

recent advances in modelling fully coupled magma dynamics

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outline

- magma dynamics
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- code implementation
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- benchmarks & results

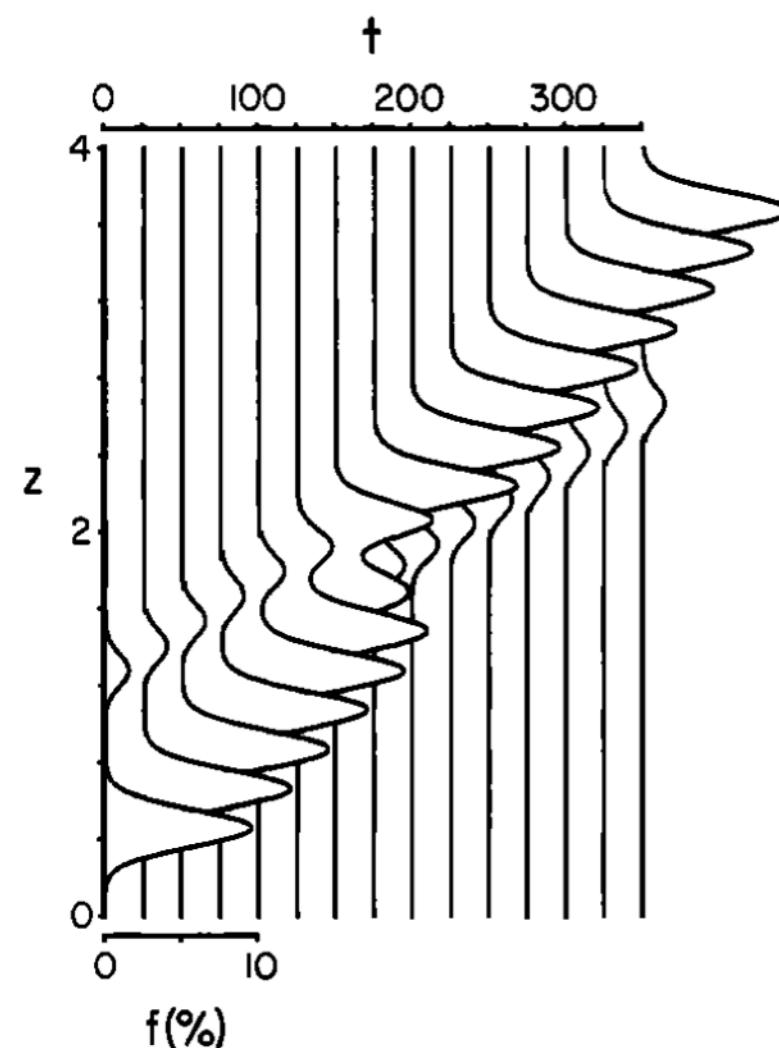


| magma dynamics

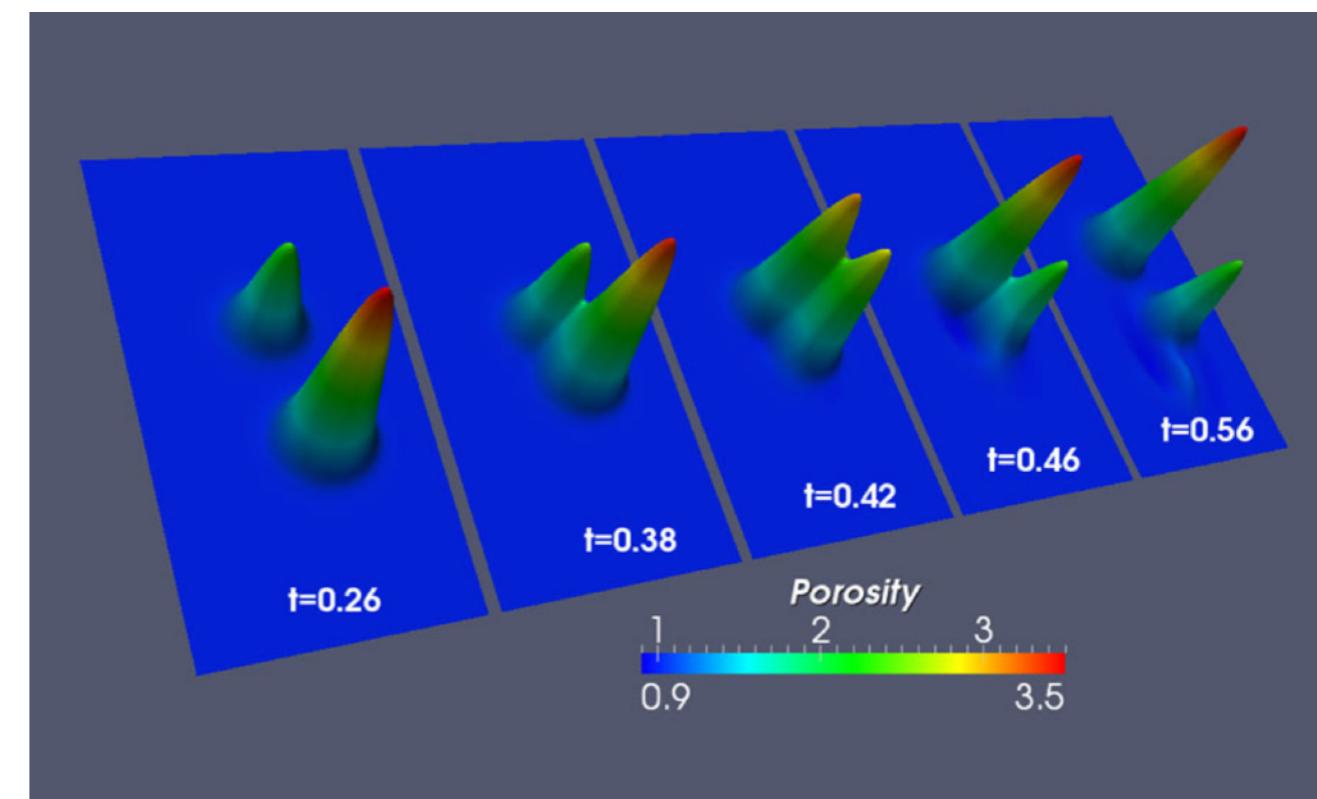
- silicate melt flowing through a deforming host rock
- multi-phase multi-component flow problem
- geodynamics: magmatism in subduction zones, arc volcanism, batholith intrusions, mid-oceanic ridges, ocean island volcanism, continental rift magmatism, monogenetic volcanic fields, ...
- physical model: McKenzie (1984), Bercovici et al. (2001,2003), Simpson et al. (2010), ...

| compaction waves

viscous compaction



Scott & Stevenson (1984)



Simpson & Spiegelman (2011)

| asthenosphere

- compaction waves relevant in the **asthenosphere**?
 - no direct observation...
 - scaling for mantle properties
- likely yes!

$$l_c \sim \sqrt{k_\phi \zeta / \mu}$$

$$p_c \sim \Delta \rho g l_c$$

$$\mathcal{C}_c \sim \Delta \rho g l_c / \zeta$$

$$l_c \approx 1 - 10 \text{ km}$$

$$p_c \approx 5 - 50 \text{ MPa}$$

$$\mathcal{C}_c \approx 10^{-17} - 10^{-15} \text{ s}^{-1}$$

lithosphere

- what about compaction waves
in the lithosphere?

- scaling for
lithosphere properties

$$l_c \approx 100 - 1000 \text{ km}$$

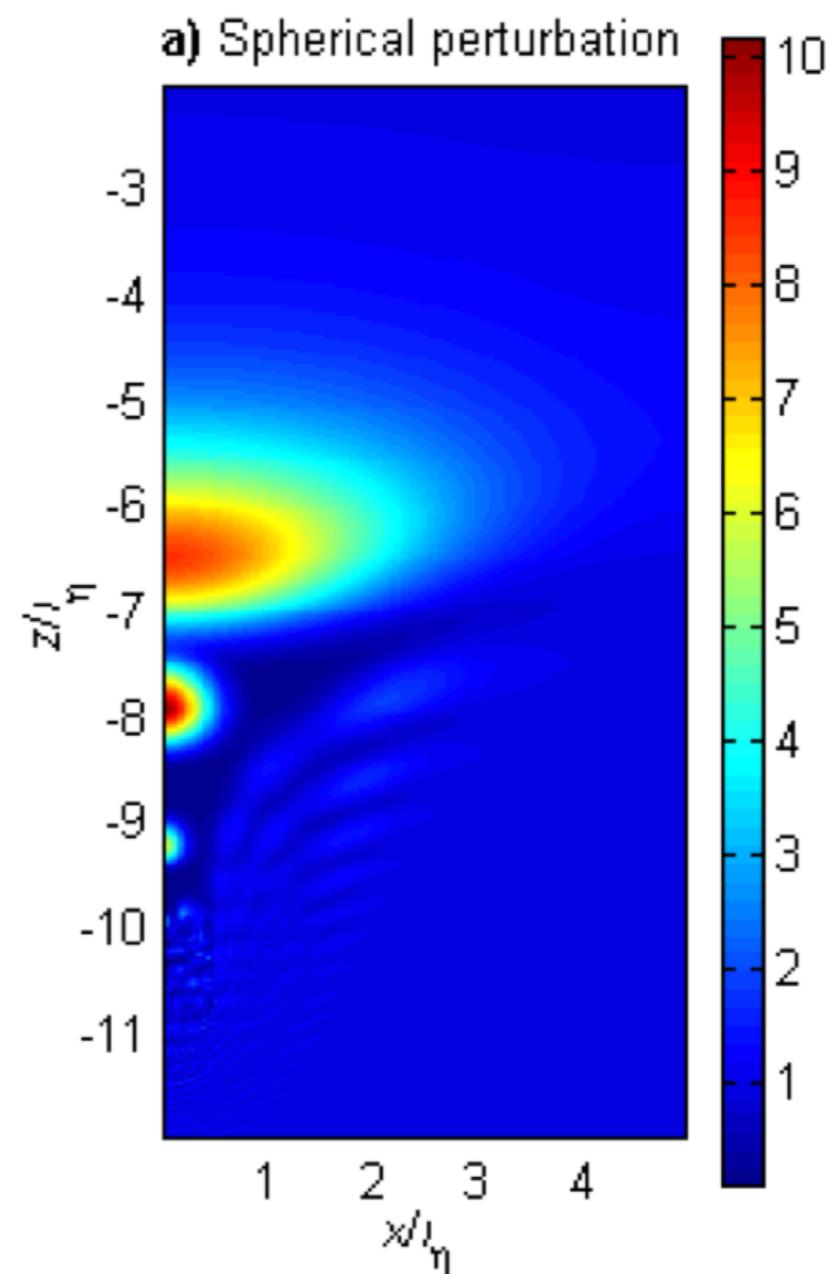
$$p_c \approx 500 - 5000 \text{ MPa}$$

$$\mathcal{C}_c \approx 10^{-19} - 10^{-17} \text{ s}^{-1}$$

- note p_c !

lithosphere

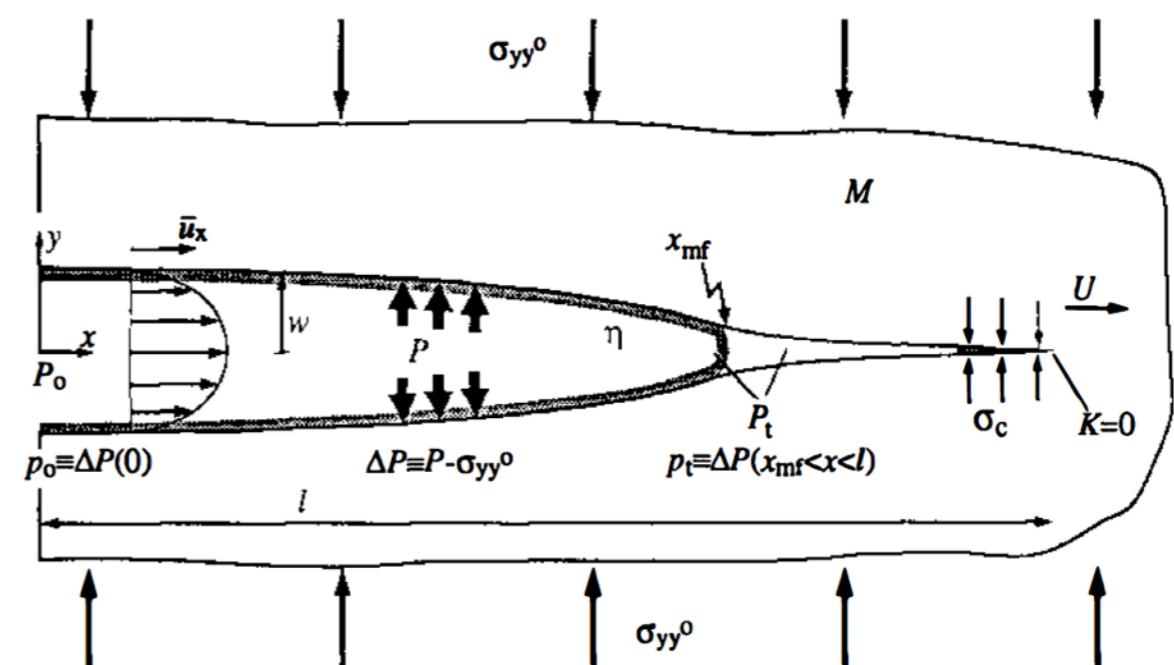
- what about compaction waves in the lithosphere?
 - scaling for lithosphere properties
- probably not...
- other mechanisms?



