



# Anelastic rotating convection modelling with Magic and Rayleigh: Zonal Flow and Vortices on giant planets.

Moritz Heimpel, Nick Featherstone and Jonathan Aurnou



CIIG all hands meeting, Davis, June 23, 2016

# Acknowledgments

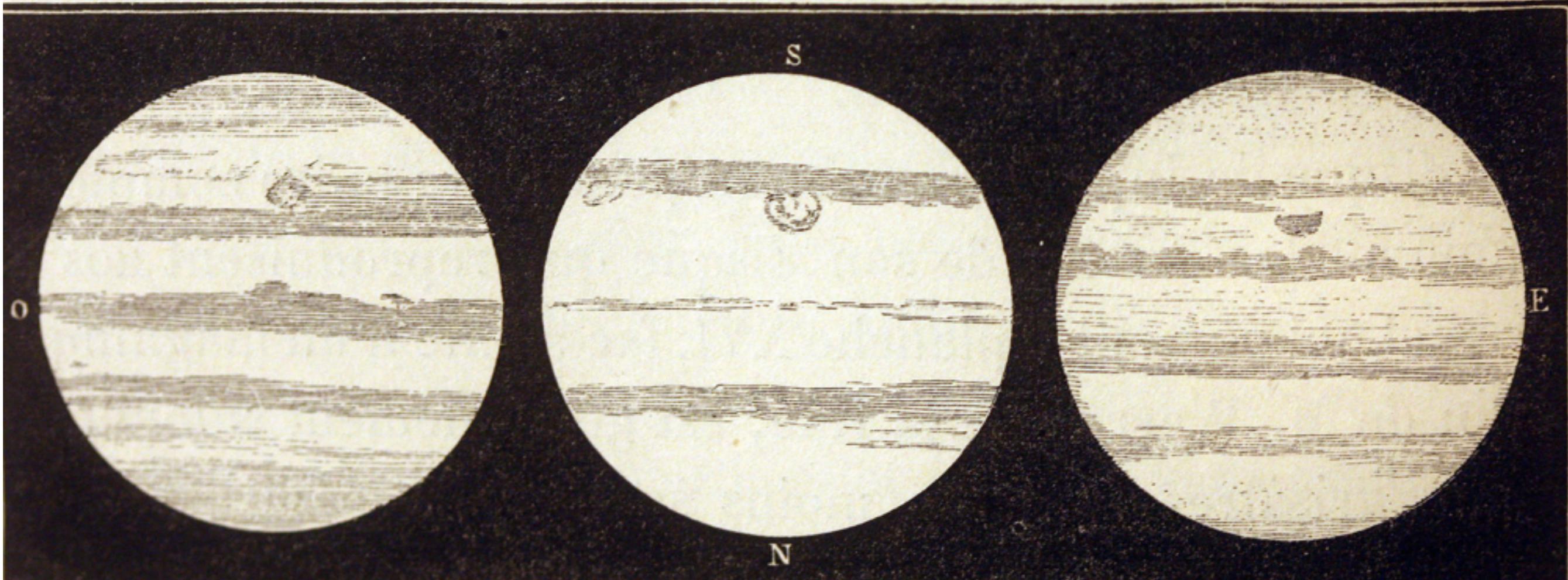
- **Krista Soderlund** (University of Texas at Austin)
- **Thomas Gastine & Johannes Wicht**  
(MPI for Solar System Research)
- **Keith Cuff** (University of Alberta)
  
- **Compute Canada**
- **Westgrid**
- **Argonne National Laboratory (Mira)**
- **Computational Infrastructure for Geodynamics (CIG)**

# Giovanni Domenico Cassini's drawings of bands and a “permanent spot” on Jupiter

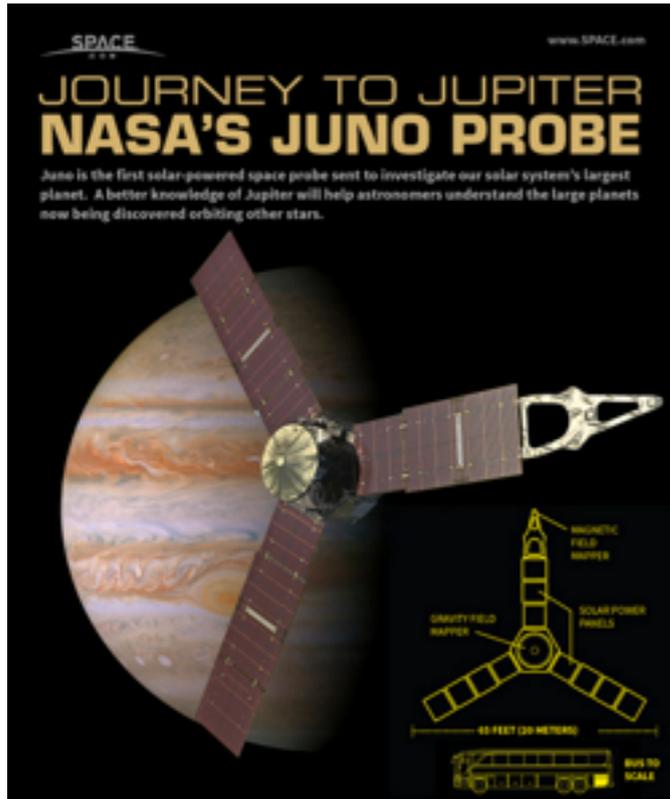
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1672

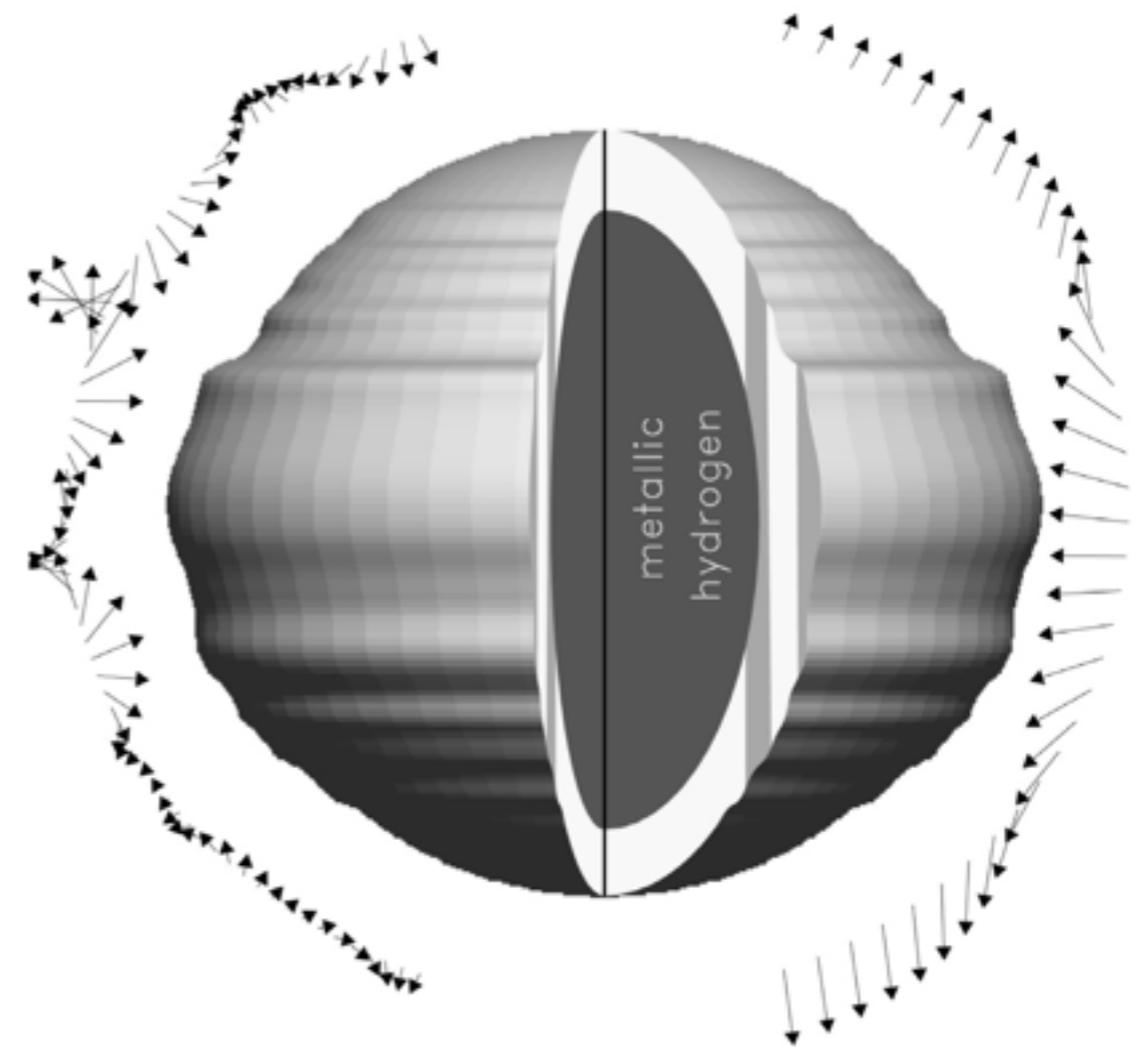
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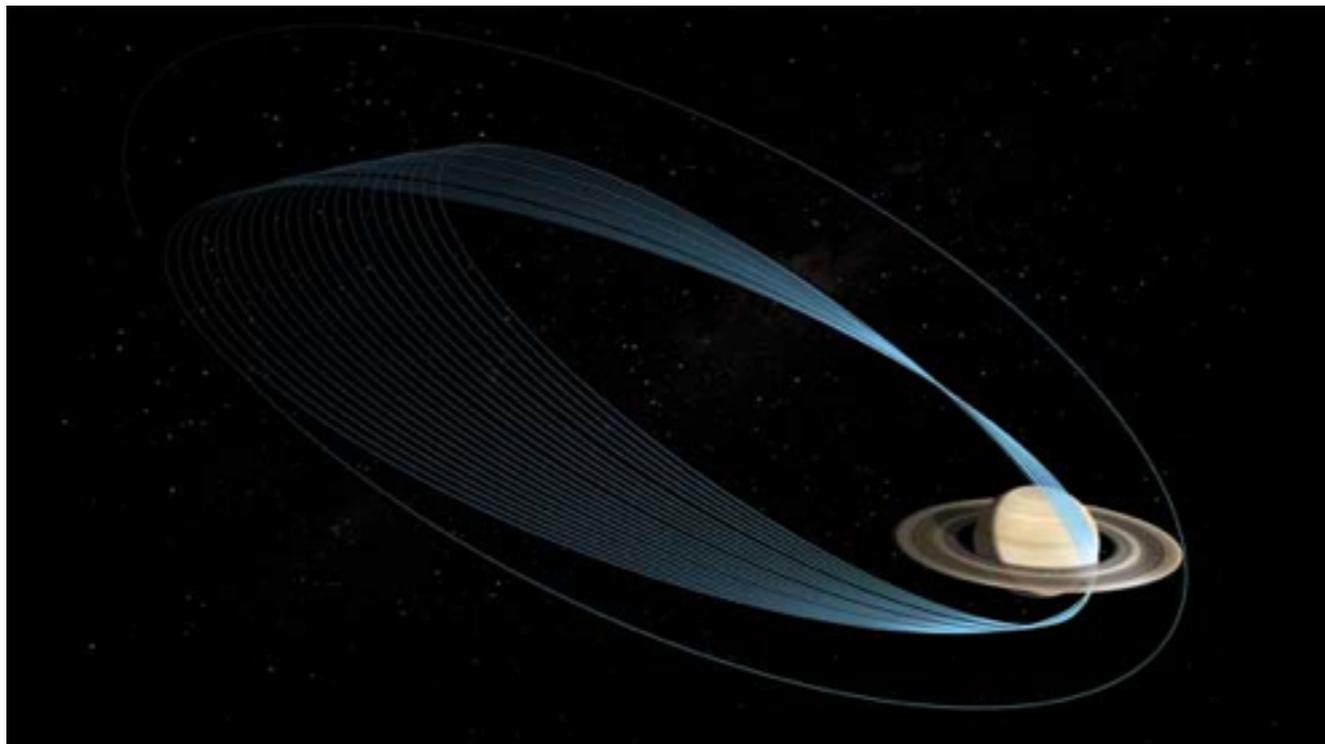
Jupiter: Juno (Orbit insertion July 4, 2016)



Future Observations will reveal the depth of zonal and vortical flows



Saturn: Cassini Proximal (Late 2016 - 2017)



Guillot et al., 2004

# Outline

**Part I (Very deep):** Rotating convection, Jupiter's interior and the jovian dynamo.

**Part IIa (Pretty deep):** Zonal flows on Jupiter, Saturn, and in models of rotating convection for incompressible and compressible fluids

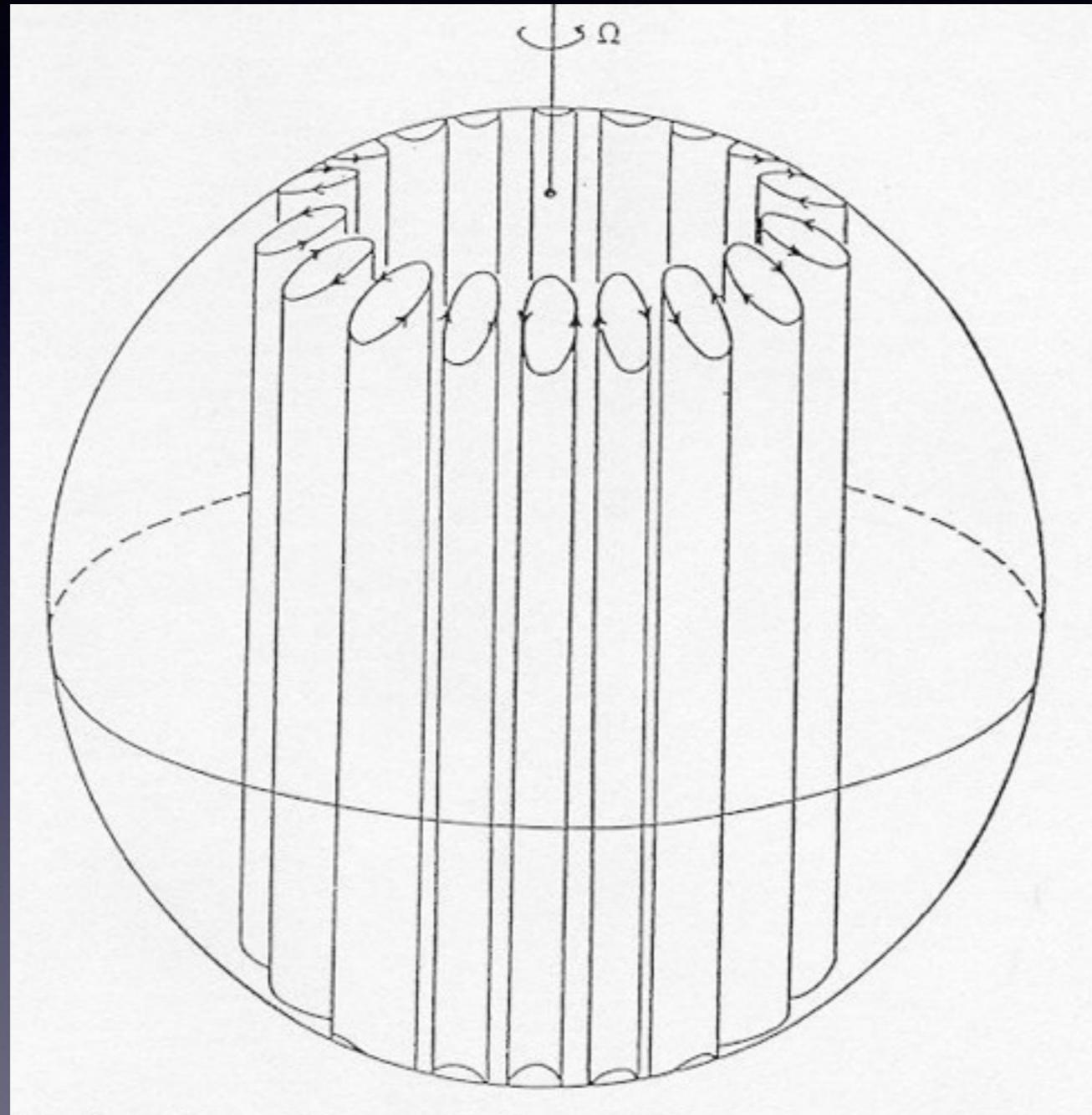
- The scale of bands and the effect of spherical geometry.

**Part IIb (Relatively shallow):** Storms and vortices on giant planets.

- Modelling the coexistence of zonal jets and vortices: The role of outer boundary stability conditions

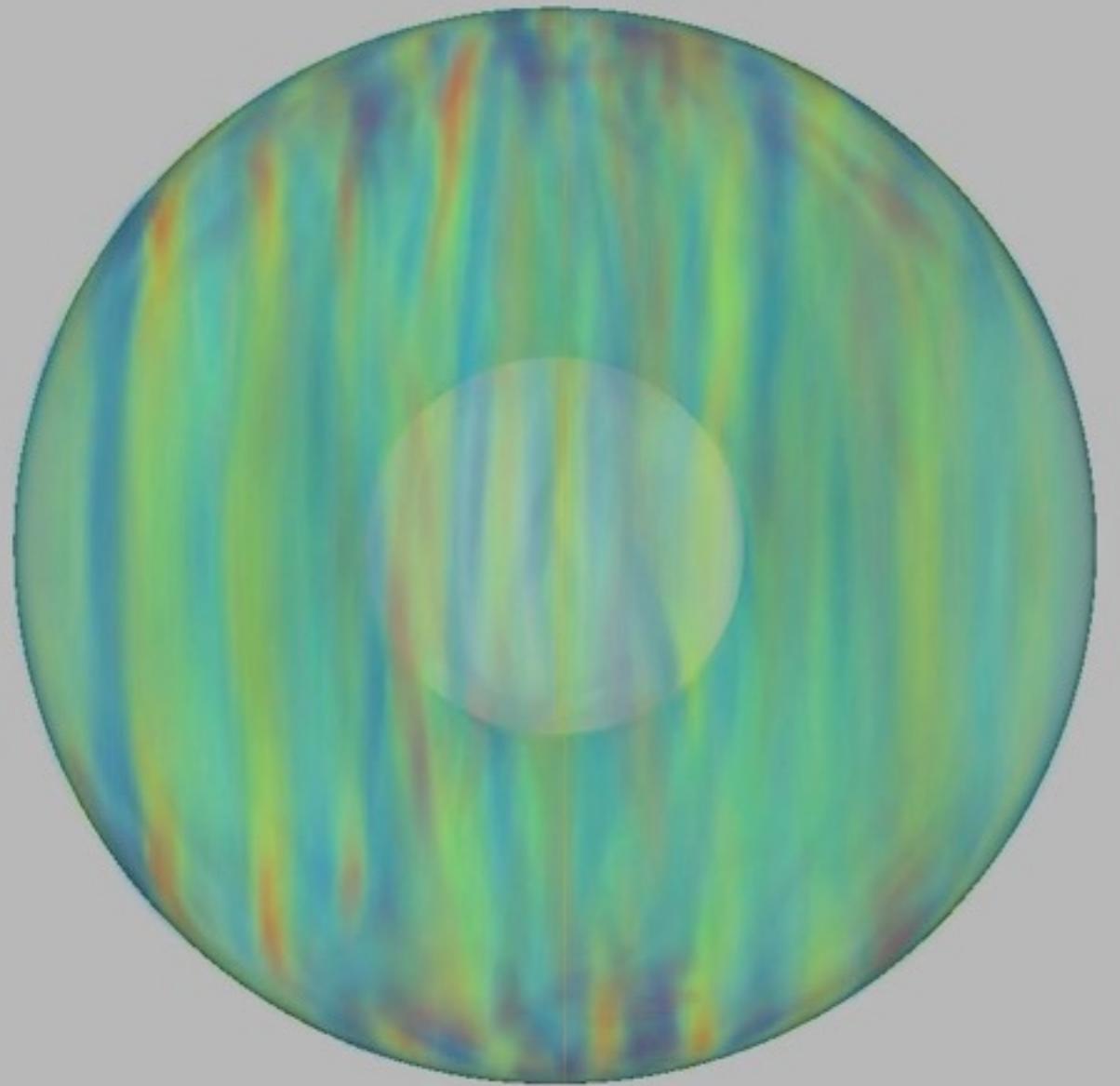
# Columnar flow in deep rotating convection

Strong rotation: axially aligned flow structures (Taylor columns)

