Westfälische Wilhelms-Universität Münster

Geophysik WWU Münster

Differentiation Processes in the Early Earth and their Impact on the Evolution of Mantle Convection

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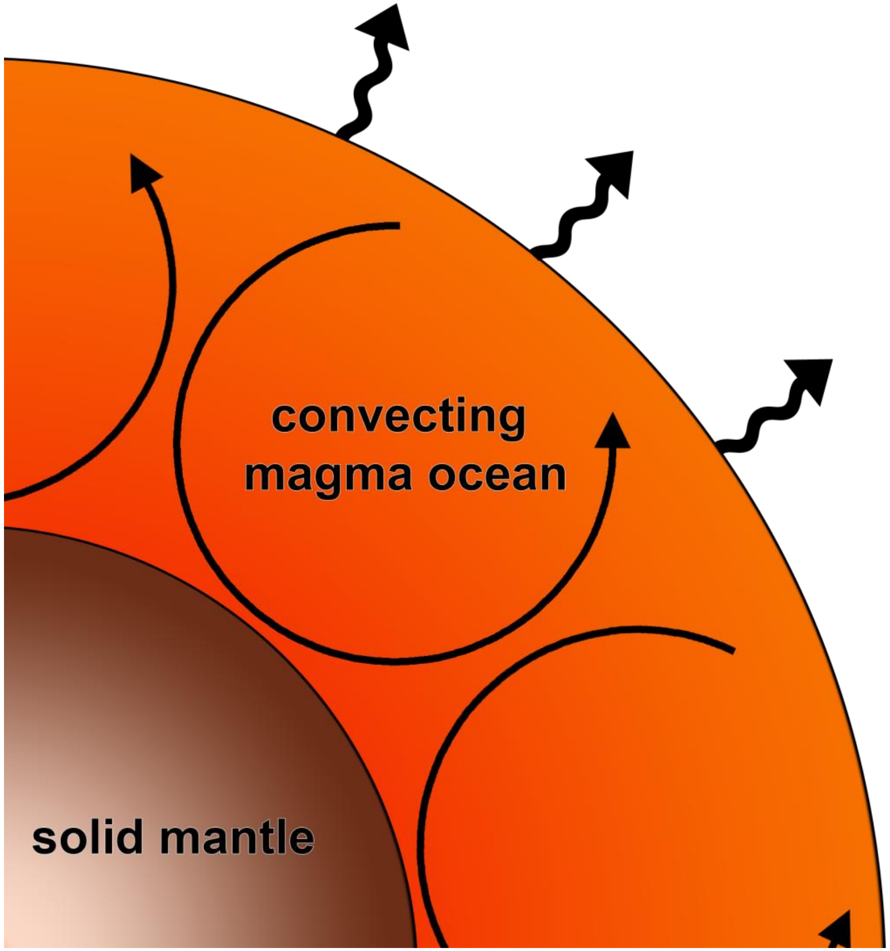
Outline

- 1) Basics:
  - Magma Ocean
    - Crystallization
  - Basic Equations
  - Numerical Model
- 2) First Results:
  - Influence of rotation on:
    - Crystal settling
    - Dependence on density
  - Summary & Implications

1) Basics

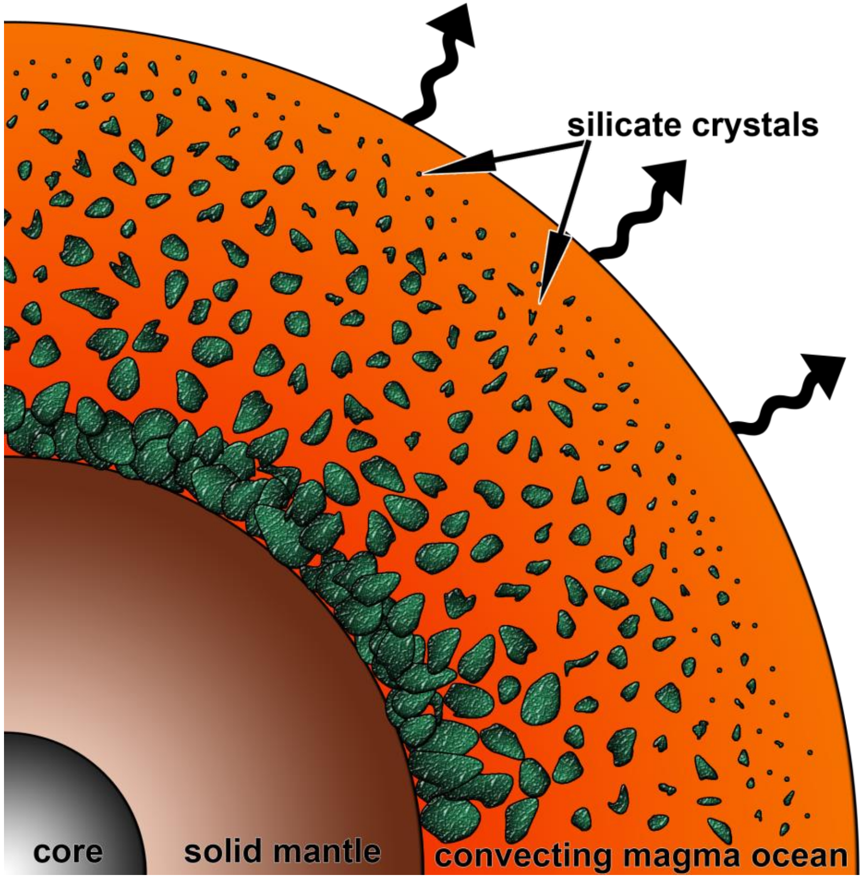
Magma Ocean on Earth

- ≈4.5 billion years ago
  - Developed due to giant impact
- Early Magma Ocean
  - ≈1000 km deep
  - Small viscosity
  - Very strong convection
  - Fast earth rotation (2-5 h)
- Metal and silicate separate
  - → core formation
- After core formation
  - → crystallization



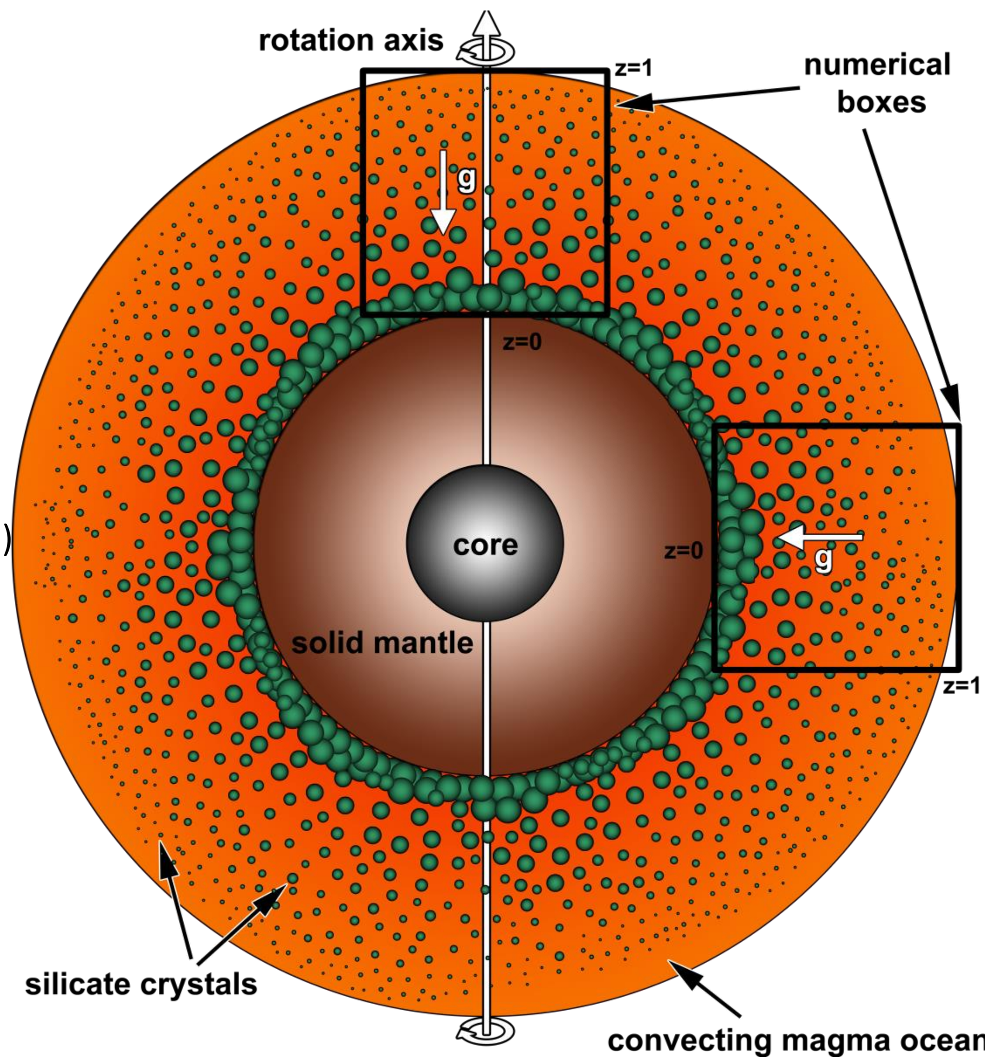
Crystallization of Magma Ocean

- Silicate Crystals:
  - Fractional crystallization?
    → Strongly differentiated mantle
  - Equilibrium crystallization?
    → Undifferentiated mantle
- Influence of rotation?
- Implications for the earth today
  - Onset of Plate tectonics
  - Mantle development until today
  - Mantle differentiation?