Connolly & Podladchikov (1998)

tensile failure

brittle plasticity

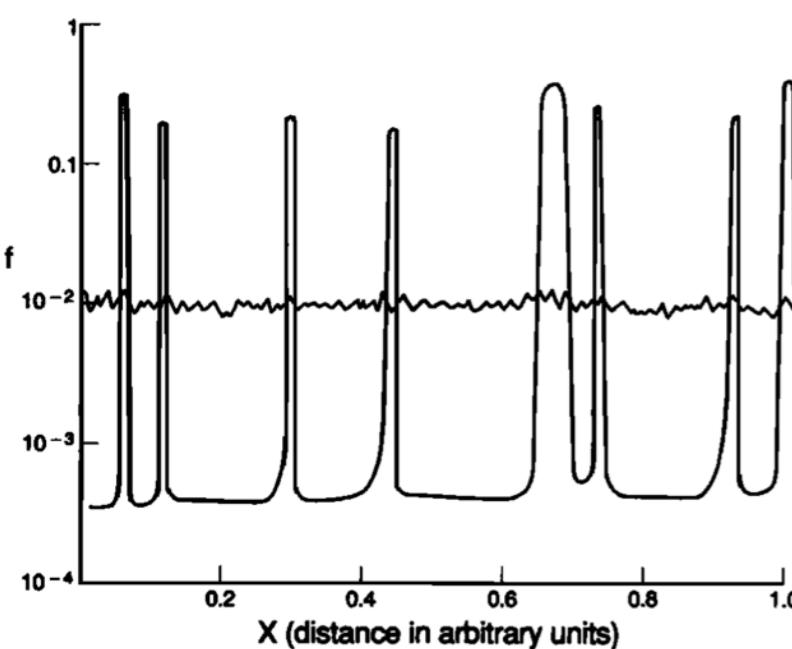


[en.wikipedia.org/wiki/Dike_\(geology\)](http://en.wikipedia.org/wiki/Dike_(geology))

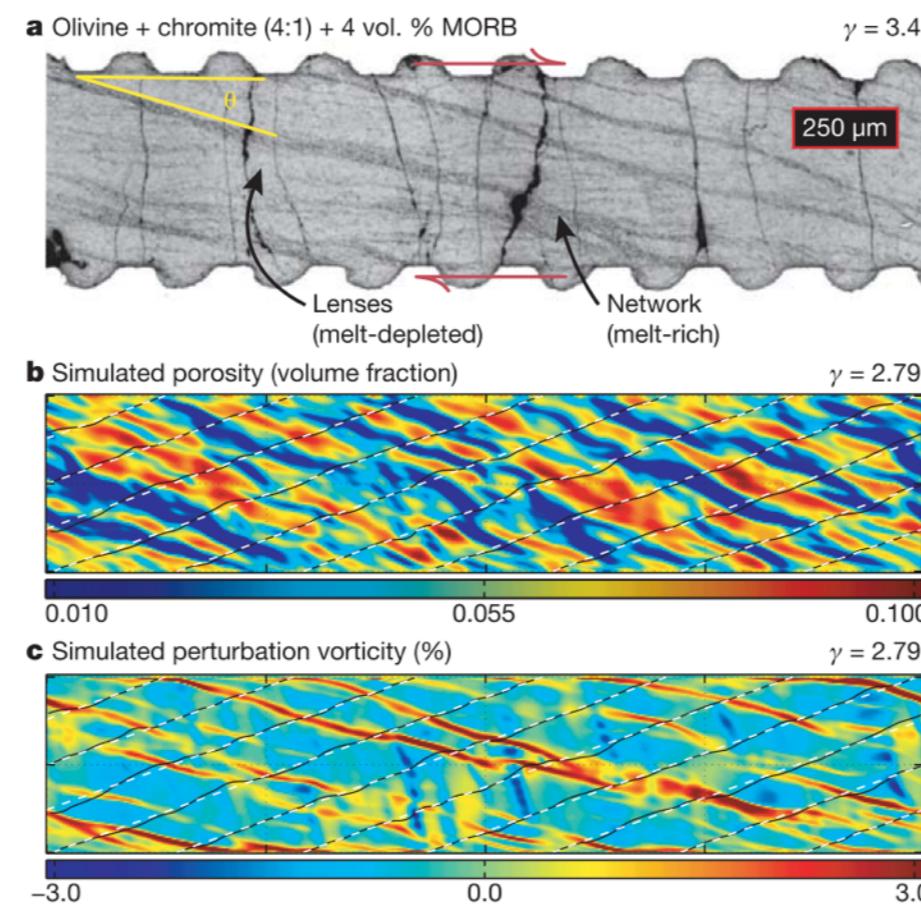
Rubin (1995)

melt channelling

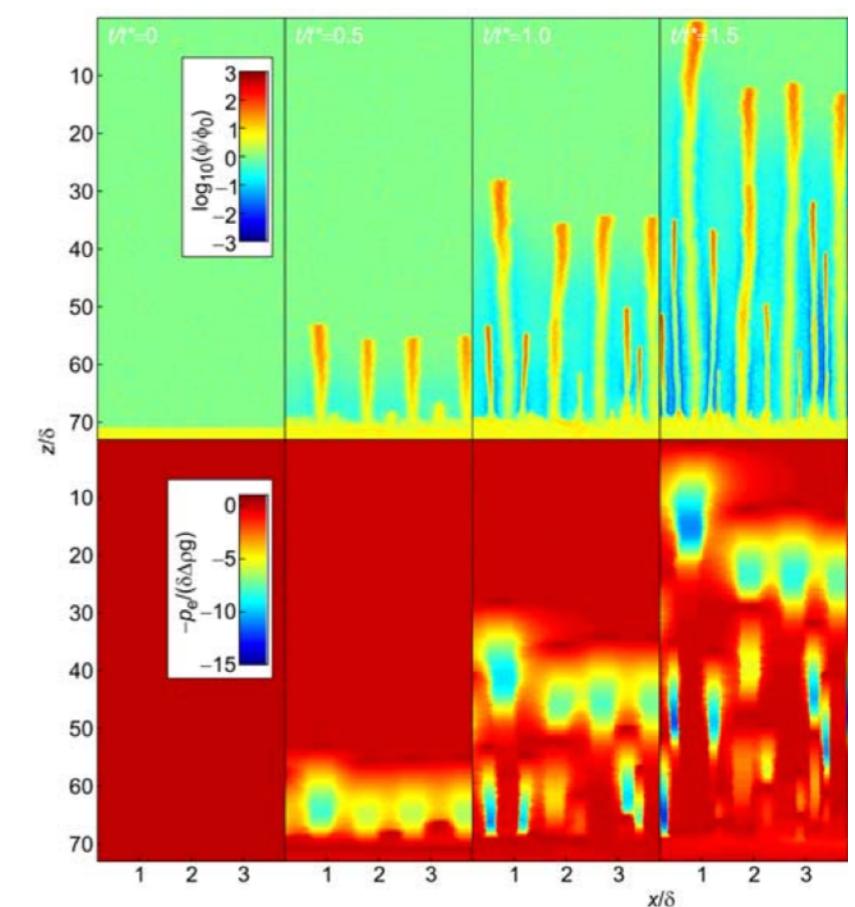
non-linear compaction



Stevenson (1989)



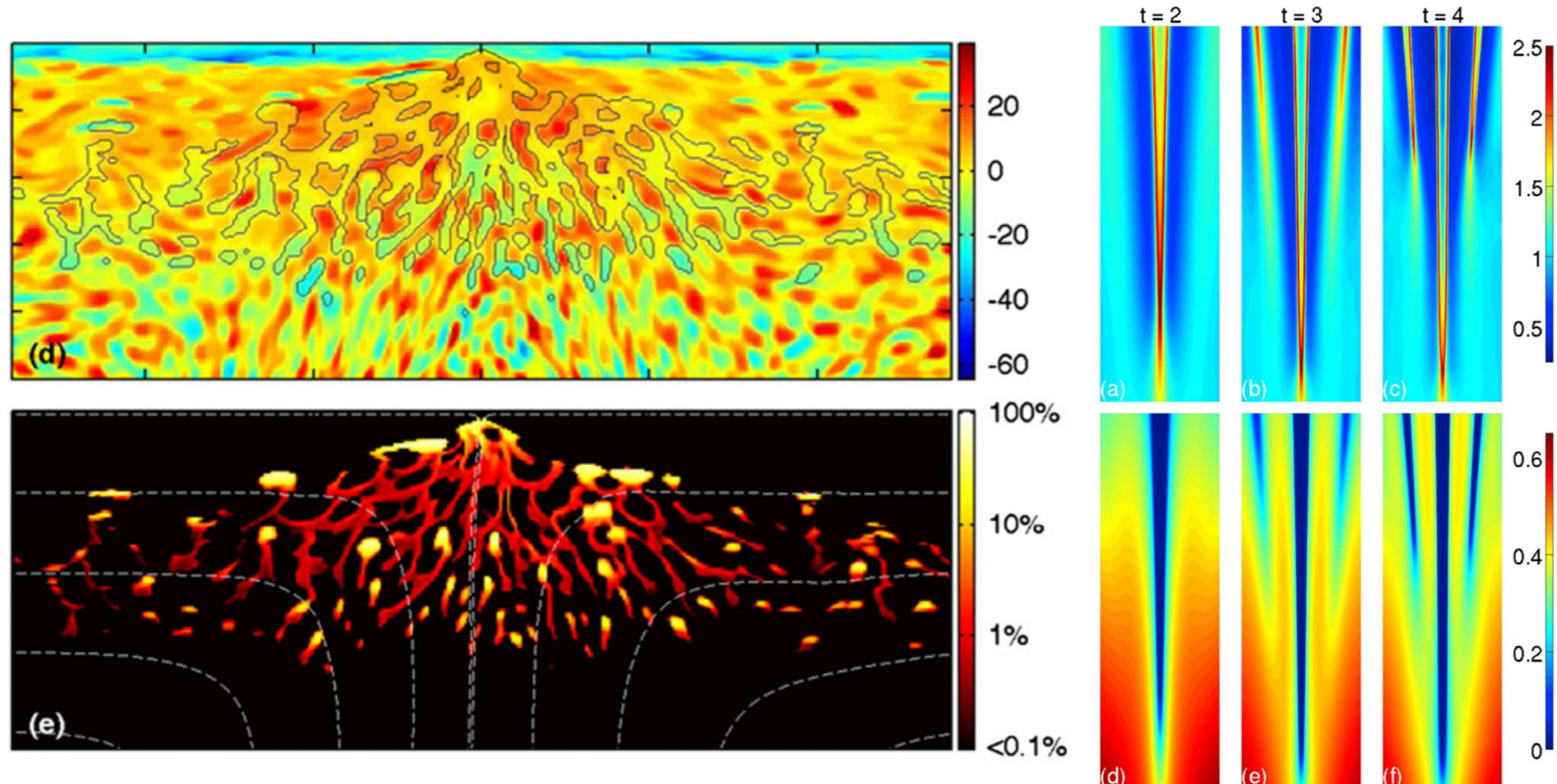
Katz et al. (2006)



Connolly & Podladchikov (2007)

| phase reactions

thermo-chemical coupling



Katz & Weatherley (2012)

Liang et al. (2010)

objective

- add visco-elasto-plastic rheology to allow emergence of **channelling instabilities** and **tensile fractures** as pathways for melt into lithosphere
 - add self-consistent **thermo-chemical model** to include first order effects of magmatic petrology
- develop **thermo-chemically coupled ductile-brittle magma dynamics simulation** for study of magmatic systems from asthenosphere to crust

“it seems that the interpretation of existing field observations is [...] hampered by an inadequate understanding of rock that can undergo both fracture and flow.”