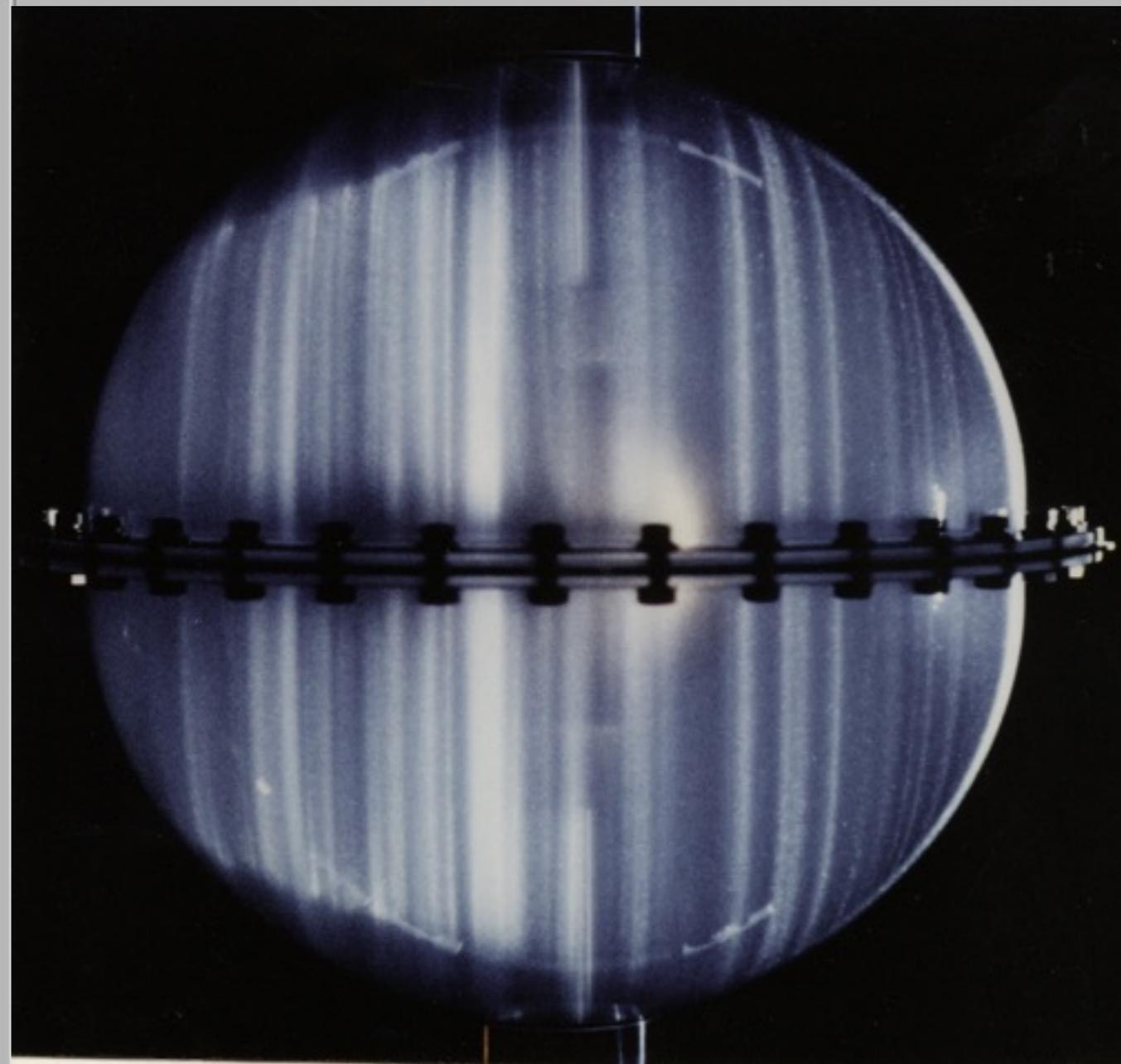


From Busse, *J. Fluid Mech.*, 1970

# Numerical dynamo driven by columnar convection and laboratory rotating convection



Low z-Vorticity High

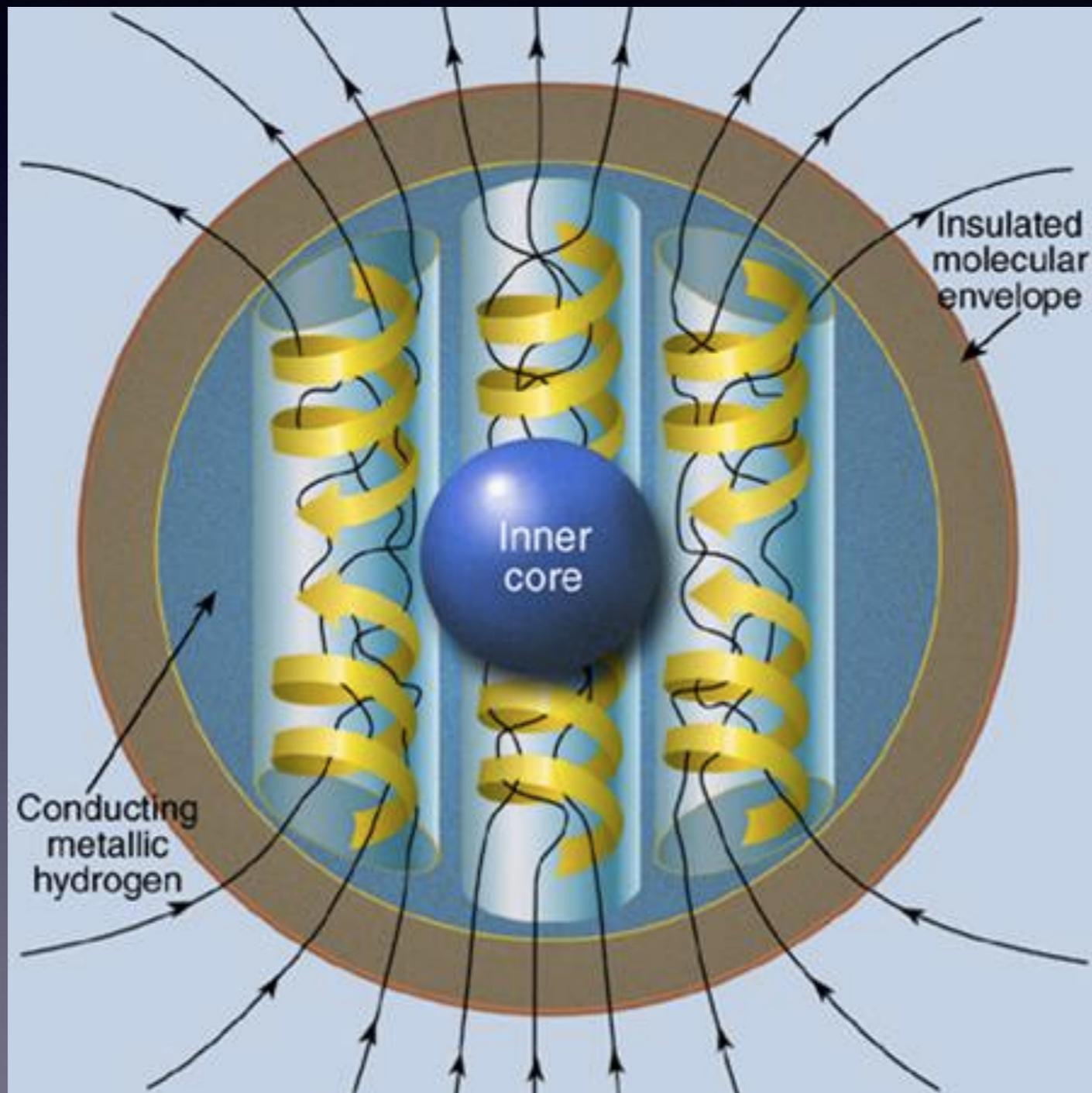


Dynamo:  $E = 10^{-4}$ ,  $Pm = 1$ ,  $Ra = 10Ra_c$

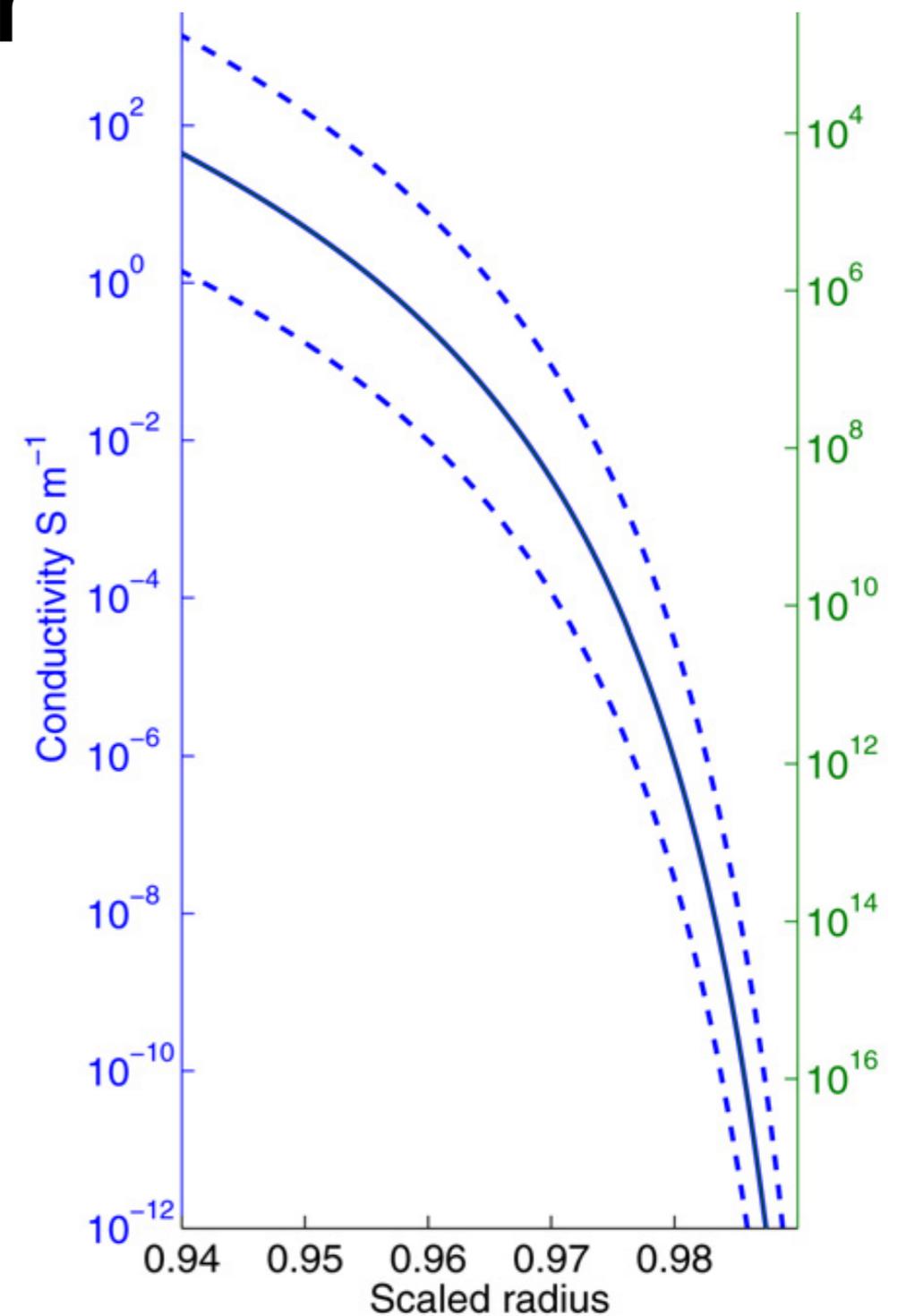
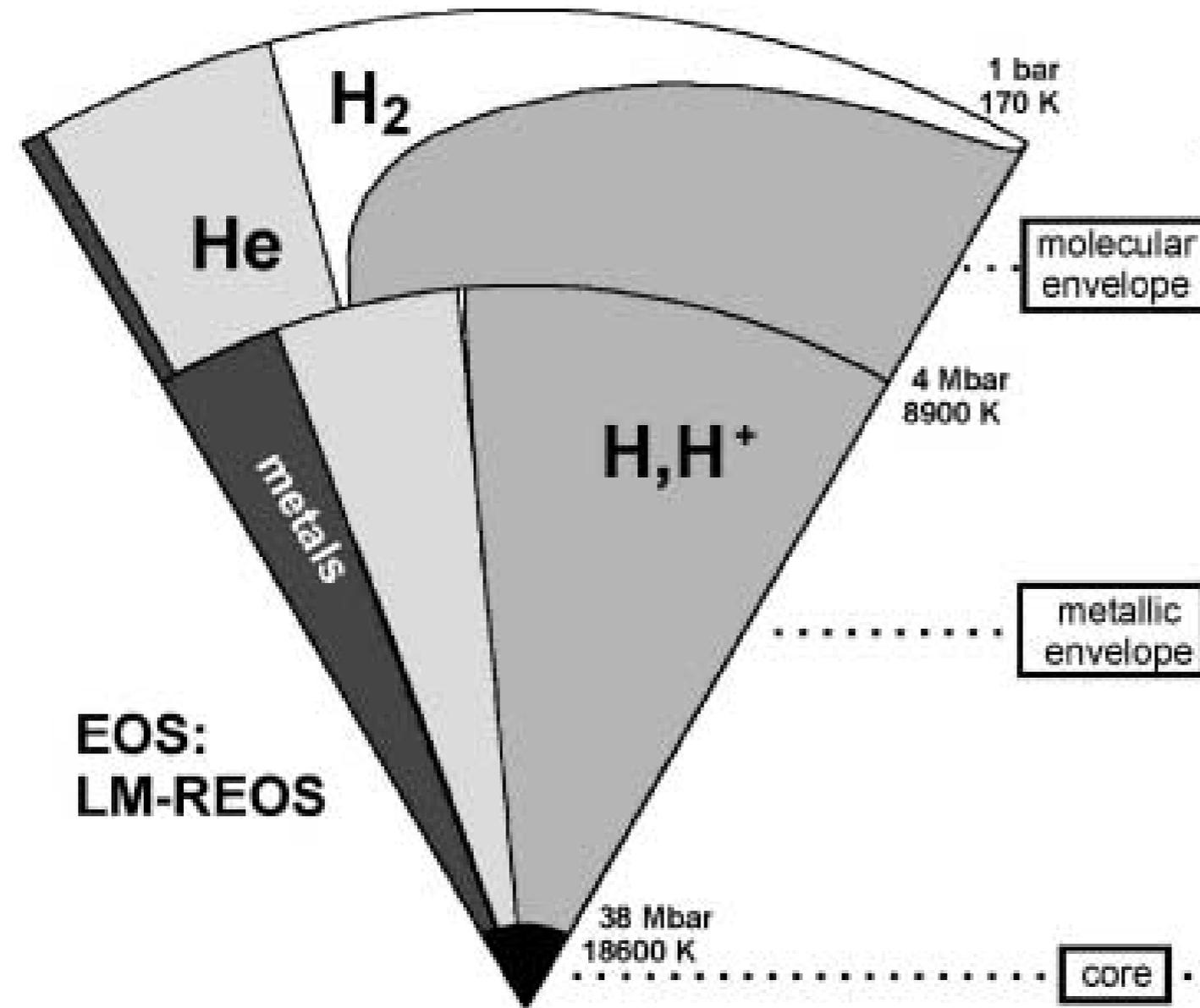
Rotating convection, cylindrical "gravity"  
*Cardin and Olson (1992)*

Side views

# Busse-Taylor columns organize the flow and the magnetic field of a dynamo



# Interior of Jupiter



Liu et al., 1998:

Electrical conductivity in the molecular envelope

Nettelmann et al., (2008):  
Ab-initio equation of state

# Periodic Table of Elements

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18																
1	<b>H</b> Hydrogen 1.00794	Atomic # Symbol Name Atomic Mass																2	<b>He</b> Helium 4.002602															
2	<b>Li</b> Lithium 6.941	<b>Be</b> Beryllium 9.012182	<div style="display: flex; justify-content: space-between;"> <div style="width: 15%;"> <b>C</b> Solid <b>Hg</b> Liquid <b>H</b> Gas <b>Rf</b> Unknown                 </div> <div style="width: 60%; text-align: center;"> <table border="1"> <tr> <th colspan="5">Metals</th> <th colspan="3">Nonmetals</th> </tr> <tr> <td style="background-color: #FFD700;">Alkali metals</td> <td style="background-color: #FFD700;">Alkaline earth metals</td> <td style="background-color: #FFDAB9;">Lanthanoids</td> <td style="background-color: #FFDAB9;">Transition metals</td> <td style="background-color: #FFDAB9;">Actinoids</td> <td style="background-color: #90EE90;">Poor metals</td> <td style="background-color: #90EE90;">Other nonmetals</td> <td style="background-color: #ADD8E6;">Noble gases</td> </tr> </table> </div> <div style="width: 15%;"></div> </div>										Metals					Nonmetals			Alkali metals	Alkaline earth metals	Lanthanoids	Transition metals	Actinoids	Poor metals	Other nonmetals	Noble gases	<b>B</b> Boron 10.811	<b>C</b> Carbon 12.0107	<b>N</b> Nitrogen 14.0067	<b>O</b> Oxygen 15.9994	<b>F</b> Fluorine 18.9984032	<b>Ne</b> Neon 20.1797
Metals					Nonmetals																													
Alkali metals	Alkaline earth metals	Lanthanoids	Transition metals	Actinoids	Poor metals	Other nonmetals	Noble gases																											
3	<b>Na</b> Sodium 22.98976928	<b>Mg</b> Magnesium 24.3050	<b>Sc</b> Scandium 44.955912	<b>Ti</b> Titanium 47.887	<b>V</b> Vanadium 50.9415	<b>Cr</b> Chromium 51.9961	<b>Mn</b> Manganese 54.938045	<b>Fe</b> Iron 55.845	<b>Co</b> Cobalt 58.933195	<b>Ni</b> Nickel 58.6934	<b>Cu</b> Copper 63.546	<b>Zn</b> Zinc 65.38	<b>Ga</b> Gallium 69.723	<b>Ge</b> Germanium 72.64	<b>As</b> Arsenic 74.92160	<b>Se</b> Selenium 78.96	<b>Br</b> Bromine 79.904	<b>Kr</b> Krypton 83.798																
4	<b>K</b> Potassium 39.0983	<b>Ca</b> Calcium 40.078	<b>Sc</b> Scandium 44.955912	<b>Ti</b> Titanium 47.887	<b>V</b> Vanadium 50.9415	<b>Cr</b> Chromium 51.9961	<b>Mn</b> Manganese 54.938045	<b>Fe</b> Iron 55.845	<b>Co</b> Cobalt 58.933195	<b>Ni</b> Nickel 58.6934	<b>Cu</b> Copper 63.546	<b>Zn</b> Zinc 65.38	<b>Ga</b> Gallium 69.723	<b>Ge</b> Germanium 72.64	<b>As</b> Arsenic 74.92160	<b>Se</b> Selenium 78.96	<b>Br</b> Bromine 79.904	<b>Kr</b> Krypton 83.798																
5	<b>Rb</b> Rubidium 85.4678	<b>Sr</b> Strontium 87.62	<b>Y</b> Yttrium 88.90585	<b>Zr</b> Zirconium 91.224	<b>Nb</b> Niobium 92.90638	<b>Mo</b> Molybdenum 95.96	<b>Tc</b> Technetium (97.9072)	<b>Ru</b> Ruthenium 101.07	<b>Rh</b> Rhodium 102.90550	<b>Pd</b> Palladium 106.42	<b>Ag</b> Silver 107.8682	<b>Cd</b> Cadmium 112.411	<b>In</b> Indium 114.818	<b>Sn</b> Tin 118.710	<b>Sb</b> Antimony 121.760	<b>Te</b> Tellurium 127.60	<b>I</b> Iodine 126.90447	<b>Xe</b> Xenon 131.293																
6	<b>Cs</b> Caesium 132.9054519	<b>Ba</b> Barium 137.327	57-71	<b>Hf</b> Hafnium 178.49	<b>Ta</b> Tantalum 180.94788	<b>W</b> Tungsten 183.84	<b>Re</b> Rhenium 186.207	<b>Os</b> Osmium 190.23	<b>Ir</b> Iridium 192.217	<b>Pt</b> Platinum 195.084	<b>Au</b> Gold 196.966569	<b>Hg</b> Mercury 200.59	<b>Tl</b> Thallium 204.3833	<b>Pb</b> Lead 207.2	<b>Bi</b> Bismuth 208.98040	<b>Po</b> Polonium (208.9824)	<b>At</b> Astatine (209.9871)	<b>Rn</b> Radon (222.0176)																
7	<b>Fr</b> Francium (223)	<b>Ra</b> Radium (226)	89-103	<b>Rf</b> Rutherfordium (261)	<b>Db</b> Dubnium (262)	<b>Sg</b> Seaborgium (266)	<b>Bh</b> Bohrium (264)	<b>Hs</b> Hassium (277)	<b>Mt</b> Meitnerium (268)	<b>Ds</b> Darmstadtium (271)	<b>Rg</b> Roentgenium (272)	<b>Uub</b> Ununbium (286)	<b>Uut</b> Ununtrium (284)	<b>Uuq</b> Ununquadium (289)	<b>Uup</b> Ununpentium (288)	<b>Uuh</b> Ununhexium (292)	<b>Uus</b> Ununseptium	<b>Uuo</b> Ununoctium (294)																

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

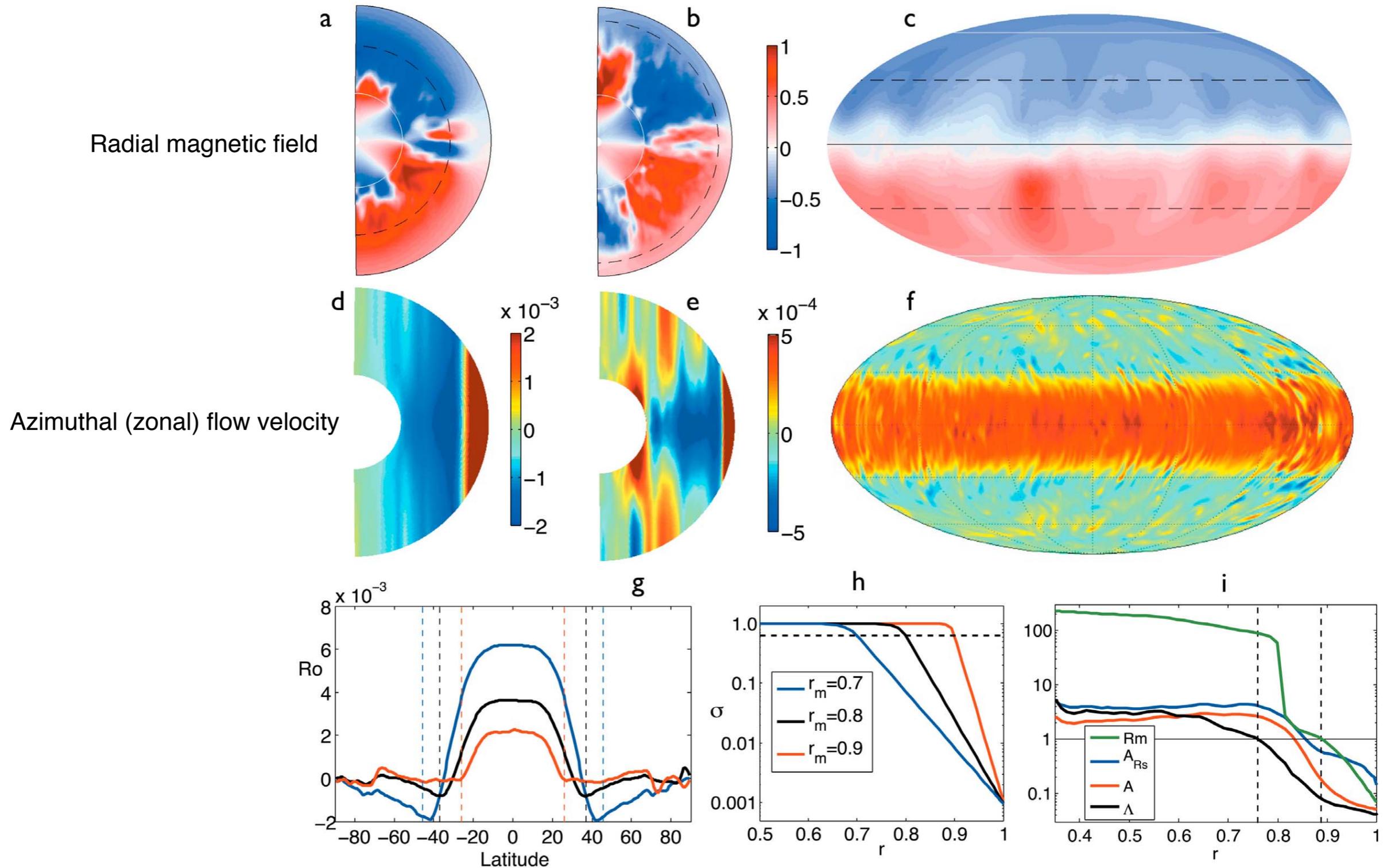
Design and Interface Copyright © 1997 Michael Dayah (michael@dayah.com). <http://www.ptable.com/>



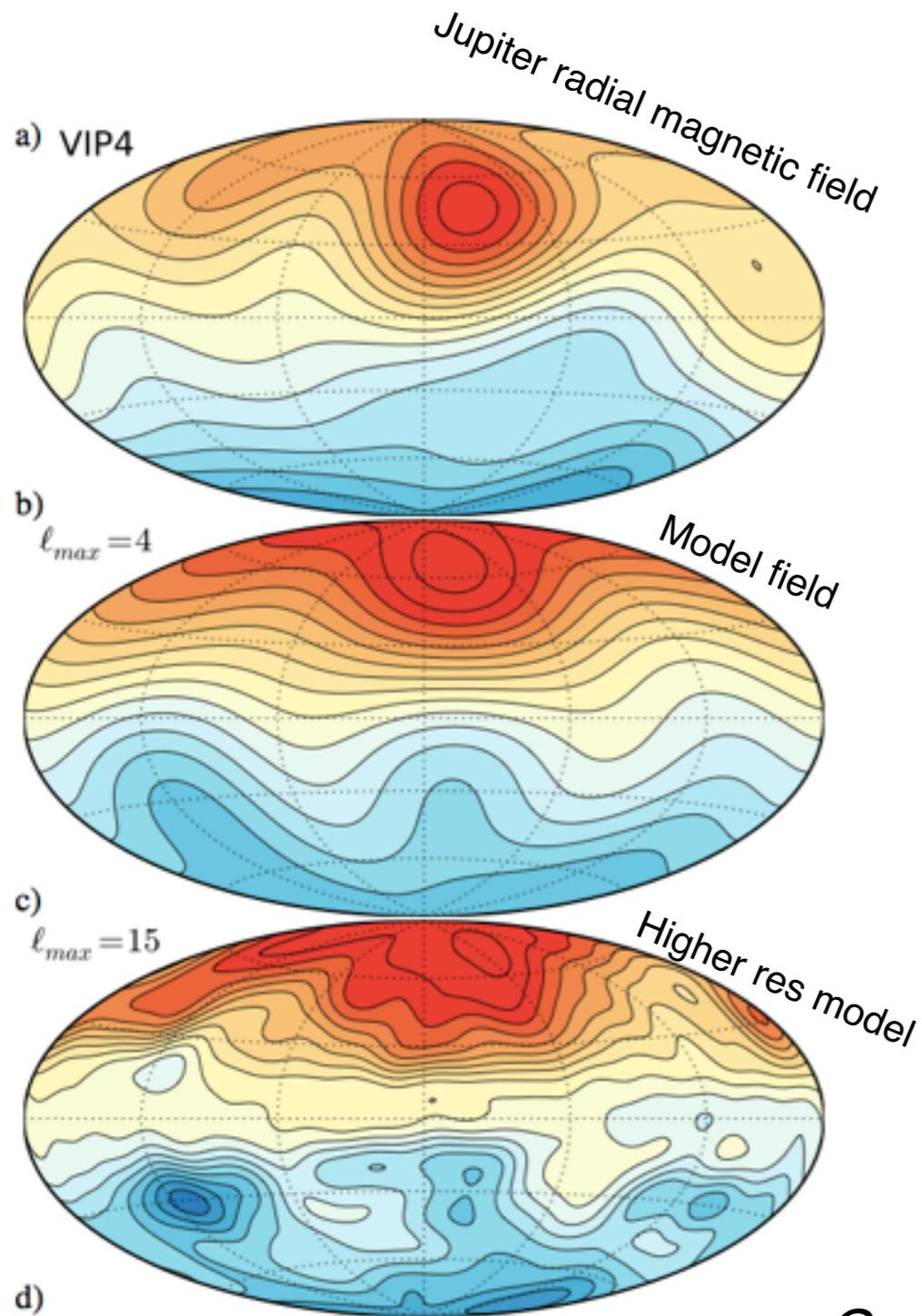
<b>57 La</b> Lanthanum 138.90547	<b>58 Ce</b> Cerium 140.116	<b>59 Pr</b> Praseodymium 140.90768	<b>60 Nd</b> Neodymium 144.242	<b>61 Pm</b> Promethium (145)	<b>62 Sm</b> Samarium 150.36	<b>63 Eu</b> Europium 151.964	<b>64 Gd</b> Gadolinium 157.25	<b>65 Tb</b> Terbium 158.92535	<b>66 Dy</b> Dysprosium 162.500	<b>67 Ho</b> Holmium 164.93032	<b>68 Er</b> Erbium 167.259	<b>69 Tm</b> Thulium 168.93421	<b>70 Yb</b> Ytterbium 173.054	<b>71 Lu</b> Lutetium 174.9668
<b>89 Ac</b> Actinium (227)	<b>90 Th</b> Thorium 232.03806	<b>91 Pa</b> Protactinium 231.03688	<b>92 U</b> Uranium 238.02891	<b>93 Np</b> Neptunium (237)	<b>94 Pu</b> Plutonium (244)	<b>95 Am</b> Americium (243)	<b>96 Cm</b> Curium (247)	<b>97 Bk</b> Berkelium (247)	<b>98 Cf</b> Californium (251)	<b>99 Es</b> Einsteinium (252)	<b>100 Fm</b> Fermium (257)	<b>101 Md</b> Mendelevium (258)	<b>102 No</b> Nobelium (259)	<b>103 Lr</b> Lawrencium (262)

# Variable conductivity $\sigma(r)$ dynamo:

High  $\sigma$  inside  $r_m$ , expon. decreasing  $\sigma$  outside  $r_m$ .