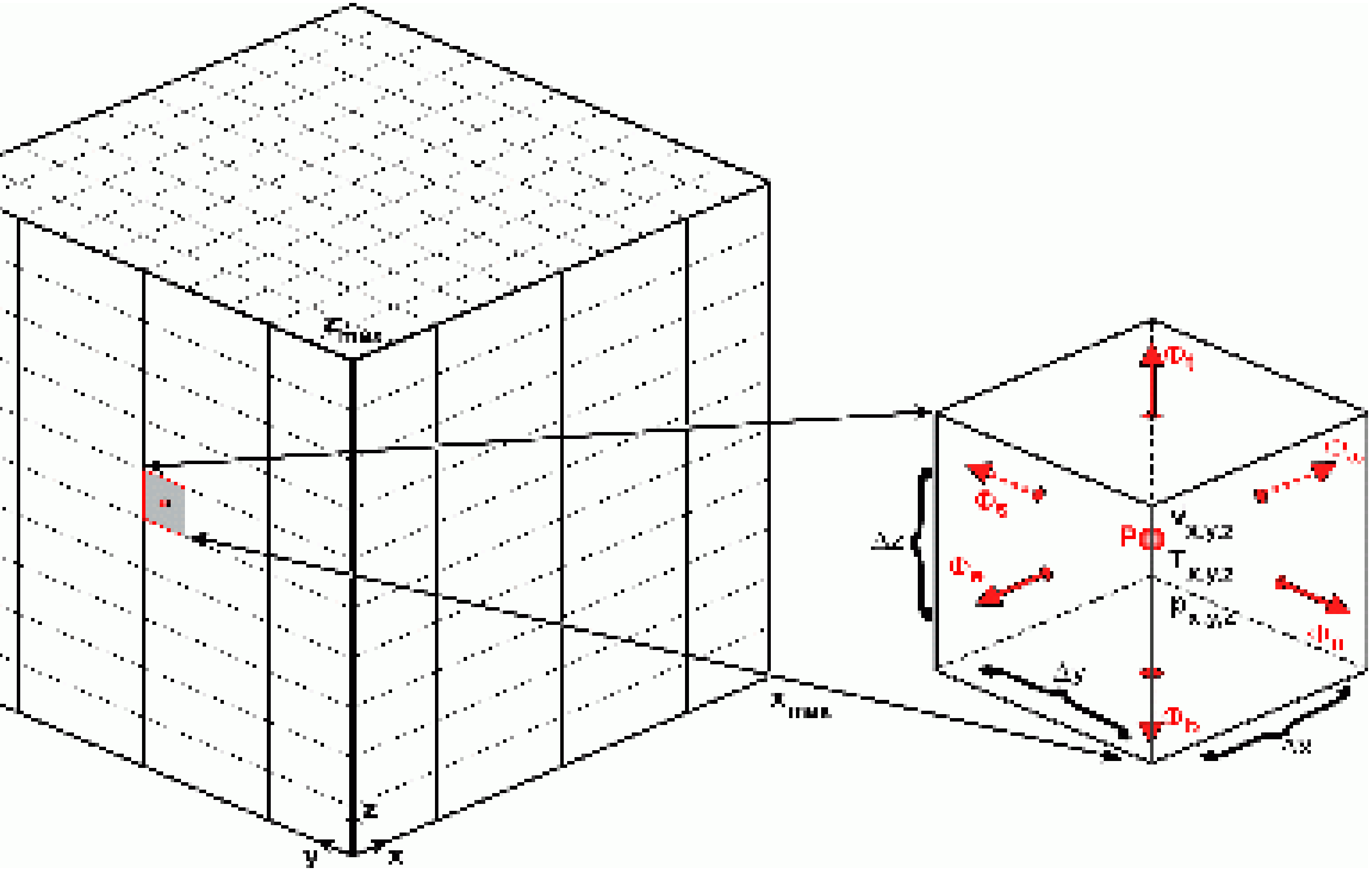


Numerical Model

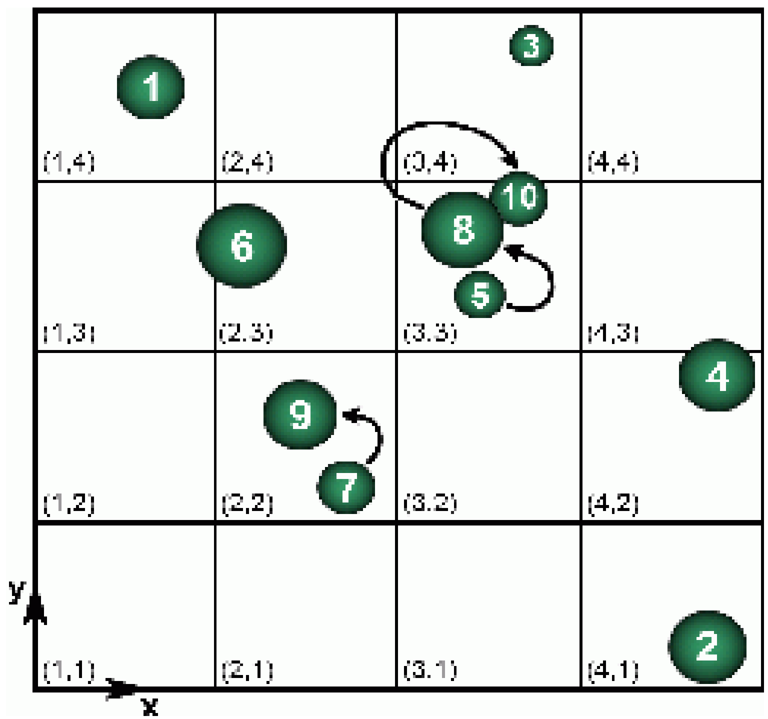
- Fluid model: (Schmalzl and Hansen, 2000)
  - 3D - Finite Volume
  - rotation axis can be tilted
- Particle model:
  (Verhoeven & Schmalzl, 2009; Möller and Hansen, 2013)
  - Discrete Element method
  - Spherical shape
  - Collision algorithm
  - Forces: Gravity, Coriolis, Friction



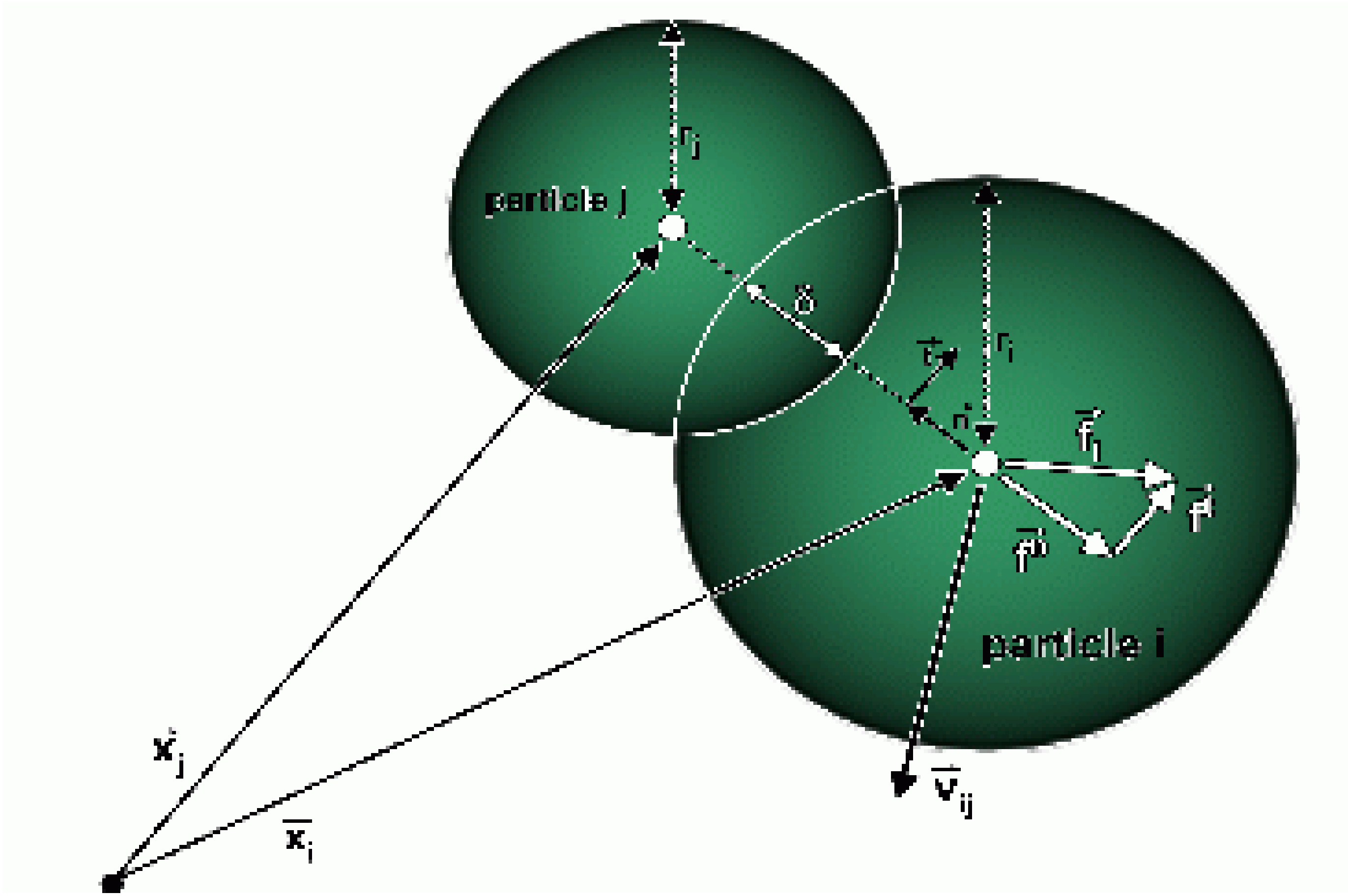
The Fluid Model



The Linking- Cell Algorithm



The Discrete Element Method



Equations

- Continuity equation: $$\nabla \cdot \vec{v} = 0$$
- Heat transport equation: $$\frac{\partial T}{\partial t} + (\vec{v} \cdot \nabla)\, T = \nabla^2 T$$
- Momentum equation:

$$\frac{1}{Pr}\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v}\right) = -\nabla p - \sqrt{Ta}(\overrightarrow{e_\omega} \times \vec{v}) + \nabla^2 \vec{v} + Ra(T - BC)\overrightarrow{e_z}$$

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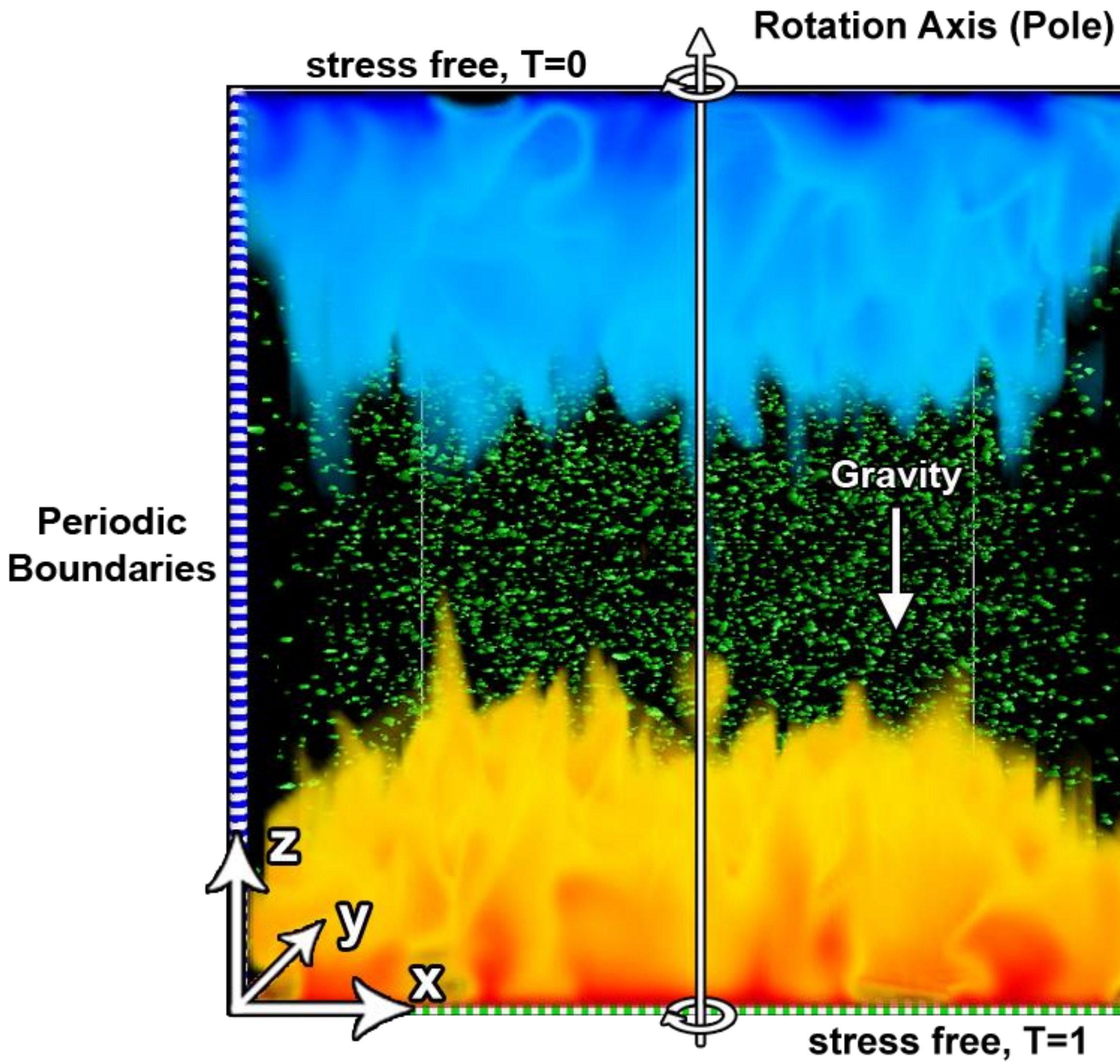
- Parameter:
  - Rayleigh: $Ra = \frac{\alpha g \Delta T d^3}{\kappa \nu_0} \rightarrow Ra = 10^8$
  - Taylor: $Ta = \frac{4\Omega^2 d^4}{{\nu_0}^2} \rightarrow 0 \leq Ta \leq 10^{10}$
  - Prandtl: $Pr = \frac{\nu_0}{\kappa} \rightarrow Pr = 1$
  - Buoyancy: $B = \frac{(\rho_p - \rho_{fl})}{\rho_{fl} \alpha \Delta T} \rightarrow B_{silicat} \approx 2.5$
  - Rossby: $Ro = \sqrt{\frac{Ra}{Pr Ta}} \rightarrow 0.1 \leq Ro \leq \infty$

