q+Vp}{nRT}\right]$$

- Non-Linear (Frictional) Plastic Rheology

$$(J_2^{/})^{1/2} = c + \alpha \cdot p \cdot Sin\,\phi$$

$$\eta_{eff}^{p} = (J_2^{/})^{1/2} / 2(\dot{I}_2^{/})^{1/2}$$

Viscous-Elastic Flows

- Compressible Visco-Elastic Flow

$$-\frac{\partial p}{\partial x_i}+\eta_{eff}^{ve}\frac{\partial}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right)=-\rho g_j - \frac{\partial}{\partial x_j}\left(\eta\theta\,\tau_{ij}^{n-1}\right)$$

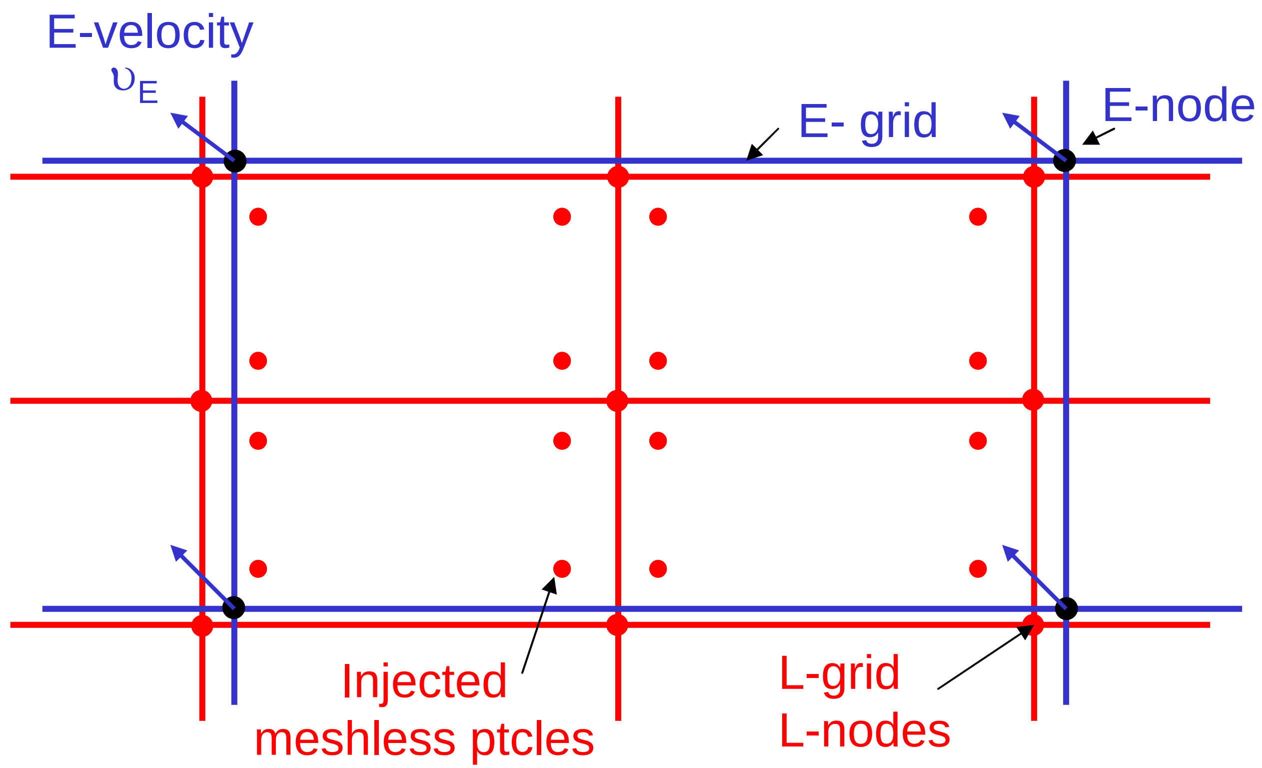
- Effective visco-elastic viscosity

$$\eta_{eff}=\frac{1}{1/(2\mu)+1/(2\Delta t G)},\quad \text{and}\quad \theta=\frac{1}{2\Delta t G}$$

- Memory terms:

$$\tau_{ij}=\eta\dot{\varepsilon}_{ij}^{n-1}+\eta\theta\tau_{ij}^{n-1},\quad \text{and}\quad P=P^{n-1}-\frac{\Delta t K}{3}\frac{\partial v_j^{n-1}}{\partial x_j}$$

The ALE Techniques: E and L Grids



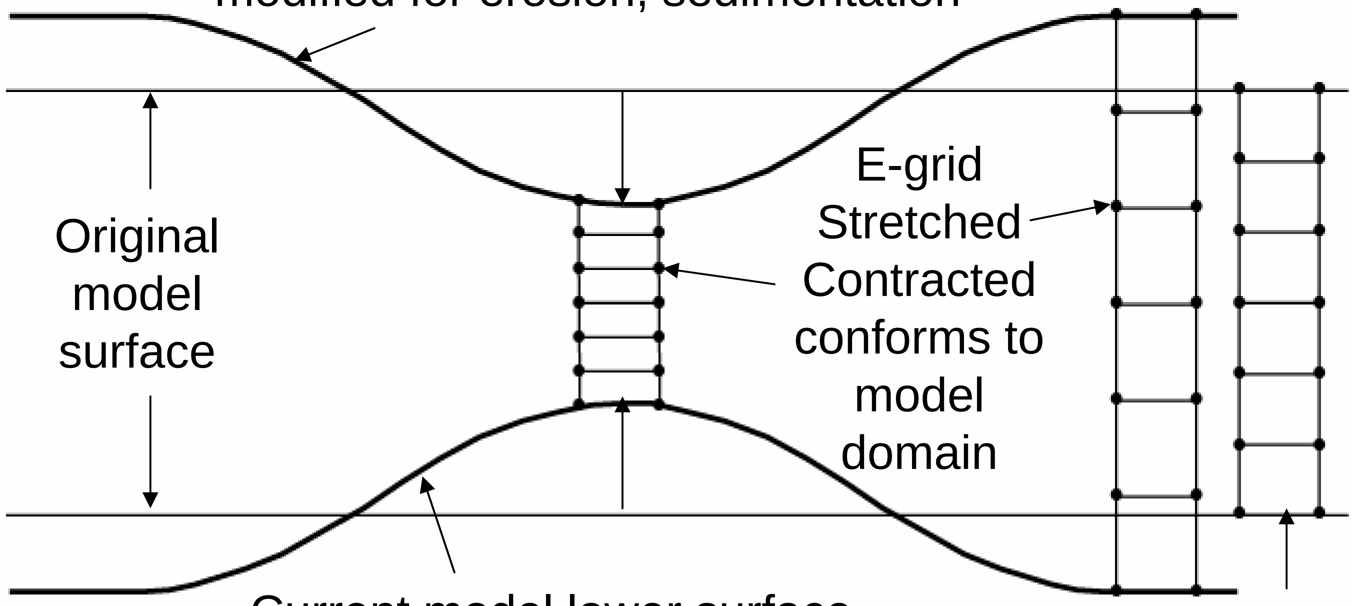
Finite element problem is solved on the E-grid

L-ptcles are located at the L-grid nodes and are injected within the E-elements

L-ptcles act as a moving ‘cloud’ to advect information on material type, strain, temperature, etc.

This information is re-interpolated back on the E-elements.

E – Remeshing to Conform to Model Domain



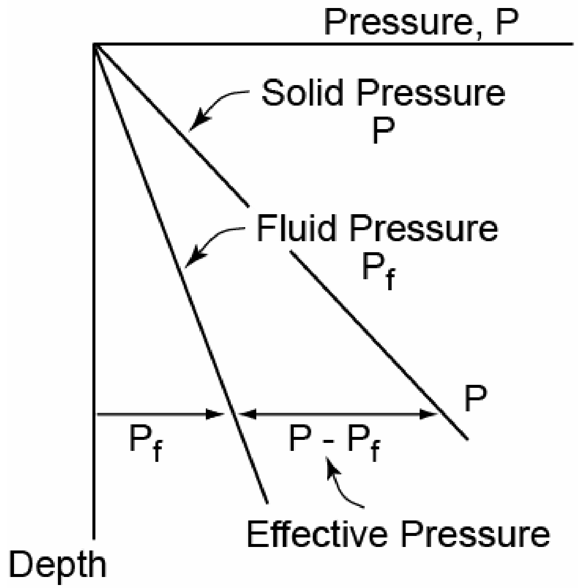
Finite element problem is solved on the E- grid.
The E – grid is stretched/contracted vertically to conform to the material domain.