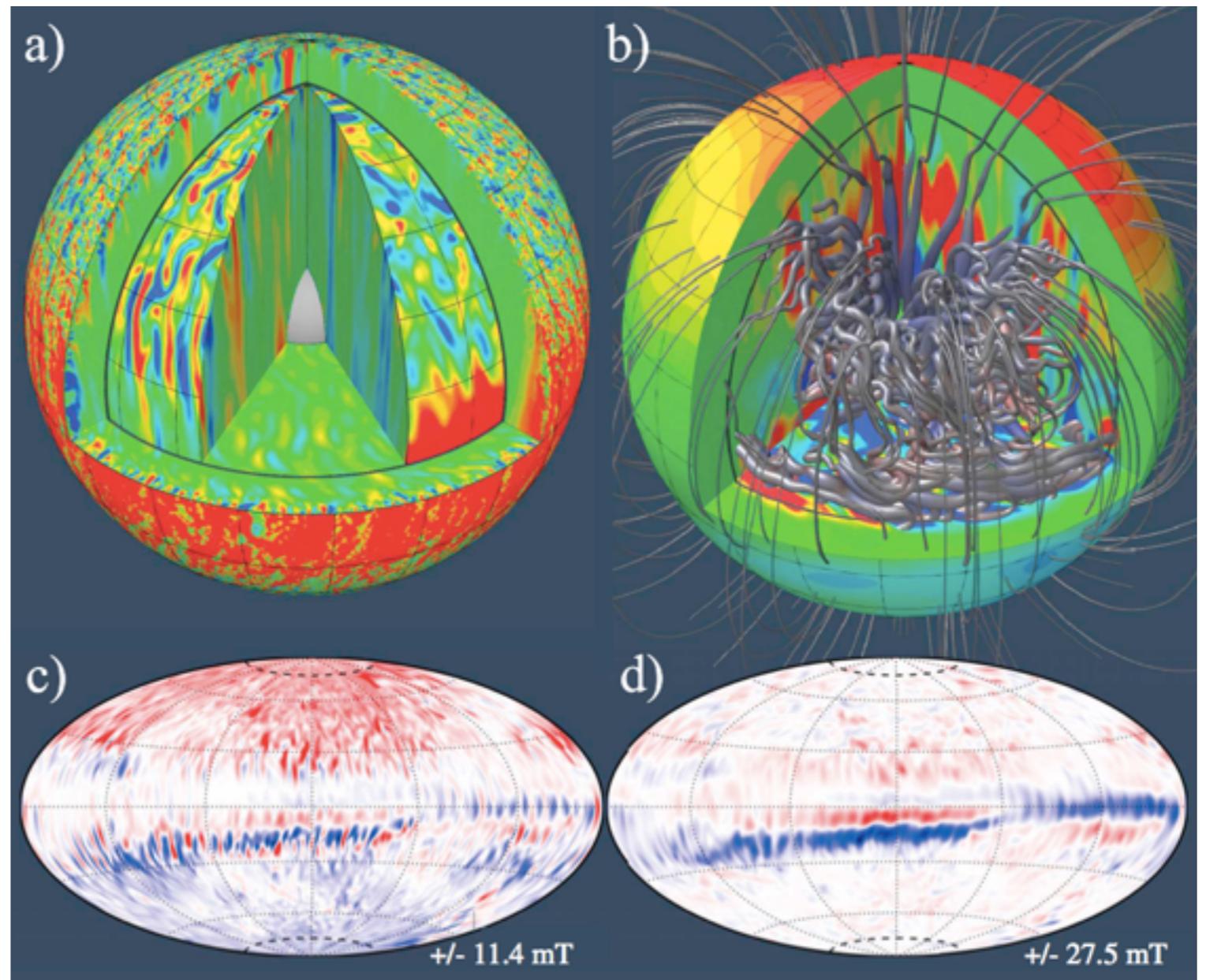


# Jupiter vs compressible dynamo model (anelastic dynamo code Magic, $E=10^{-5}$ , $Pr=1$ , $Pm=0.6$ )



Flow field (deeper = slower)

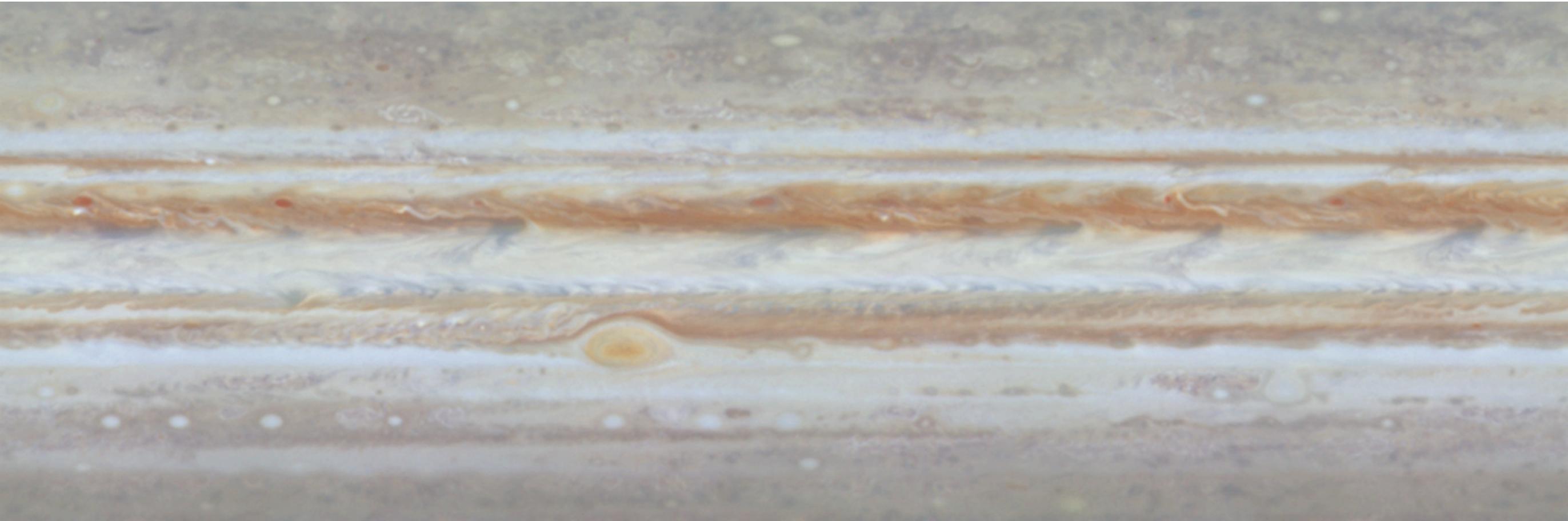
Magnetic field (deeper = stronger)



Radial magnetic field at  $r=0.87$

Azimuthal magnetic field at  $r=0.87$

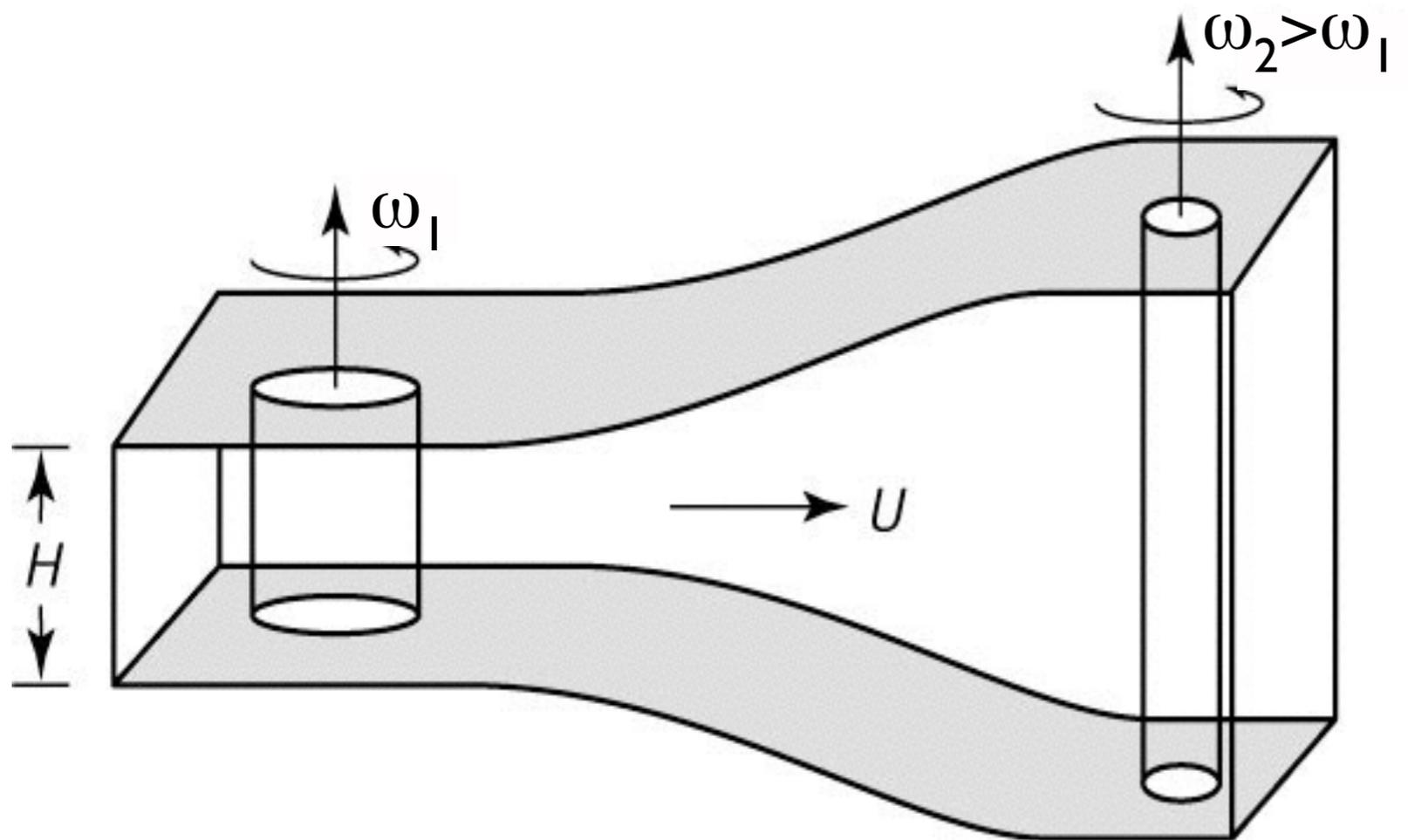
# Part IIa Zonal flows



NASA Movie PIA02863  
(Cassini flyby of Jupiter: Oct 31-Nov 9, 2000)

# Conservation of potential vorticity

## Topographic effect



For inviscid flow:

$$\Pi = (\omega_z + f)/H.$$

The motion of fluid columns conserves  $\Pi$

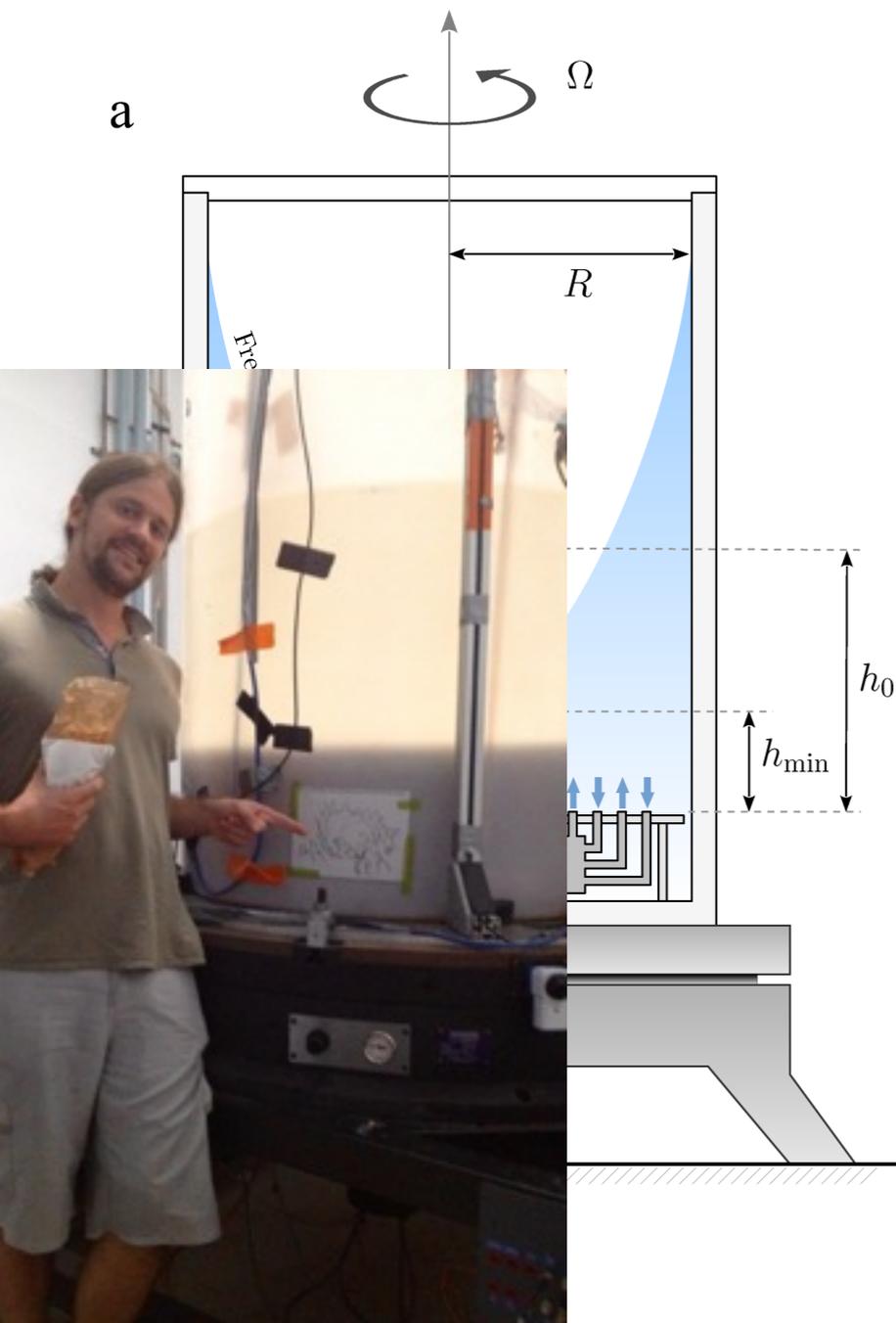
Leads to a *topographic* parameter

$$\beta = -2\Omega(dH/dx)/H$$

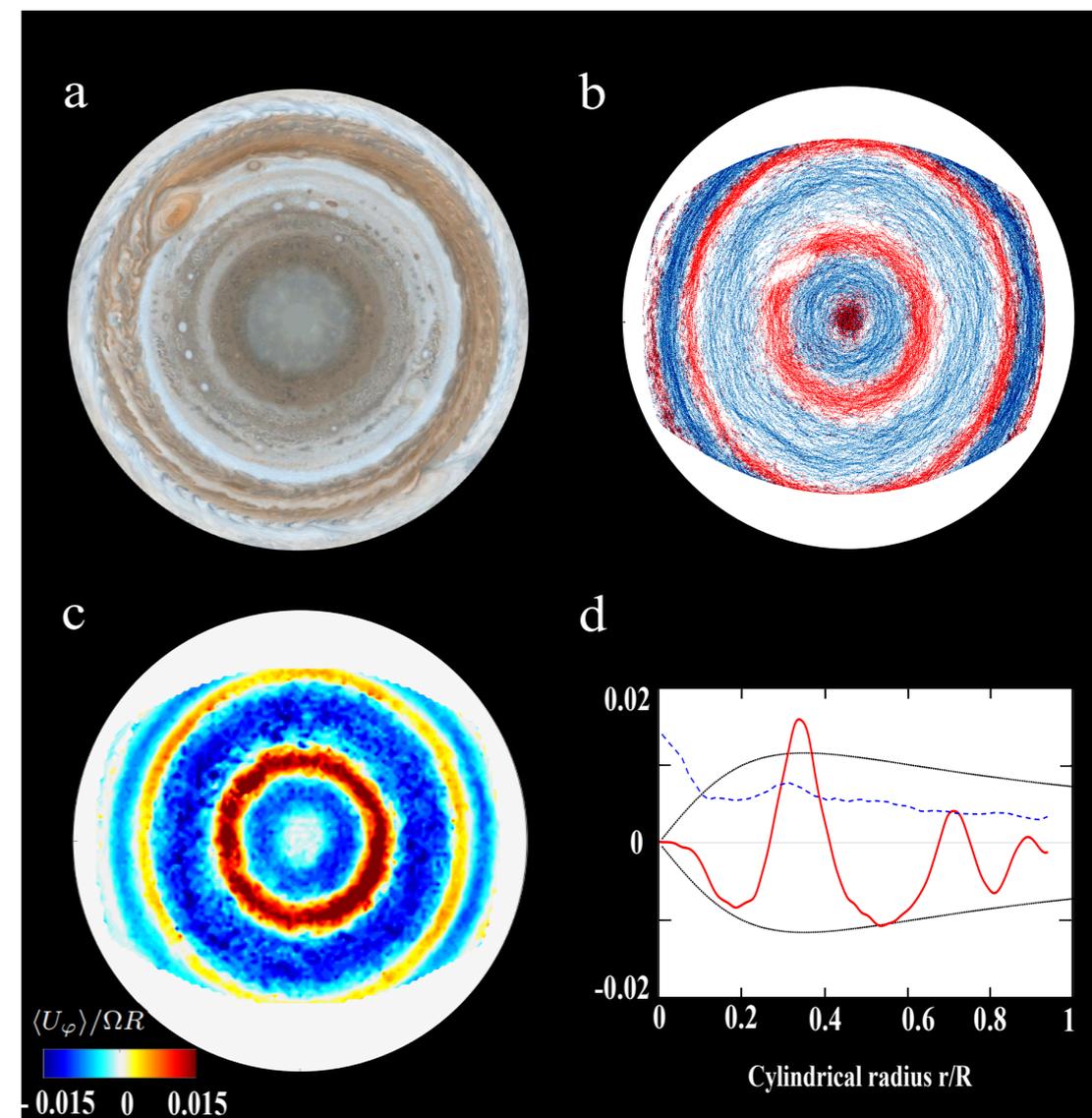
R.L. Stewart, 2004

# Laboratory zonal jets

(Cabanes et al., 2016, in review)



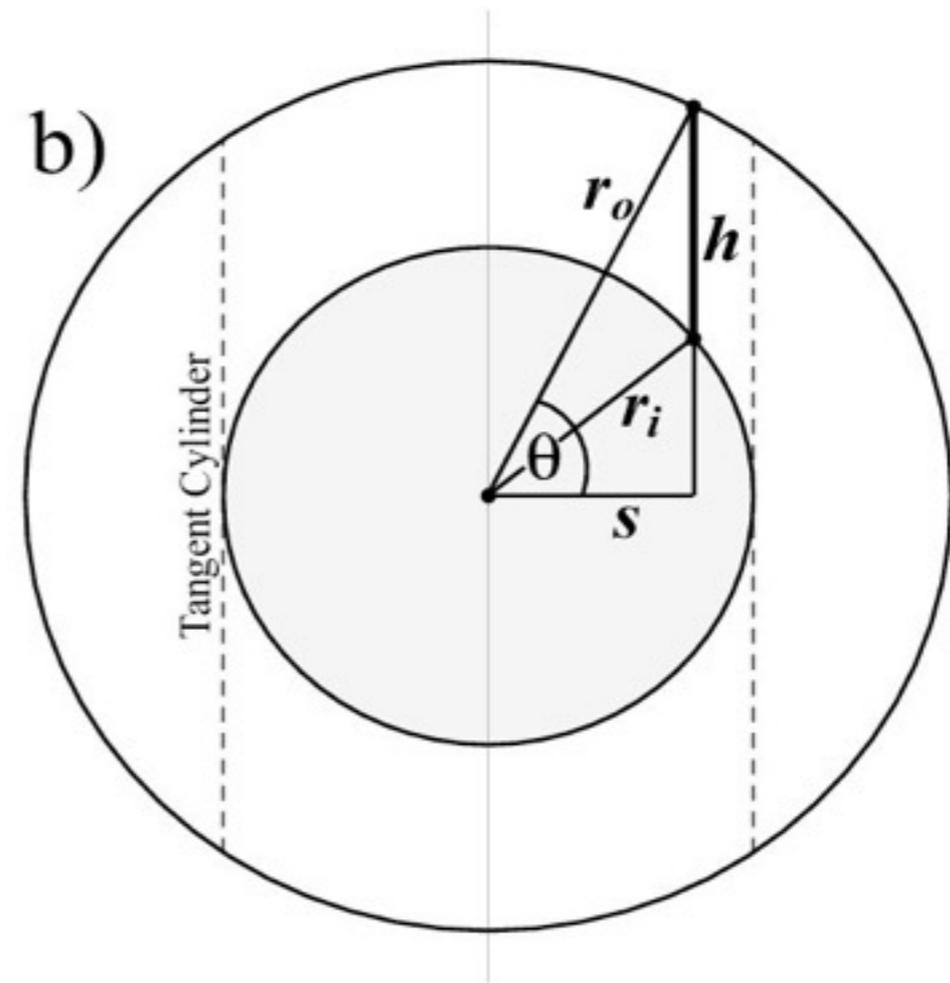
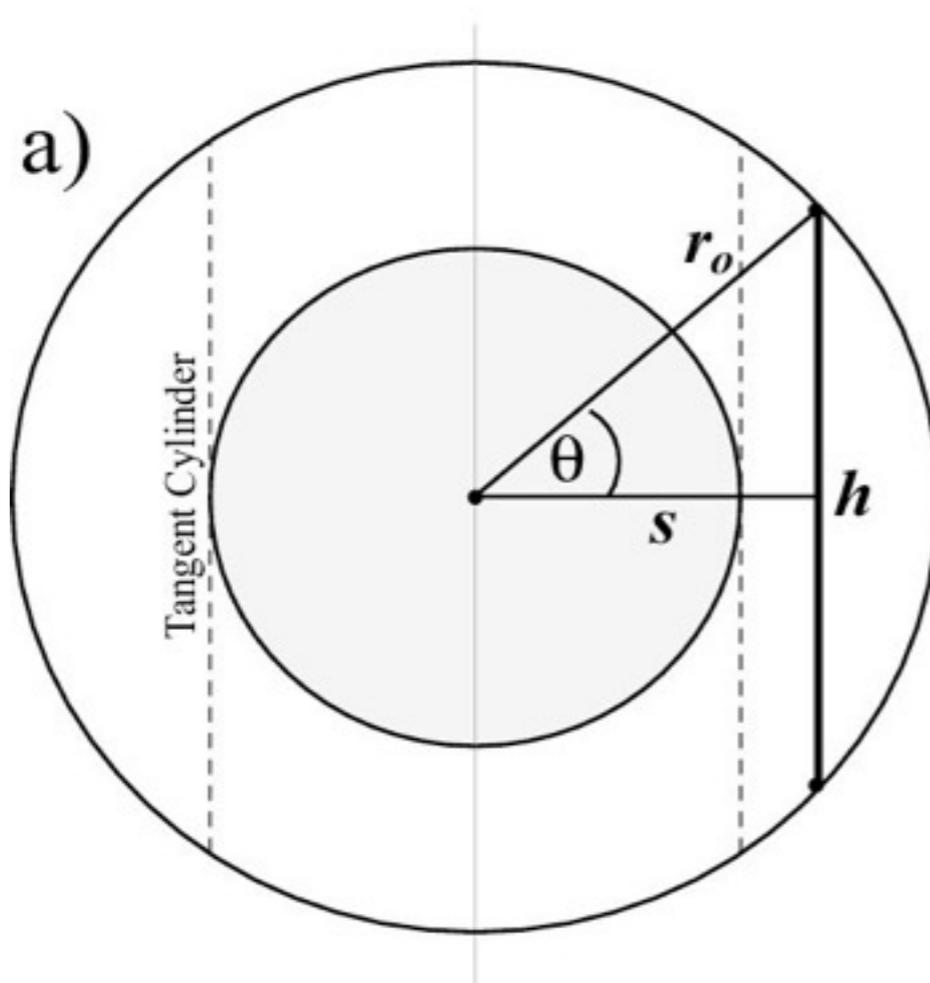
Jupiter vs. Lab