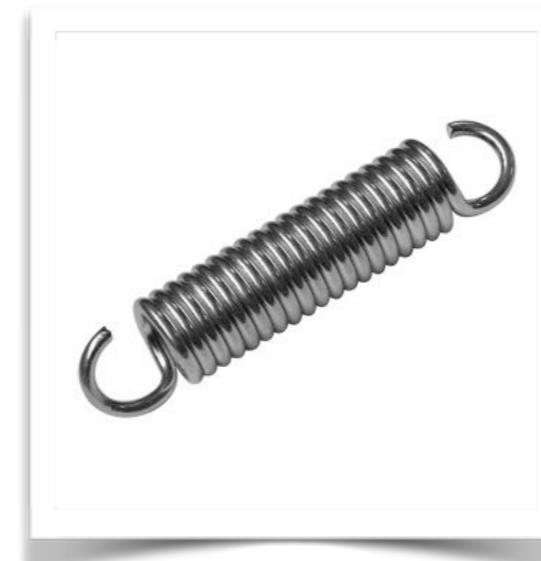
Rubin (1993)

| host rock rheology

- visco-elasto-plastic shear & compaction deformation



+



+



viscous creep

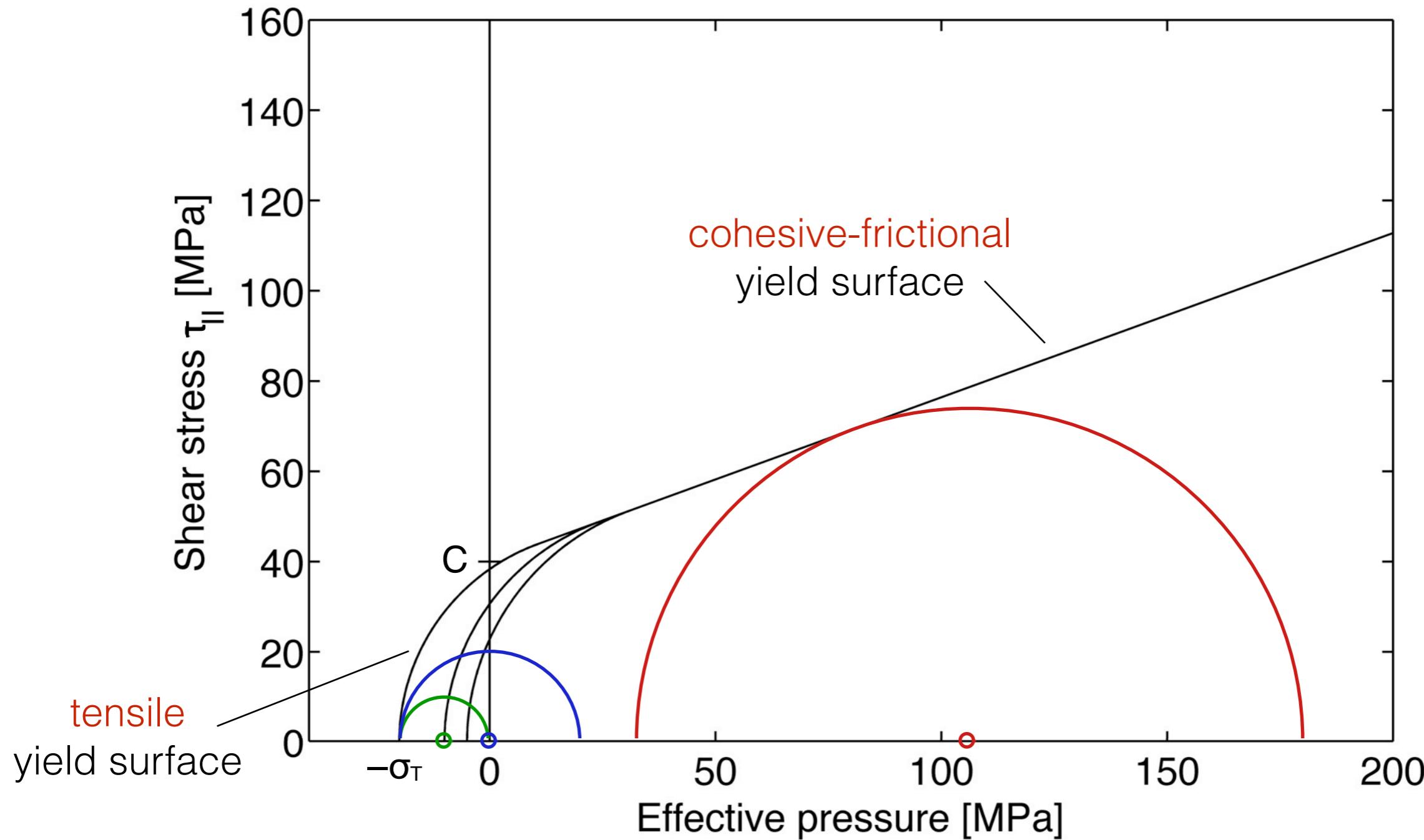
elastic strain

plastic failure

- melt-weakened diffusion & dislocation creep
- Maxwell-type visco-elastic model
- combined Drucker-Prager / Griffith-Murrell failure criterion

host rock rheology

Mohr-Coulomb-Griffith failure criterion



| host rock rheology

- visco-elasto-plastic **shear stress**

$$\bar{\tau} = 2\eta_{eff}^* \dot{\epsilon}_s' + \chi_\tau^* \bar{\tau}^o$$

η_{eff}^* effective **shear** visco-elasto-plasticity

χ_τ^* visco-elastic **shear** stress evolution parameter

- visco-elasto-plastic **compaction pressure**

$$P_c = -\zeta_{eff}^* \nabla \cdot \mathbf{v}_s + \chi_p^* P_c^o$$

ζ_{eff}^* effective **compaction** visco-elasto-plasticity

χ_p^* visco-elastic **compaction** pressure evolution parameter



see Keller et al. (2013), GJI, for derivation

I governing equations

- Stokes: conservation of mass & momentum

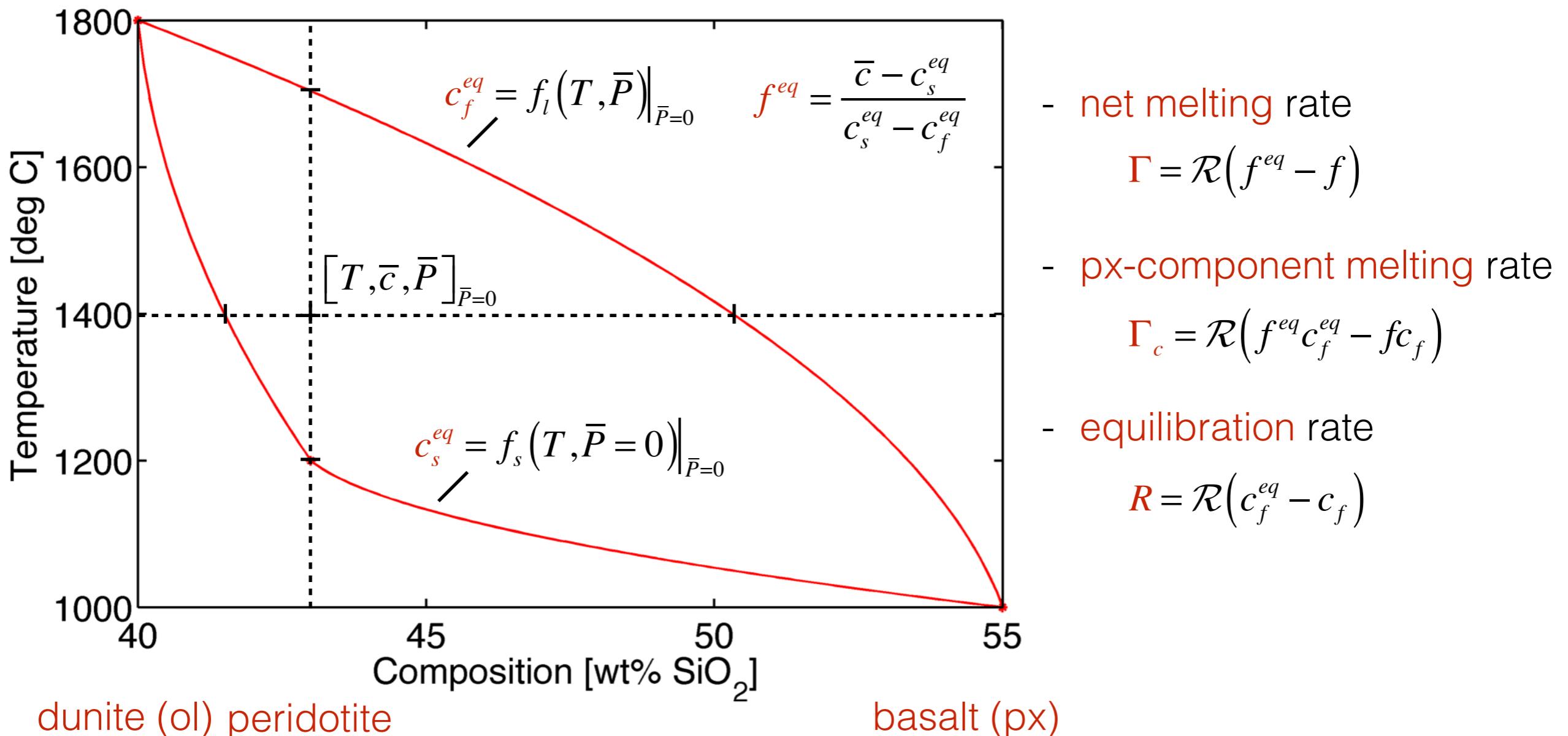
$$\begin{aligned}\mathbf{v} \quad -\nabla \cdot \bar{\boldsymbol{\tau}} + \nabla \bar{P} - \bar{\rho} \mathbf{g} &= 0 \\ P \quad \nabla \cdot \bar{\mathbf{v}} &= 0\end{aligned}$$

- McKenzie: conservation of mass & momentum

$$\begin{aligned}\mathbf{v}_s \quad -\nabla \cdot [\bar{\boldsymbol{\tau}} + \mathbf{P}_c \mathbf{I}] + \nabla P_f - \bar{\rho} \mathbf{g} &= 0 \\ P_f \quad \nabla \cdot \mathbf{v}_s - \nabla \cdot K_D (\nabla P_f - \rho_f \mathbf{g}) &= 0 \\ f \quad \frac{D_s f}{Dt} - (1-f) \nabla \cdot \mathbf{v}_s &= 0\end{aligned}$$

petrological model

thermodynamic equilibrium and equilibration rates



I governing equations II

- conservation of mass & momentum

$$\mathbf{v}_s \quad -\nabla \cdot [\boldsymbol{\tau} - P_c \mathbf{I}] + \nabla P_f = \bar{\rho} \mathbf{g}$$

$$P_f \quad \nabla \cdot \mathbf{v}_s - \nabla \cdot \mathbf{q} = 0$$

- conservation of energy, px-component & melt mass

$$T \quad \bar{\rho} c_p \frac{\bar{D}T}{Dt} - \nabla \cdot \bar{k} \nabla T = -L\Gamma + \Psi$$

$$c_s \quad \bar{\rho}(1-f) \frac{D_s c_s}{Dt} = -c_s \Gamma - \Gamma_c - R$$

$$c_f \quad \bar{\rho} f \frac{D_f c_f}{Dt} - \nabla \cdot d \nabla c_f = -c_f \Gamma + \Gamma_c + R$$

$$f \quad \bar{\rho} \frac{D_s f}{Dt} = (1-f) \bar{\rho} \nabla \cdot \mathbf{v}_s + \Gamma$$

II code implementation

- FEM2PHASE
 - FE code developed in Matlab
 - weak form discretised on linear Q1 elements
 - full non-linear problem solved by Picard iterations
 - split elliptic from hyperbolic problem
 - two linearised problems solved by direct solver
 - marker-in-cell advection with reseeding

||

v-P solver

- standard formulation

$$\mathbf{v}_s - \nabla \cdot [\eta_{eff}^* \dot{\boldsymbol{\epsilon}}' - \zeta_{eff}^* \nabla \cdot \mathbf{v}_s \mathbf{I}] + \nabla P_f = \bar{\rho} \mathbf{g} + \nabla \cdot [\chi_\tau^* \bar{\boldsymbol{\tau}}^o + \chi_p^* \mathbf{P}_c^o \mathbf{I}]$$

$$P_f \quad \nabla \cdot \mathbf{v}_s - \nabla \cdot K_D \nabla P_f = -\nabla \cdot K_D \rho_f \mathbf{g}$$