Parametric Approach to:

- Effects of fluid pressure variation on strength of rock
- Strain weakening of plastic and viscous materials
- Effects of melting on strength of rock
- Surface process models

Simplified Rheological Stratification - 4

Effect of Pore Fluid Pressure



'τ' ~ $J_2 = C + (P - P_f)\sin\phi$

Let $P_f / P = \lambda$

$J_2 = C + P(1 - \lambda)\sin\phi$

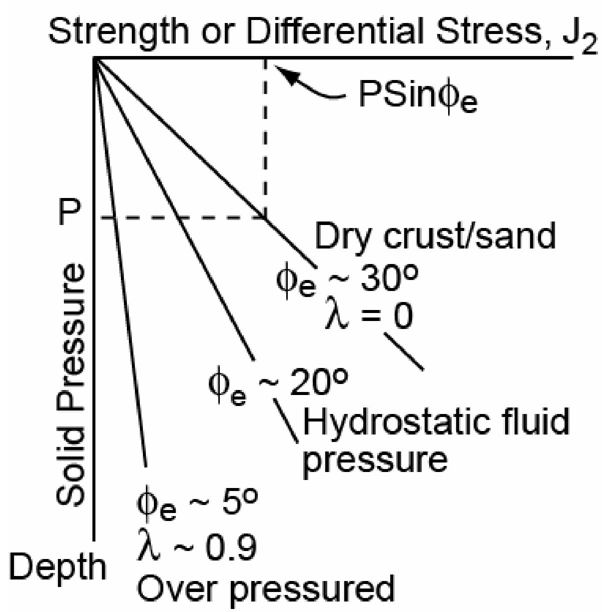
or, approximately

$J_2 = C + P\sin\phi_e$

where

$(1 - \lambda)\sin\phi = \sin\phi_e$

ϕ_e = effective ϕ

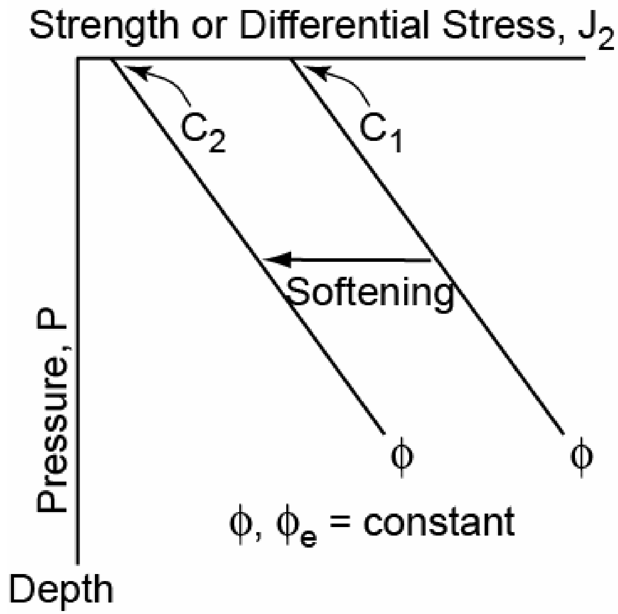


$\lambda = P_f / P$ = Hubbert-Rubey fluid pressure ratio

ϕ_e = effective ϕ modified (approx) for fluid pressure.

Simplified Rheological Stratification - 5

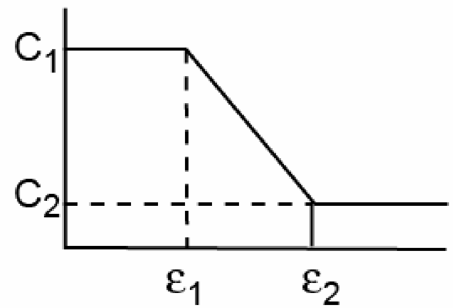
Strain - Dependent Properties - Cohesion, C



Cohesion Softening

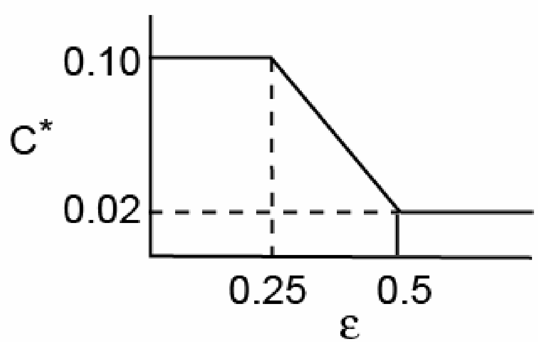
C decreases with strain, ε

(strictly f (ε_{1D}, ε_{2D},ε_{3D}))



Simple parameterization

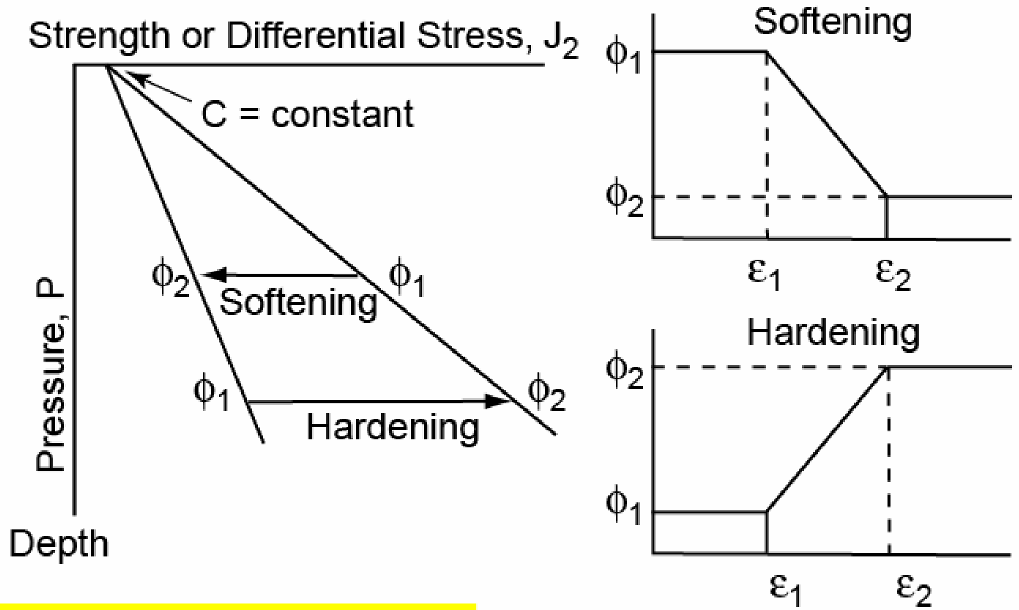
Case example:



$C^* = C / \rho gh = C / P$ = Cohesion / Pressure

Simplified Rheological Stratification - 6

Strain - Dependent Properties - Internal Angle of Friction, ϕ



Interpretation of $\phi_1 \rightleftharpoons \phi_2$

1) Effect of Pore Fluid Pressure

$\phi_{e1} \longrightarrow \phi_{e2}$, because $\lambda = \frac{P_f}{P}$ Increases or Decreases