# Spherical Shell Rhines Scale

Outside TC

Inside TC

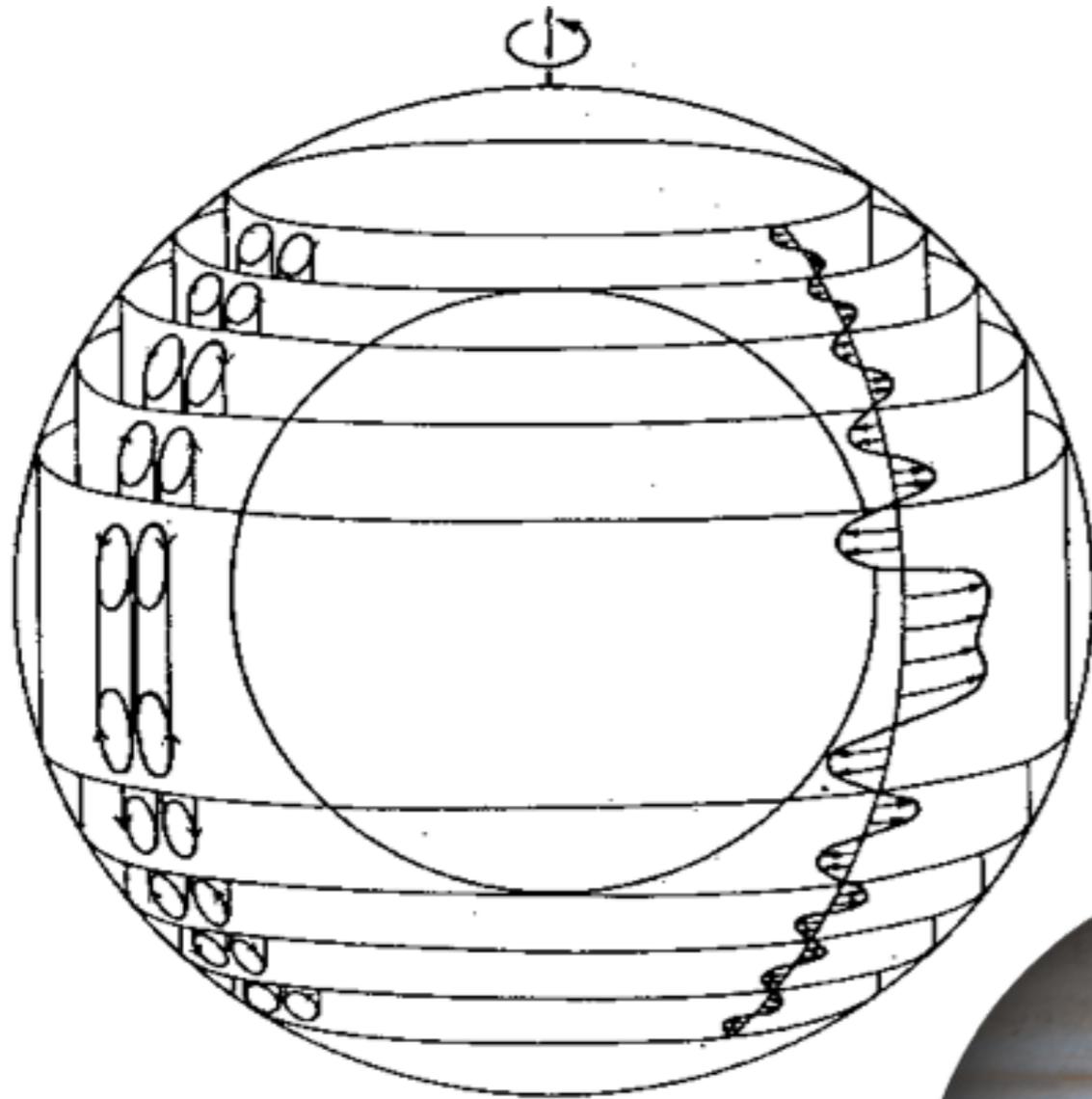


$$\lambda_g = 2\pi \sqrt{\frac{U}{r_o \Omega \cos \theta}}$$

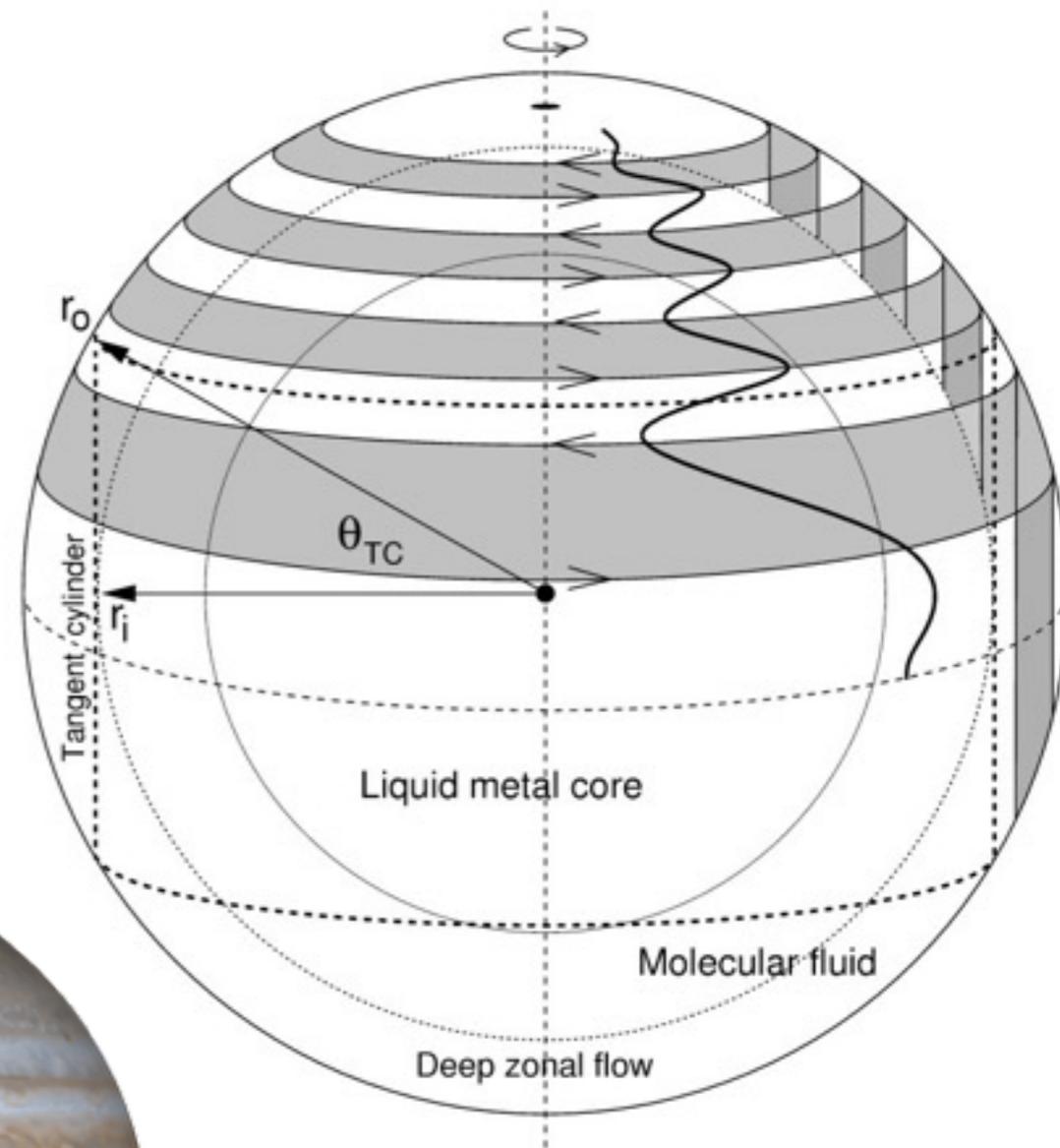
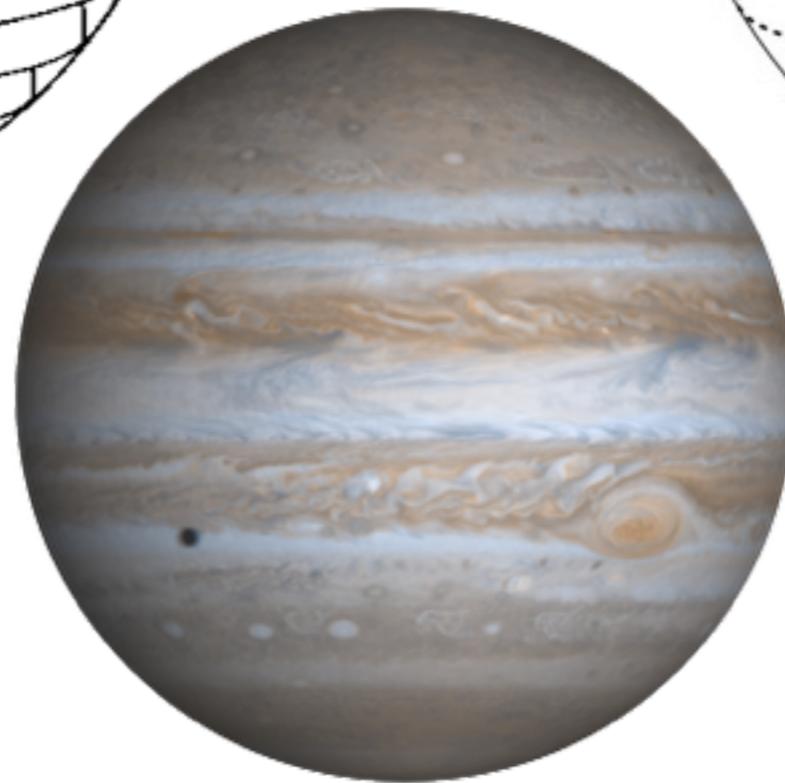
$$\lambda_g = 2\pi \sqrt{\frac{U}{r_o \Omega} \left| \frac{(\chi^2 - \cos^2 \theta)^{1/2}}{\sin \theta \cos \theta} \right|}$$

(Notes: 1. Scaling discontinuity across TC; 2.  $\lambda$  depends on U)

# Zonal flow in spherical shells: Reynolds stress spins columnar convection (Busse columns) into zonal flows.



Busse, 1976



Heimpel, Aurnou & Wicht., 2005

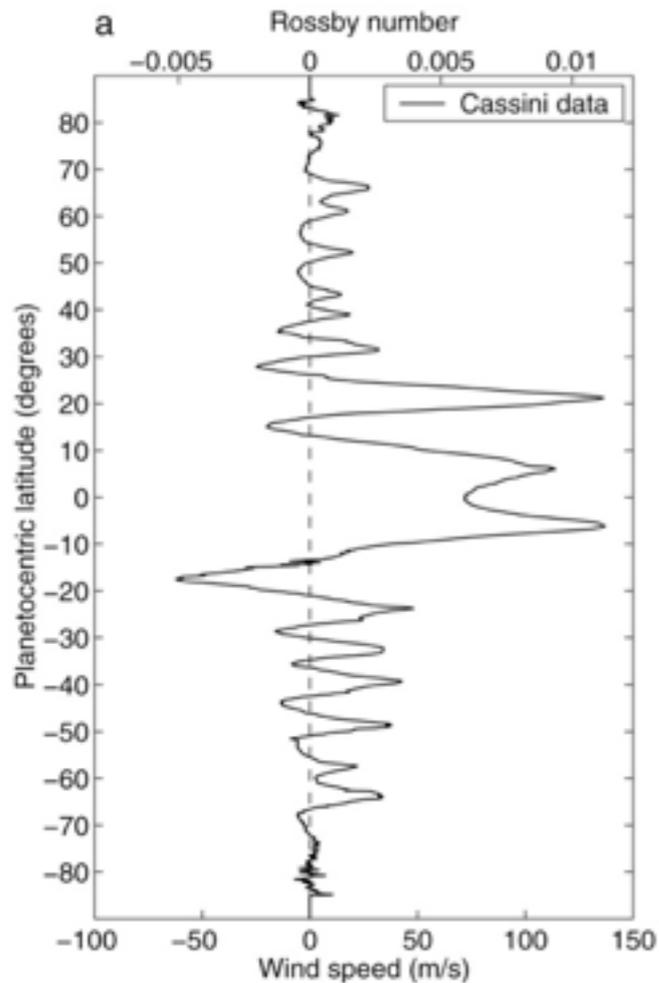
# Jupiter & Saturn winds, and numerical model

Model:  $E=3 \times 10^{-6}$ ,  $Ra=5.6 \times 10^8$ ,  $Pr=0.1$ ,  $r_i/r_o=0.9$ , 8-fold symmetry  
Boussinesq approximation.

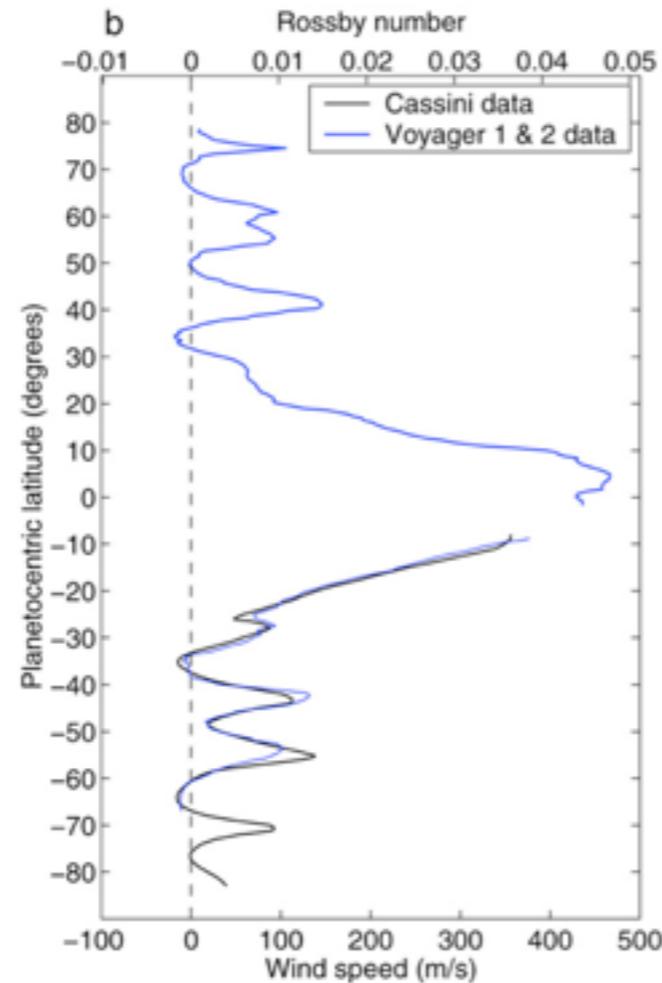
Constant top-bottom temperature difference drives convection.

Free-slip top and bottom boundary conditions

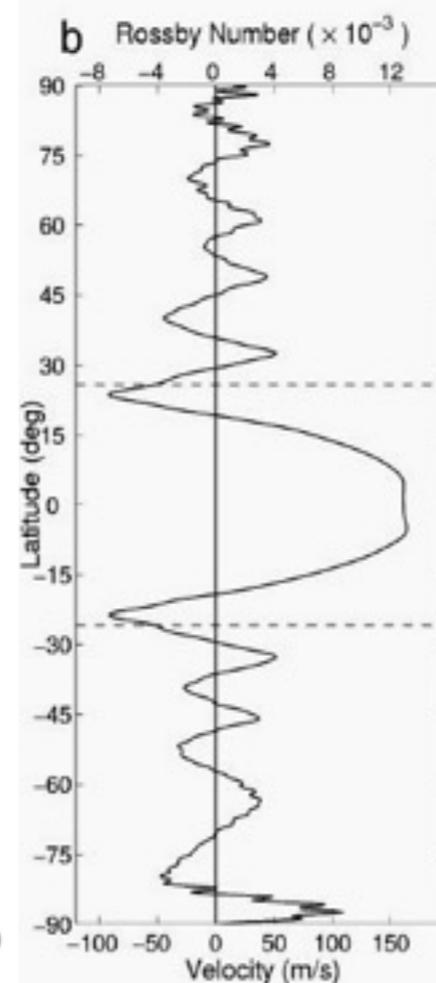
Jupiter



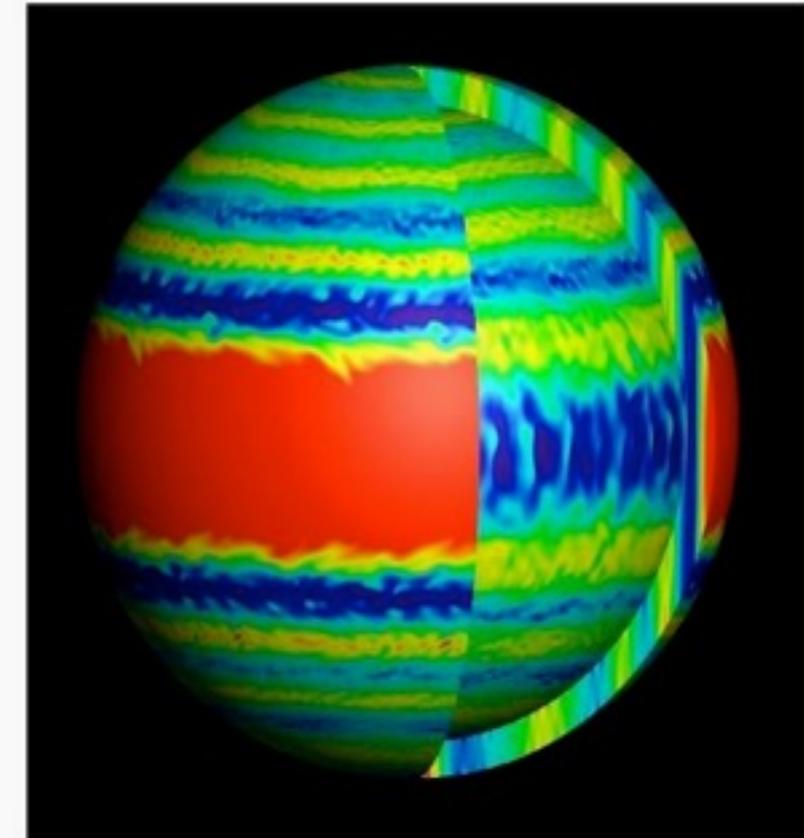
Saturn



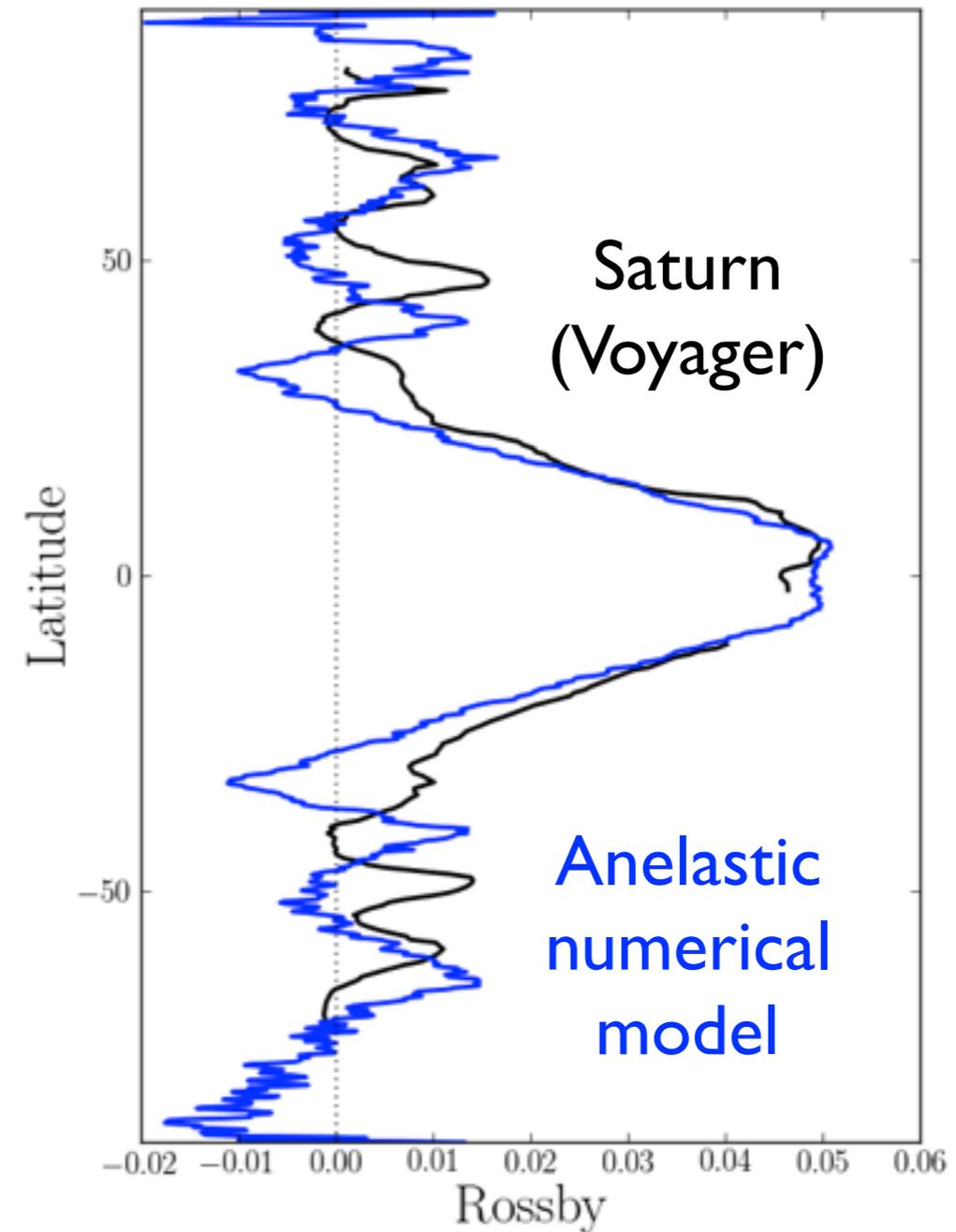
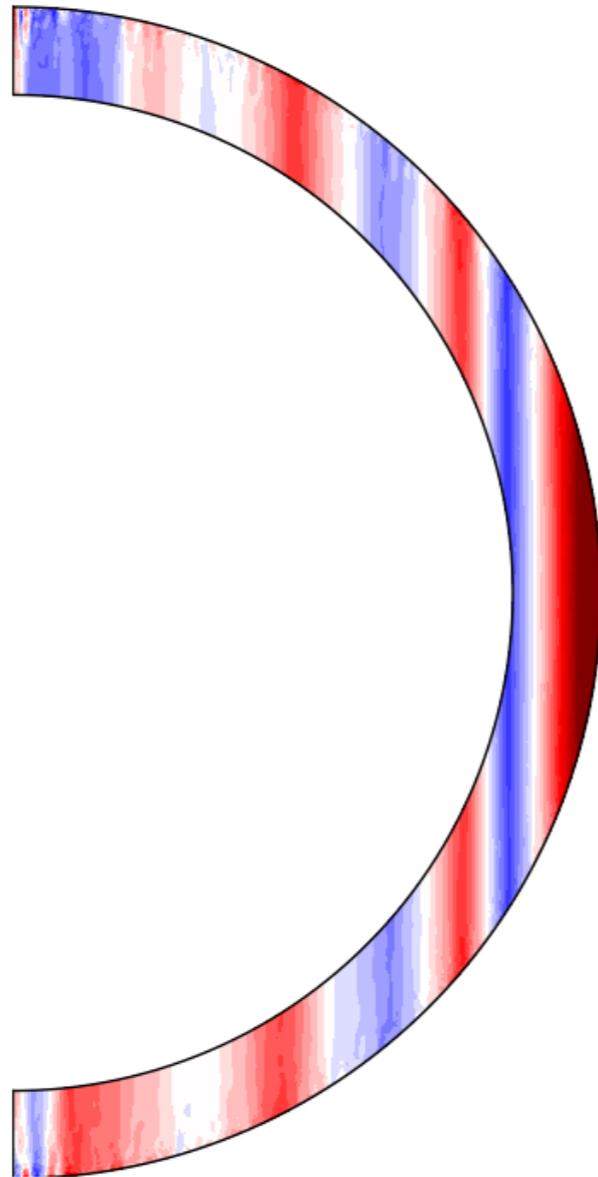
Numerical model



c

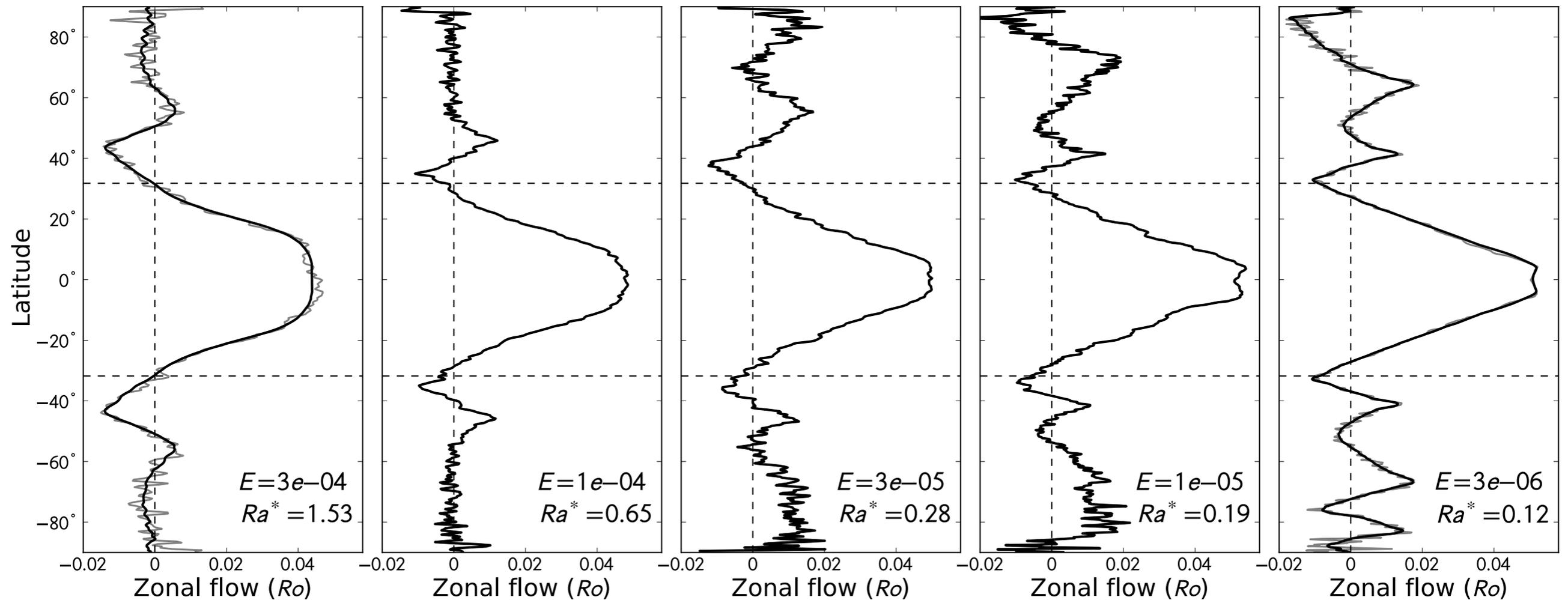


Anelastic model (Magic) with  $E=3 \times 10^{-6}$ ,  $Ra=1.5 \times 10^9$ ,  $N_\rho=5$ ,  $Pr=0.1$ , and  $r_i/r_o=0.85$ . Comparison of zonal flow to cloud layer zonal wind of Saturn.