- linear system

$$\begin{bmatrix} \mathbf{K}_c & \mathbf{G} \\ \mathbf{G}^T & \Delta \end{bmatrix} \begin{bmatrix} \mathbf{v}_s \\ \mathbf{p}_f \end{bmatrix} = \begin{bmatrix} \mathbf{r}_v \\ \mathbf{r}_p \end{bmatrix}$$

for $\zeta_{eff}^* \gg \eta_{eff}^*$, momentum eq is dominated by $\zeta_{eff}^* \nabla \cdot \mathbf{v}_s$;



v-P solver

- compaction pressure formulation

$$\mathbf{v}_s \quad -\nabla \cdot \eta_{eff}^* \dot{\boldsymbol{\epsilon}}' + \nabla P_f + \nabla P_c = \bar{\rho} \mathbf{g} + \nabla \cdot \chi_\tau^* \bar{\boldsymbol{\tau}}^o$$

$$P_f \quad \nabla \cdot \mathbf{v}_s - \nabla \cdot K_D \nabla P_f = -\nabla \cdot K_D \rho_f \mathbf{g}$$

$$P_c \quad \nabla \cdot \mathbf{v}_s + \frac{1}{\zeta_{eff}^*} P_c = \frac{\chi_p^*}{\zeta_{eff}^*} P_c^o$$

- linear system

$$\begin{bmatrix} \mathbf{K} & \mathbf{G} & \mathbf{G} \\ \mathbf{G}^T & \Delta & \mathbf{0} \\ \mathbf{G}^T & \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{v}_s \\ \mathbf{p}_f \\ \mathbf{p}_c \end{bmatrix} = \begin{bmatrix} \mathbf{r}_v \\ \mathbf{r}_p \\ \mathbf{r}_c \end{bmatrix}$$

compaction pressure

$$P_c = -\zeta_{eff}^* \nabla \cdot \mathbf{v}_s + \chi_p^* P_c^o$$

II solution stability

- stability issues with Q1-Q1 elements
 - stabilisation requires a Laplacian op. on pressure
- two-phase equations already include such a term

$$-\nabla \cdot [\boldsymbol{\tau} - P_c \mathbf{I}] + \nabla P_f = \bar{\rho} \mathbf{g}$$

$$\nabla \cdot \mathbf{v}_s - \nabla \cdot K_D (\nabla P_f + \rho_f \mathbf{g}) = 0$$

- stability requires $K_D^{stab} = \max\left(\frac{k_\phi}{\mu}, \frac{\nu_{el}}{\eta}\right)$ (e.g. Elman 2005)

|| zero melt limit

- problem reduces to Stokes flow for solid phase
- fluid pressure becomes equal to solid pressure
- phase separation flux goes to zero
- use third equation to fix P_c at zero

$$\begin{aligned}
 -\nabla \cdot \eta_{eff} \dot{\boldsymbol{\varepsilon}}'_s + \nabla P_{f=s} + \nabla 0 &= \rho_s \mathbf{g} + \nabla \cdot \chi_\tau \bar{\boldsymbol{\tau}}^o \\
 \nabla \cdot \mathbf{v}_s + \nabla \cdot 0 &= 0 \\
 P_c &= 0
 \end{aligned}
 \quad
 \begin{bmatrix}
 \mathbf{K} & \mathbf{G} & \mathbf{0} \\
 \mathbf{G}^T & \Delta^{stab} & \mathbf{0} \\
 \mathbf{0} & \mathbf{0} & 1
 \end{bmatrix}
 \begin{bmatrix}
 \mathbf{v}_s \\
 \mathbf{p}_{f=s} \\
 \mathbf{p}_c
 \end{bmatrix}
 = \begin{bmatrix}
 \mathbf{r}_v \\
 \mathbf{r}_p \\
 \mathbf{0}
 \end{bmatrix}$$

|| full melt limit

- $1 - \phi < \phi_{crit}$
- recovering Stokes flow for fluid phase for
- shear viscosity cut off at $\sim 1e17$ Pas for stability
- Darcy coefficient kept non-zero for stability

$$\begin{aligned}
 -\nabla \cdot \eta^{stab} \dot{\boldsymbol{\varepsilon}}'_{s=f} + \nabla P_f &= \rho_f \mathbf{g} \\
 \nabla \cdot \mathbf{v}_{s=f} - \nabla \cdot K_D^{stab} \nabla P_f &= 0 \\
 P_c &= 0
 \end{aligned}
 \quad
 \begin{bmatrix}
 \mathbf{K}^{stab} & \mathbf{G} & \mathbf{0} \\
 \mathbf{G}^T & \Delta^{stab} & \mathbf{0} \\
 \mathbf{0} & \mathbf{0} & 1
 \end{bmatrix}
 \begin{bmatrix}
 \mathbf{v}_{s=f} \\
 \mathbf{p}_f \\
 \mathbf{p}_c
 \end{bmatrix}
 = \begin{bmatrix}
 \mathbf{r}_v \\
 \mathbf{r}_p \\
 \mathbf{0}
 \end{bmatrix}$$

||

matrix scaling

- coefficient matrix scales with **viscosity** & **permeability**

$$\mathbf{L}(\eta, k_\phi, \dots) \mathbf{x} = \mathbf{r}$$

- range of magnitudes causes **ill-conditioned** system
- we apply element-wise symmetric **matrix scaling**

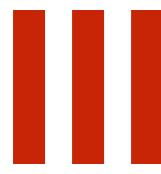
$$\Lambda = \frac{1}{\text{diag}(\sqrt{|\mathbf{L}|})} \mathbf{I}$$

$$\Lambda \mathbf{L} \Lambda \hat{\mathbf{x}} = \Lambda \mathbf{r} \quad \hat{\mathbf{x}} = \Lambda \mathbf{x}$$

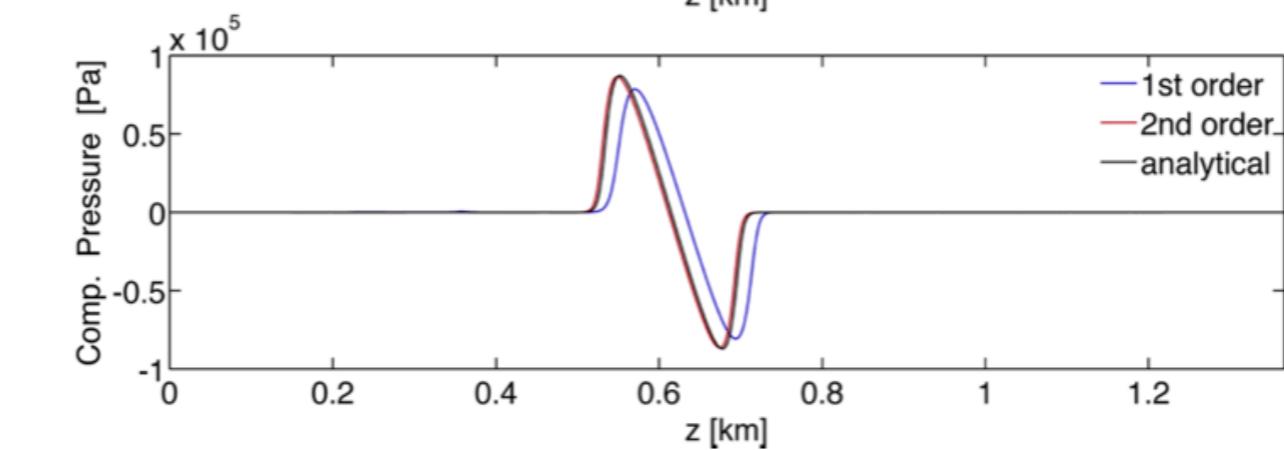
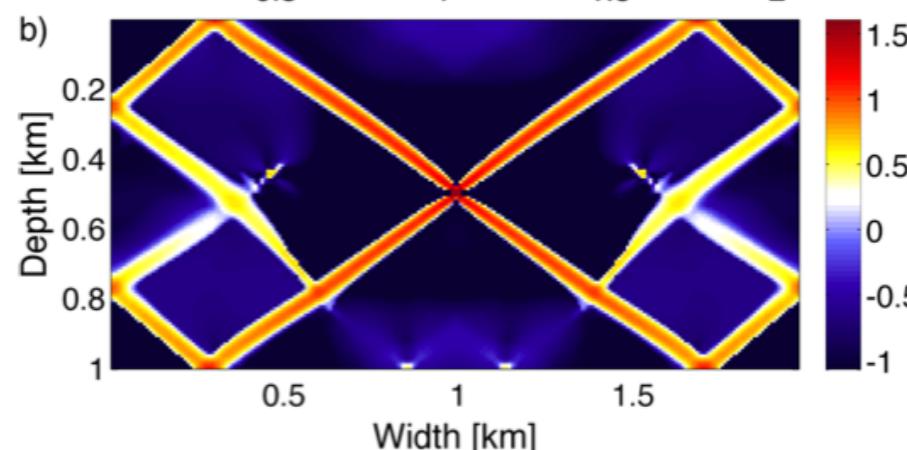
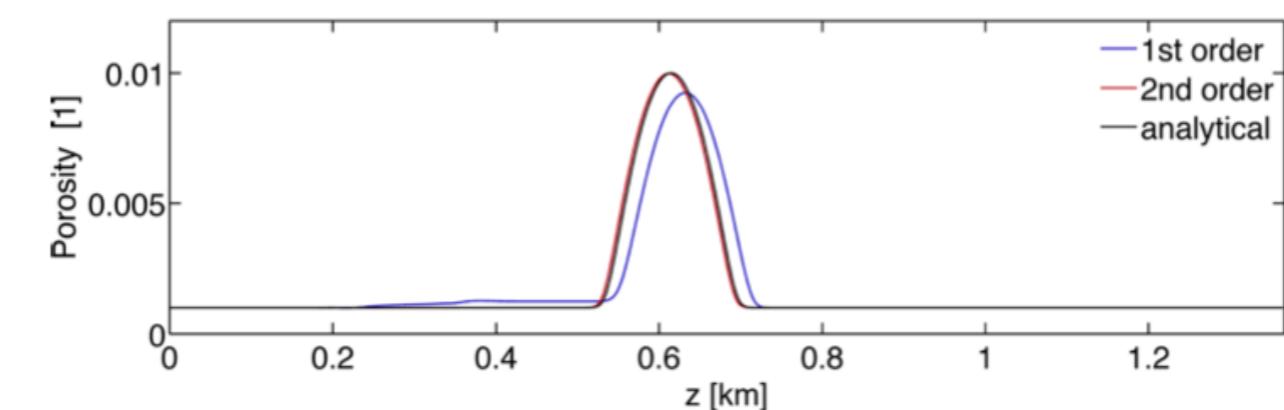
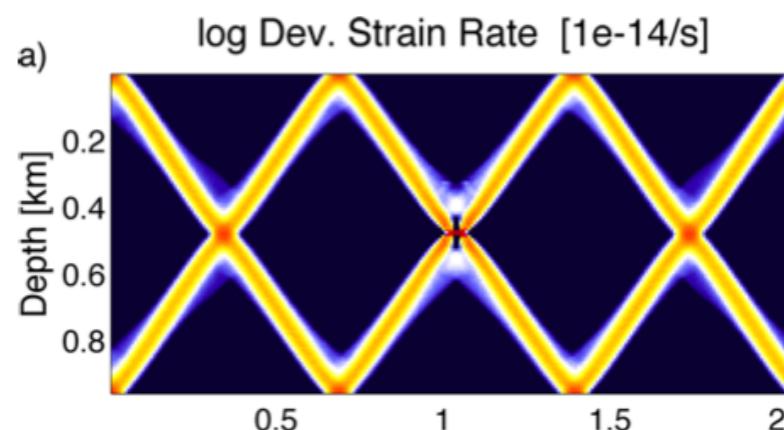
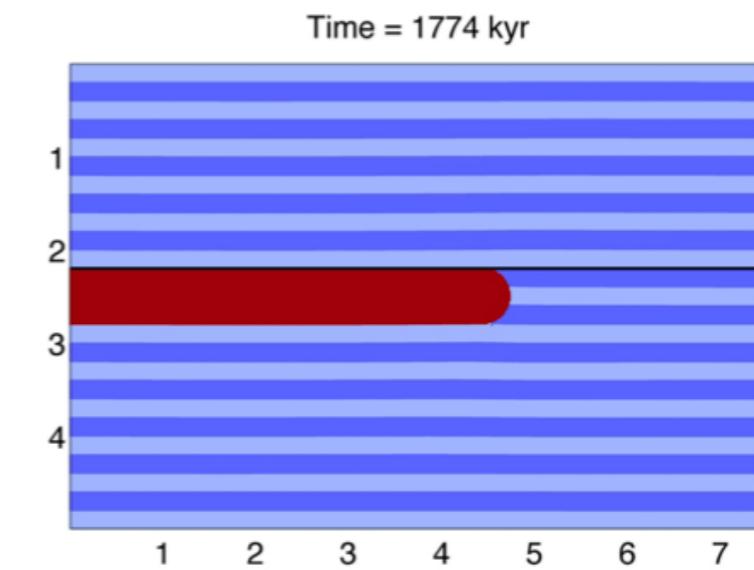
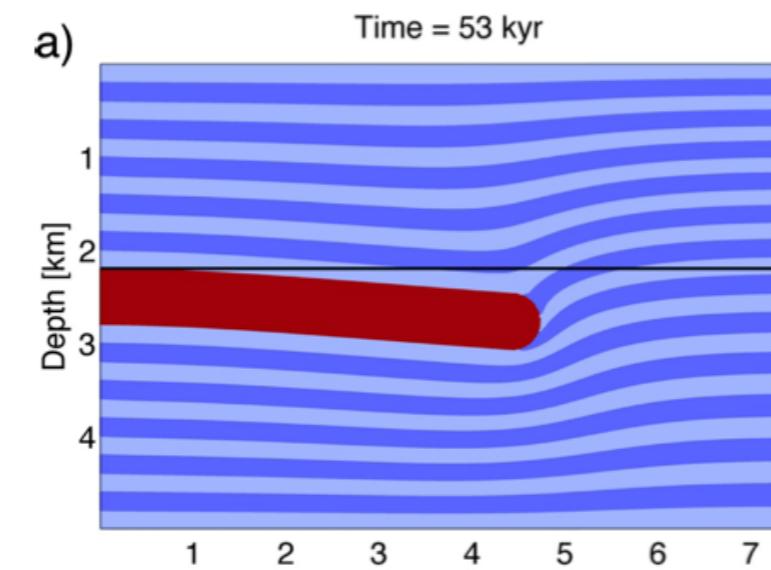
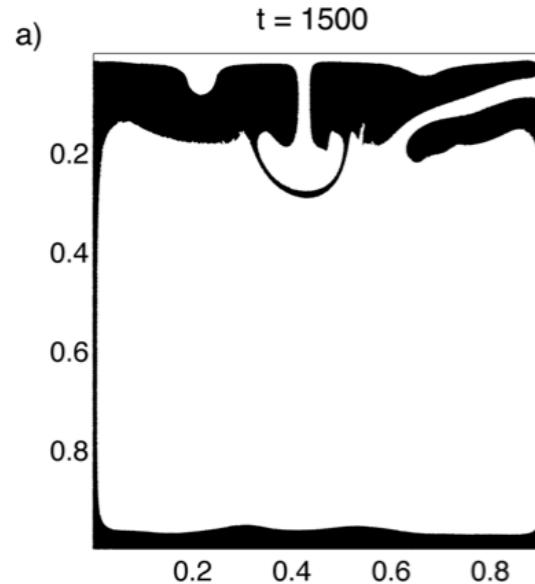
- generally **Stokes** preconditioners/solver applicable