Boundary Conditions

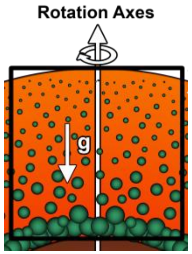


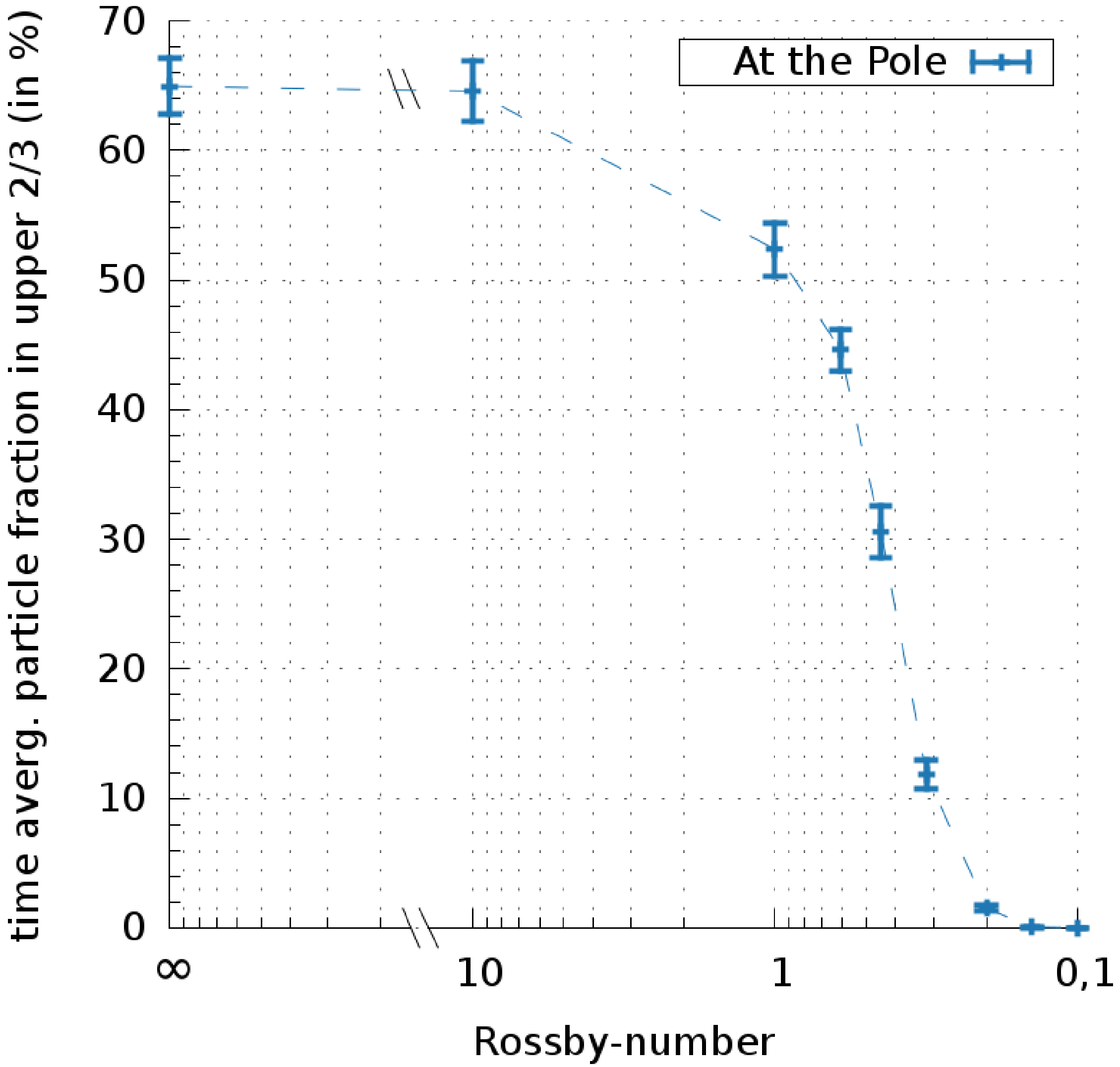
2) Results

14

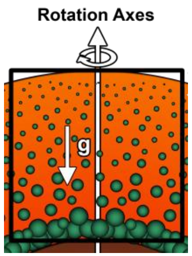
2.1) Polar Scenario

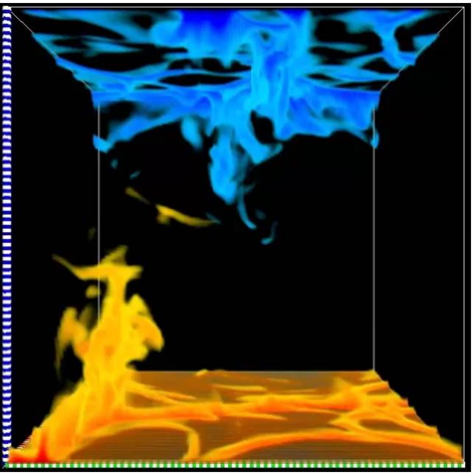
Crystal Settling: Pole



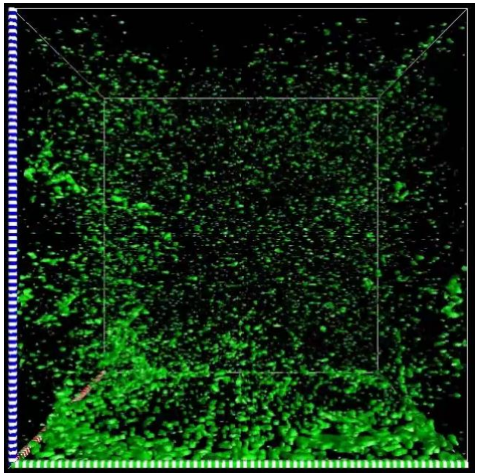


Crystal Settling: Pole

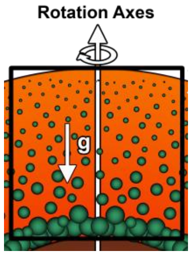


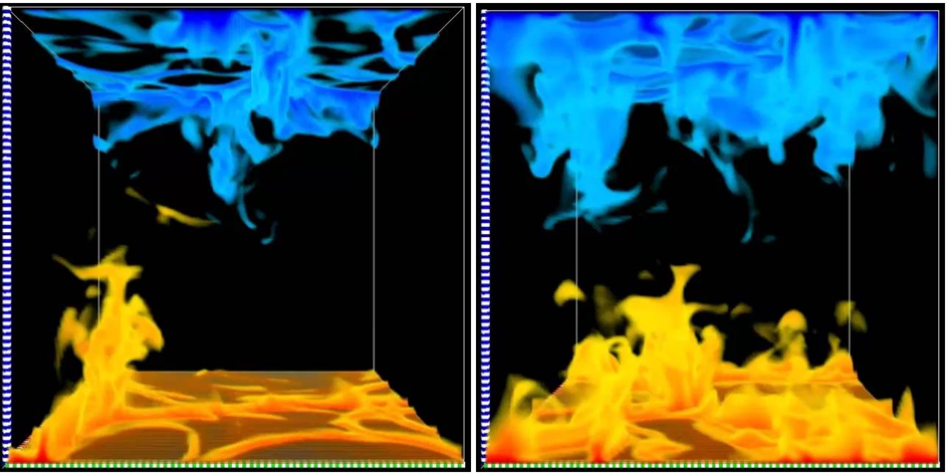


$Ro = \infty$

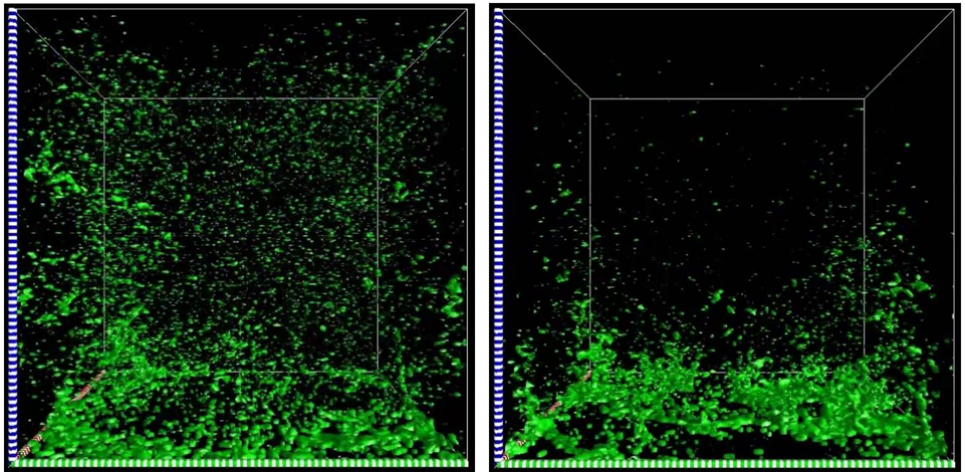


Crystal Settling: Pole

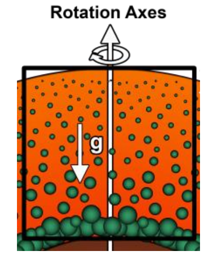


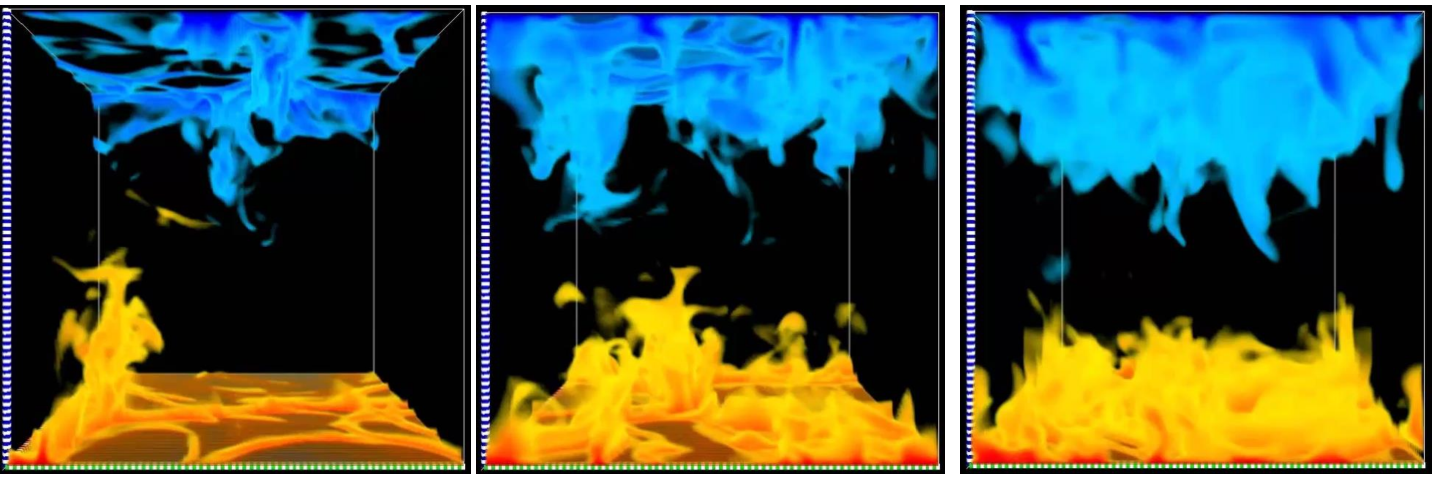


$Ro = \infty$ $\quad$ $Ro = 1$

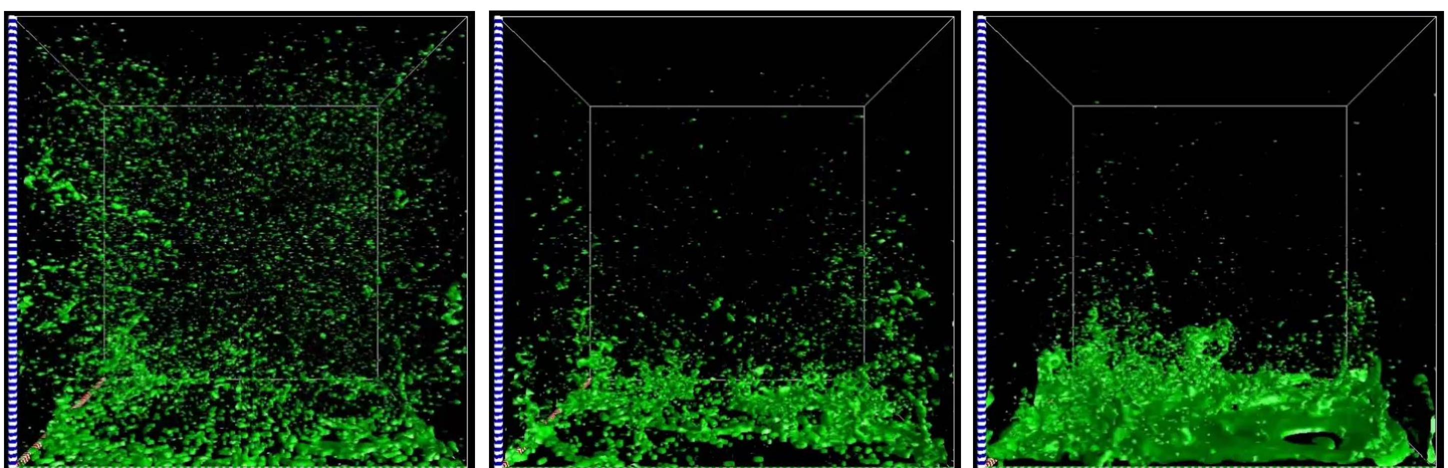


Crystal Settling: Pole

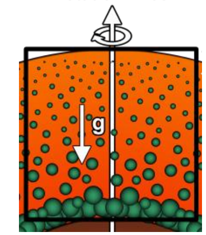


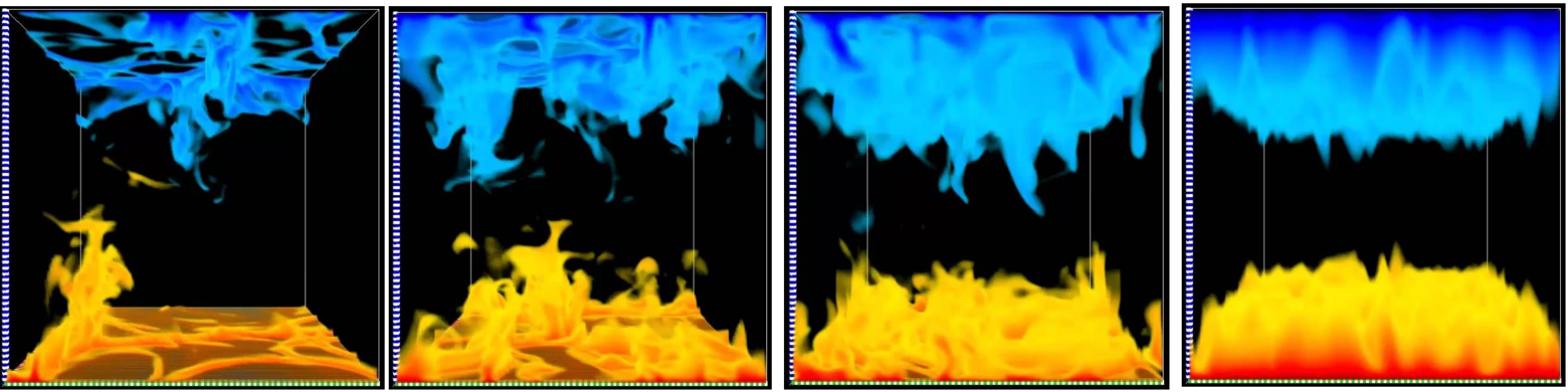


$Ro = \infty$ $Ro = 1$ $Ro = 0{,}32$

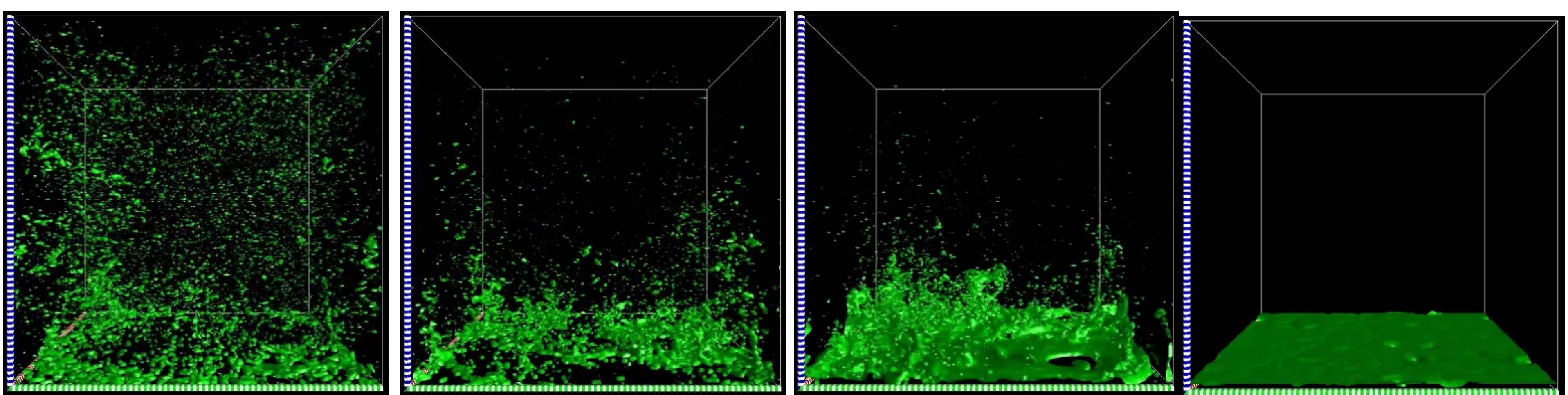


Crystal Settling: Pole

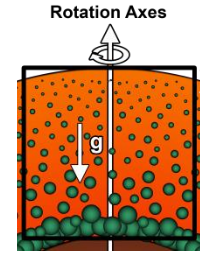




$Ro = \infty$ | $Ro = 1$ | $Ro = 0{,}32$ | $Ro = 0{,}1$