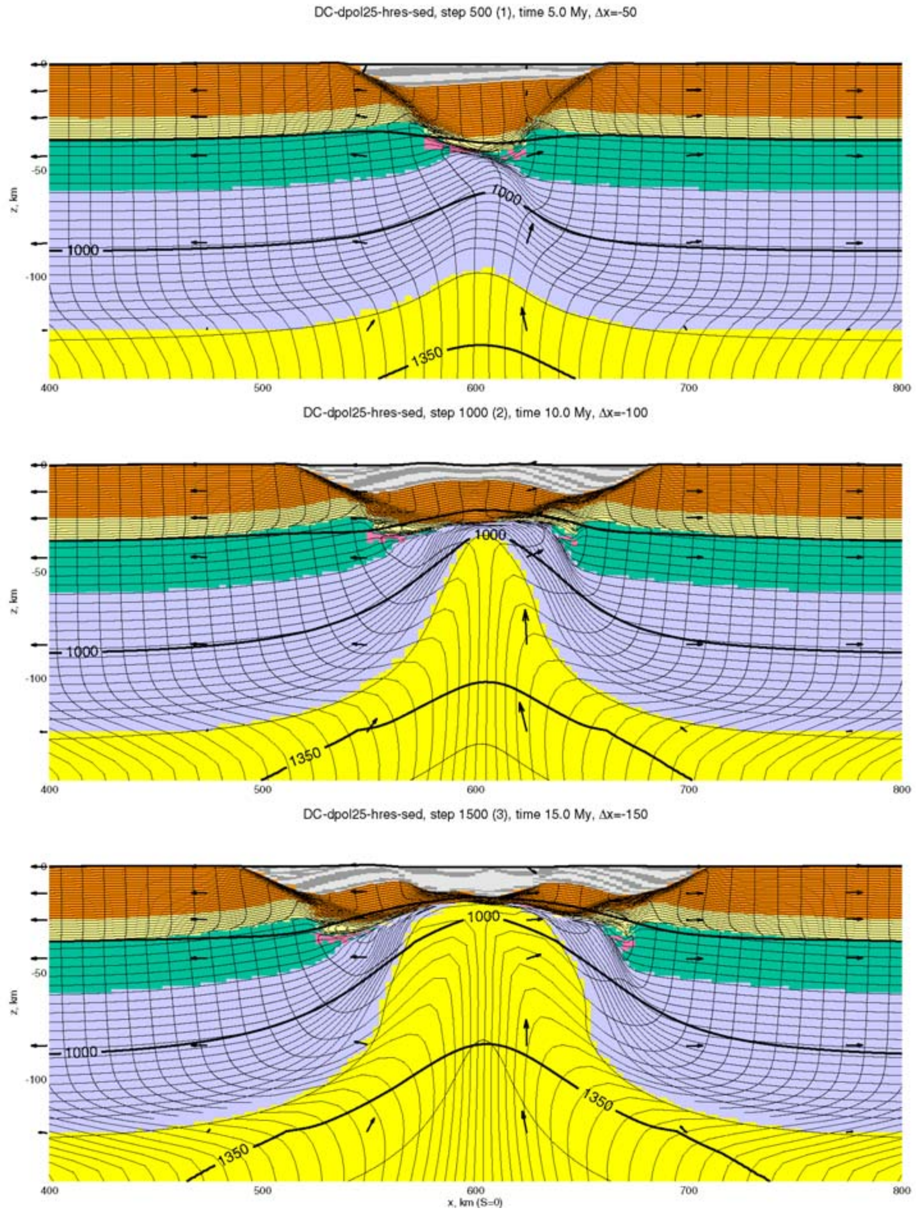
2) Mineral Reactions

$\phi_{e1}(37^o) \longrightarrow \phi_{e2}(17^o)$

by creation of weak phyllosilicates (Bos and Spiers, JGR, 2002)

3) In 'sandboxes' $\phi_1 \longrightarrow \phi_2$ caused by 'packing' differences - dilatant shears, dense pack where P is high

Weak Crust, Full Sedimentation&Erosion, Local Refinement



Current and Future Directions
2D Approaches

- Bridging of scales - improving resolution
  - Parallel solution
  - Nested model approaches
- Coupling fluid flow – deformation
- Magmatism – prediction and migration
- Phase changes
- Sediment interfaces

Questions

- Importance of Elasticity
- Dilatational Plasticity
- …

Gcube
A New Three-Dimensional Modelling Tool

Jean Braun, Philippe Fullsack & Marthijn de Kool

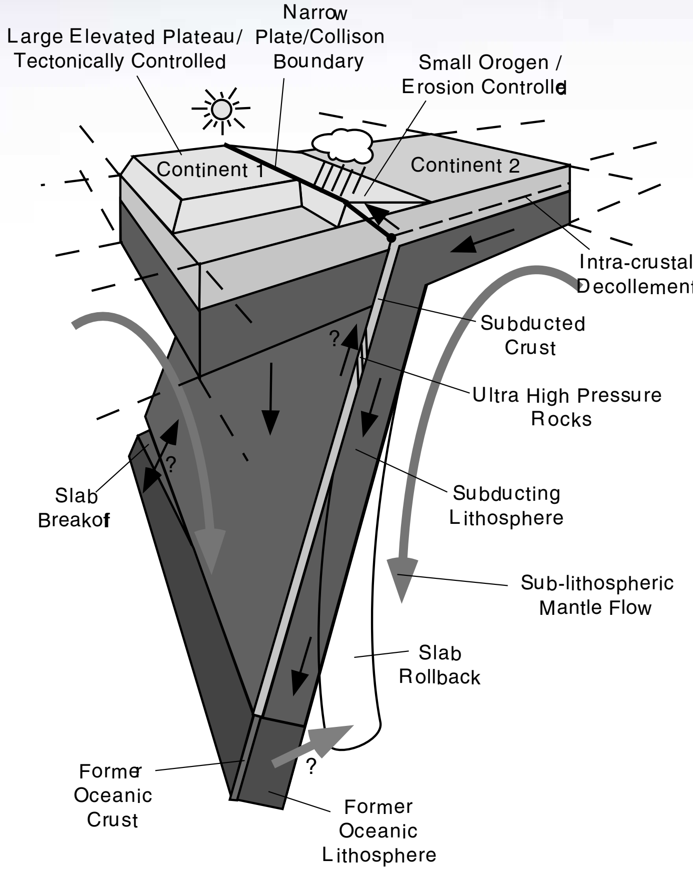
A research project co-funded by the Australian National University

and

Dalhousie University

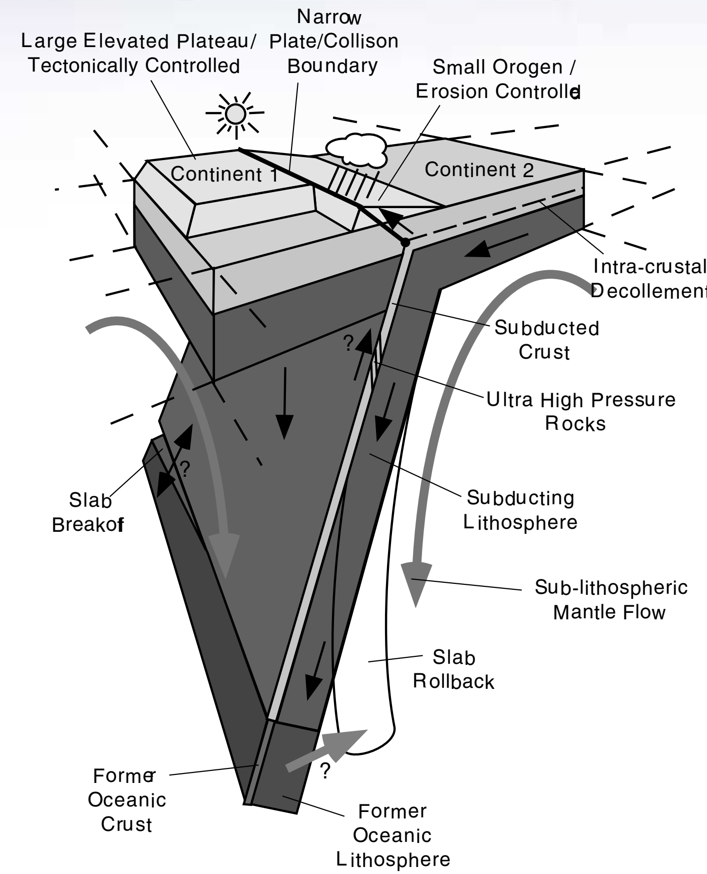
Purpose

- ☞ To develop a 3D version of our crustal deformation model



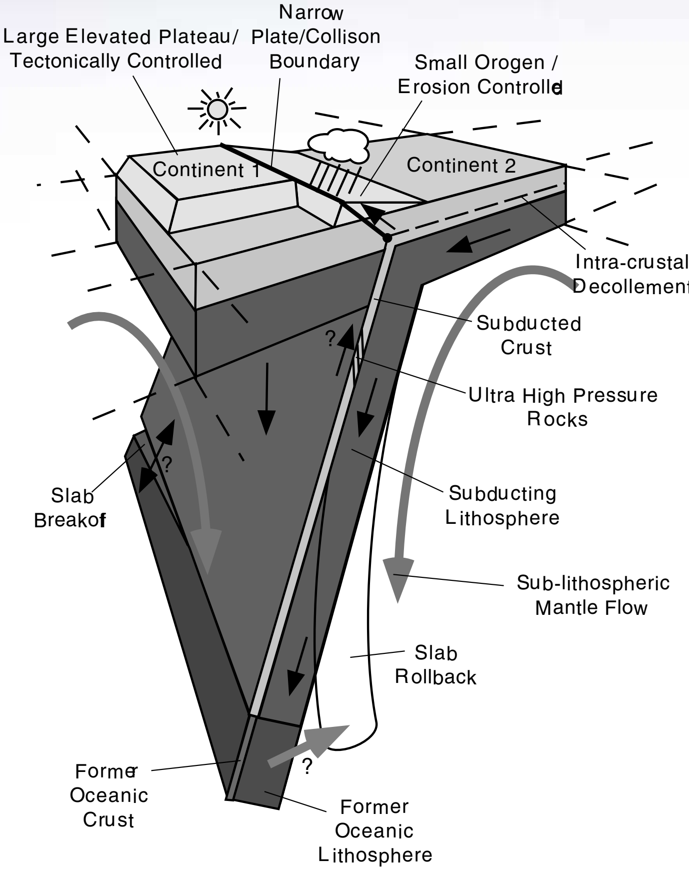
Purpose

- To develop a 3D version of our crustal deformation model
- Interactions with hydrosphere (erosion)