II code performance

- MILAMIN-type optimized matrix assembly
- kdtree nearest neigh. algorithm for marker handling
- sufficient convergence ~10 non-linear iterations
- <10 min per time step for $>10^5$ dof, 10^6 markers
- bottleneck: direct solver (matlab "\")



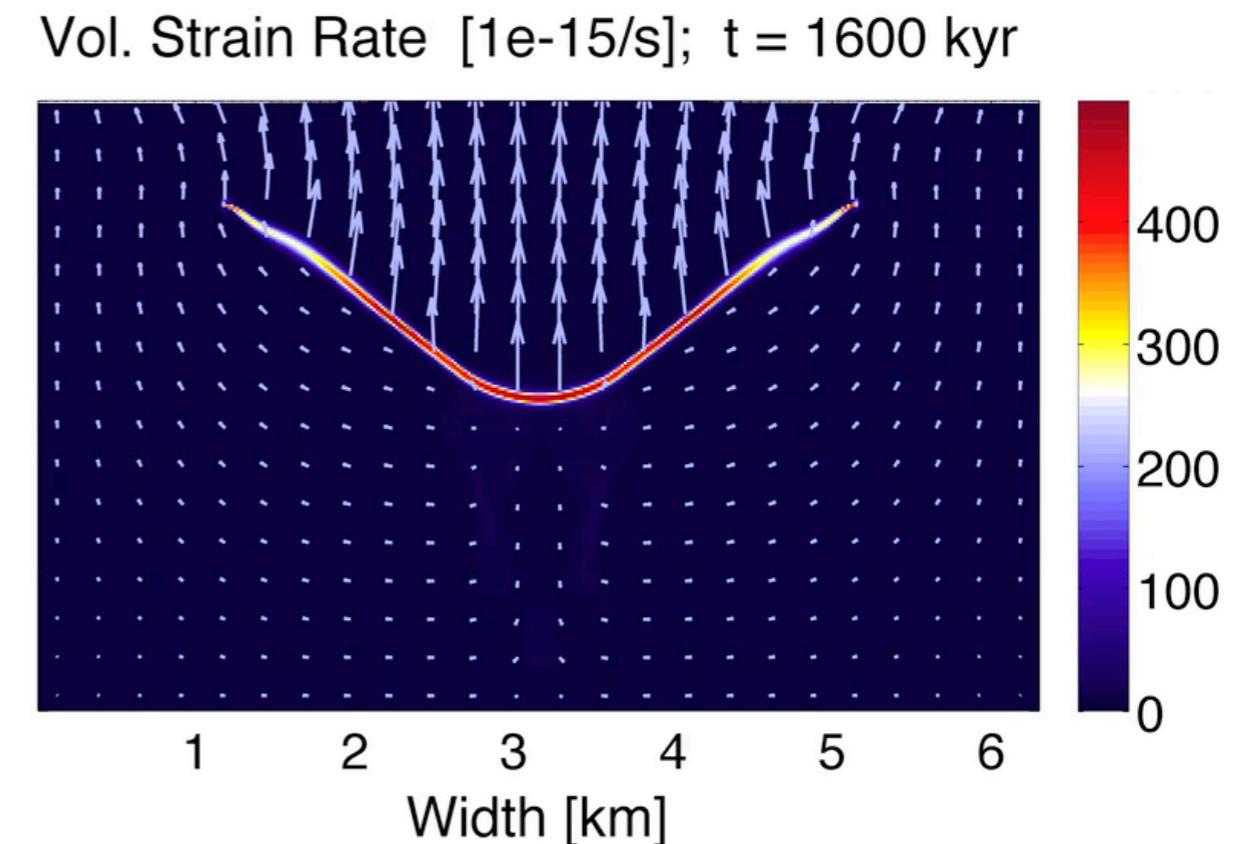
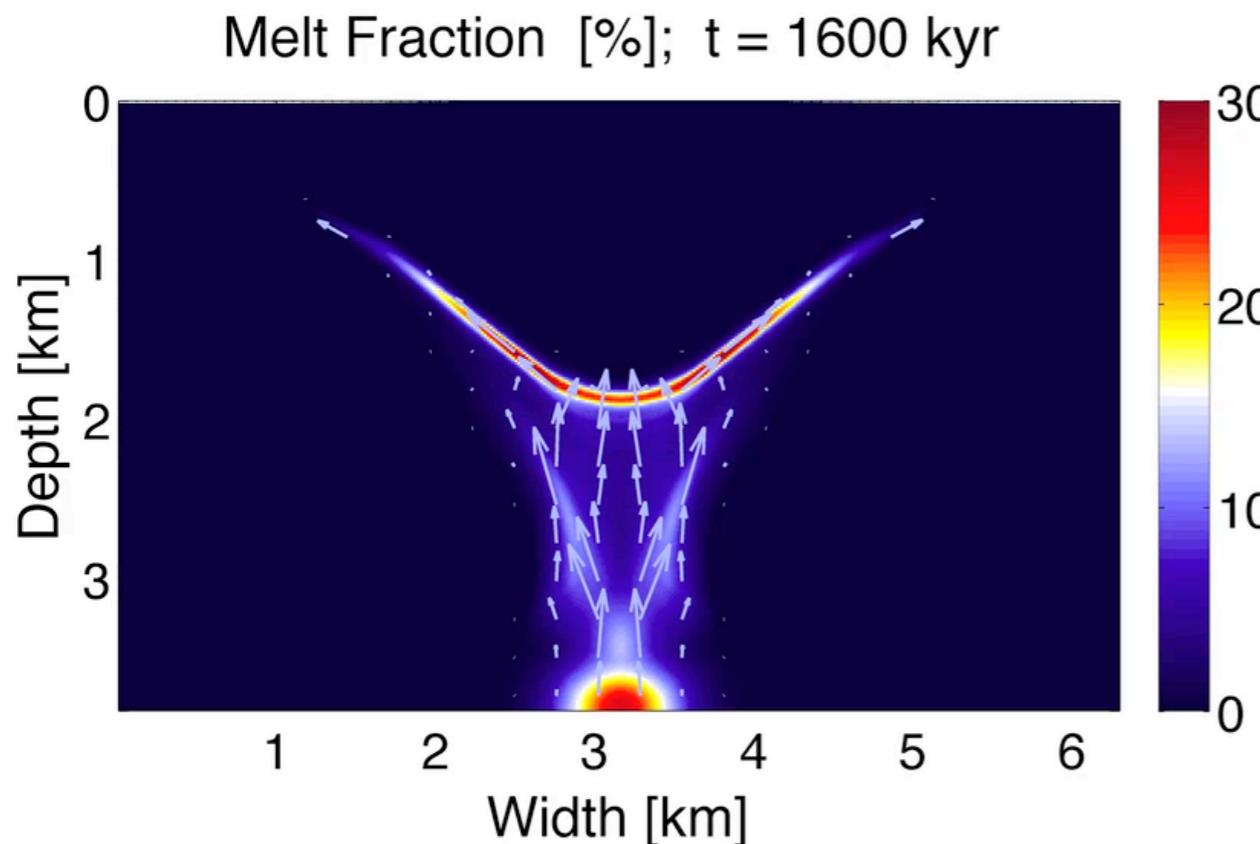
benchmarks



III modelling results I

melt channeling: melt percolates through localized channels

rock viscosity = $1\text{e}21$ Pas; tensile strength = 5 MPa.