Taylor Proudman Theorem



$$\frac{\partial u}{\partial t}+\mathrm{u}\nabla \mathrm{u} = -\frac{1}{\rho}\nabla \mathrm{P} - 2\boldsymbol{\Omega} \times u + \nu\nabla^2 \mathrm{u}$$

-

$$2\boldsymbol{\Omega}\times \mathrm{u} = -\frac{1}{\rho}\nabla \mathrm{P}$$

Neglecting viscous and assuming
Rotation dominates inertia

$$\nabla \times (\boldsymbol{\Omega} \times u) = 0$$

$$\boldsymbol{\Omega} \bullet \nabla u - u \bullet \nabla \boldsymbol{\Omega} + u(\nabla \bullet \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\nabla \bullet u) = 0$$

Taylor –Proudman Theorem

Since $\boldsymbol{\Omega}$ is independent on position , and for an
incompressible fluid $\nabla \bullet u$ holds, the statement

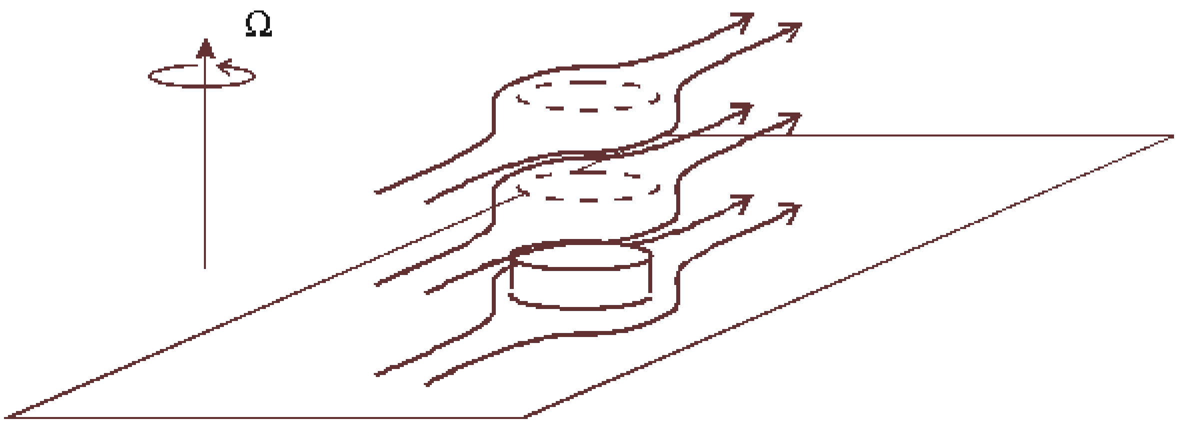
$$\boldsymbol{\Omega} \bullet \nabla u - u \bullet \nabla \boldsymbol{\Omega} + u(\nabla \bullet \boldsymbol{\Omega}) - \boldsymbol{\Omega}(\nabla \bullet u) = 0$$