# Why do we need high resolution models?

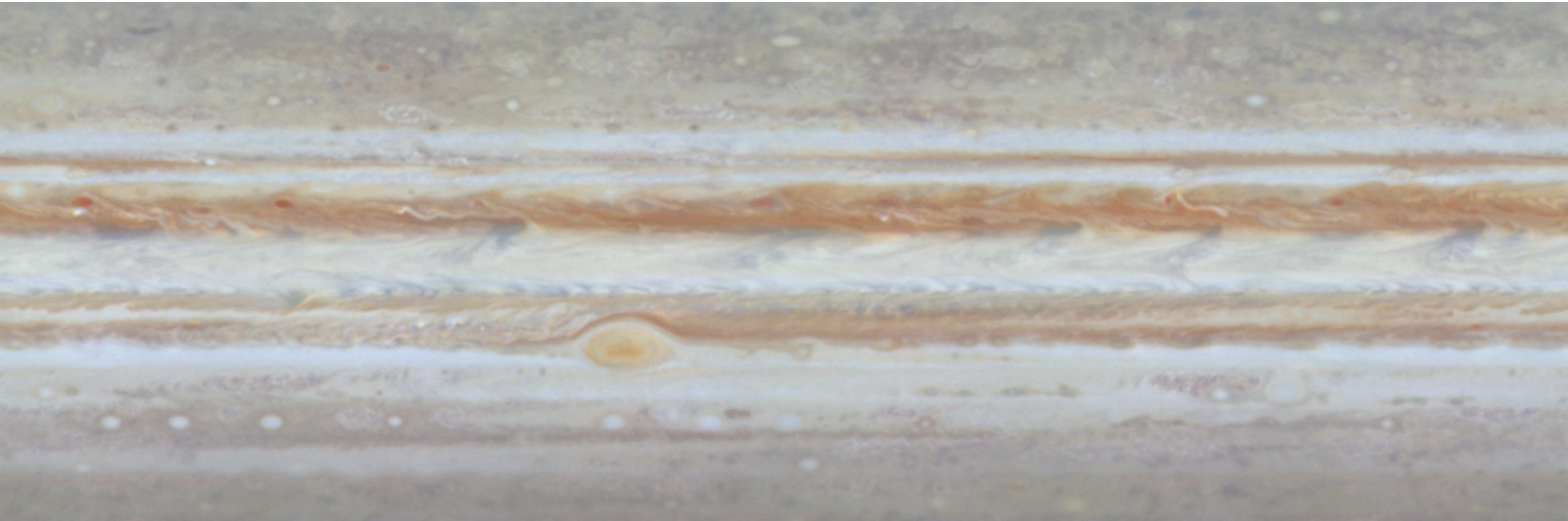
Well-defined multiple jets inside the tangent cylinder ( $N_\rho = 5$ ,  $r_i/r_o = 0.85$ )



high viscosity or weak rotation

lower viscosity or strong rotation

# Part IIb: Jets, Vortices and Storms



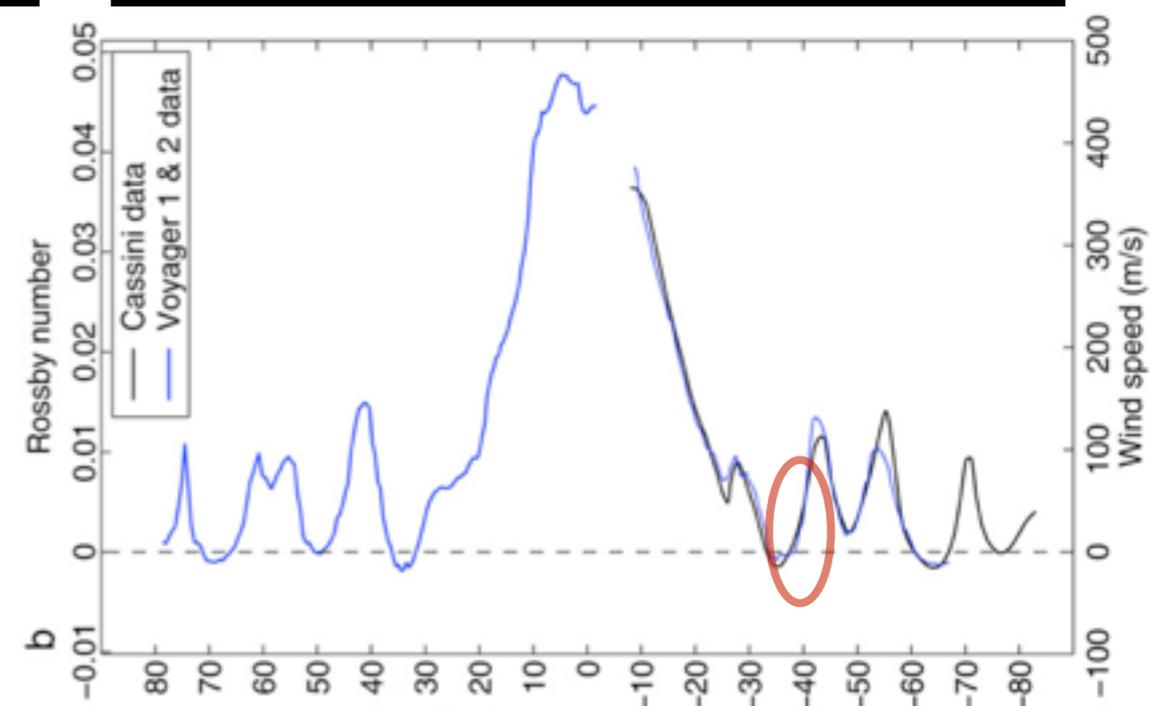
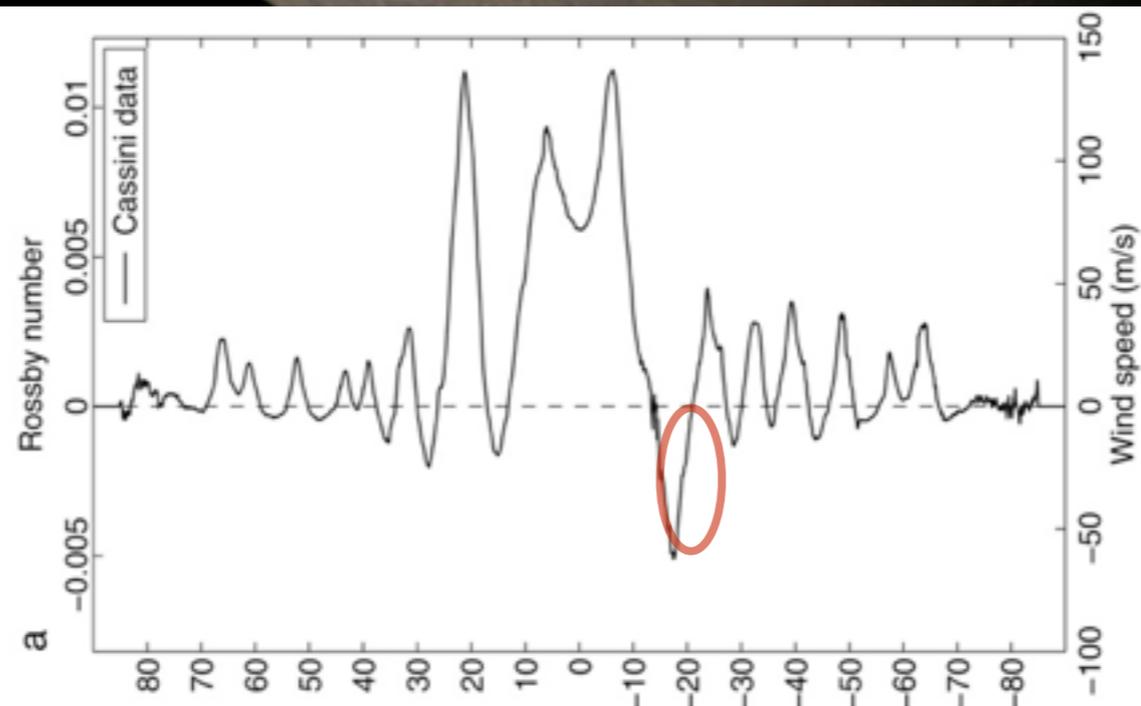
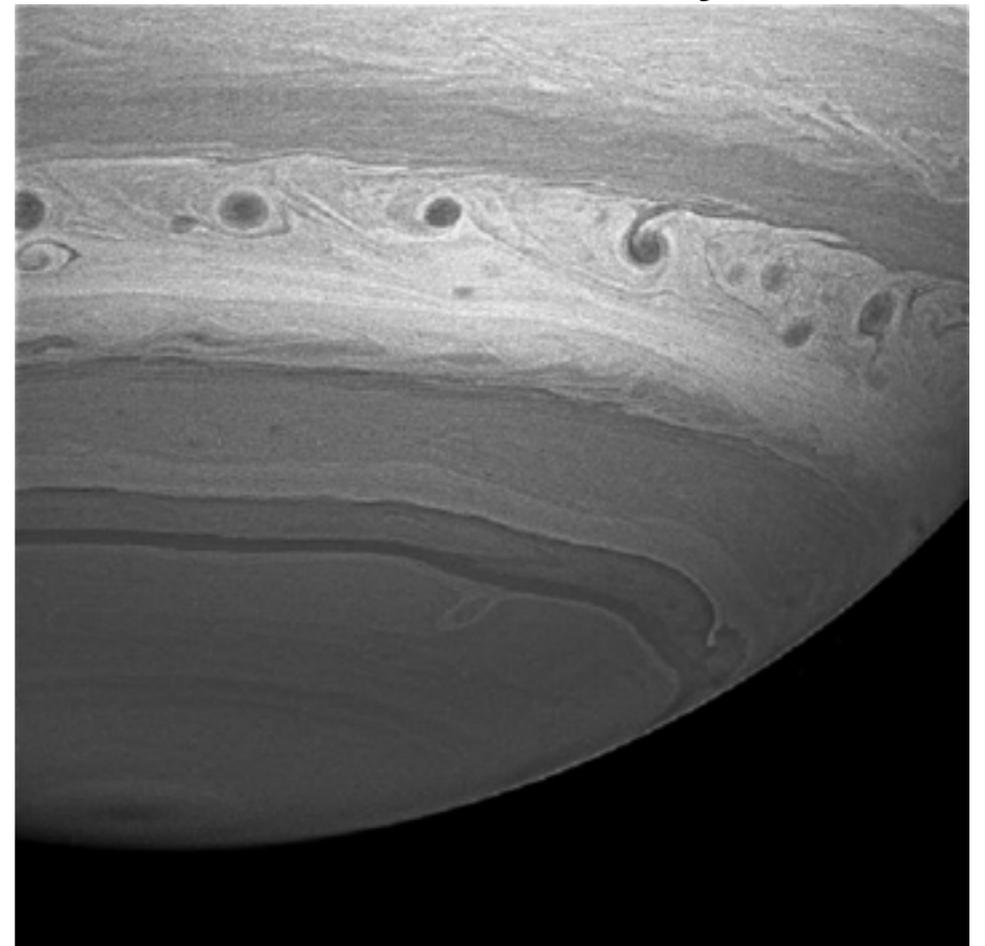
NASA Movie PIA02863

# Vortices on Jupiter and Saturn

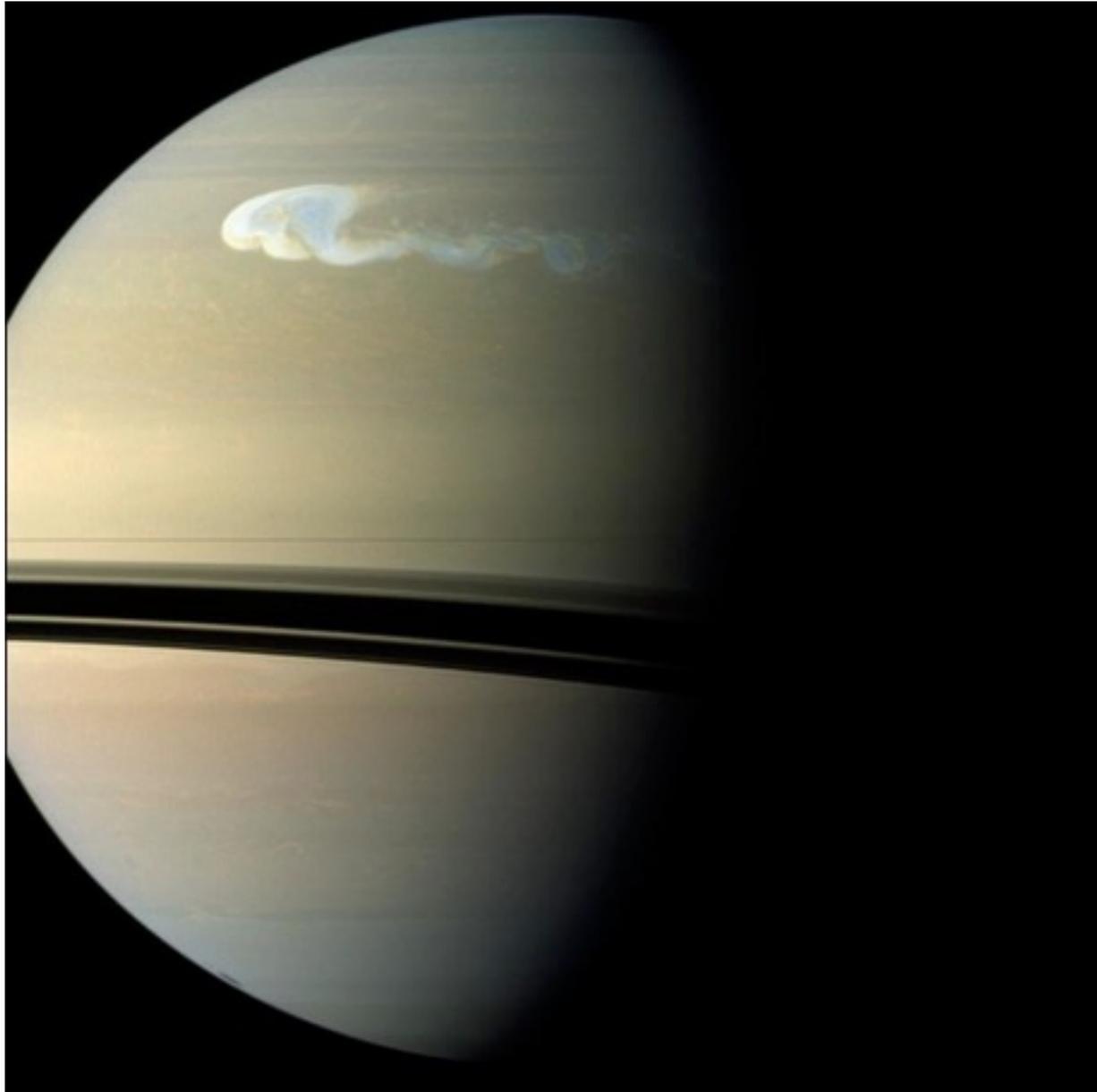
Jupiter Great Red Spot and White Ovals



Saturn "Storm Alley"



# Saturn's great storm of 2010/2011



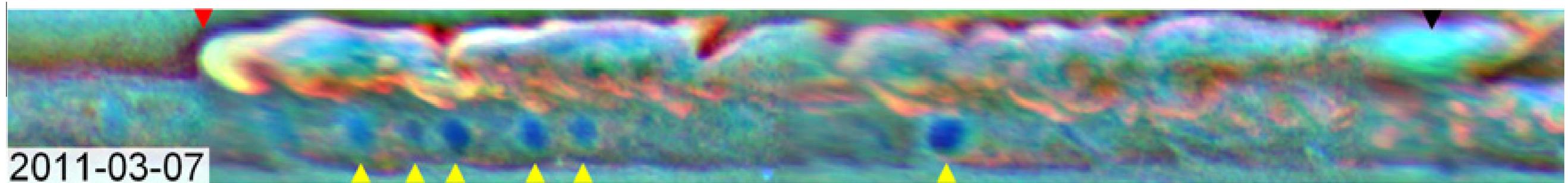
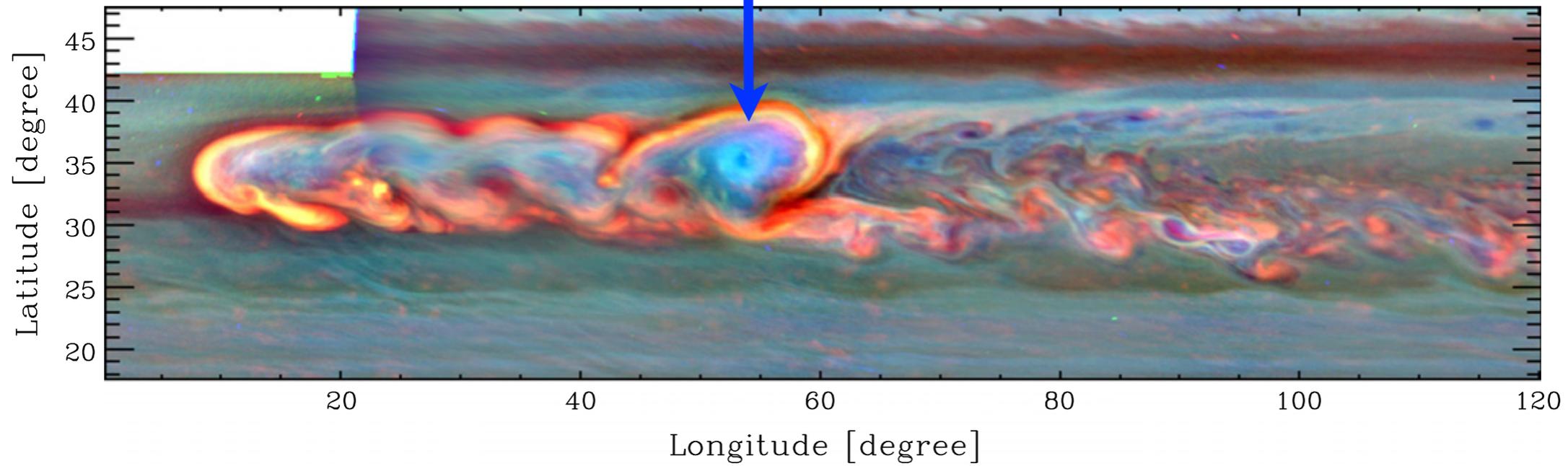
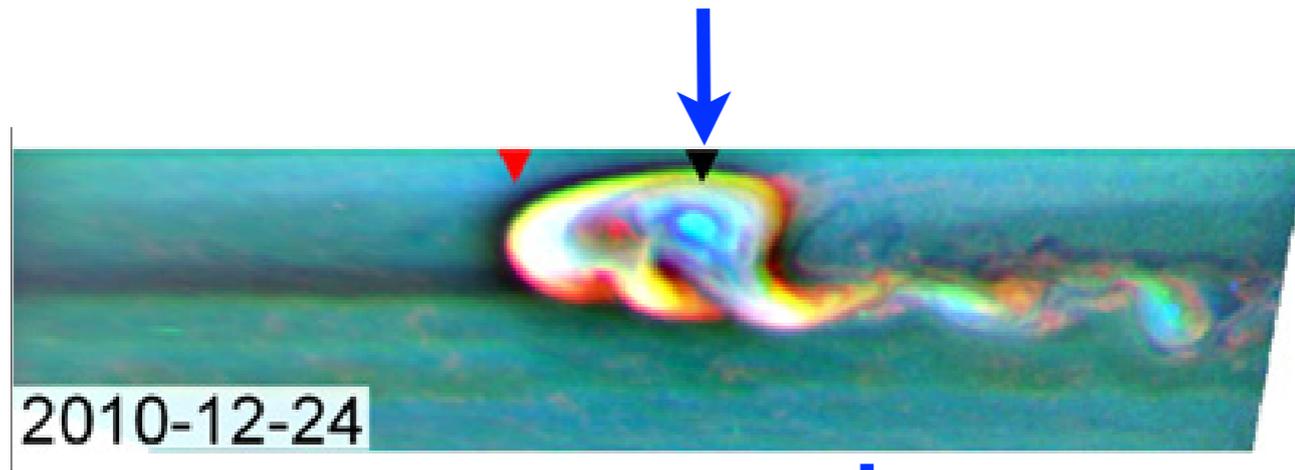
Dec 5, 2010



Feb 25, 2011

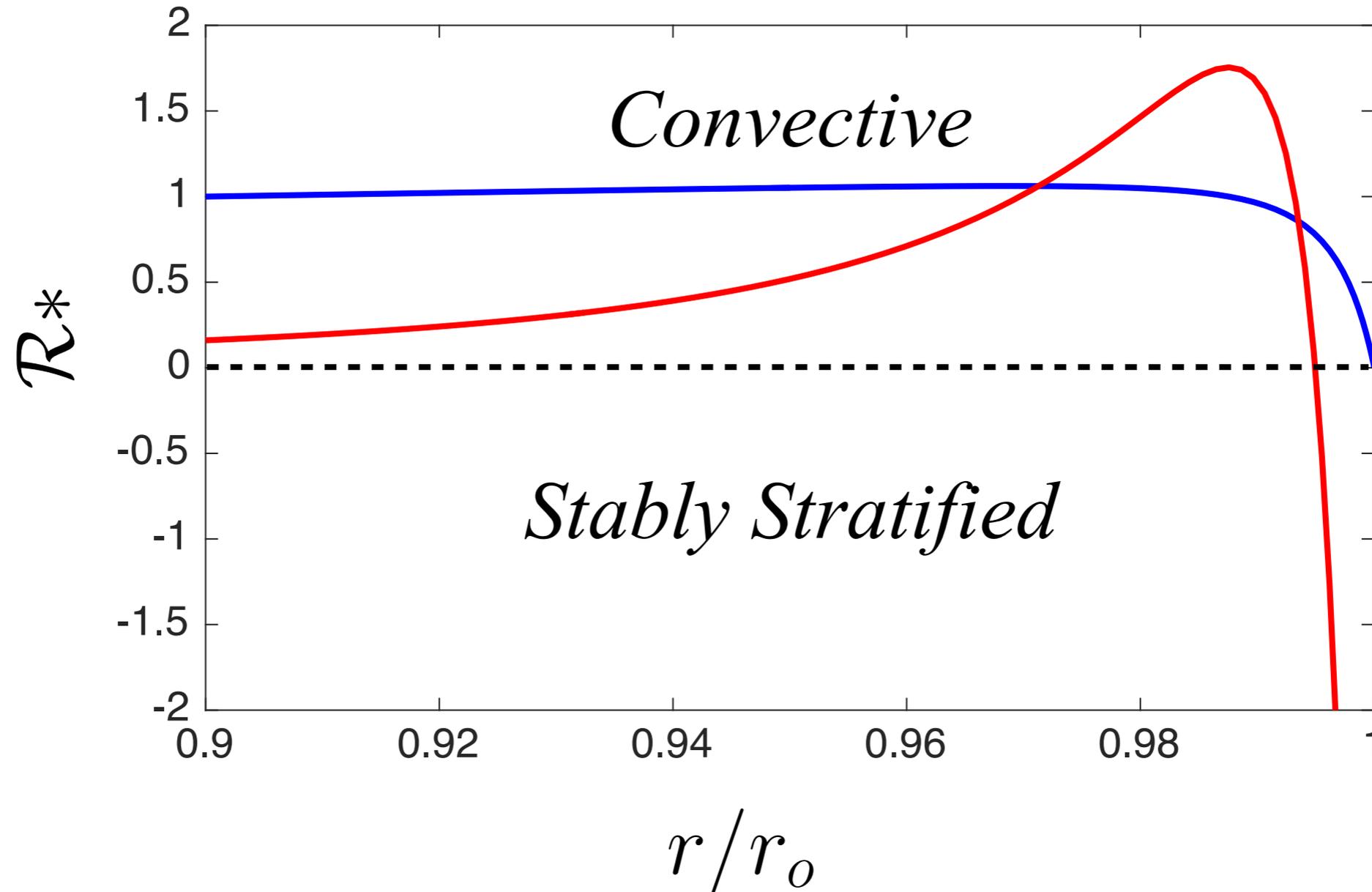
*NASA Cassini images PIA 12824 and PIA 12826*