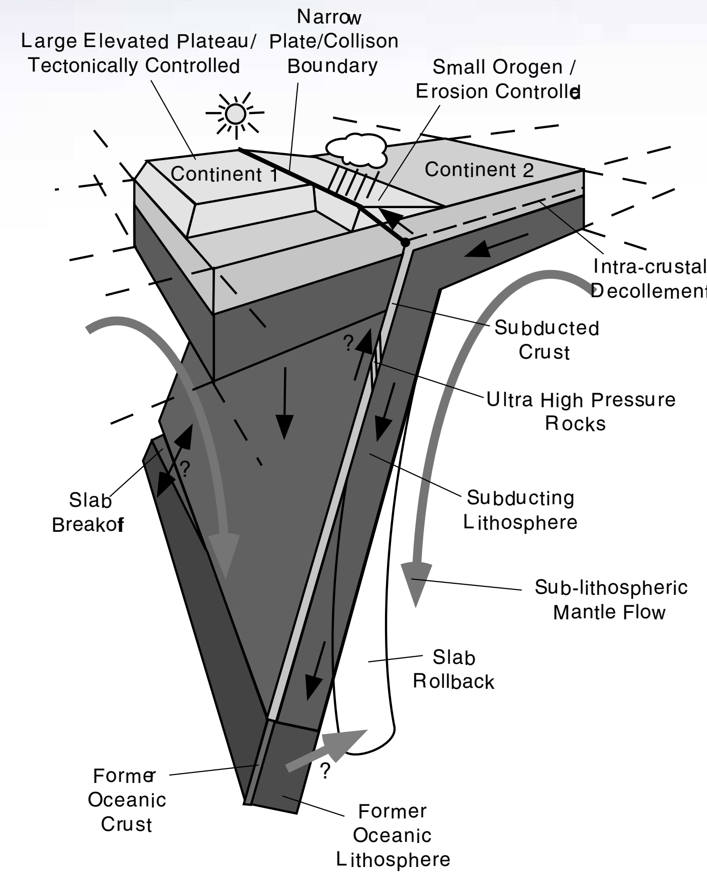
Purpose

- To develop a 3D version of our crustal deformation model
- Interactions with hydrosphere (erosion)
- Include lithospheric mantle



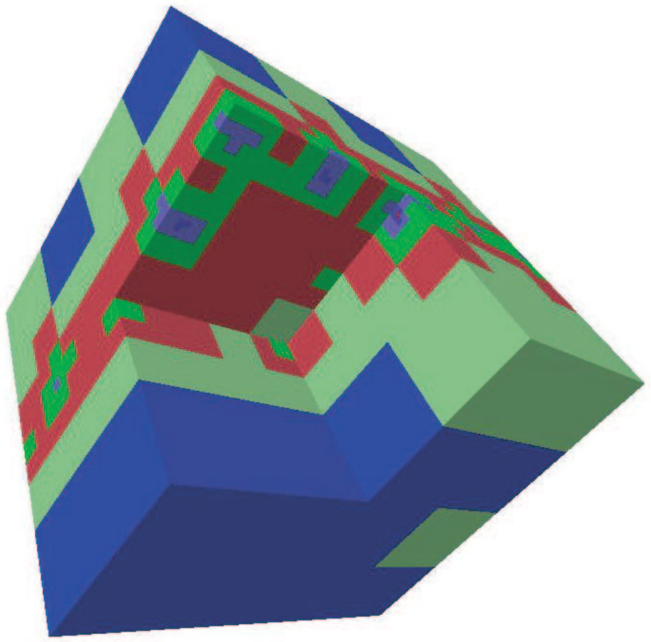
Purpose

- To develop a 3D version of our crustal deformation model
- Interactions with hydrosphere (erosion)
- Include lithospheric mantle
- Address mantle flow as driving force