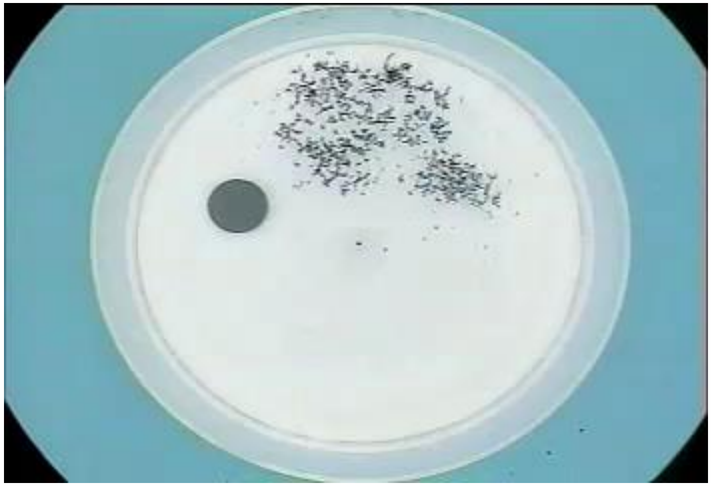
reduces to $\boldsymbol{\Omega} \bullet \nabla u = 0$

$$For\ \boldsymbol{\Omega} in\ z - direction \frac{\partial u}{\partial z} = 0 \Rightarrow \frac{\partial u}{\partial z} = 0$$

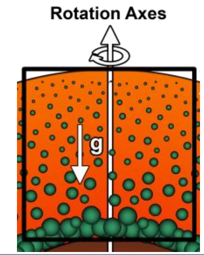
No Variation of velocity in direction paralle to Rotational axis - Taylor – Proudman theorem

Taylor-Proudman 3





Crystal Settling: Pole

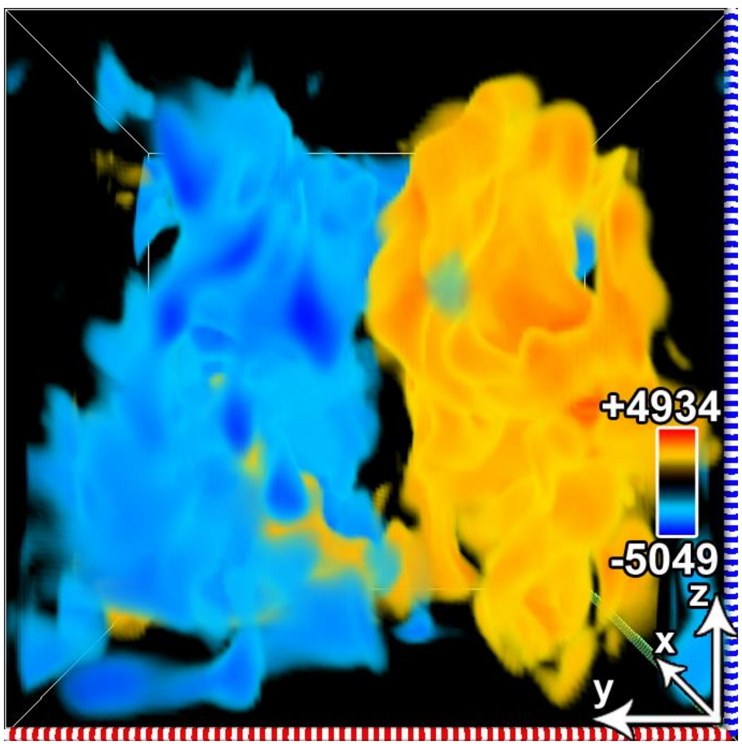


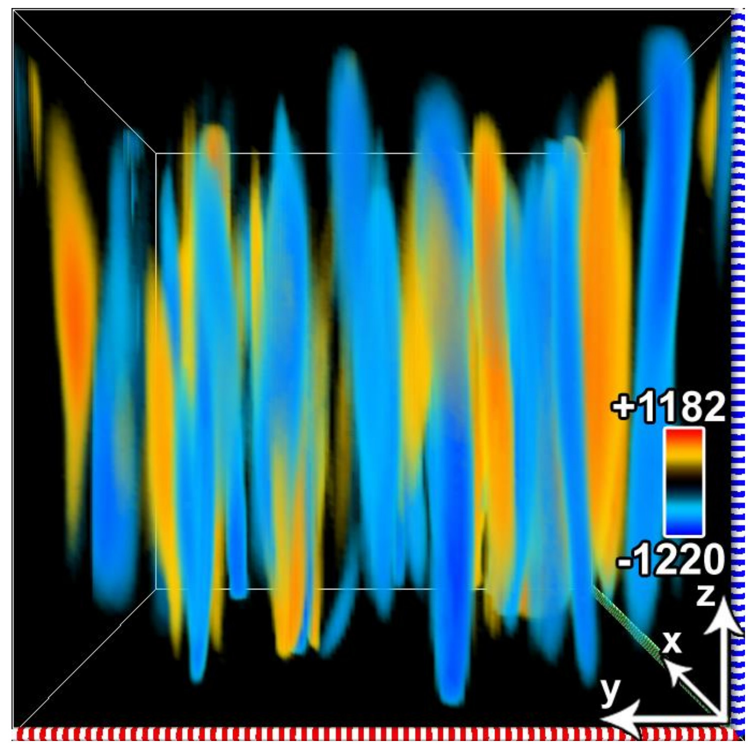
- Taylor-Proudman Theorem: Vertical velocity

$Ro = \infty$:

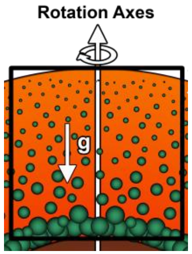
$Ro = 0{,}1$:

v_z:



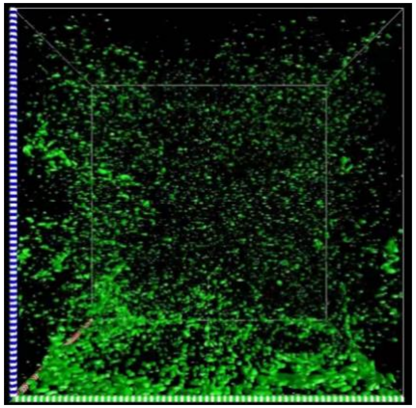


Influence of density: Pole

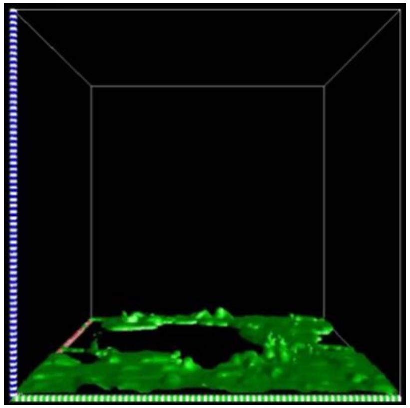


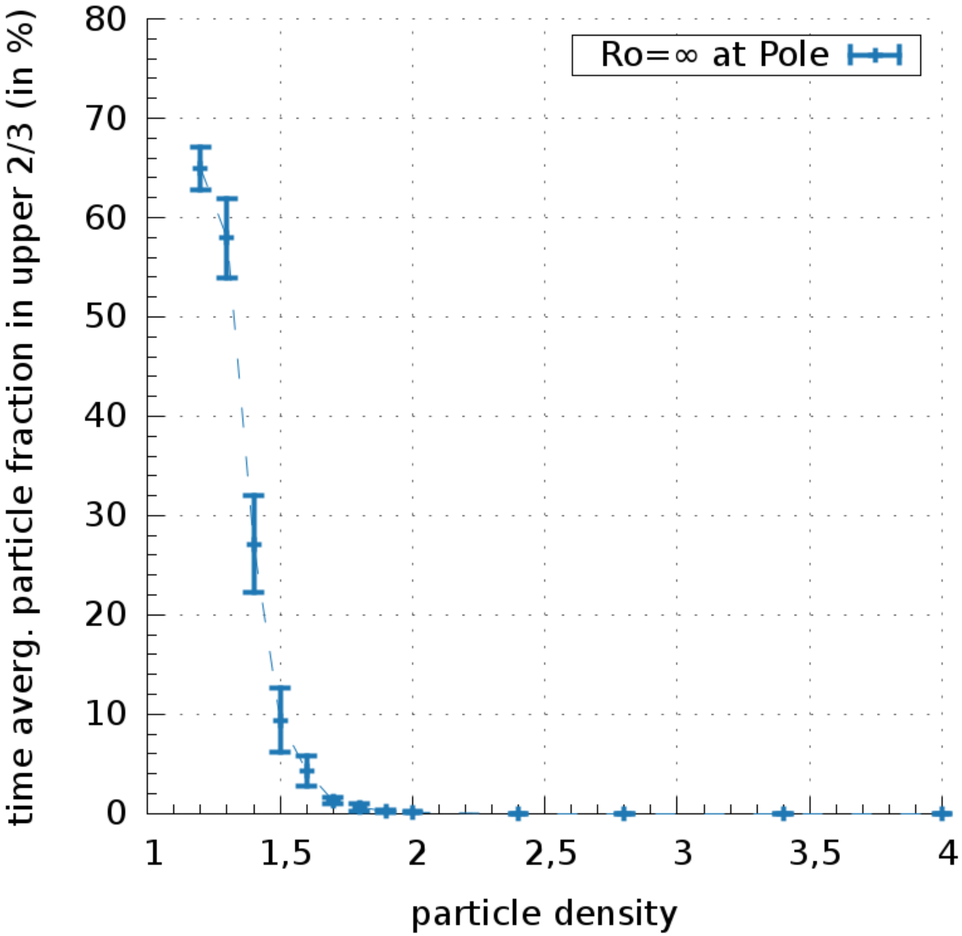
- Without rotation ($Ro = \infty$):
  - $\rho < 1{,}9$: suspension
  - $\rho \geq 1{,}9$: sedimentation