# Anticyclonic vortex initiated Saturn GWS



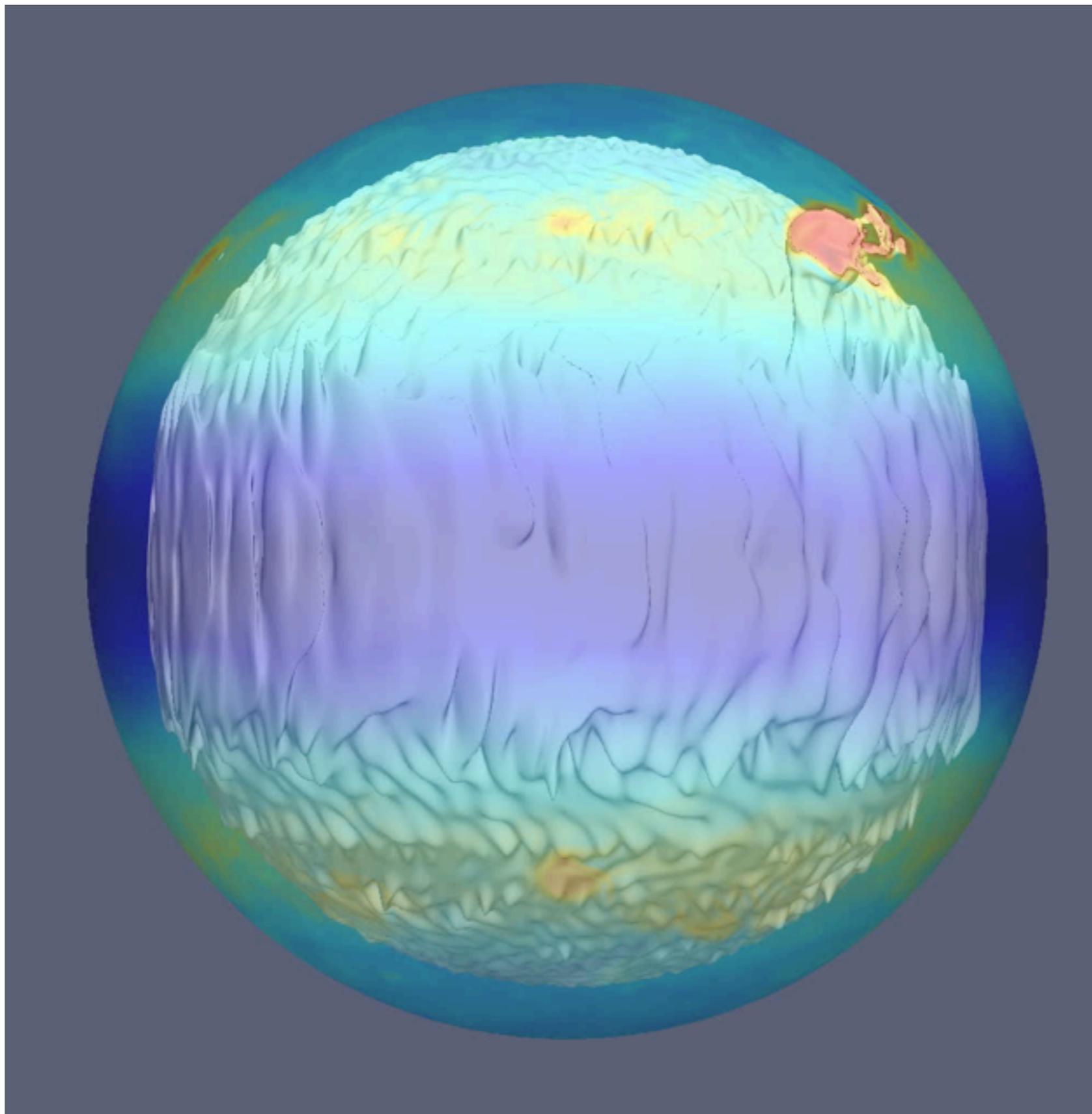
*Sayanagi et al., Icarus, 2013*

Compressible fluid (ideal gas, anelastic approximation). Constant entropy flux BC with stably stratified or neutral buoyancy outer boundary condition



- $R^* = \text{Ra}E^2/\text{Pr}$
- Dimensionless  $\text{N}_{\text{BV}}^2 = R^*\Omega^2$
- Rossby radius =  $H_\rho (R^*)^{1/2}$

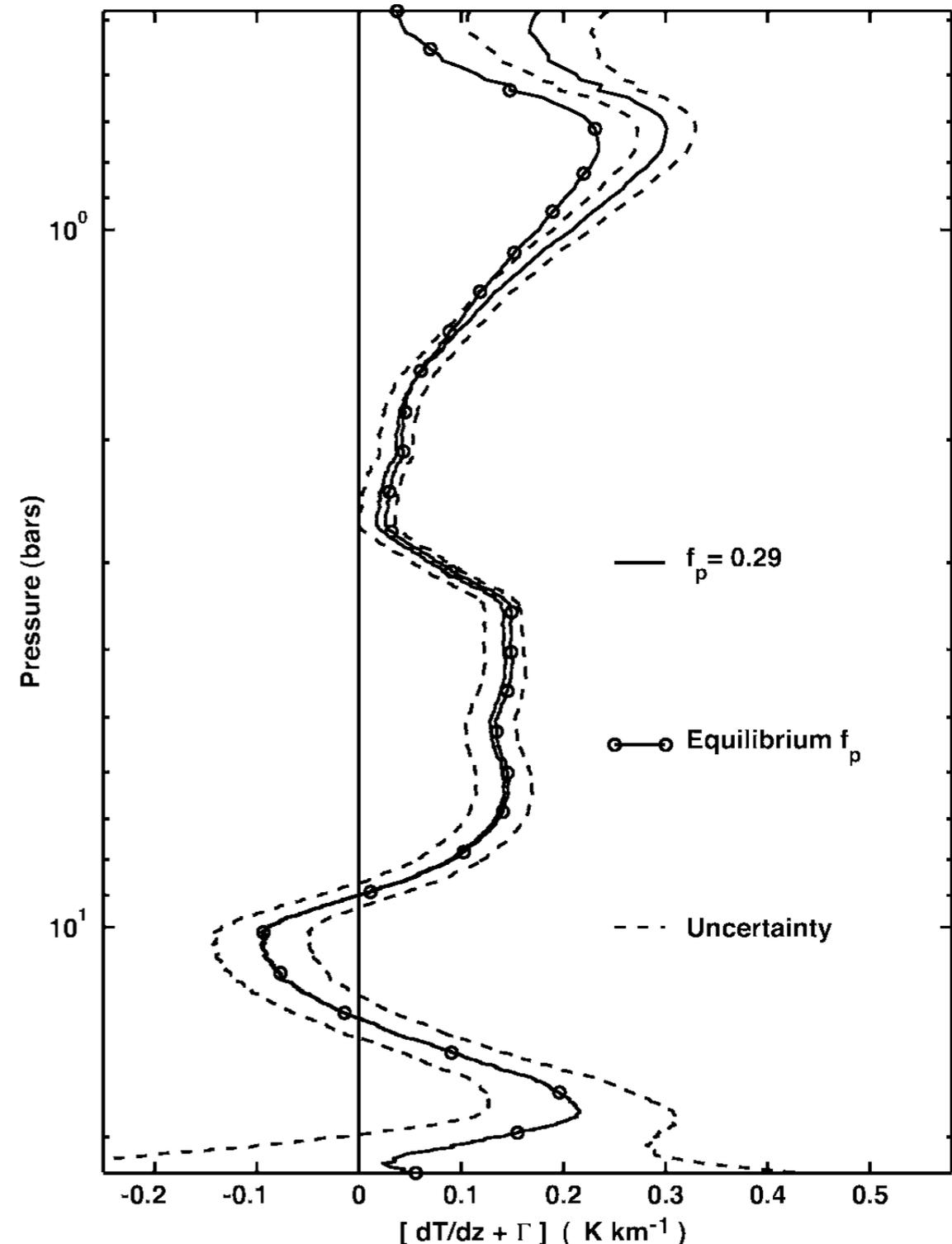
Vortices fed by rising plumes. Neutral stability outer boundary  
(visualization: Keith Cuff)



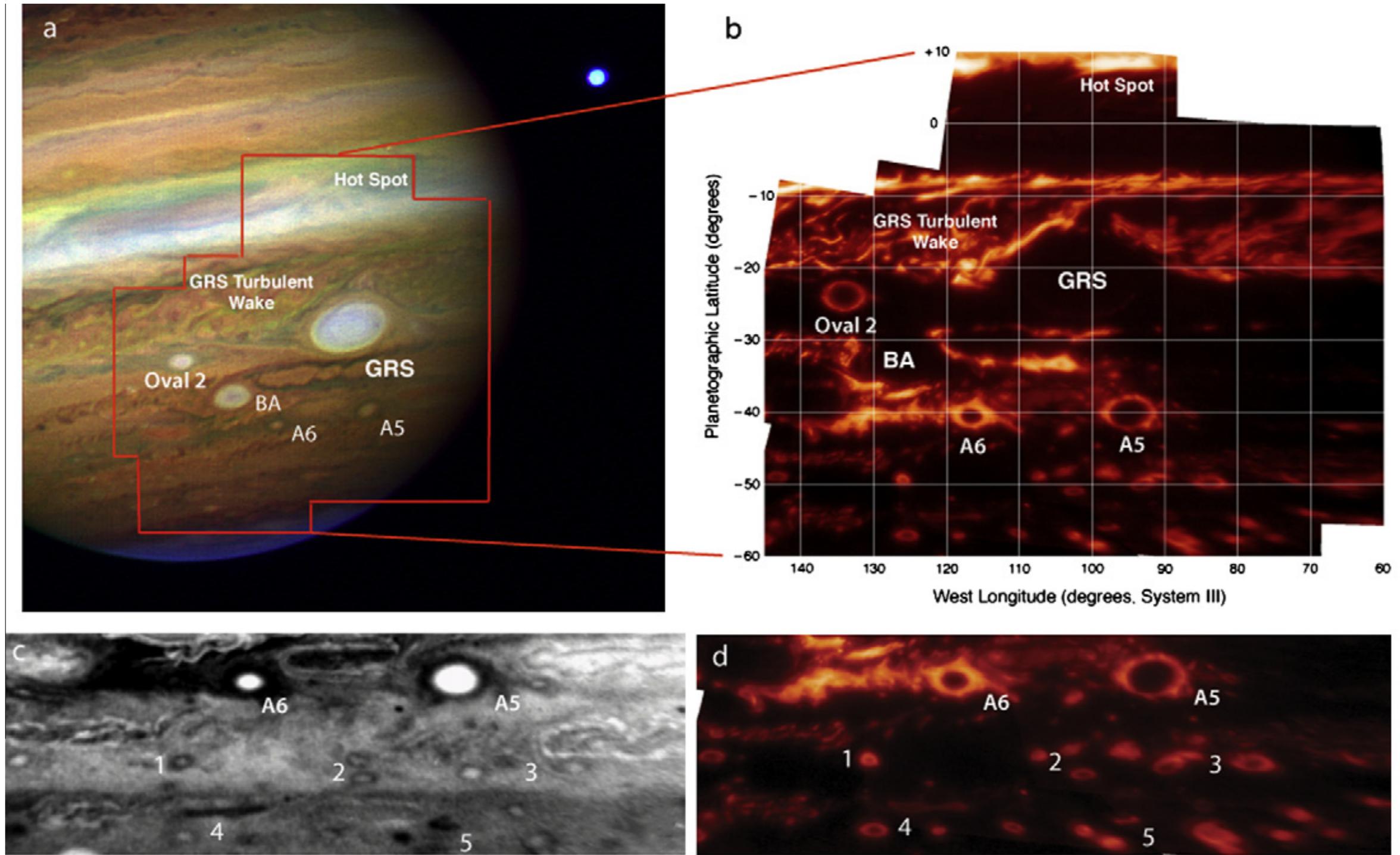
# Depth and strength of static stability in Jupiter's deep atmosphere

- Magalhaes et al., 2002: Static stability layer reaches to maximum depth of Galileo probe ( $\sim 22$  bar,  $\sim 150$  km).

*“Regions with static stability 0.1–0.2 K/km are found at 0.5–1.7 bars, 3–8.5 bars, and 14–20 bars. Between these layers, regions of weaker static stability are present.”*