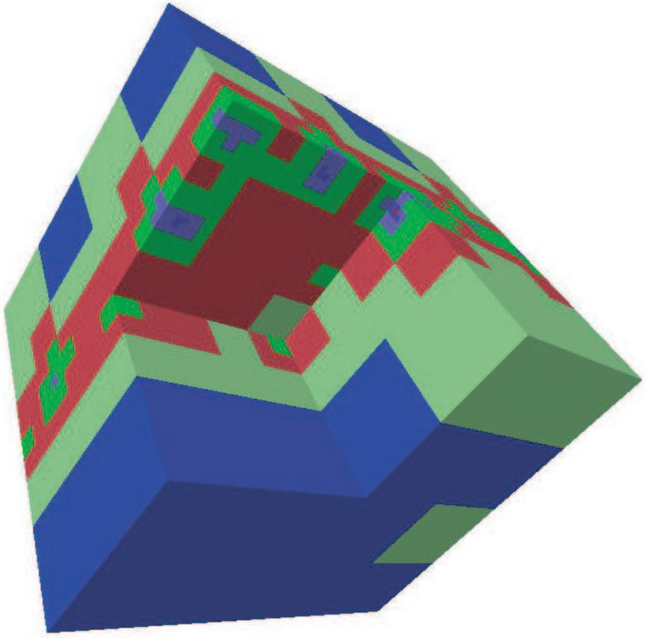
New Approach

- ‘Gobject’ oriented



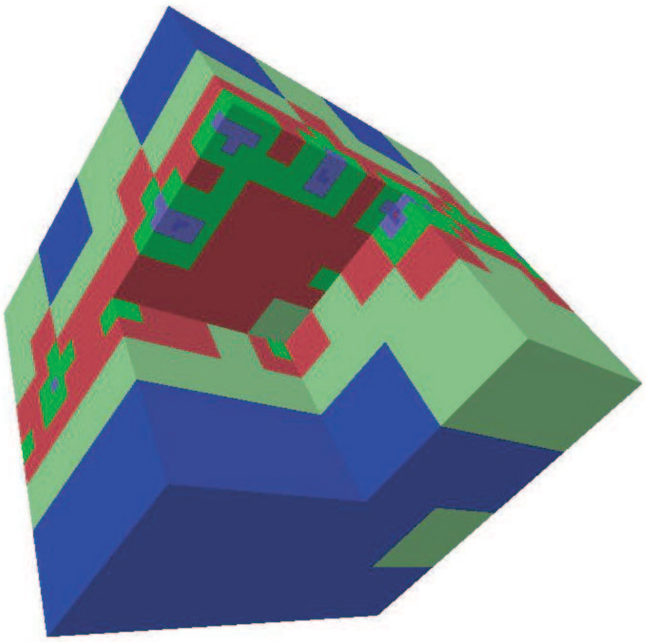
New Approach

- ‘Gobject’ oriented
- Octree division of space



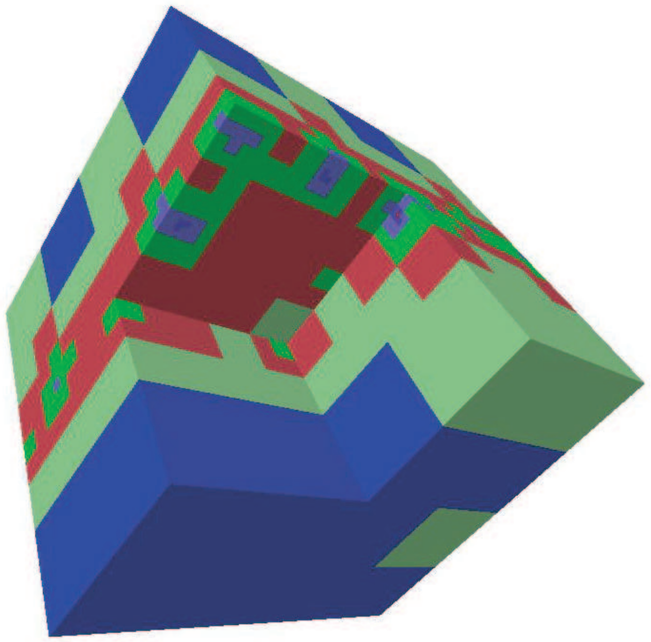
New Approach

- ‘Gobject’ oriented
- Octree division of space
- divFEM



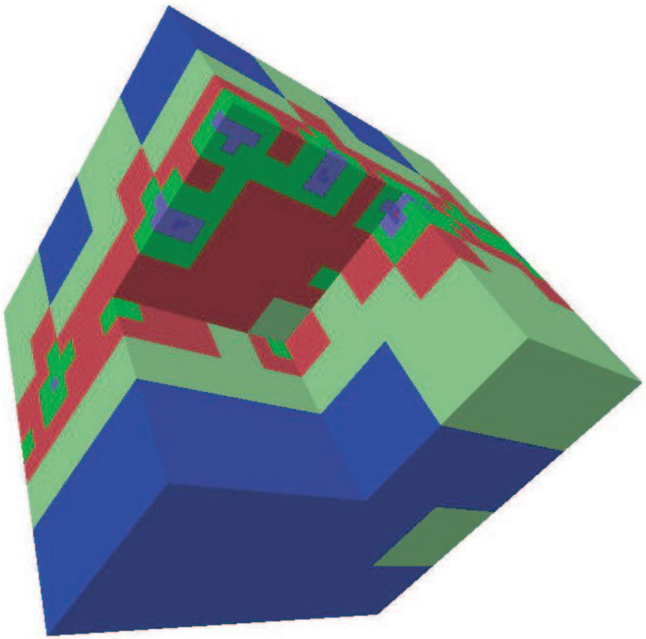
New Approach

- ‘Gobject’ oriented
- Octree division of space
- divFEM
- Direct solver



New Approach

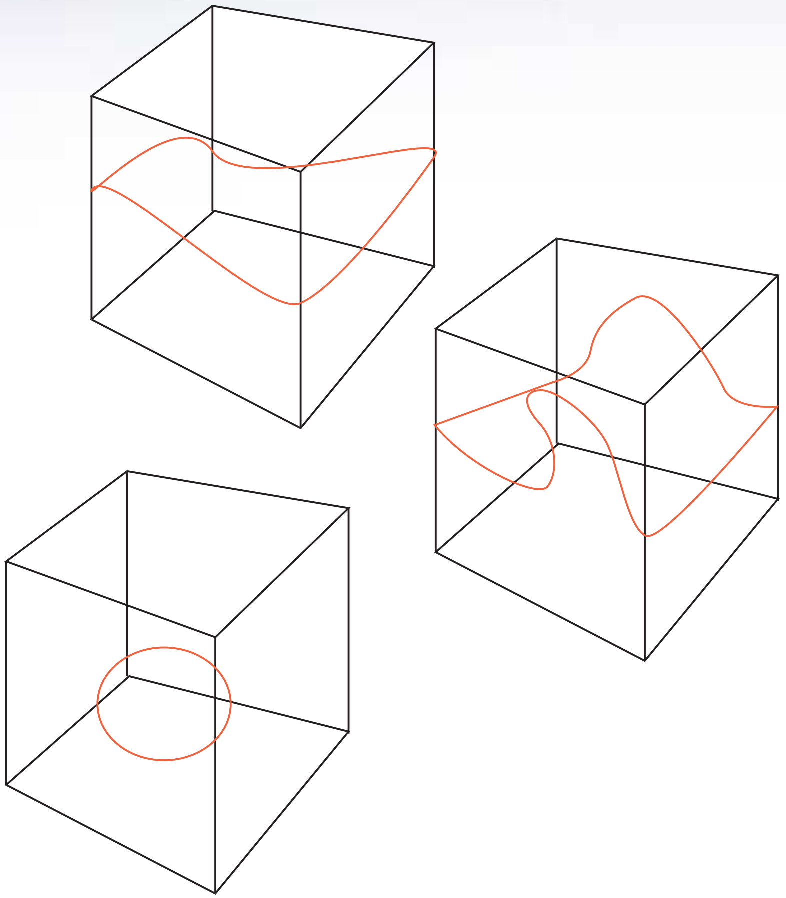
- ‘Gobject’ oriented
- Octree division of space
- divFEM
- Direct solver
- Modular/Open structure (ForTran 90)



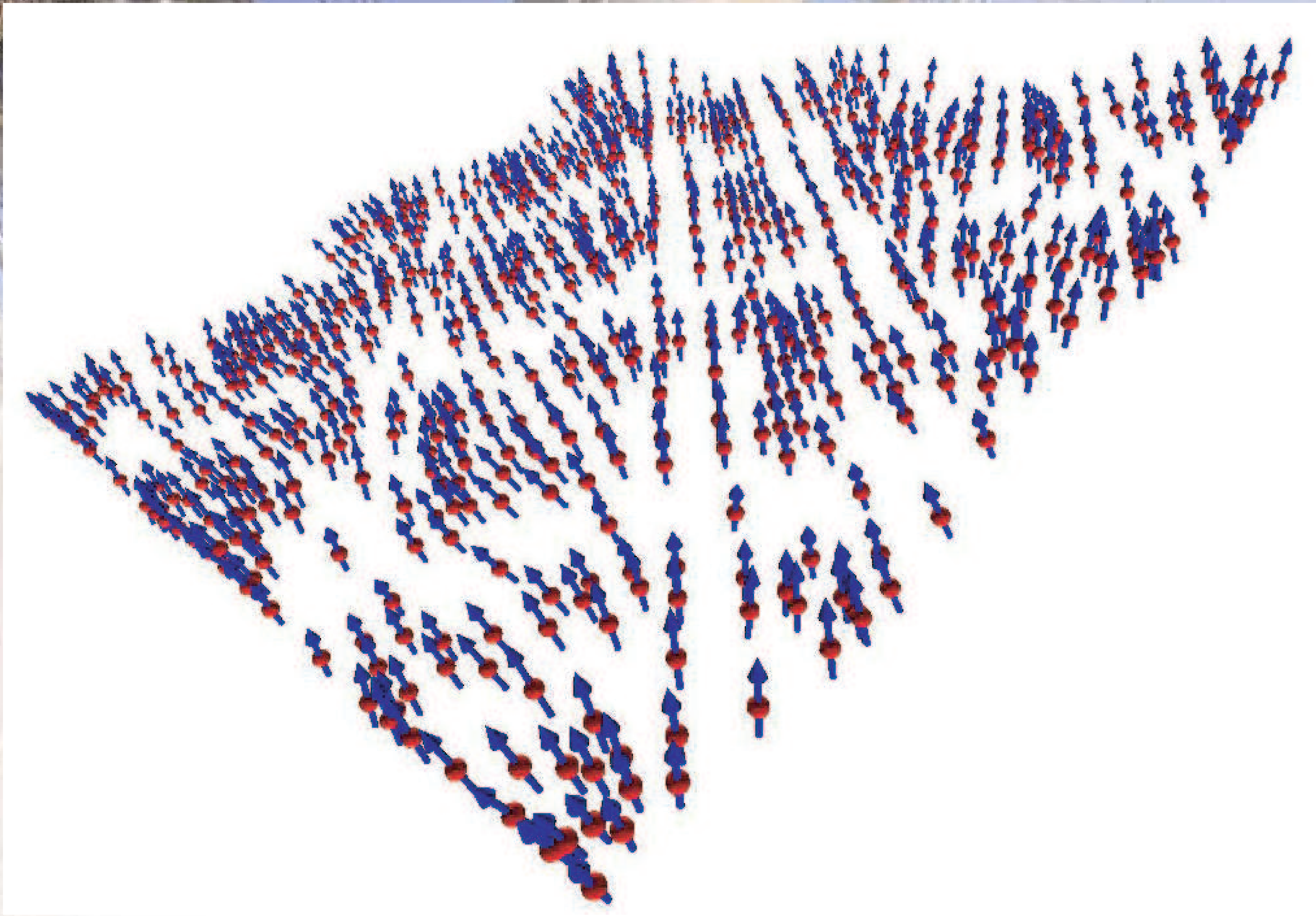
Gobjects

Gobject types:

- Free surface, $h[x, y]$
- Interface, $[u, v] \Rightarrow [x, y, z]$
- General surface, $f(x, y, z) = 0$
- 3D cloud
- Point, Line