$Ro = \infty, \rho = 1{,}2$

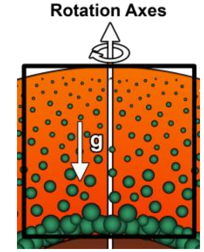


$Ro = \infty, \rho = 2{,}0$

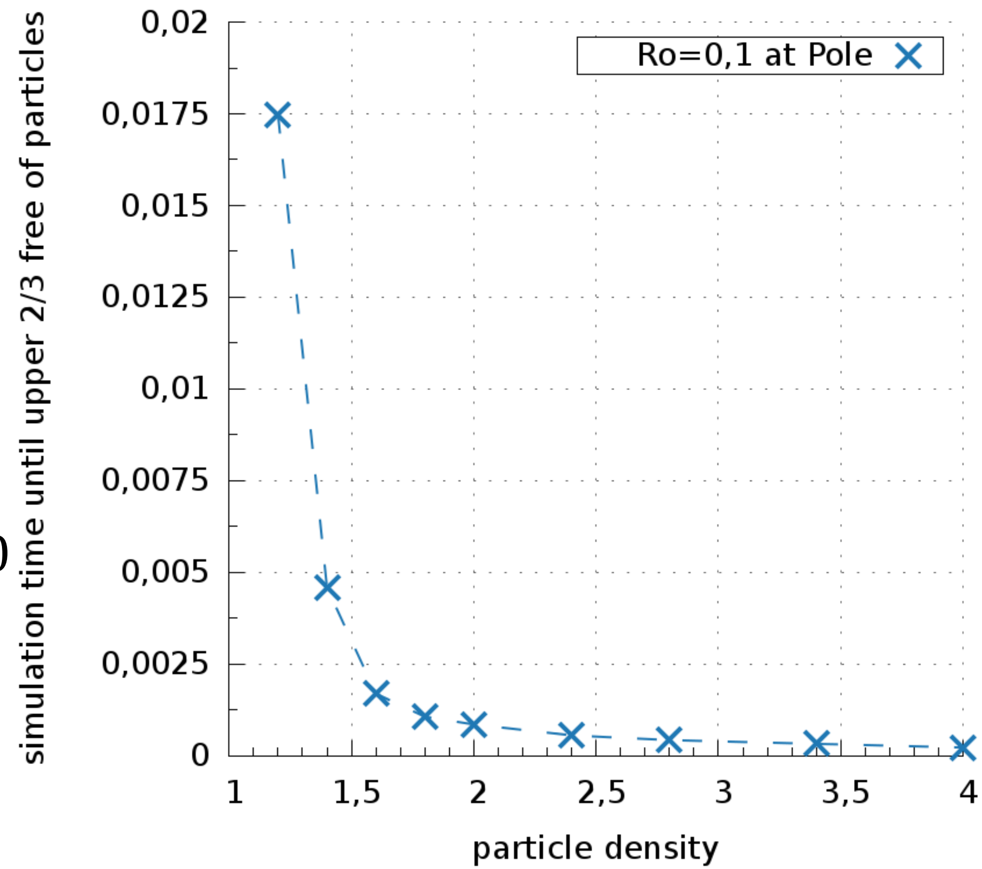




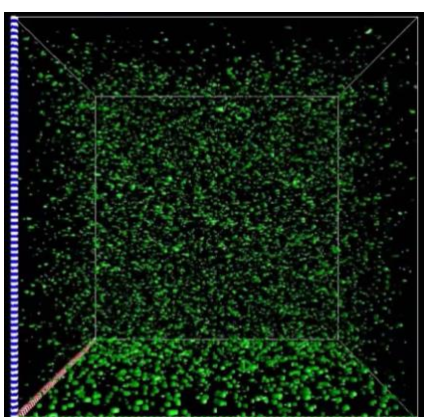
Influence of density: Pole



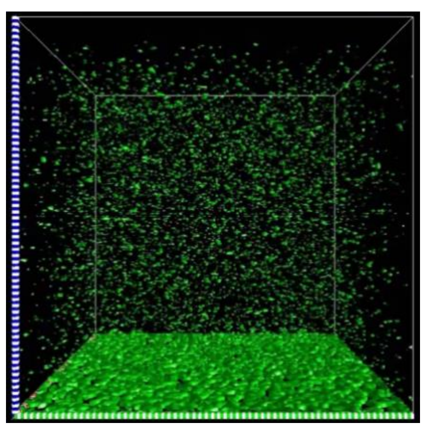
- High Rotation $(Ro = 0{,}1)$:
  - Light particle settle last
  - High density: small difference in settling time



$Ro = 0{,}1, \rho = 1{,}2$



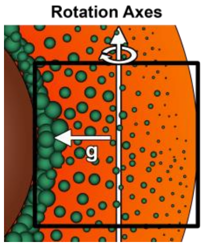
$Ro = 0{,}1, \rho = 2{,}0$

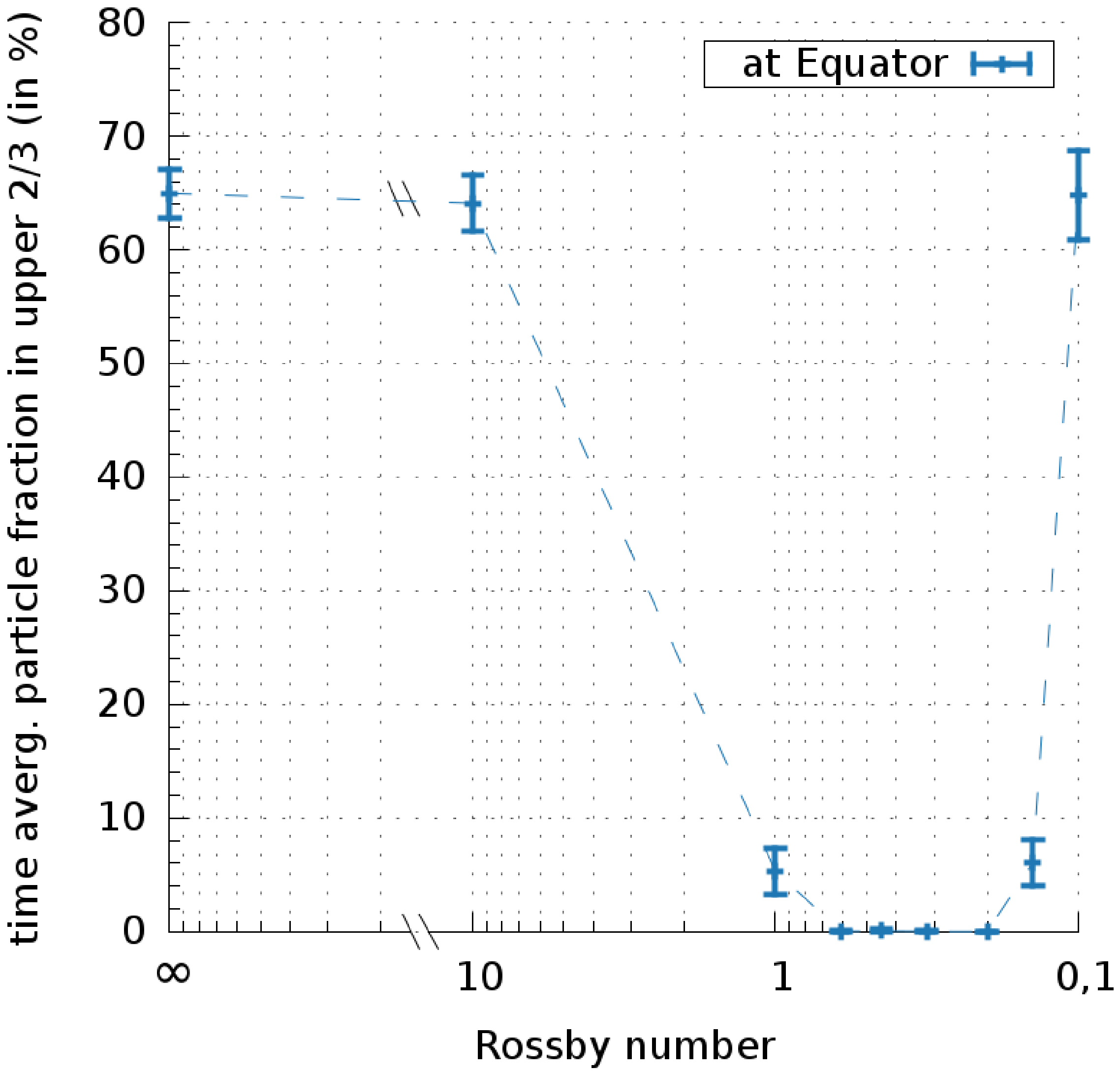


26

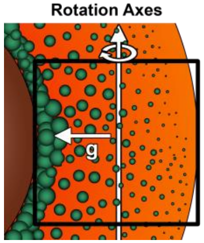
2.2) Equatorial Scenario

Crystal Settling: Equator





Crystal Settling: Equator



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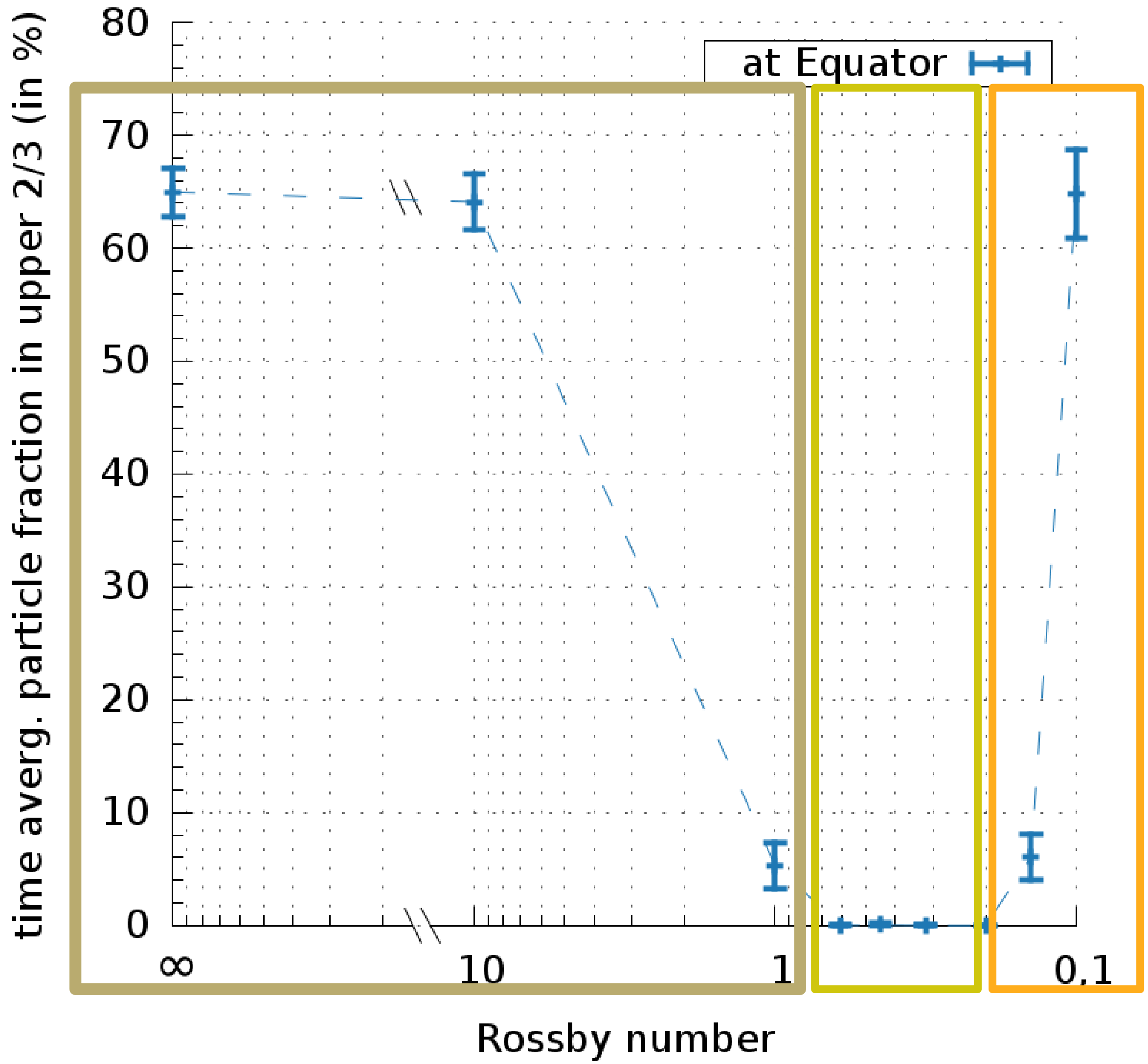
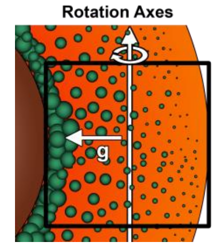
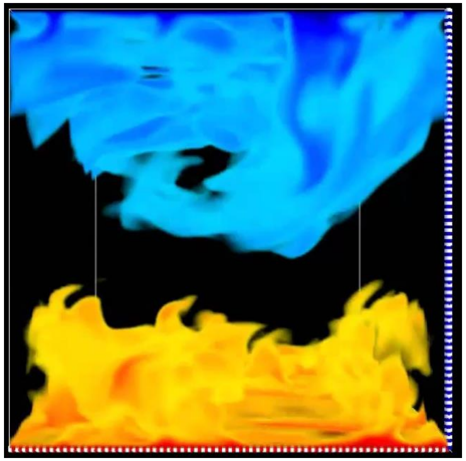