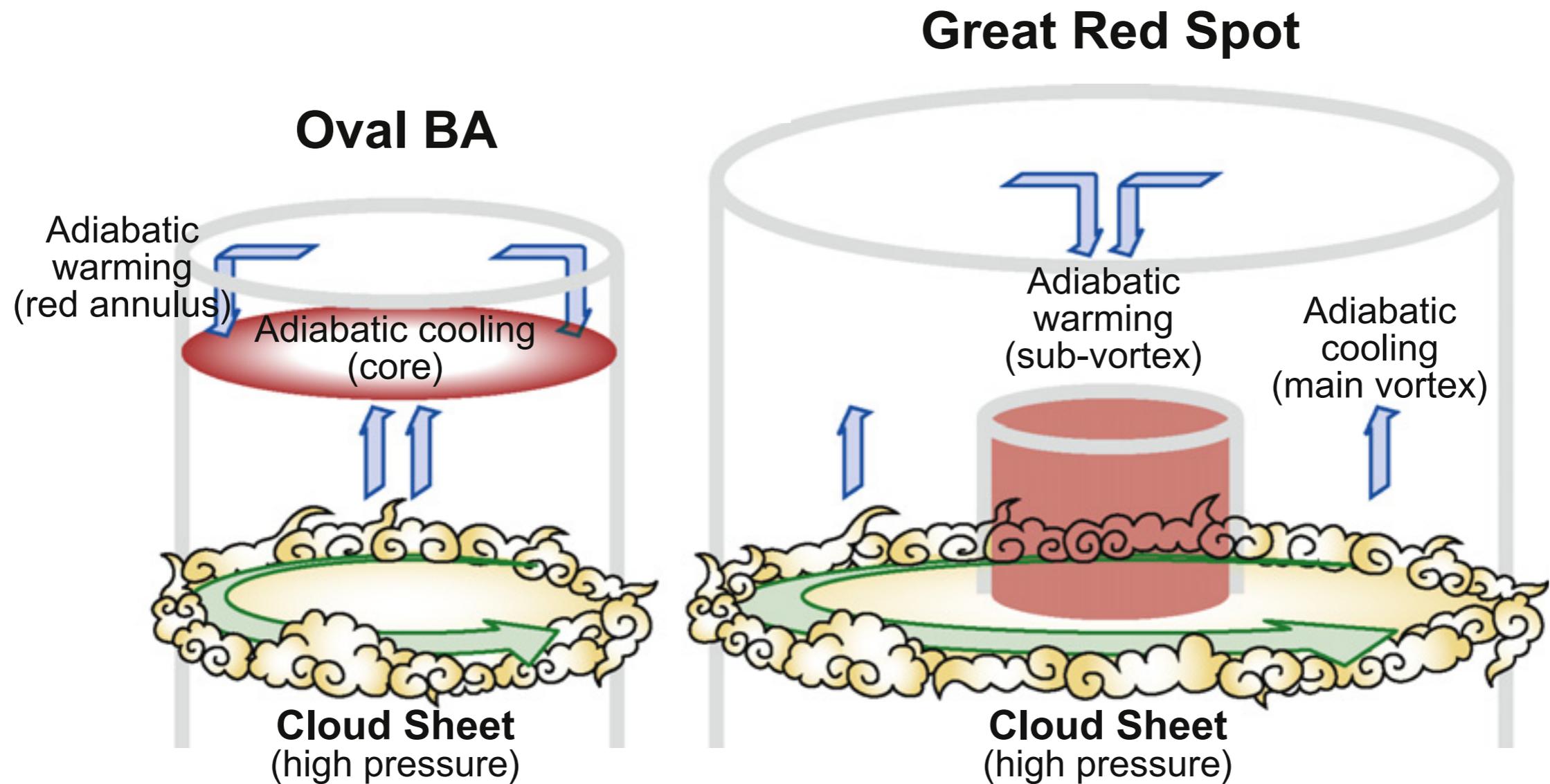


# Jupiter anticyclones, visible and 5 micron IR.



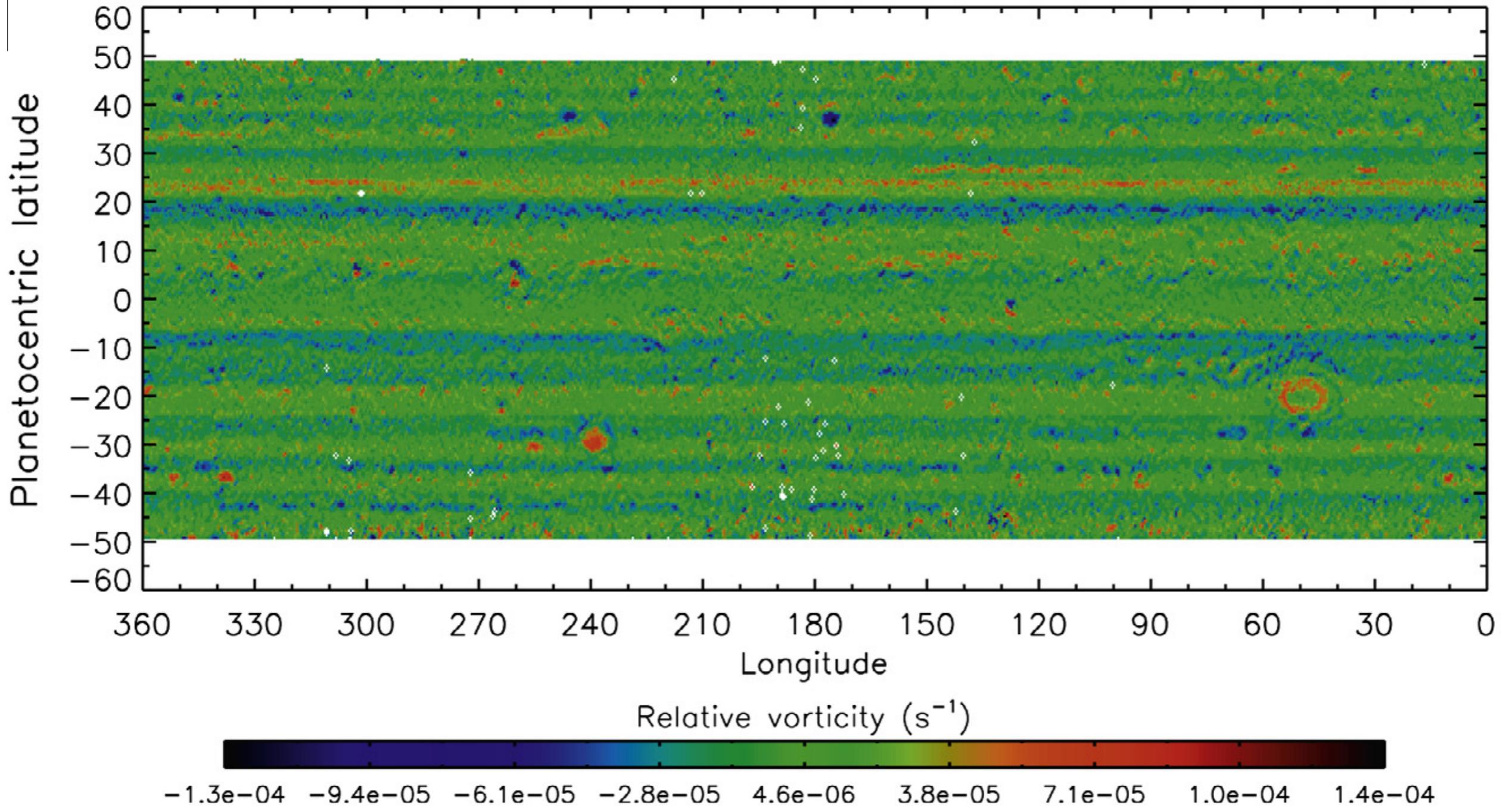
*de Pater et al., 2010*

# Interpretation of Jupiter vortices observations



*de Pater et al., 2010*

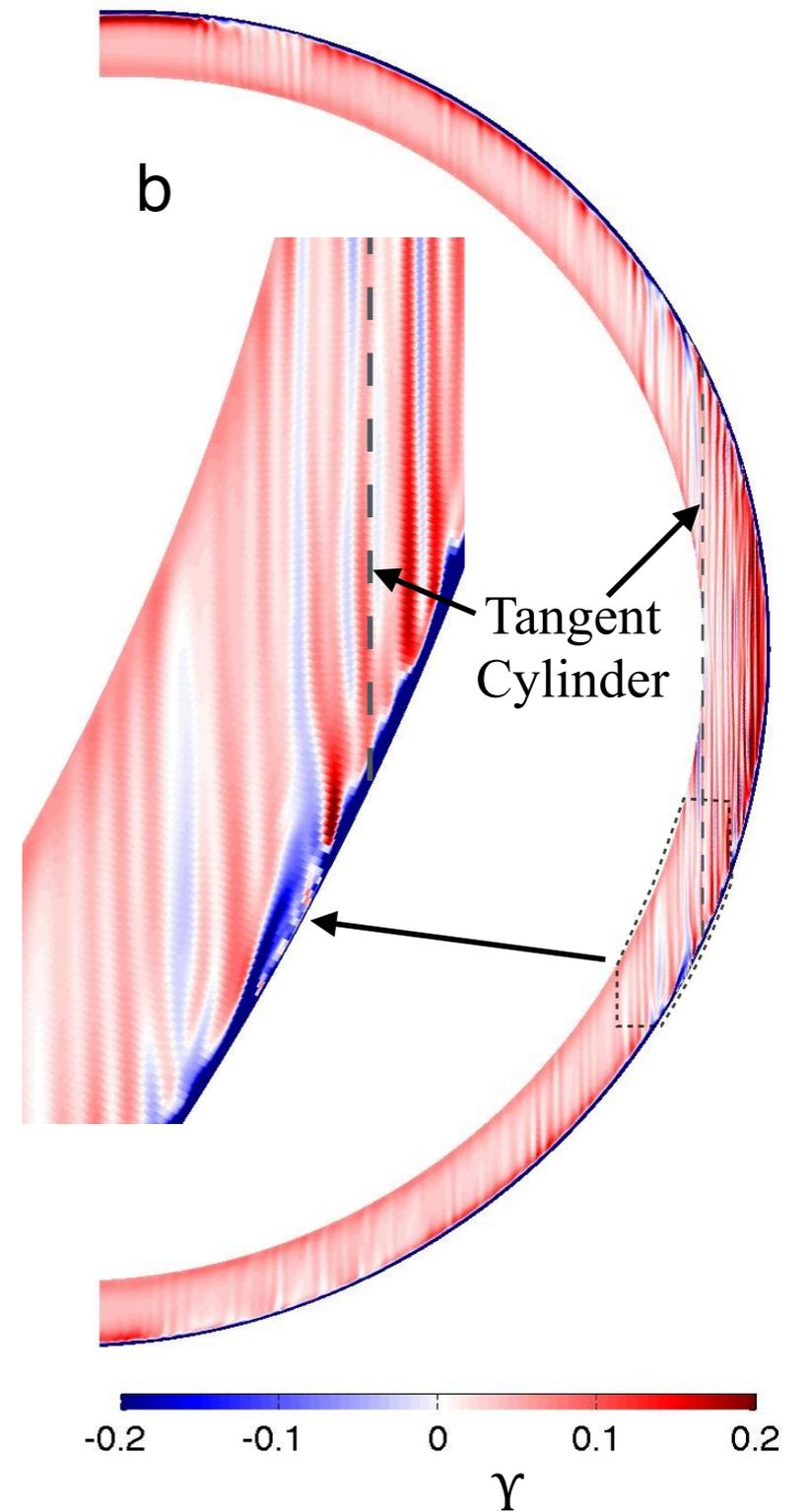
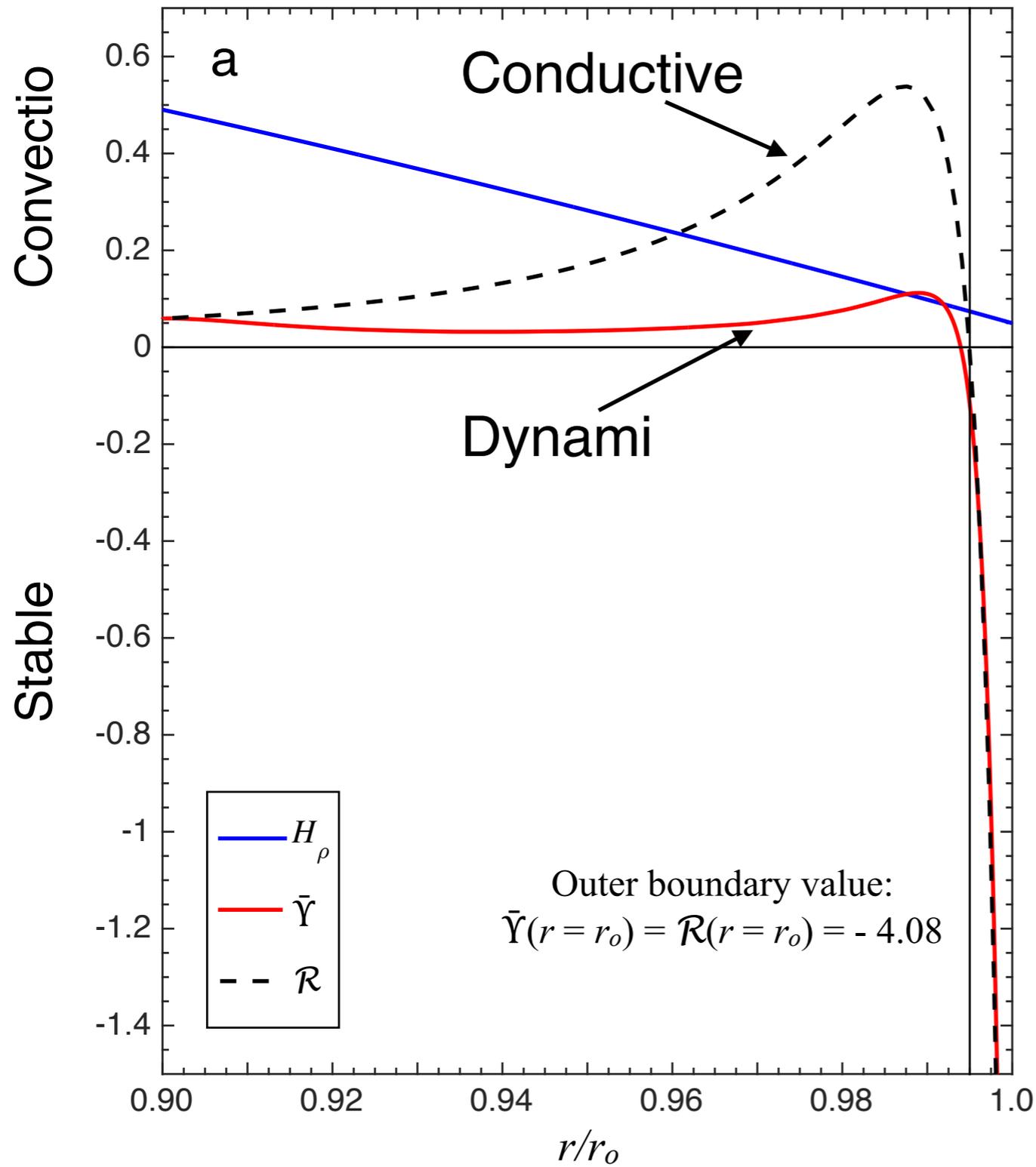
Jupiter radial vorticity at cloud level  
note: Jovian Day = 9:55:30,  $\Omega = 1.8e-4$



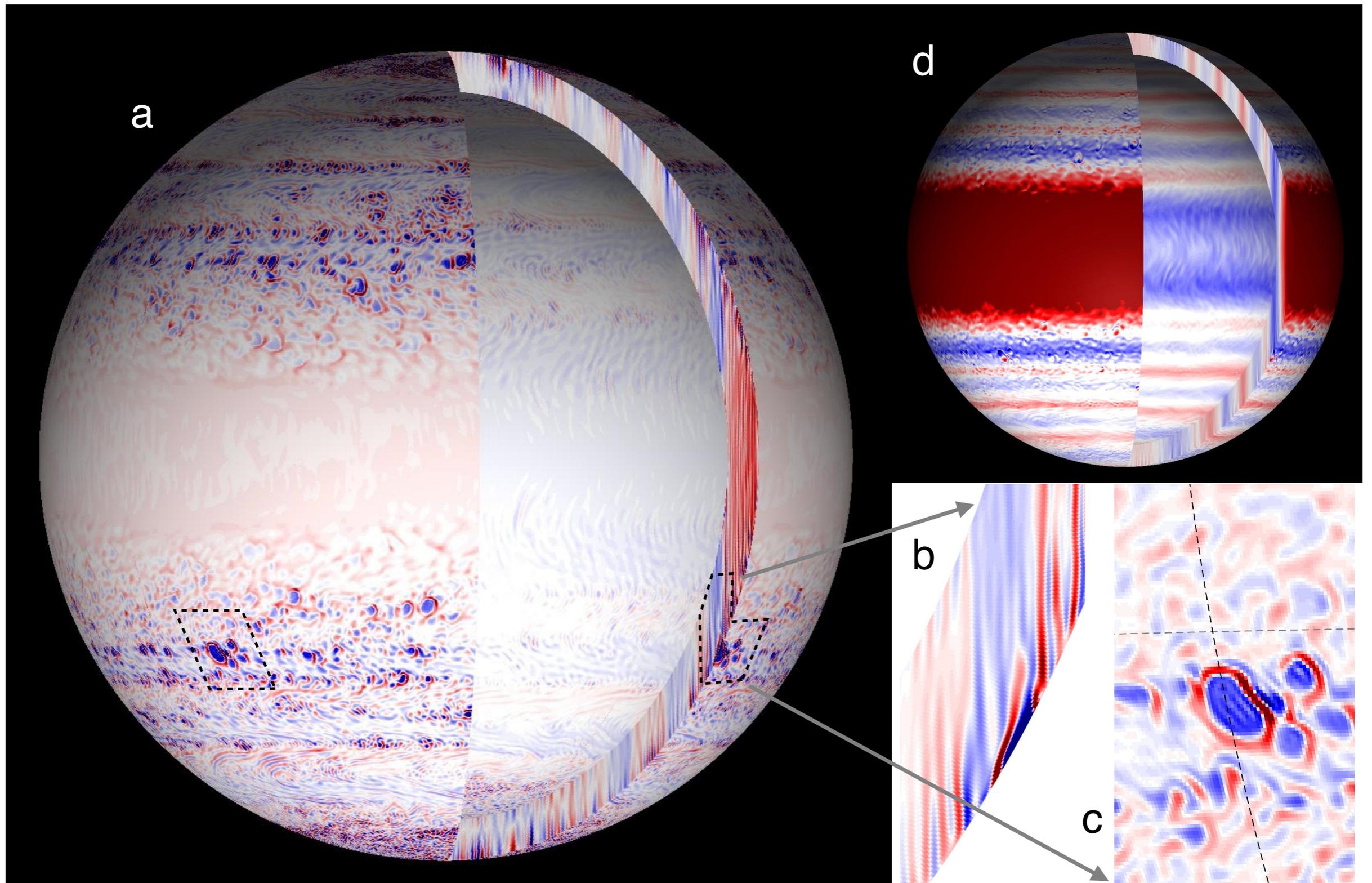
(b) Relative vorticity  $\nabla \times \mathbf{u}$

*Galperin et al., 2014*

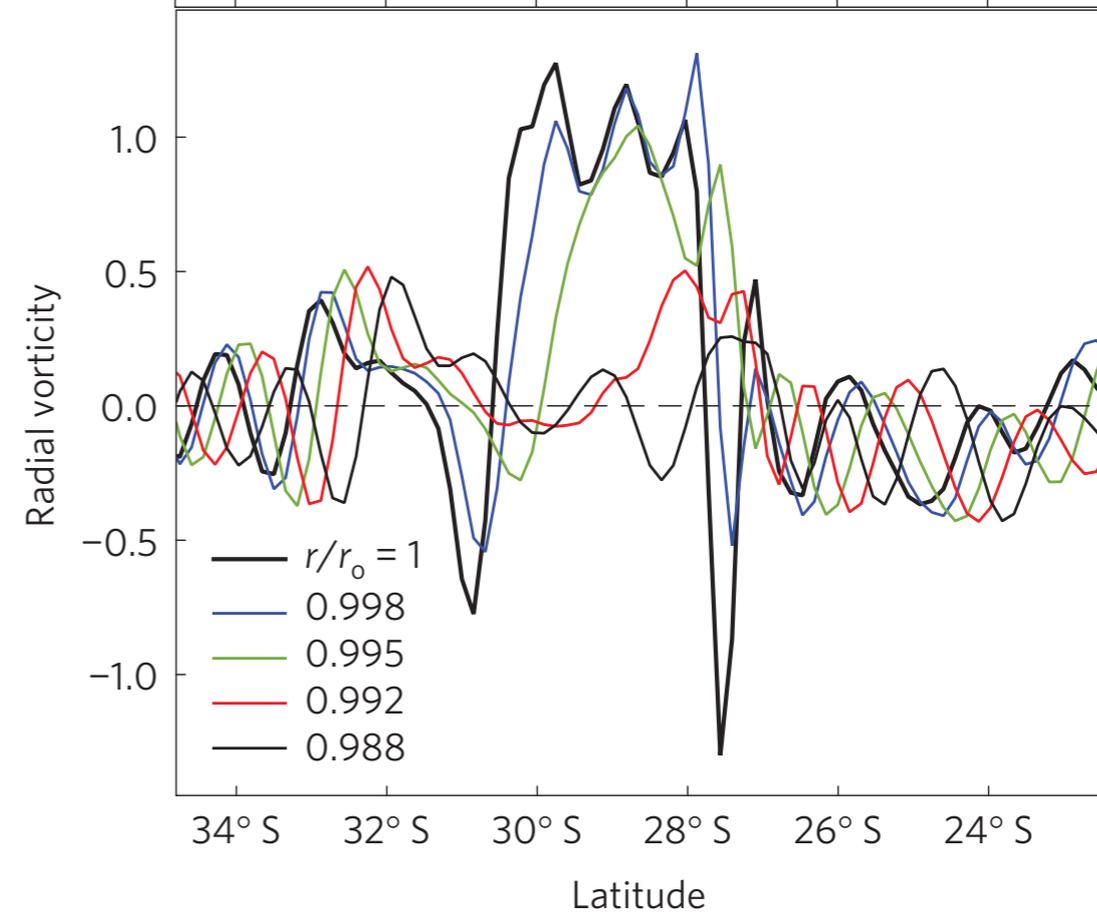
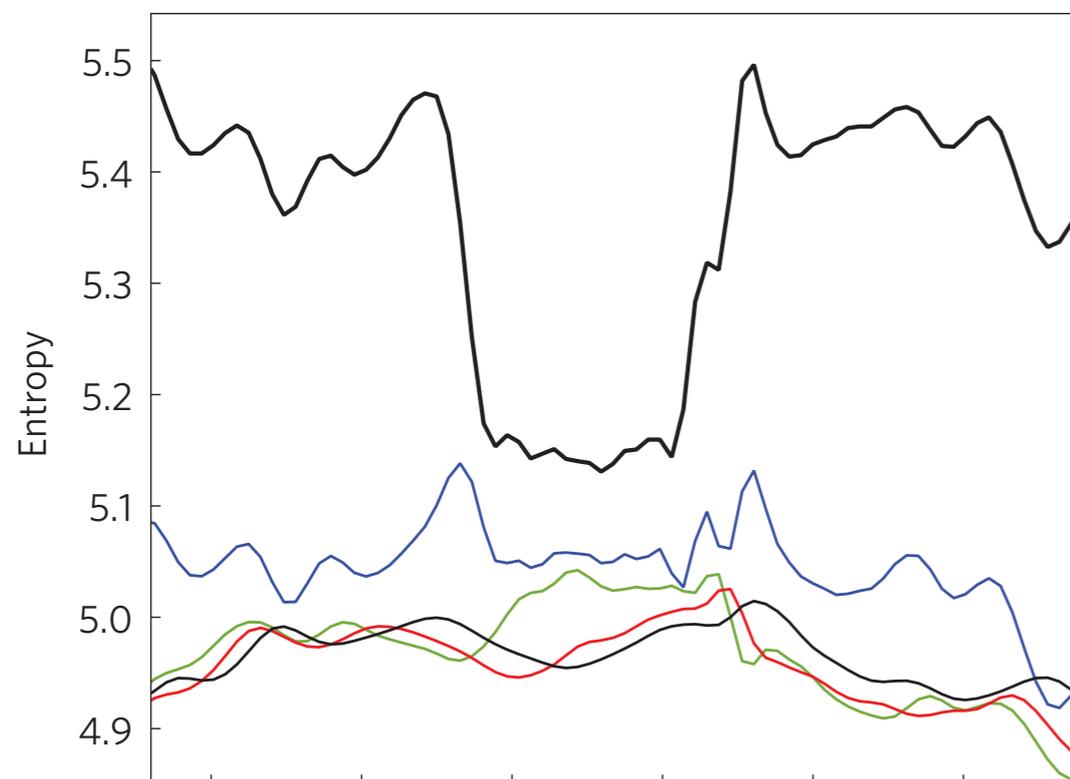
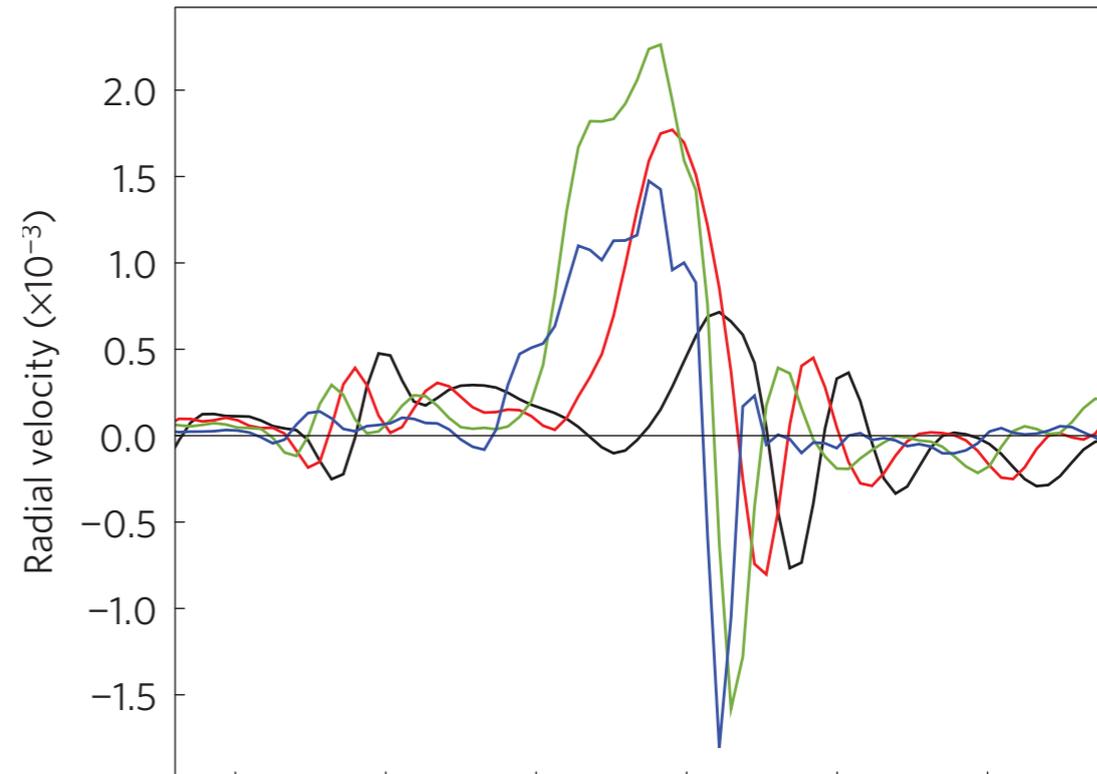
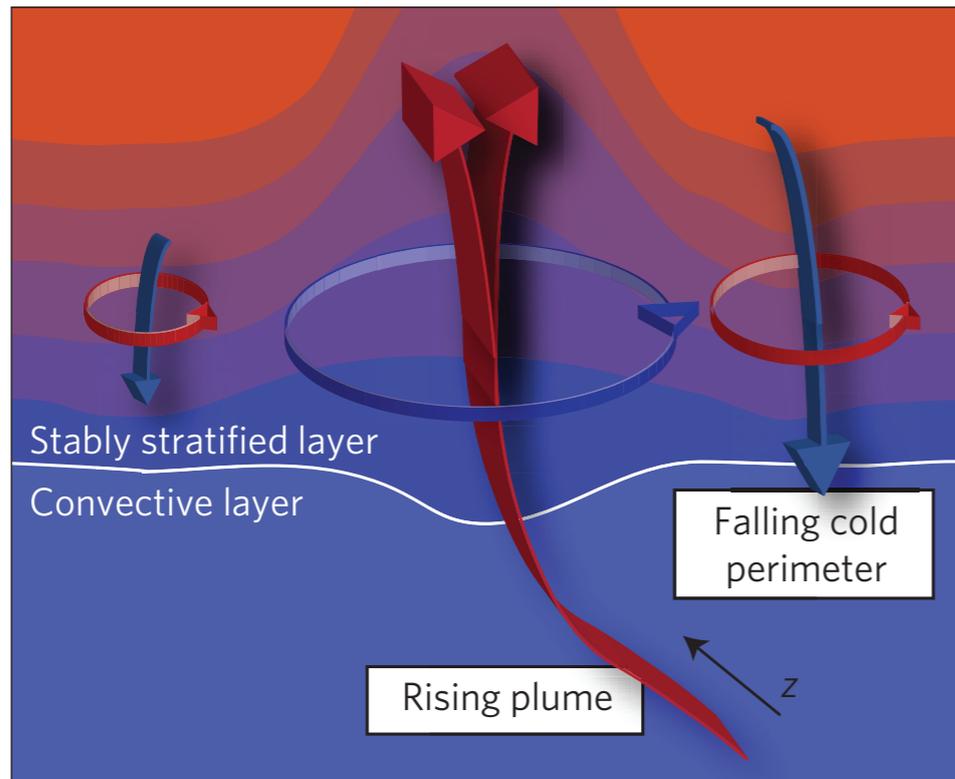
# Local stability parameter $\mathcal{R}(r)$ and stability field $\Upsilon(r, \theta, \phi, t)$



Outer & inner boundary and meridional slices of flow fields:  
a, Axial vorticity; b & c, zoom-in details of a large vortex;  
d, Eastward zonal velocity (Magic on MP2, Compute Canada)

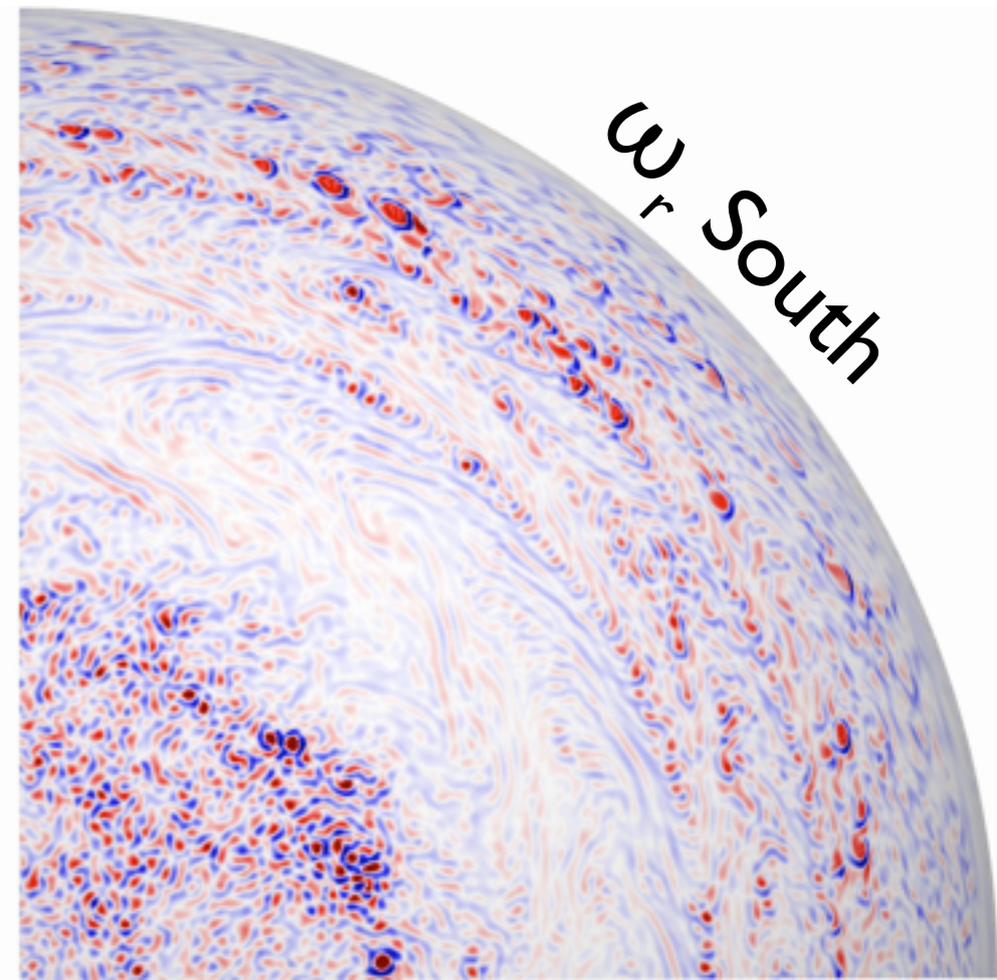
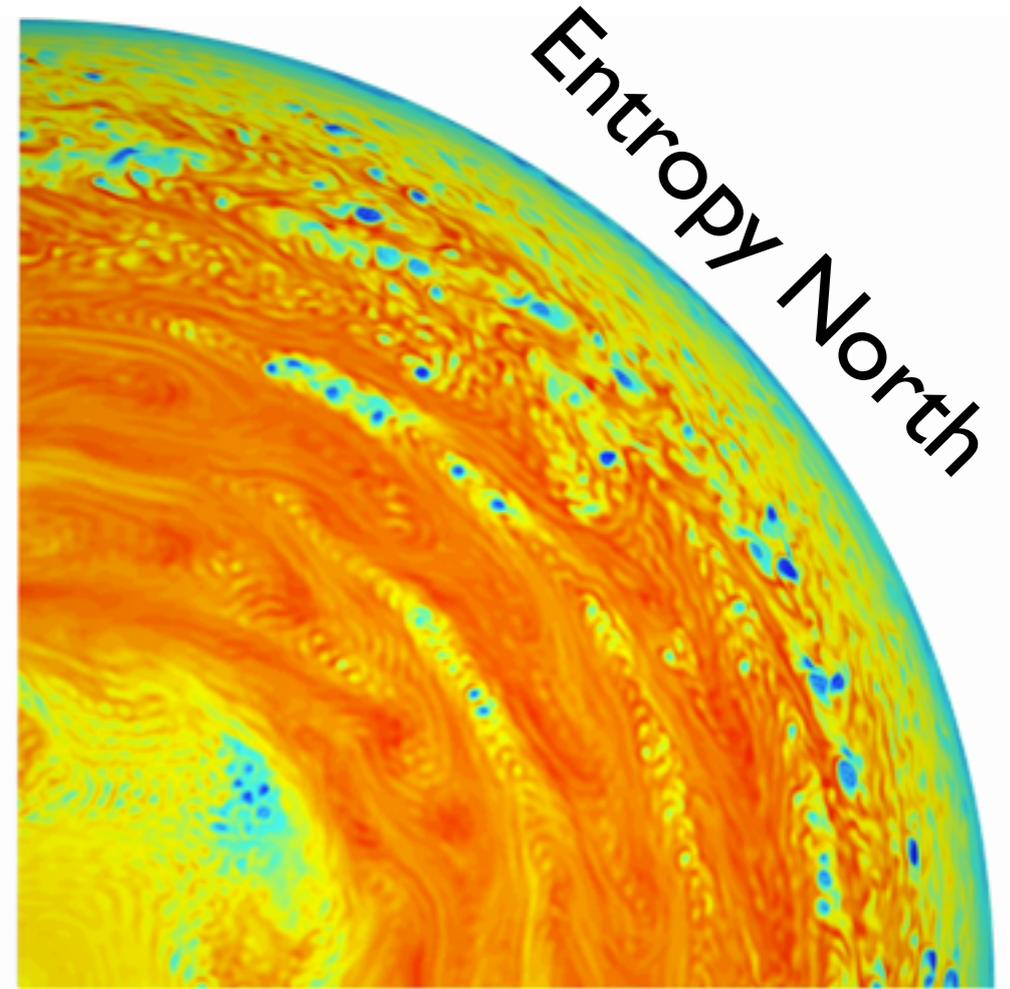
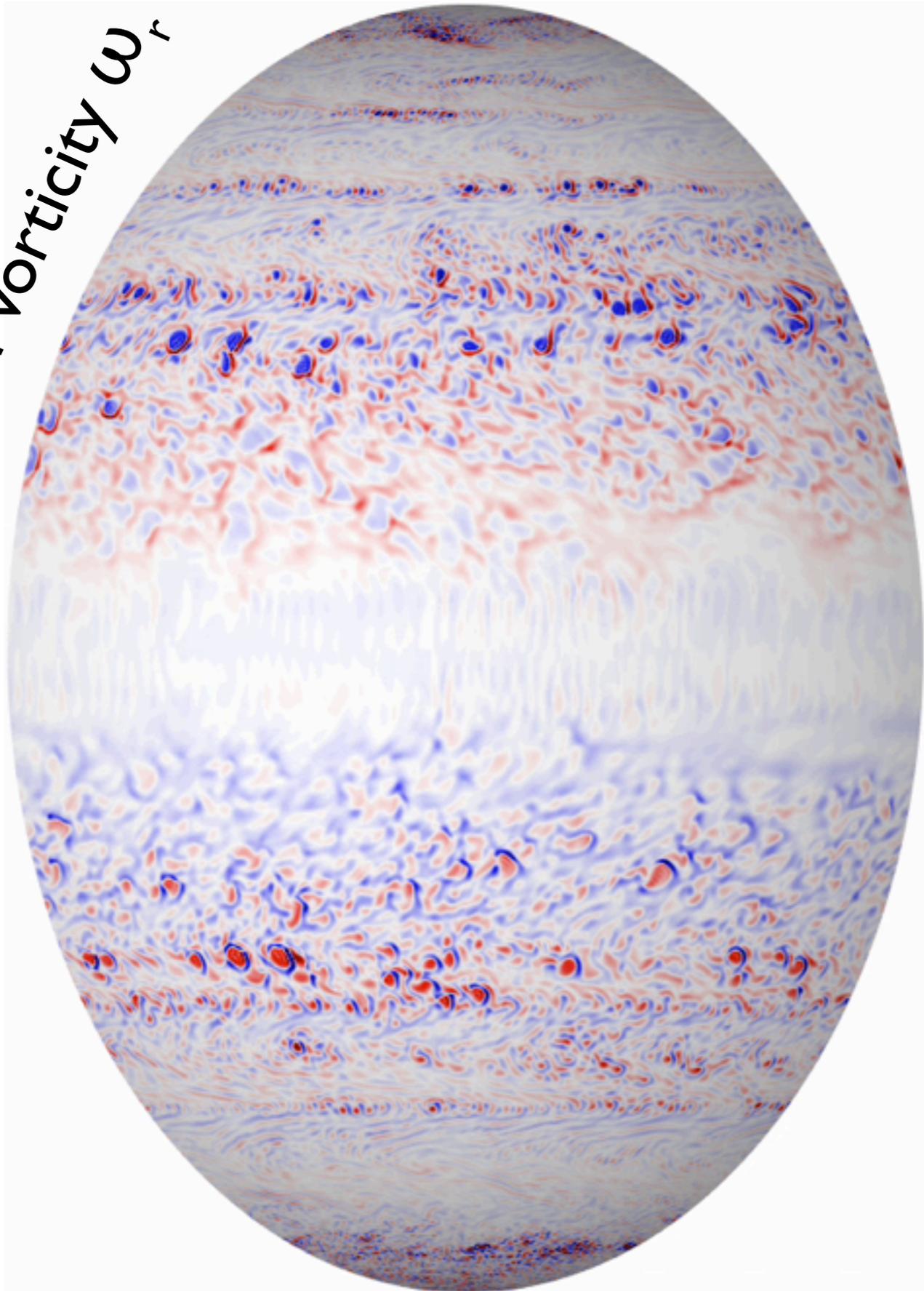