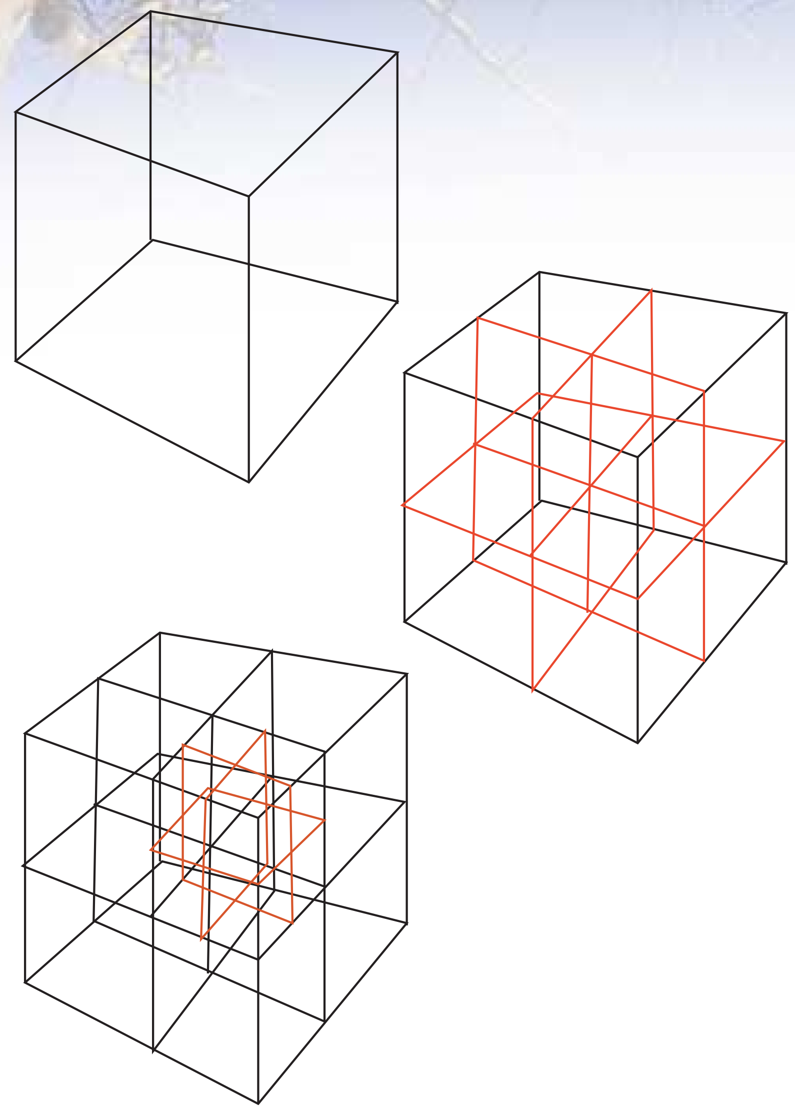


Surface Representation



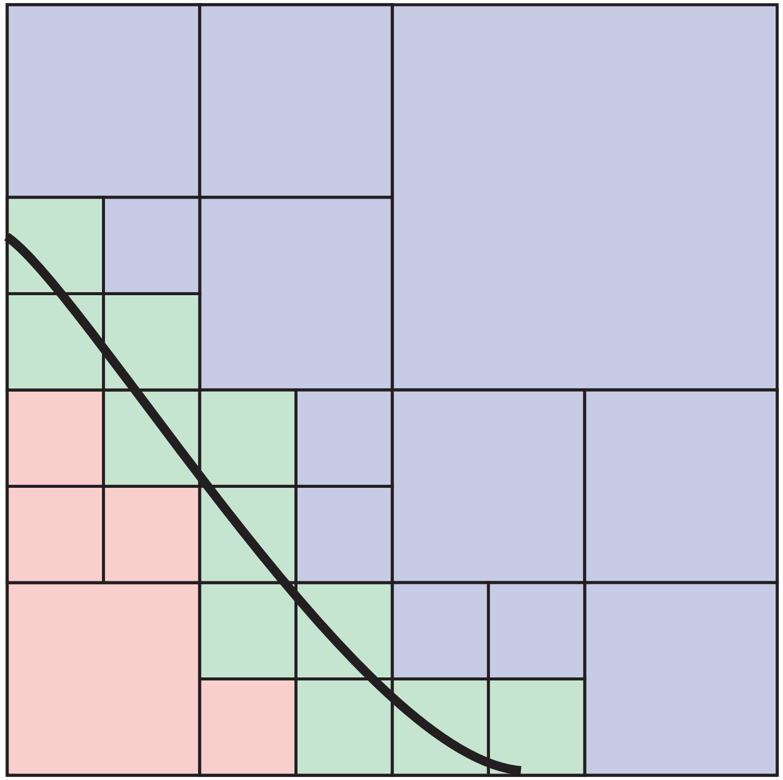
Octree Discretization

- Based on division of Unity
- Variable spatial discretization
- All elements are 8-node tri-linear cubes
- Hanging nodes/faces dealt with by linear constraints



Gobjects & Octree

- Gobjects are represented by Level Set Functions (LFS), on their own ‘gobtree’
- LFS’s gobtrees merged into FEM ‘octreeV’
- LFS’s on FEM octree used to partition space/elements
- divFEM: $\int_{V_e} dV = \sum_{P_i} \int_{V_{P_i}} dV$
- Octree division at element level used for integration



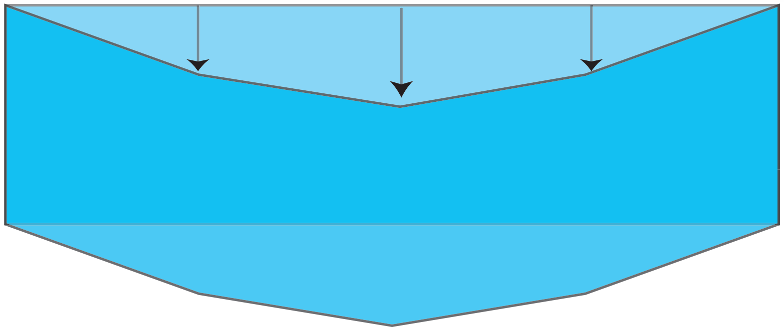
Direct Solver

- WSMP (IBM-Watson Lab: A. Gupta)
- Cholesky Factorization
- Parallel implementation
- Octrees generate ‘small’ grids
- Can solve ill-conditioned systems
- Ideal to impose incompressibility and complex materials

Problem size	Wall time
$40 \times 40 \times 40$	140s
$32 \times 32 \times 32$ + refinement	40-60s

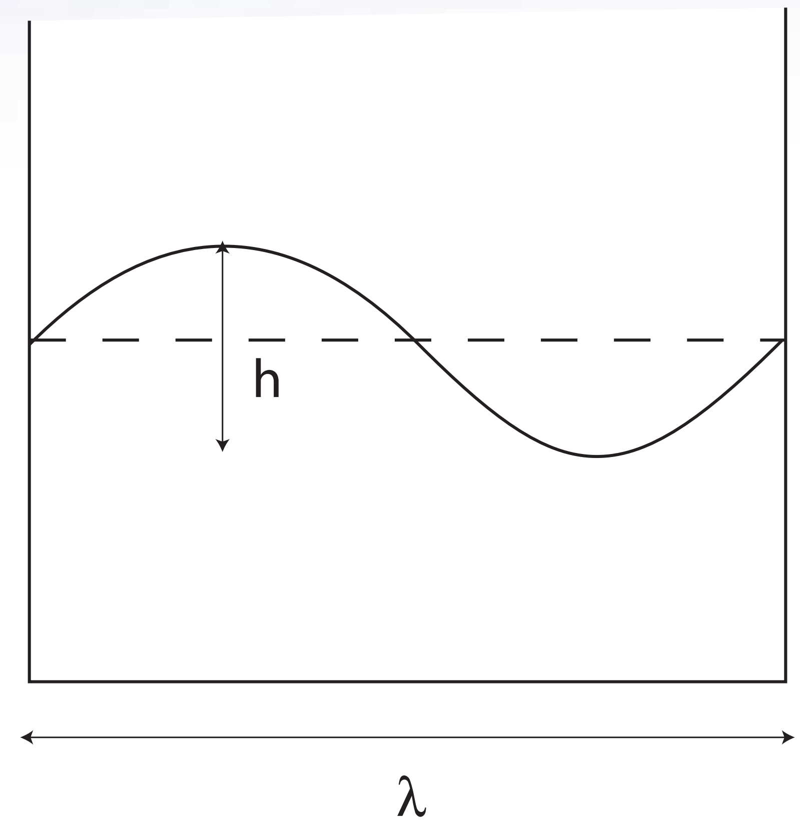
Tests - analytical

- No analytical solution for large deformation, free surface viscous problems
- One exception: the slumping bridge problem (initial V only)
- Difficult to calculate
- Central deflection reproduced numerically within < 1%