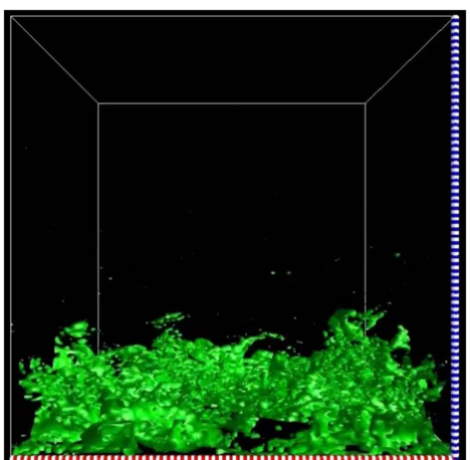
Crystal Settling: Equator



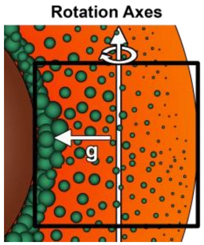


$Ro = 1$

y-z plane



Crystal Settling: Equator



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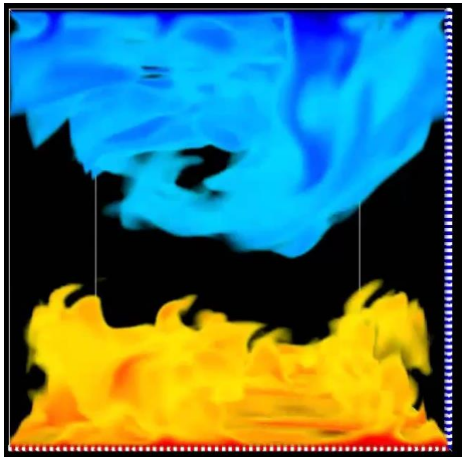
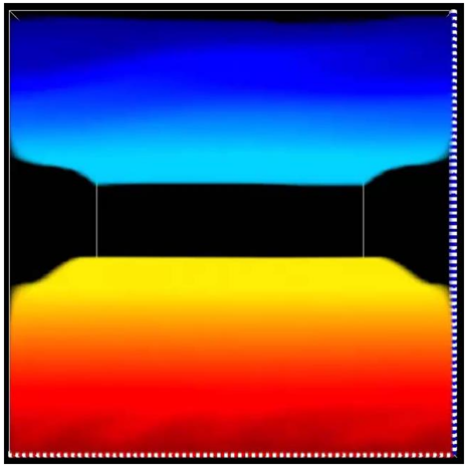