# Dynamics of a large anticyclone

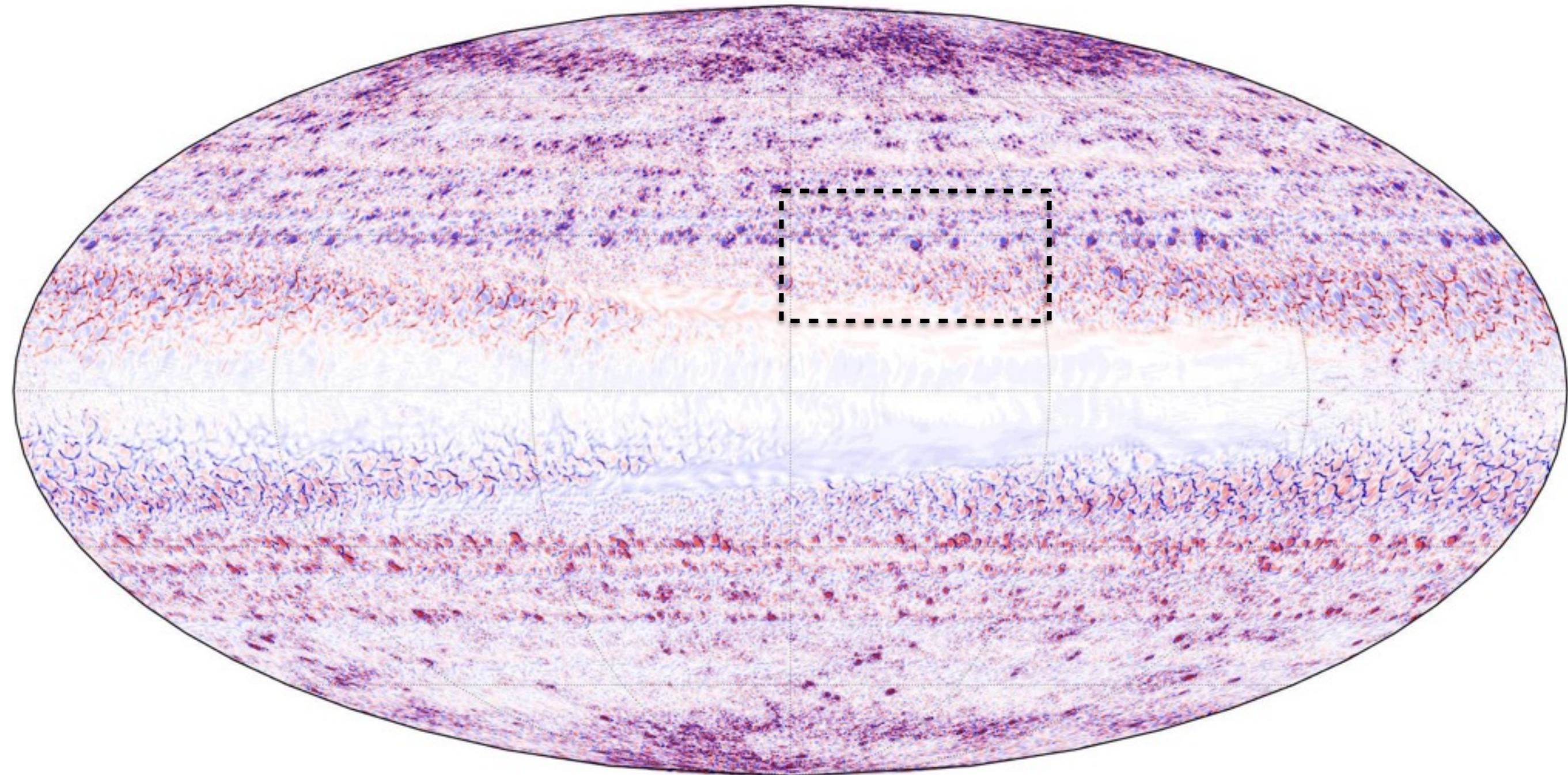


$$E = 3 \times 10^{-6}, Pr = 1, N_\rho = 5, r_i/r_o = 0.9$$

Radial vorticity  $\omega_r$

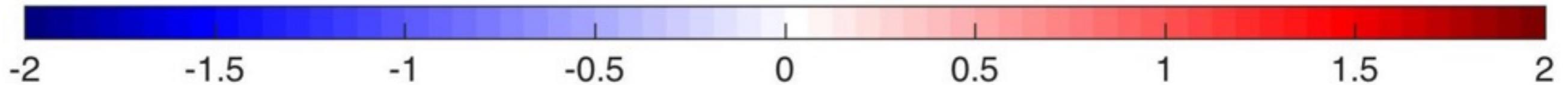
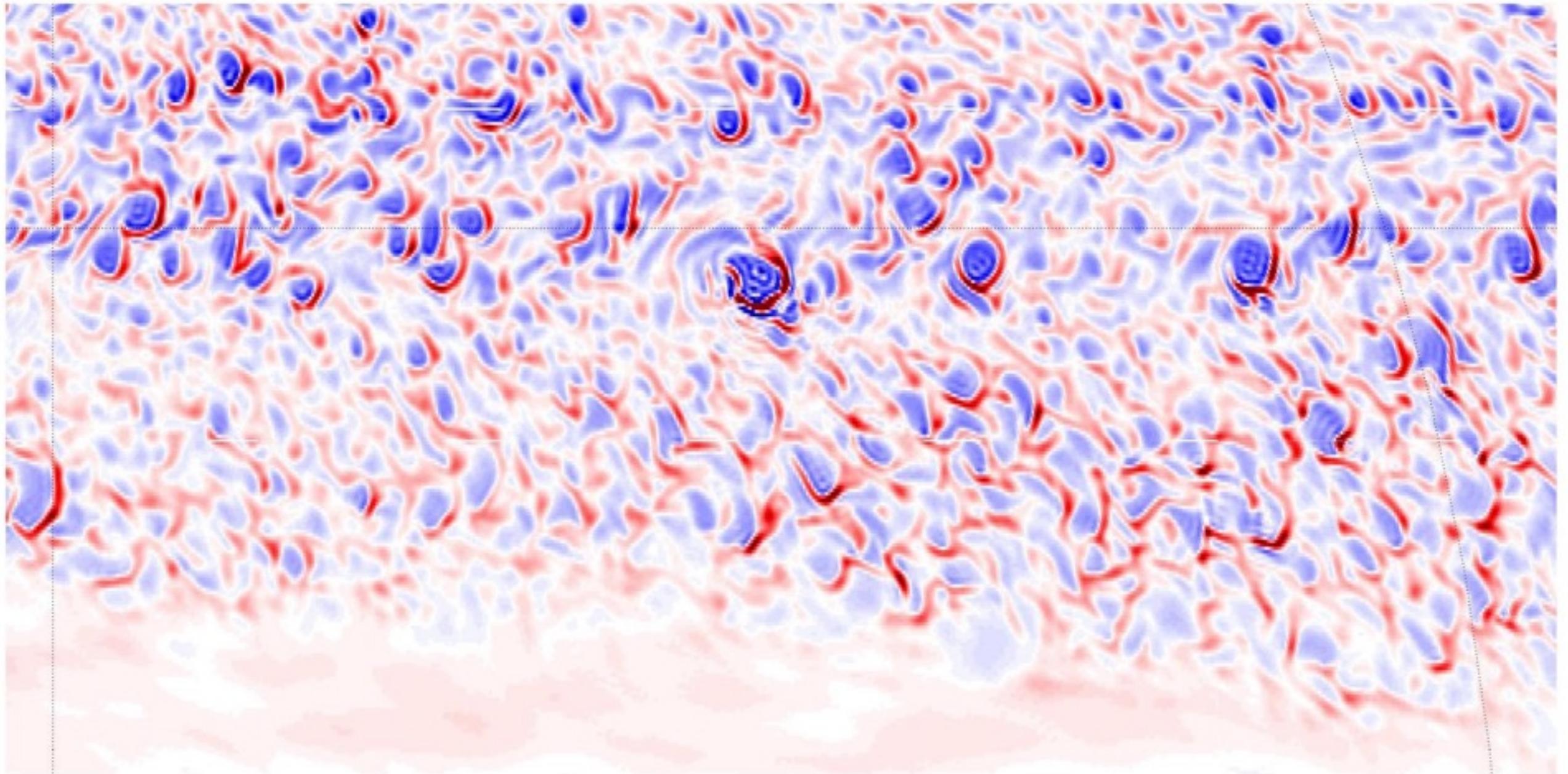


Mira (Argonne) running Rayleigh with 131072 cores  
( $l_{\max}+1 = 1024$ , 384 radial,  $1.8e9$  grid points in spherical shell)

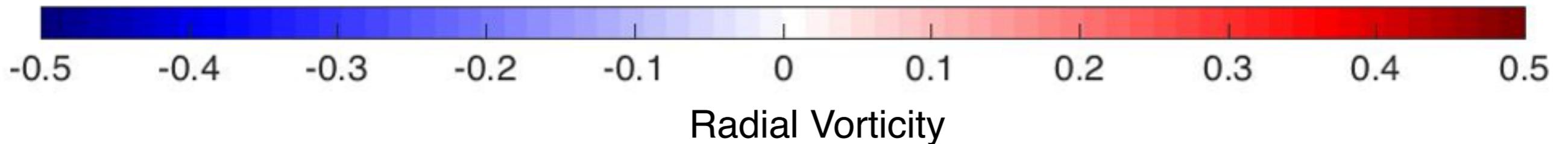
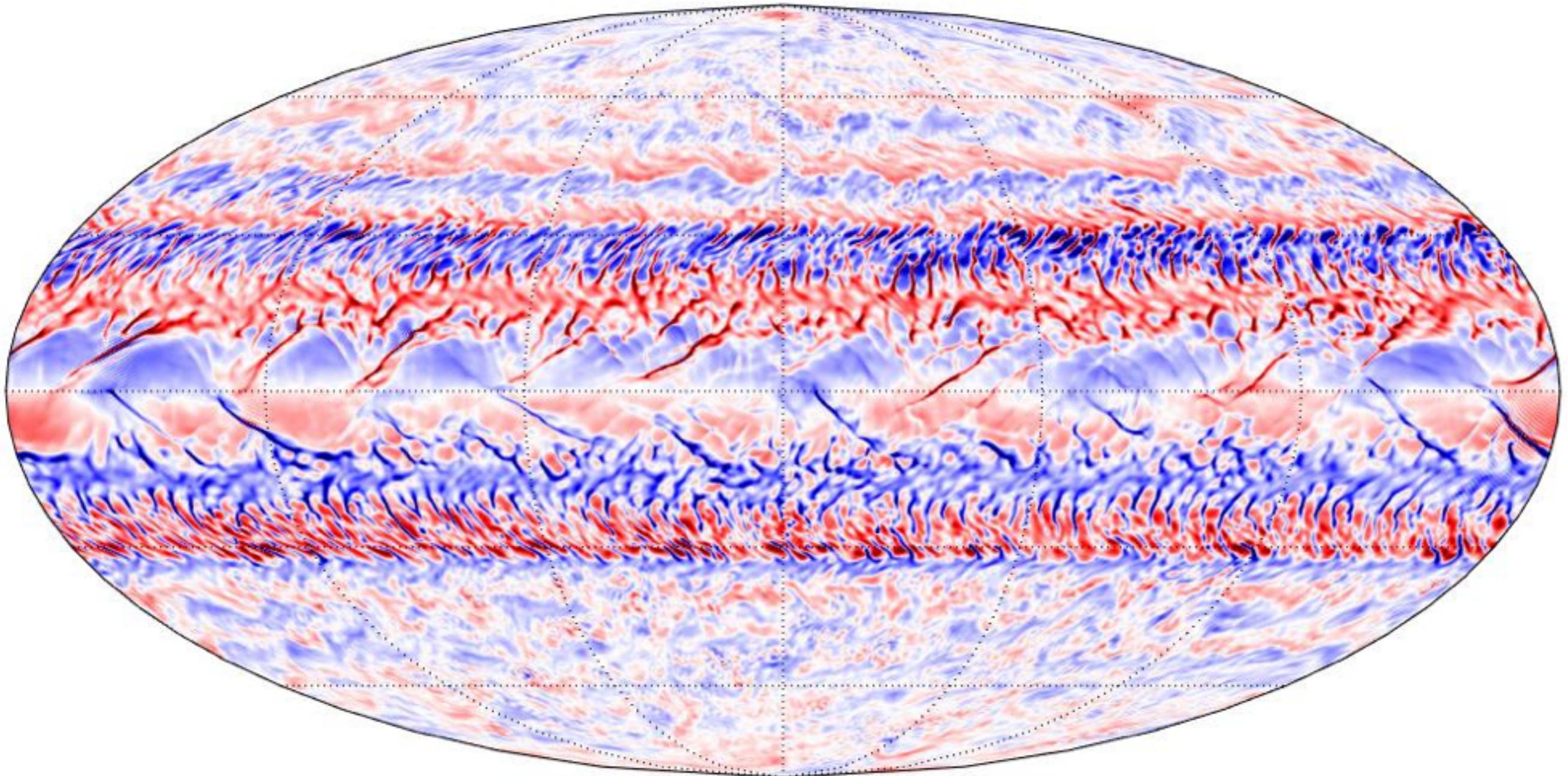


Radial Vorticity at outer boundary (Color saturation:  $-1.0 < W_r < 1.0$ )

# Zoom in of radial vorticity. Outer boundary region of first anticyclonic shear zone

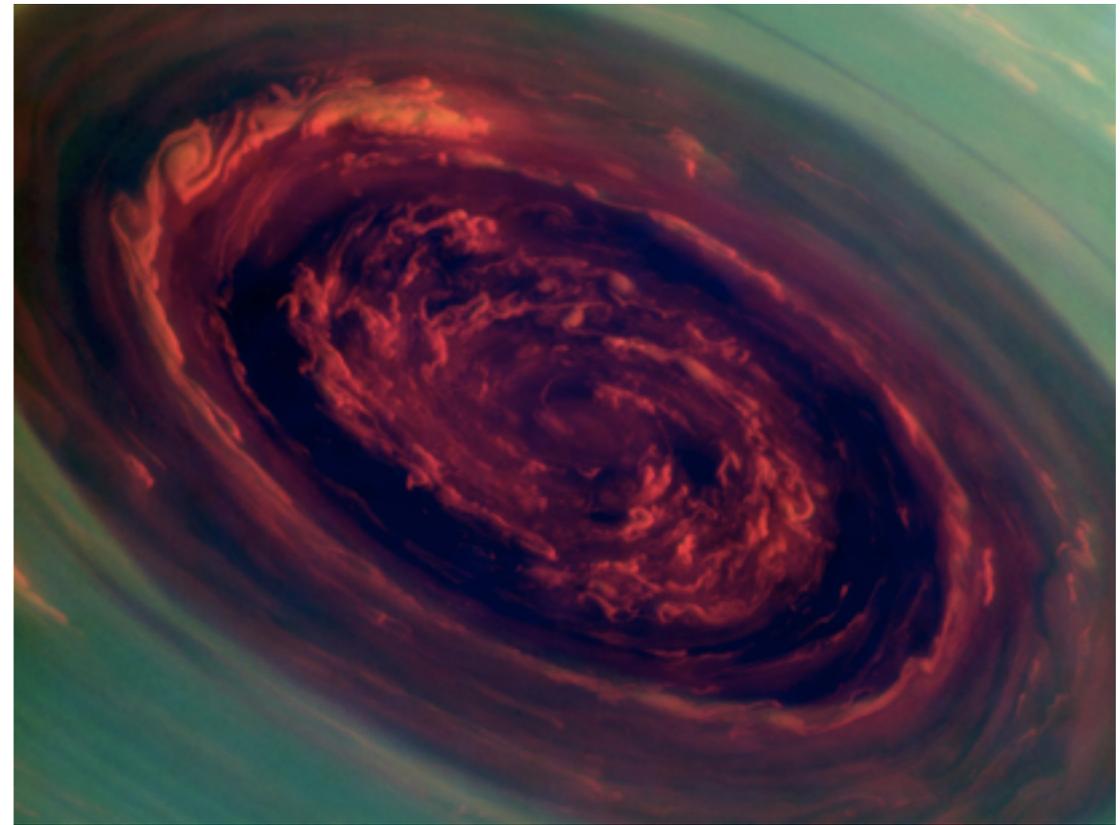
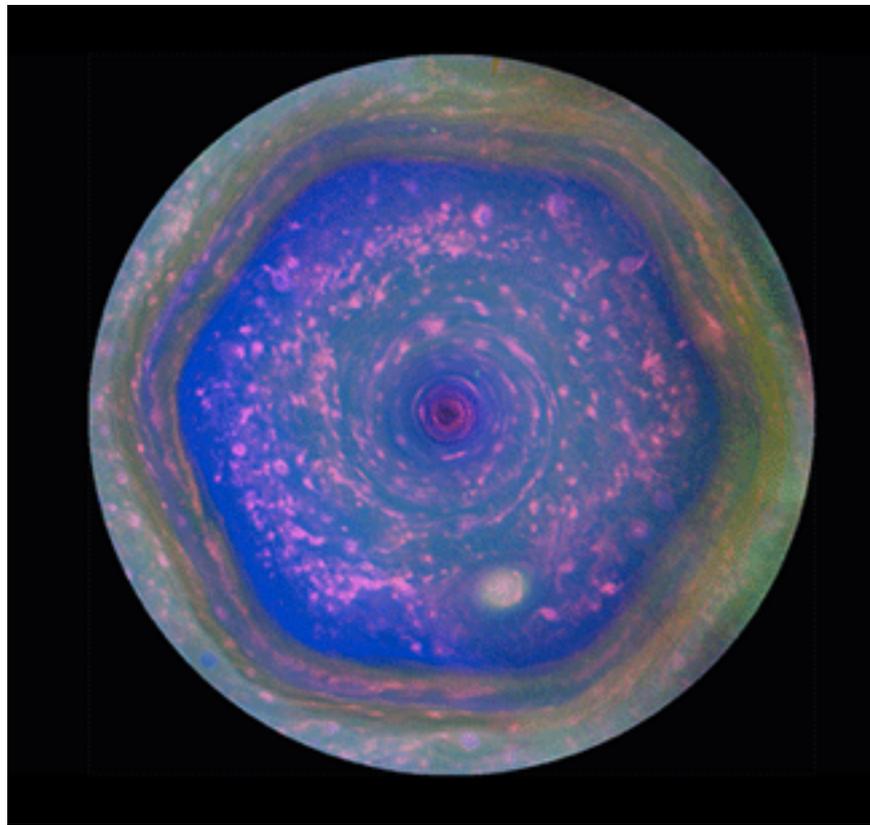


Preliminary Rayleigh run on Mira with  $r_i/r_o = 0.95$ ,  
 $E = 3 \times 10^{-5}$ ,  $N_\rho = 7$  (density drop of 1097)



# Why do we need High Performance Computing (Rayleigh/Mira)?

- Scale separation: Dynamo region  $\sim 10$  x Jets region  $\sim 10$  x Vortices region. ( $\sim 10$  density scale heights from planet center to 1 bar cloud level.)
- Low viscosity planetary fluids. (Multiple jets form at low  $Ro$  and high  $Re$ . That means we need low  $E$  and High  $Ra$ ).
- Long-lived vortices favoured by strong stable stratification and large density contrast.
- Long time scale for formation and evolution of jets and vortices



## Some questions for the near future

- Why do both poles of Saturn have cyclonic vortices? And what about the hexagon? What will Juno show us at Jupiters poles?
- Do the magnetic fields of Jupiter (dipolar but complex) and Saturn (nearly axisymmetric) relate to the hemispheric asymmetry (Jupiter) or symmetry (Saturn) of their zonal flow profiles?
- How has Jupiter's great red spot persisted for hundreds of years?
- Will Cassini - Juno - Rayleigh - Mira - ClG help answer these questions? Yes!

**Thanks!**

# Internal entropy sources/sinks

Jones 2014

Energy Equation

with constant heating  $H$

$$\tilde{\rho}\tilde{T} \left( \frac{\partial s}{\partial t} + \mathbf{u} \cdot \nabla s \right) = \frac{E}{Pr} \left[ \nabla \cdot \left( \tilde{\rho}\tilde{T} \nabla s \right) + H \right] + \frac{E Di}{Ra^*} Q_v$$

$$\nabla \cdot \left( \tilde{\rho}\tilde{T} \nabla s \right) = -H$$

Heimpel et al., 2015

Energy Equation

with density dependent

heating  $H$

$$\frac{\mathcal{D}S}{\mathcal{D}t} = \frac{Pm}{Pr} \left( \frac{1}{\bar{\rho}\bar{T}} \nabla \cdot \bar{\rho}\bar{T} \nabla S + H \right) + \frac{Pr}{PmRa\bar{T}} \left[ \frac{\bar{\eta}}{E\bar{\rho}} (\nabla \times \mathbf{B})^2 + Q_v \right],$$

$$\frac{1}{\tilde{\rho}\tilde{T}} \nabla \cdot \left( \tilde{\rho}\tilde{T} \nabla s \right) = -H$$

Both of these are options in Magic & Rayleigh