→ governing physics: melt over-pressure leads to decompaction failure

III host rock rheology

- localization intensity ratio $r_{loc} = \zeta_{eff}^* / \zeta \approx 10^{-1} - 10^{-3}$
- adapted scaling for localized melt transport

$$l_c^* = \sqrt{r_{loc}} \quad l_c \approx [0.3 - 0.03] l_c$$

$$\mathcal{C}_c^* = \mathcal{C}_c / r_{loc} \approx [10 - 1000] \mathcal{C}_c$$

→ localized melt channelling instead of compaction waves

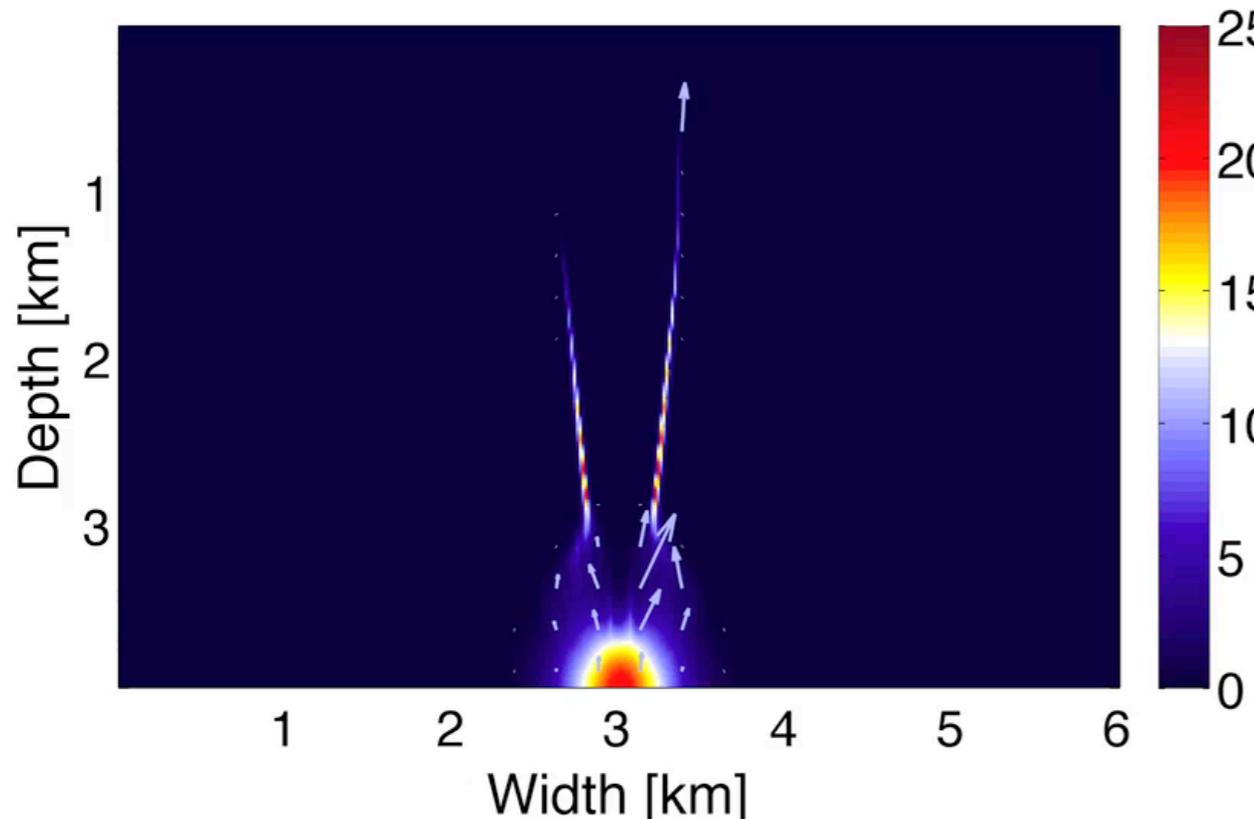


modelling results I

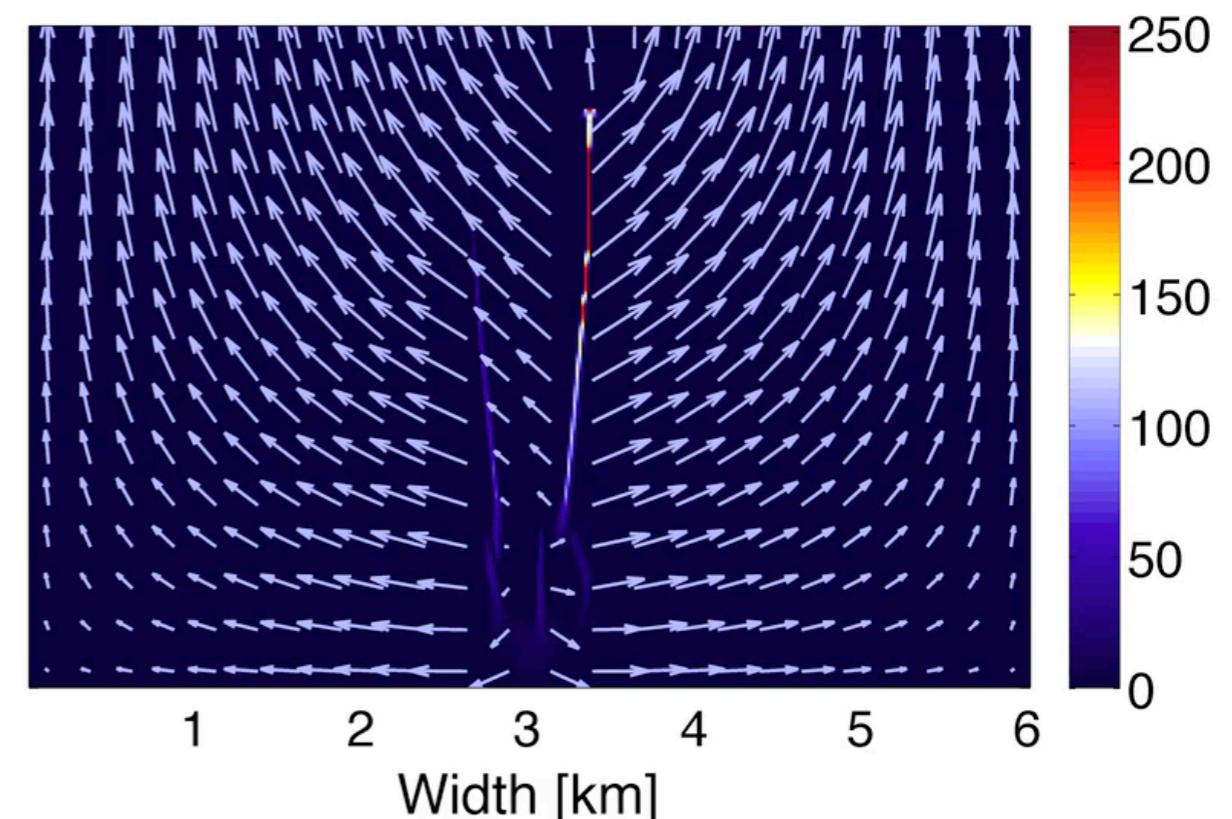
tensile fracturing: melt flows along brittle fractures

rock viscosity = $1\text{e}22$ Pas; tensile strength = 10 MPa.

Melt Fraction [%]; $t = 143$ kyr



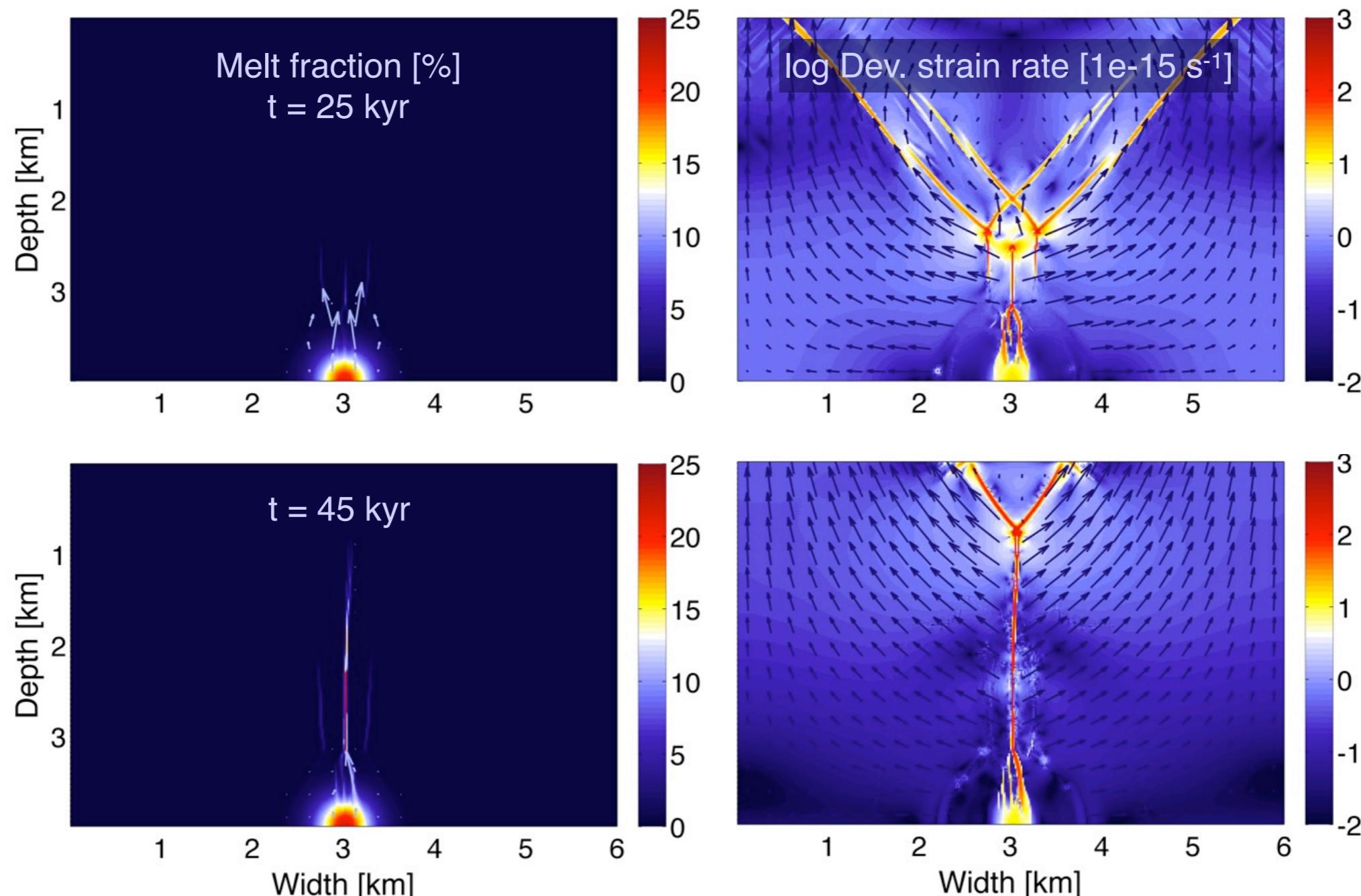
Vol. Strain Rate [$1\text{e}-15/\text{s}$]; $t = 143$ kyr



governing physics: shear stress leads to tensile failure

III | modelling results |

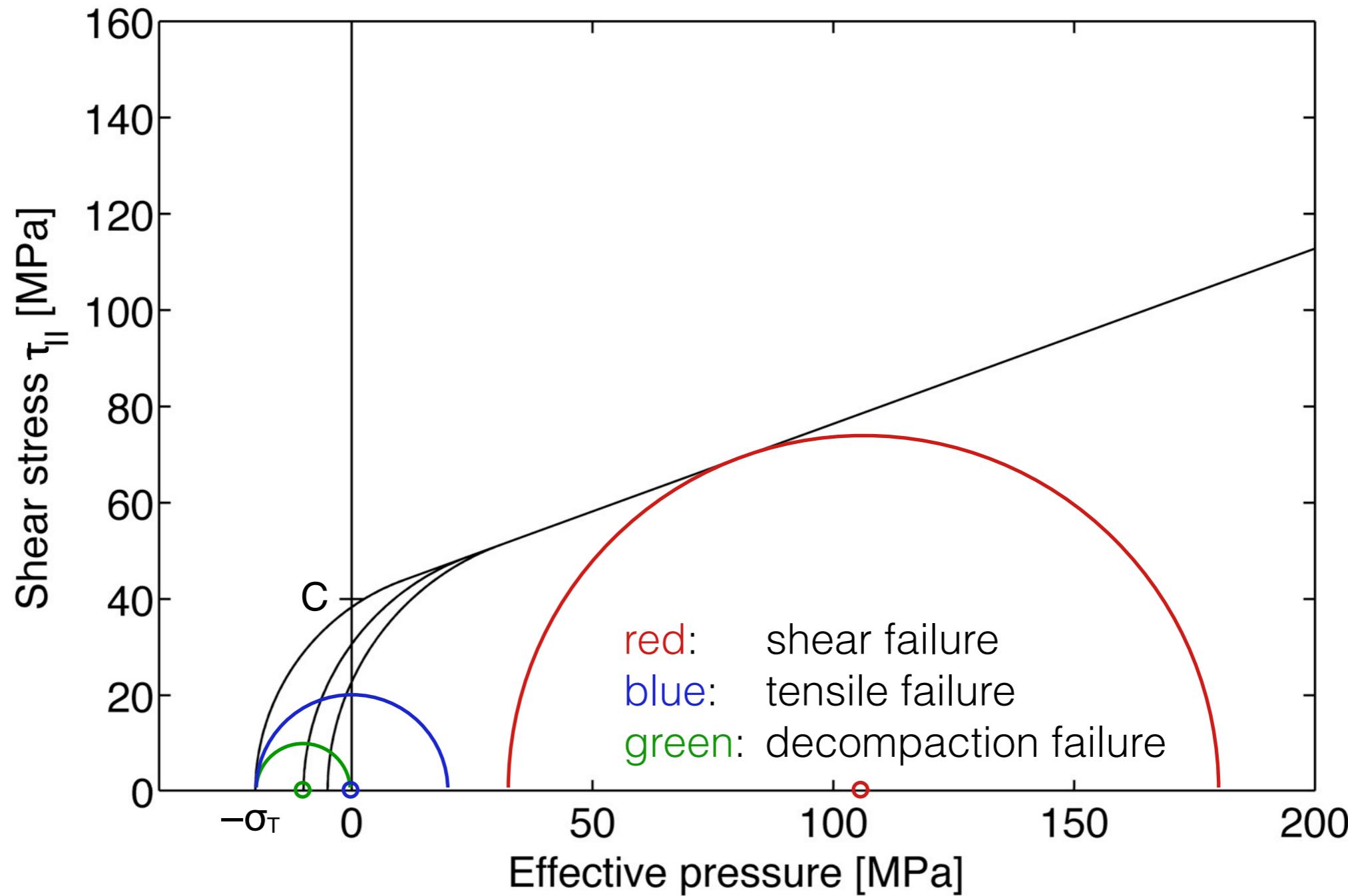
shear & tensile fracturing: melt flows along tensile fractures
rock viscosity = $1\text{e}23$ Pas; tensile strength = 10 MPa.