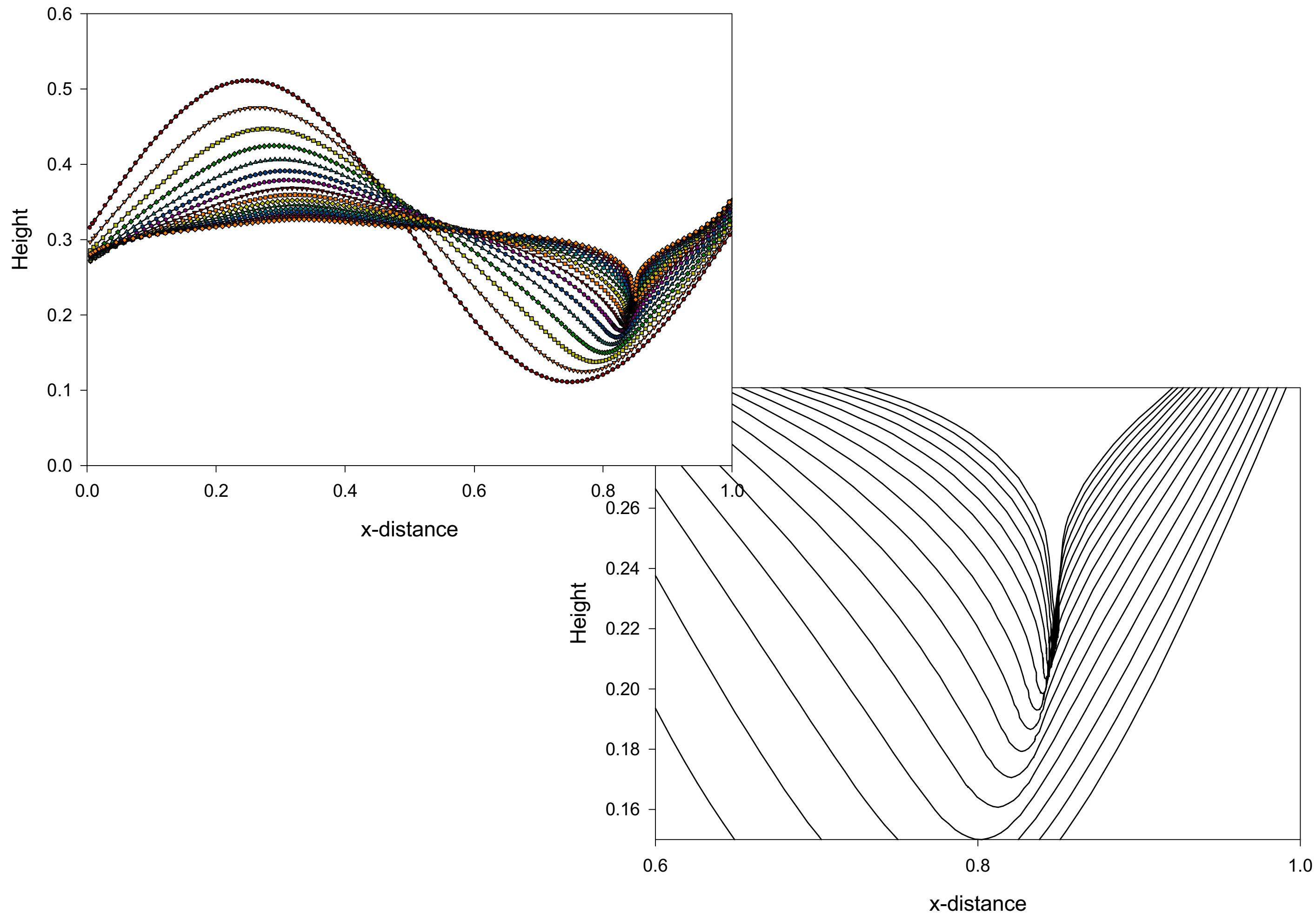


Tests - numerical

- Free relaxation of large amplitude ($h \approx \lambda$) ‘sine’ surface
- 2D FEM with conformal elements solution (uniform discretization)
- Good test for div FEM
- Solution is not intuitive