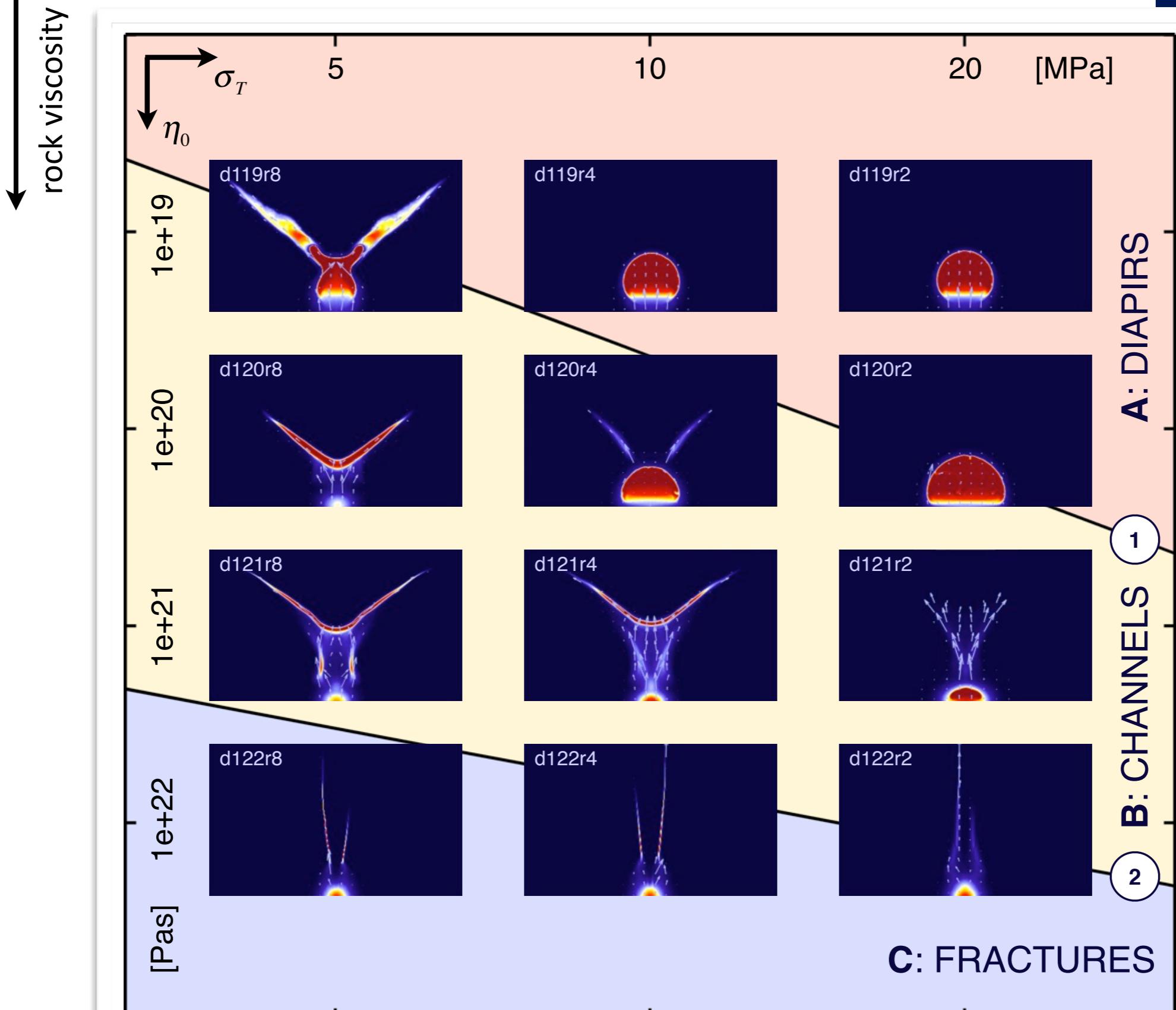
plastic failure

Mohr-Coulomb-Griffith failure criterion



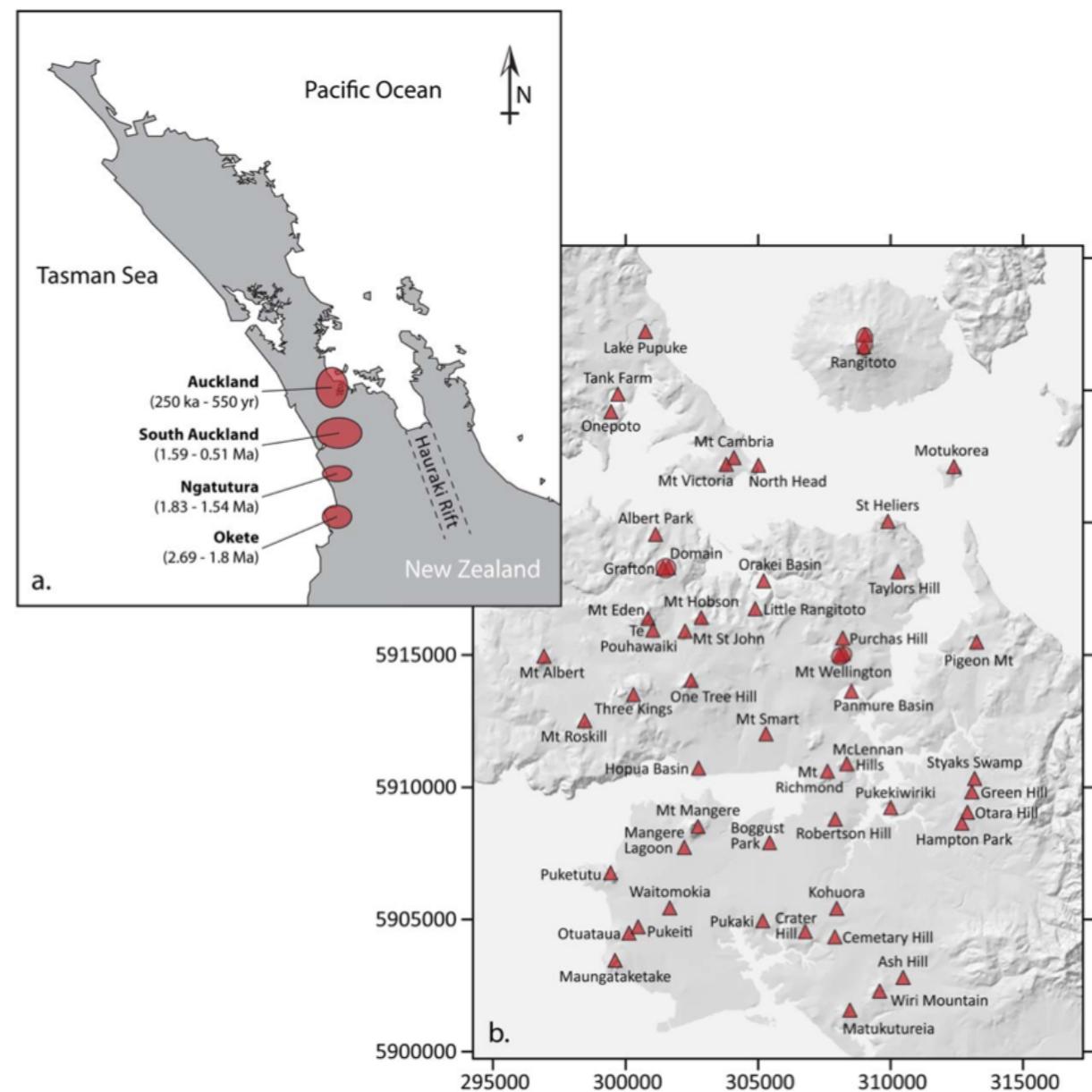


tensile strength



monogenetic fields

- motivation
Auckland Volcanic Field underlying metropolitan area of >1 Mio. inhabitants
 - big question
how are confined magma volumes collected from distributed source & transported on separate pathways into lithosphere?

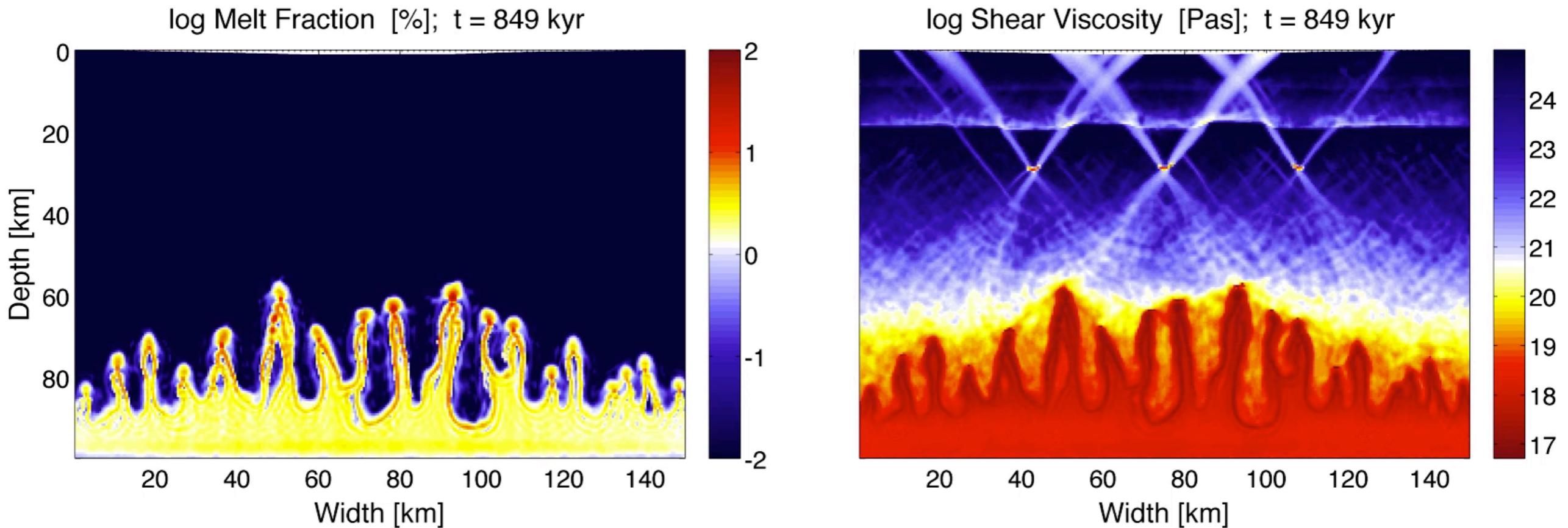


Le Corvec et al. (2013)

III modelling results II

reference case – melt fraction & shear viscosity

permeability (1% melt) = $1\text{e}-14 \text{ m}^2$; tensile strength = 10 MPa.

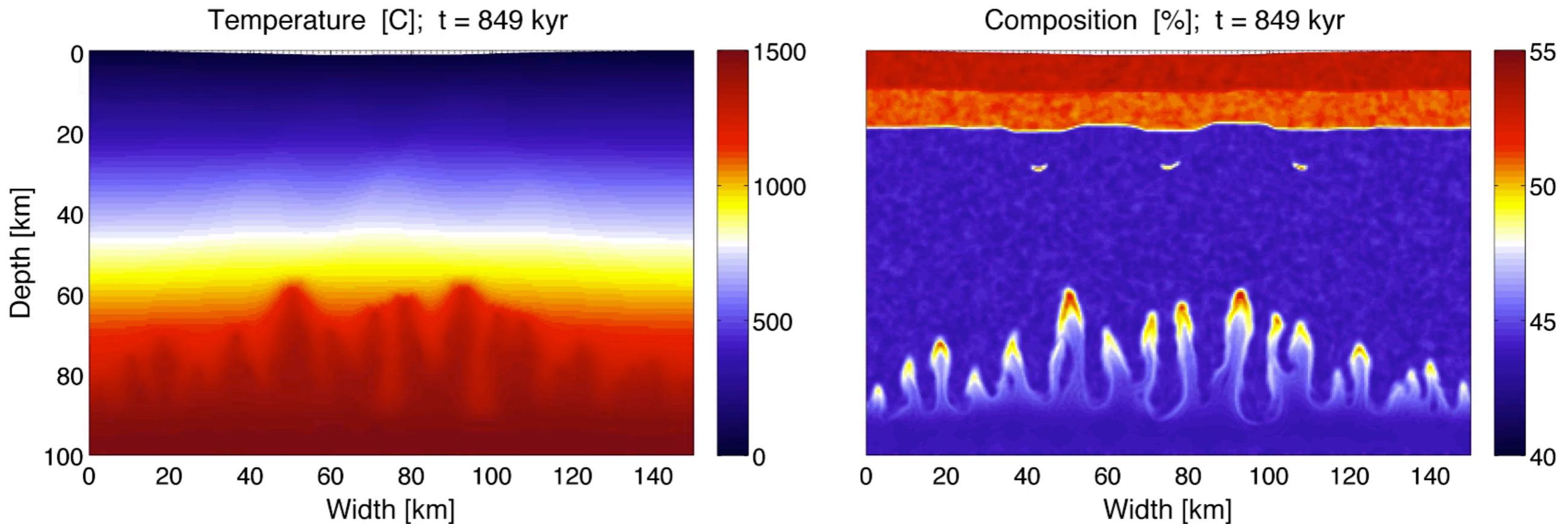


- decompaction failure leads to melt channelling across LAB
- shear failure leads to normal faulting of the lithosphere

III modelling results II

reference case – temperature & composition

permeability (1% melt) = $1\text{e}-14 \text{ m}^2$; tensile strength = 10 MPa.

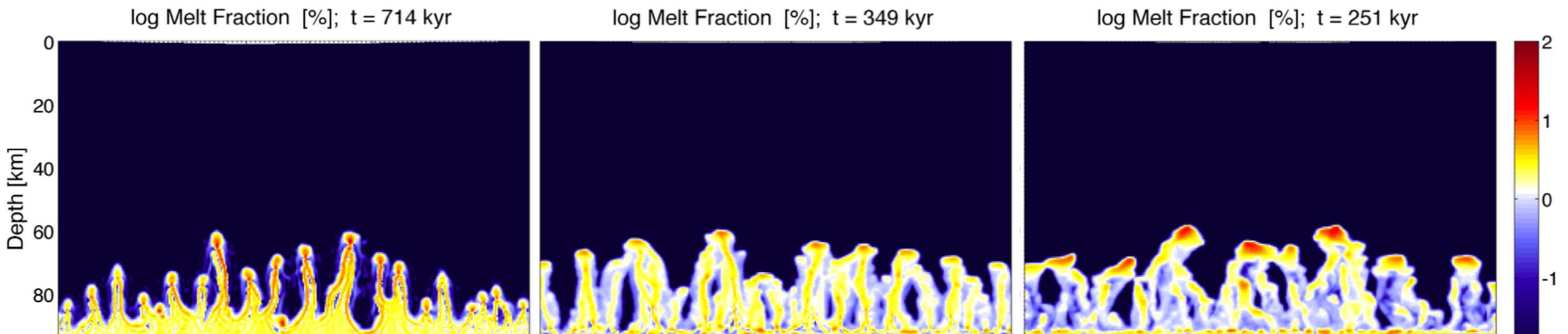


- temperature: thermal erosion by heat advection across LAB
- composition: impregnation / differentiation towards basalt

III modelling results II

effect of increasing permeability

permeability (1% melt) = $1\text{e-}14$ / $1\text{e-}13$ / $1\text{e-}12$ m 2 ; tensile strength = 10 MPa.



$$q_z \approx 5 \text{ cm yr}^{-1}$$

$$n \approx 19$$

$$V < 2 \text{ km}^3$$

$$q_z \approx 10 \text{ cm yr}^{-1}$$

$$n \approx 10$$

$$V < 8 \text{ km}^3$$

$$q_z \approx 20 \text{ cm yr}^{-1}$$

$$n \approx 7$$

$$V < 16 \text{ km}^3$$

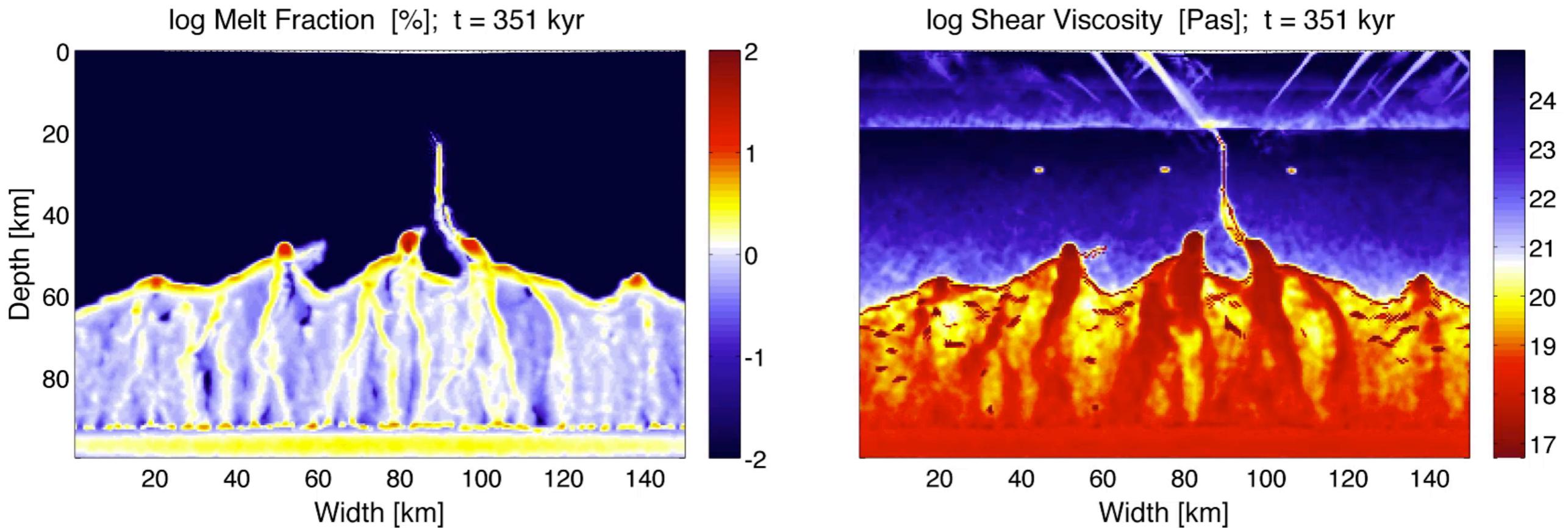


increasing melt flow, decreasing number but increasing volume of magma batches; cf. Auckland VF: $V_{\text{erupt}} \approx 1\text{e-}5 - 2.4 \text{ km}^3$; $n_{\text{erupt}} \approx 50$

III modelling results II

dike nucleation at top of melt channels?

permeability (1% melt) = $1e-12 \text{ m}^2$; tensile strength = 10 MPa.



- method is a two-phase continuum approximation of diking
- some issues with resolution and numerical stability remain

summary

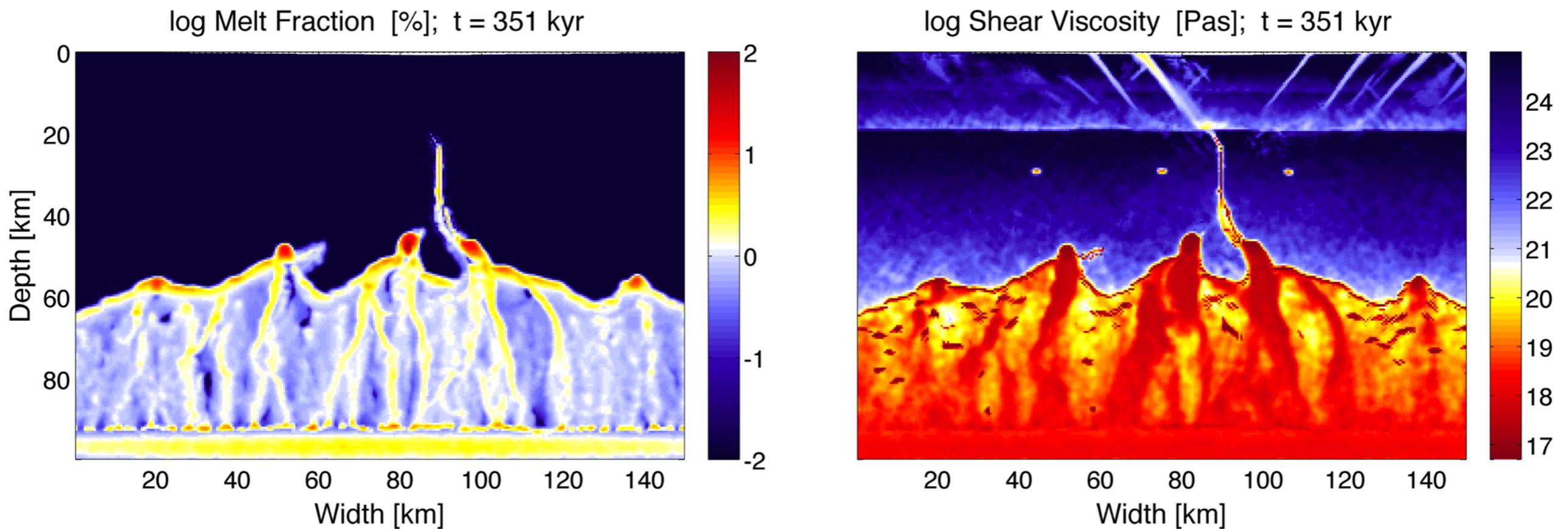
- fully coupled magma dynamics simulation code is functional and benchmarked modelling strategy similar to Stokes flow
- magma dynamics in visco-elasto-plastic host rock predicted features emerge self-consistently some issues with stability and resolution
- thermo-chemically coupled magma dynamics simplified petrology for basaltic magmatism first results demonstrate geodynamics applicability

perspectives

- parallel PETSc¹ FD-code in development
- extend petrology to eutectic ol-qz system
- extend petrology to include volatiles (H_2O , CO_2)
- add geochemical tracers to compare with data
- combine with analogue and analytical methods

¹ www.mcs.anl.gov/petsc/ (open source)

thank you



next talk: GMPV33/TS3.5 / 1 May / 08:30 / G11

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host rock rheology

- visco-elasto-plastic **shear stress**

$$\bar{\tau} = 2\eta_{eff}^* \dot{\varepsilon}'_s + \chi_\tau^* \bar{\tau}^o ; \quad \eta_{eff}^* = \begin{cases} \frac{\tau_y}{2\dot{\varepsilon}'_{s,II}} & \text{for } \bar{\tau}_{II} = \bar{\tau}_y \\ \frac{1-\phi}{\frac{1}{\eta} + \frac{1}{G\Delta t}} & \text{for } \bar{\tau}_{II} < \bar{\tau}_y \end{cases} ; \quad \chi_\tau^* = \frac{1-\phi}{1 + \frac{G\Delta t}{\eta}}$$

- visco-elasto-plastic **compaction pressure**

$$P_c = -\zeta_{eff}^* \nabla \cdot \mathbf{v}_s + \chi_p^* P_c^o ; \quad \zeta_{eff}^* = \begin{cases} \frac{-P_y}{\nabla \cdot \mathbf{v}_s} & \text{for } P_c = P_y \\ \frac{1-\phi}{\frac{1}{\zeta} + \frac{1}{K\Delta t}} & \text{for } P_c < P_y \end{cases} ; \quad \chi_p^* = \frac{1-\phi}{1 + \frac{K\Delta t}{\zeta}}$$

→ see Keller et al. (2013), GJI, for derivation