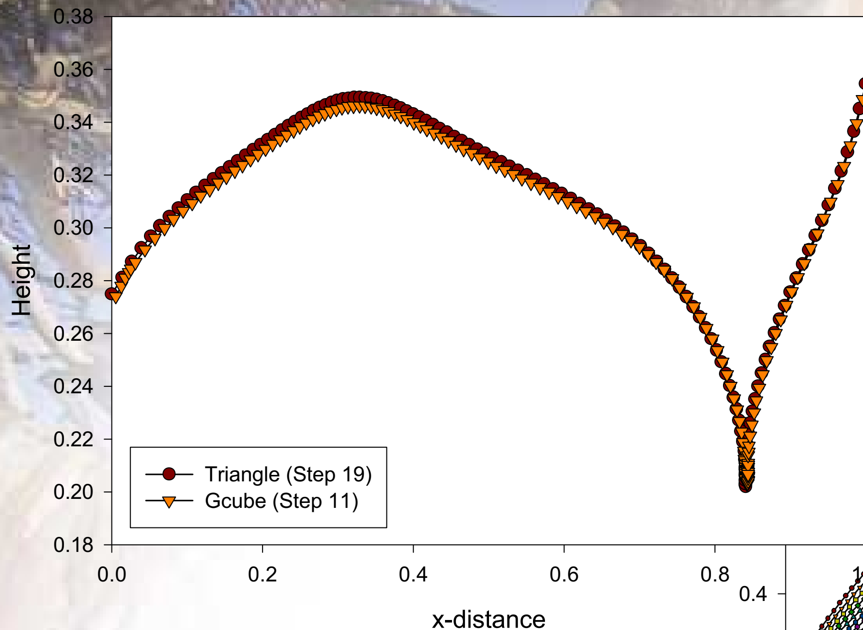


Gcube Solution

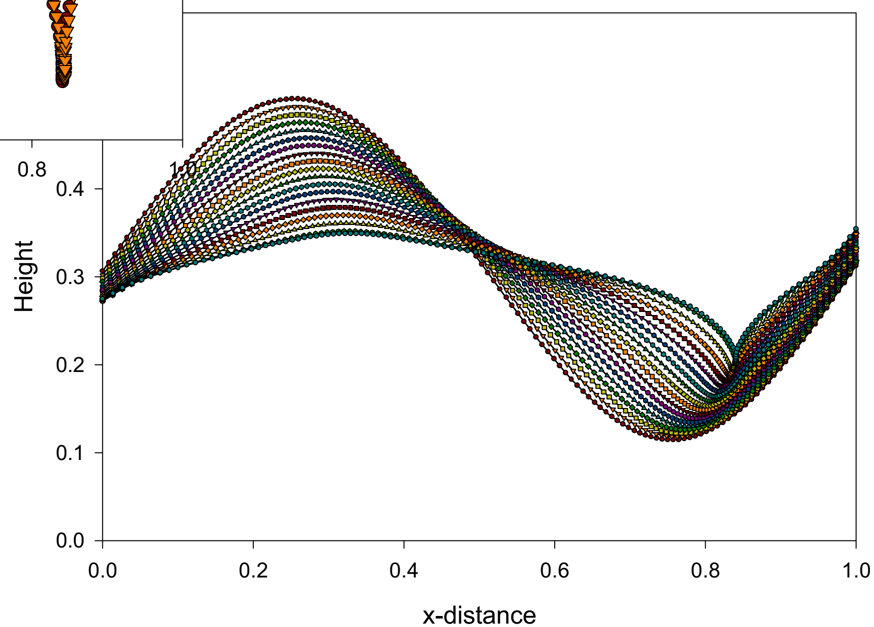


Comparison

Comparison at t = 20



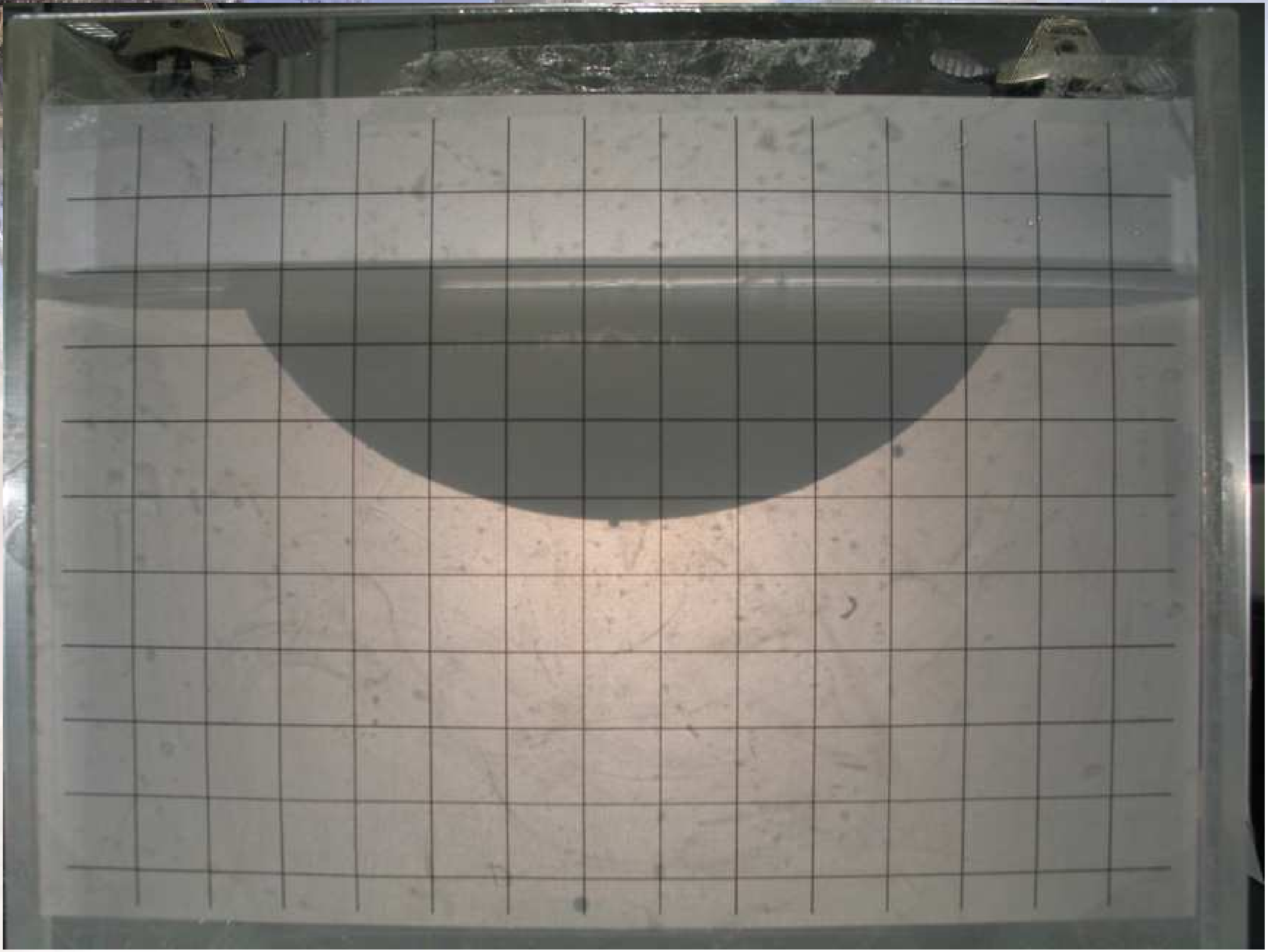
Triangular + Conformal FE



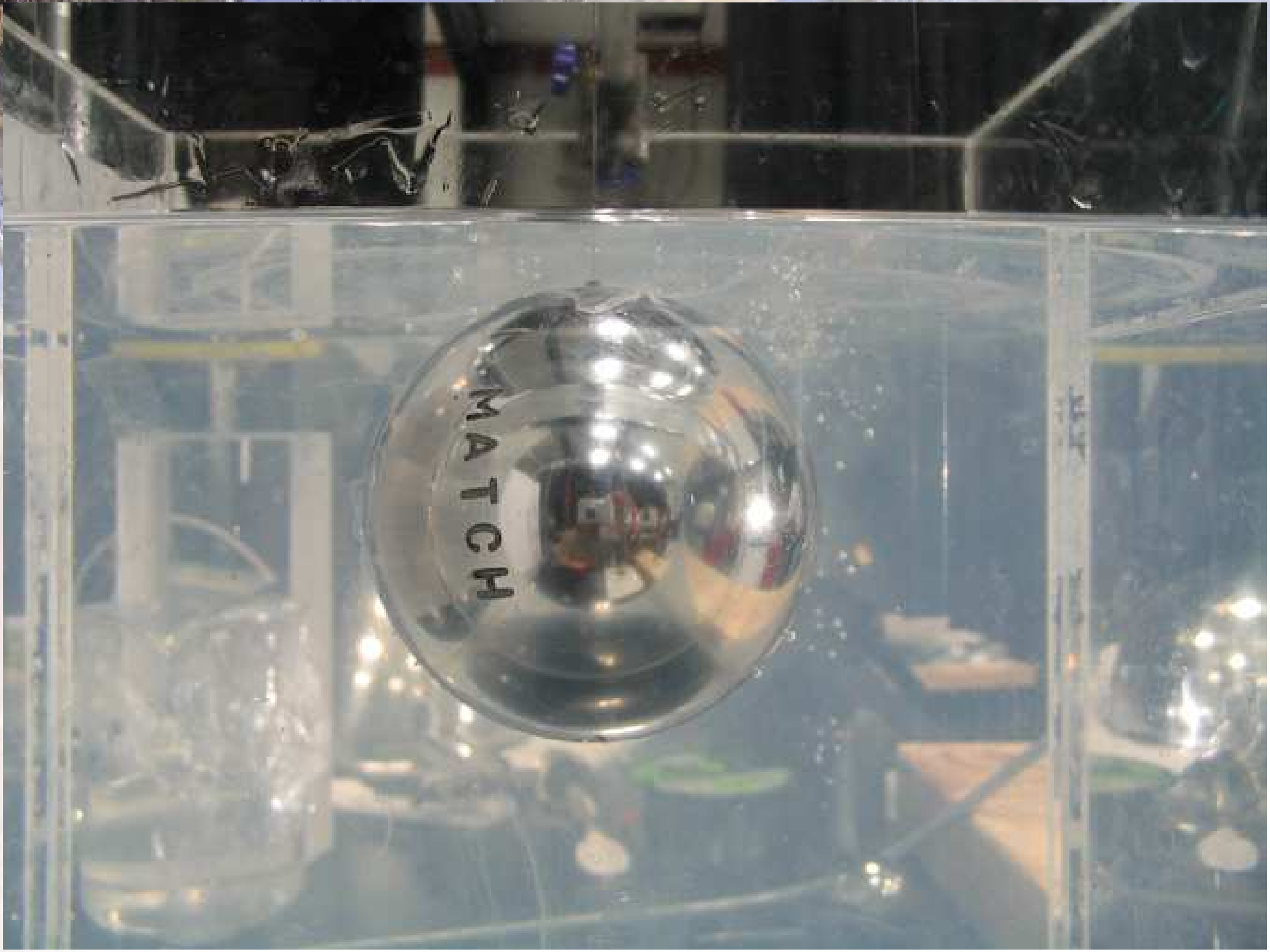
Tests - analogue

- High viscosity silicon oil (19×10^3 Pas)
- Relaxation of free surface
- Free fall of sphere

Surface Relaxation

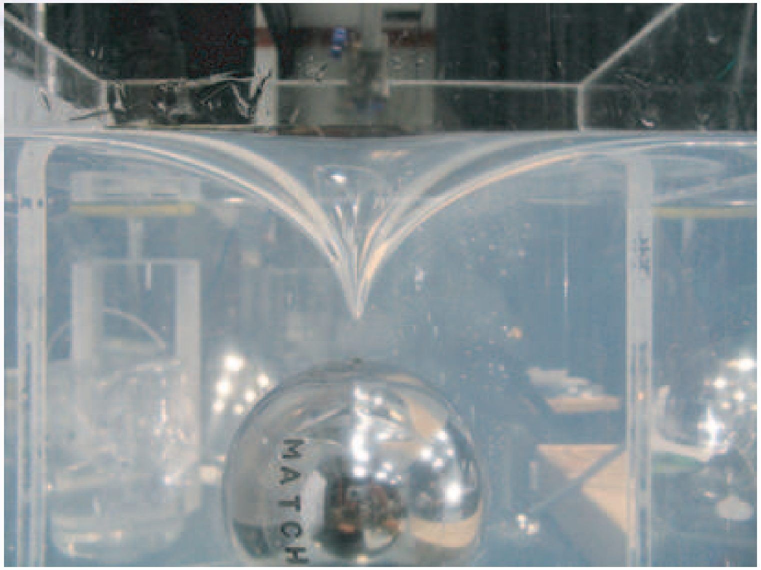


Sphere Experiment

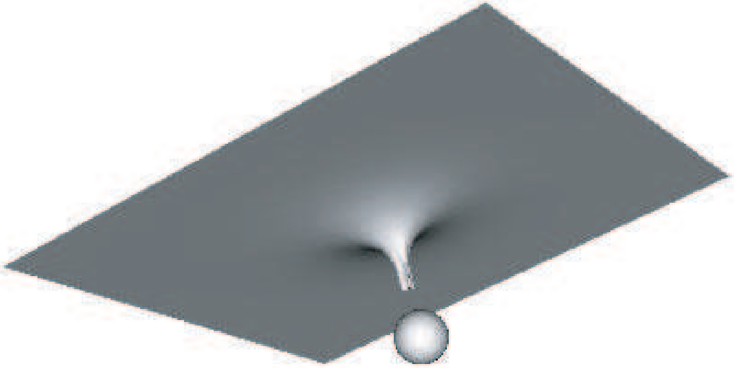


Gcube Solution

(a) Scaled laboratory experiment



(b) Numerical experiment



Future Developments

- More testing
- Non-linear rheology
- Temperature
- Graphical interface
- etc...