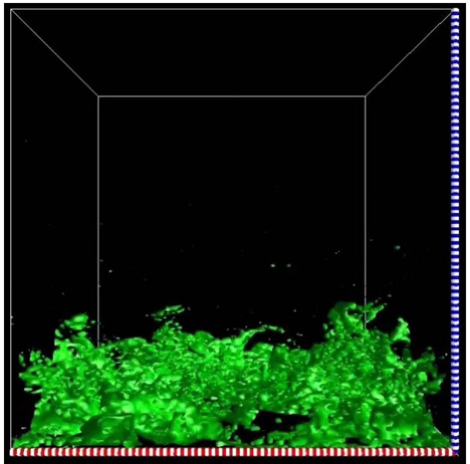
$Ro = 1$

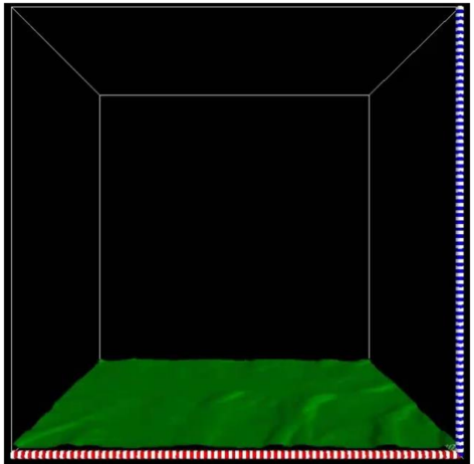
y-z plane



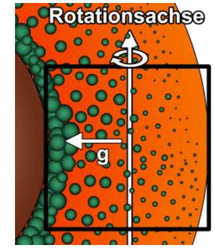
$Ro = 0{,}45$

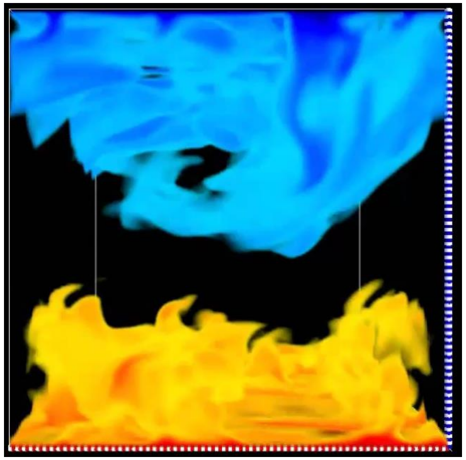
y-z plane

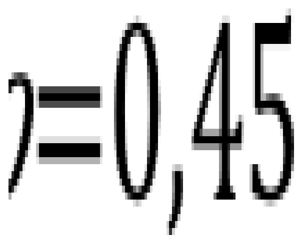


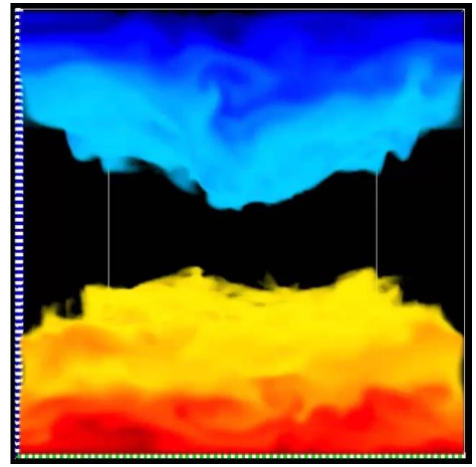


Crystal Settling: Equator





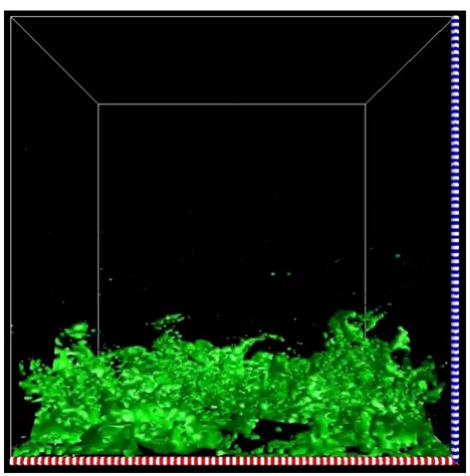


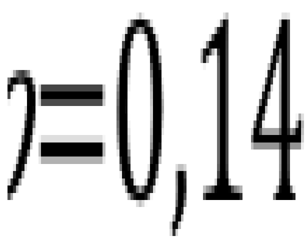


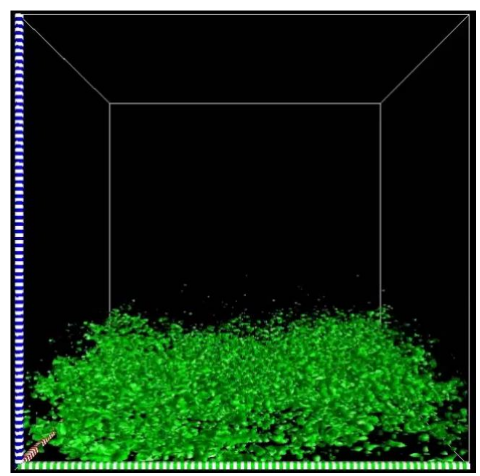
$Ro = 1$
y-z plane

$Ro = 0{,}45$
y-z plane

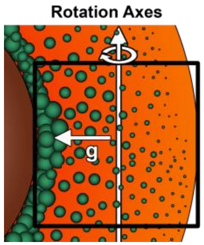
$Ro = 0{,}14$
x-z plane

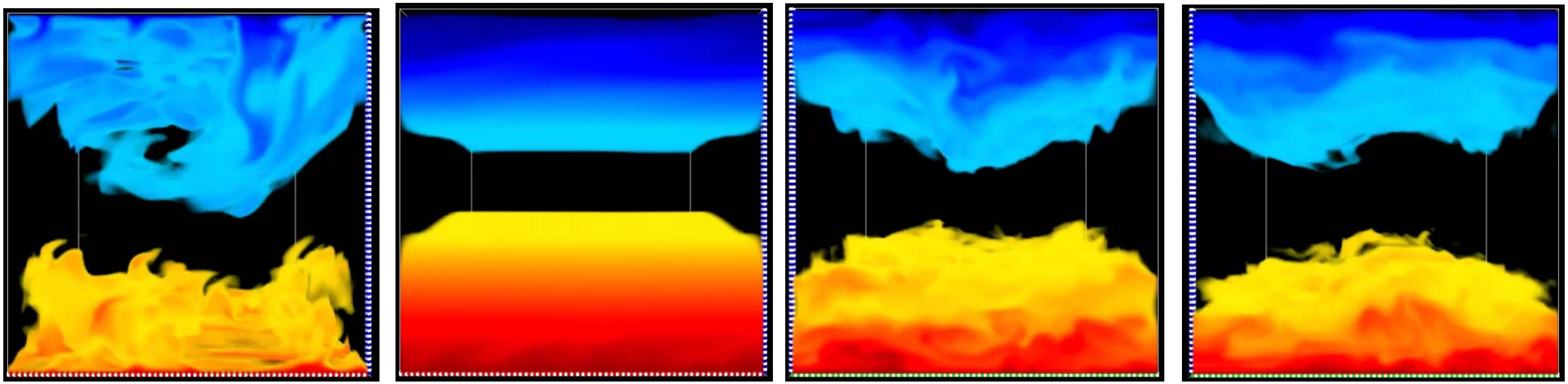




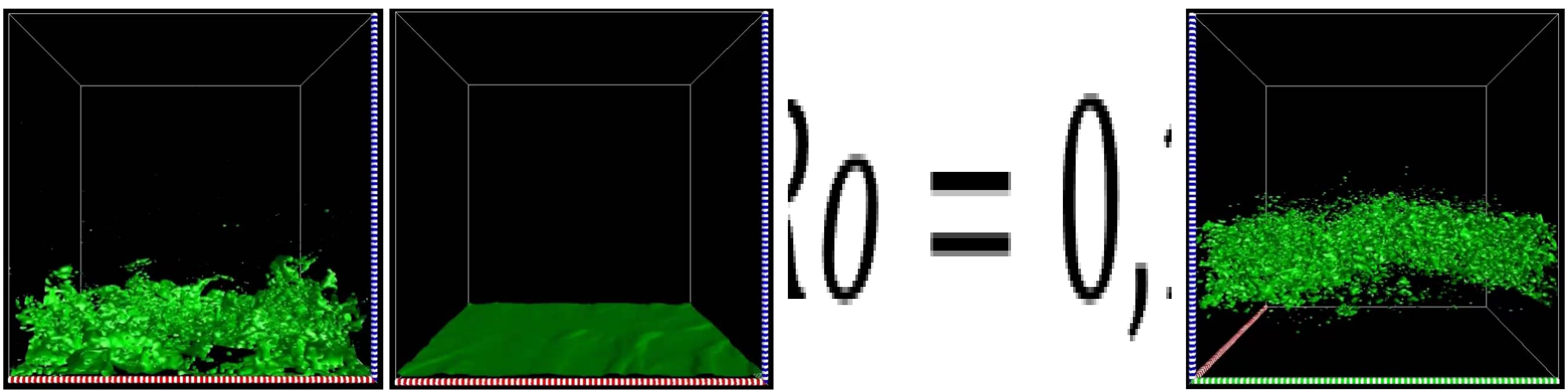


Crystal Settling: Equator

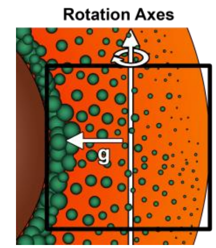


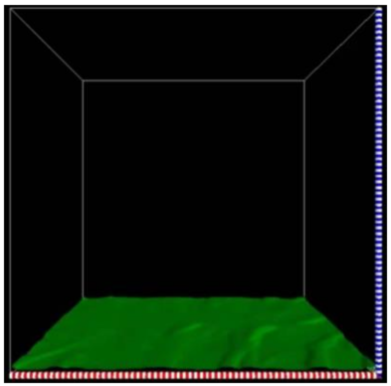


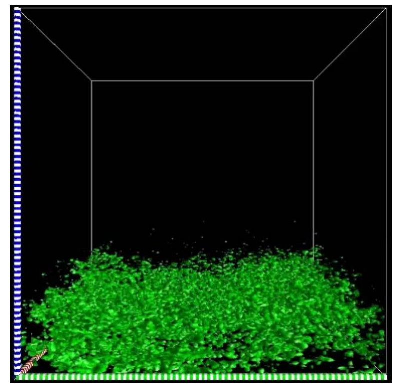
$Ro = 1$	$Ro = 0{,}45$	$Ro = 0{,}14$	$Ro = 0{,}1$
y-z plane	y-z plane	x-z plane	x-z plane

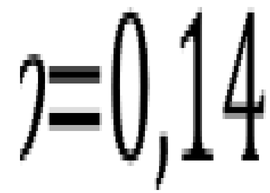


Crystal Settling: Equator

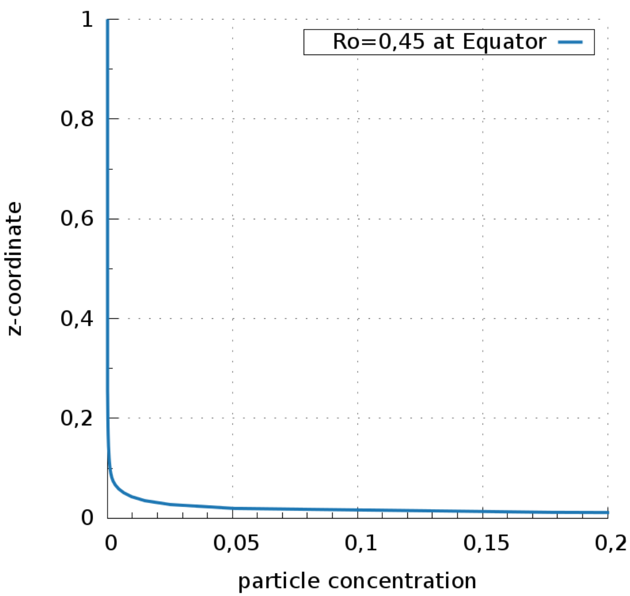


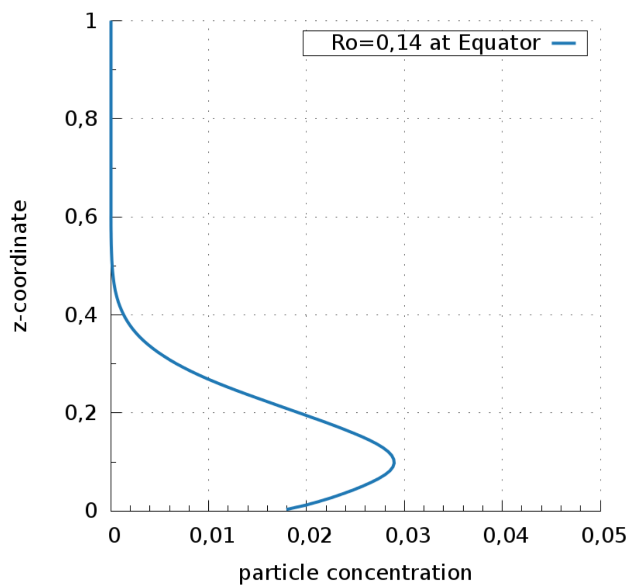


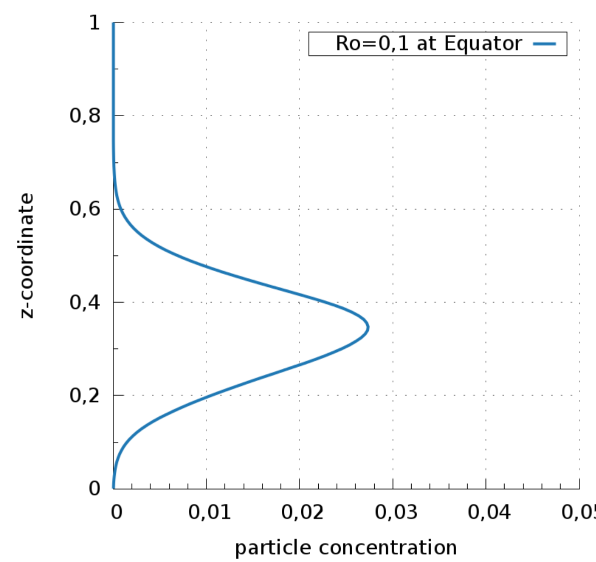




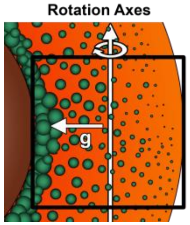
$Ro = 0{,}45$ $Ro = 0{,}14$ $Ro = 0{,}1$







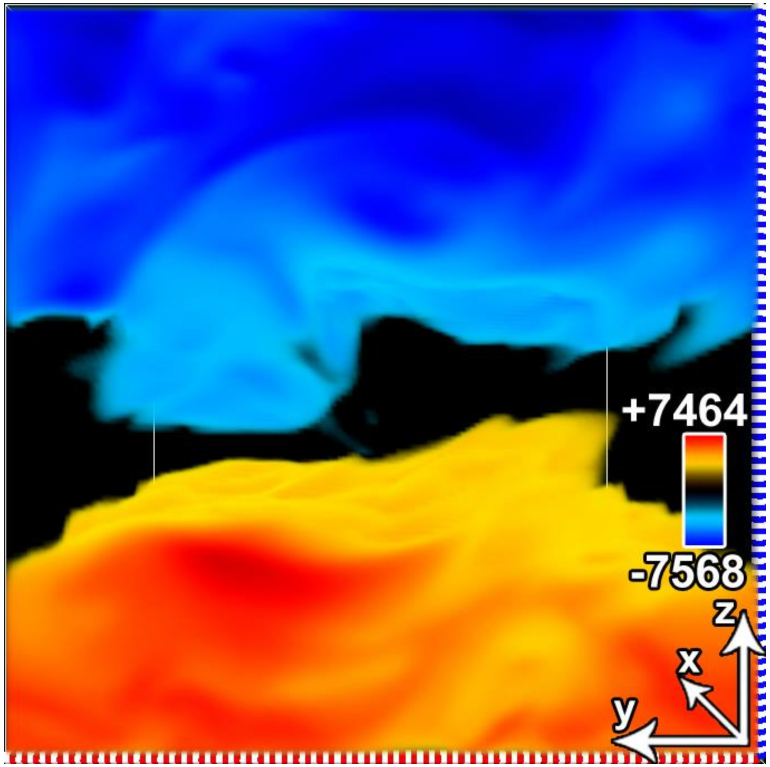
Crystal Settling: Equator

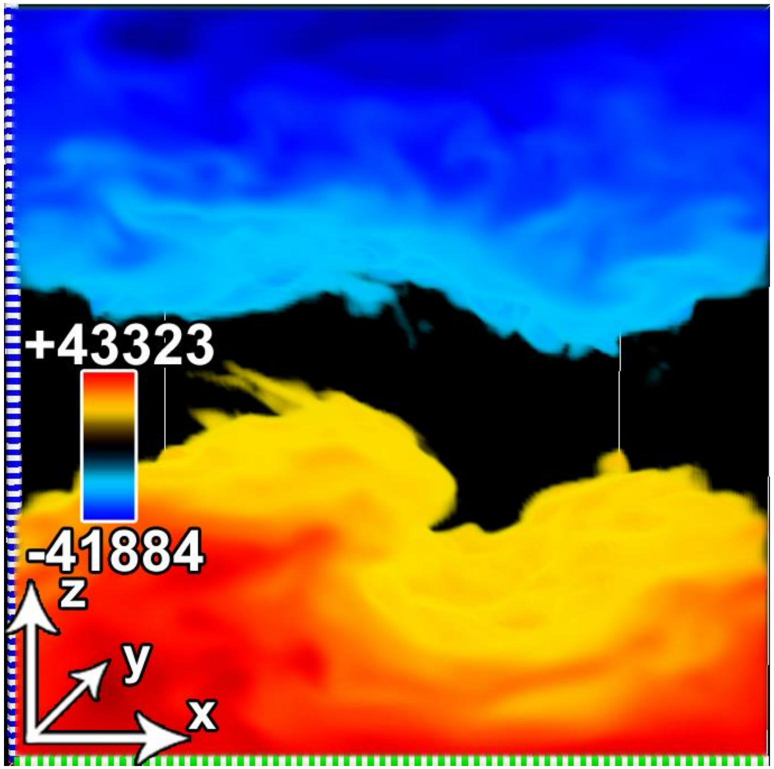


- Shear flow:

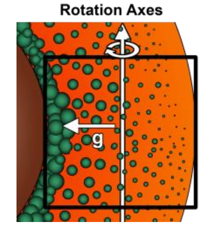
$Ro = 1$ $\qquad$ $Ro = 0{,}1$

v_y:

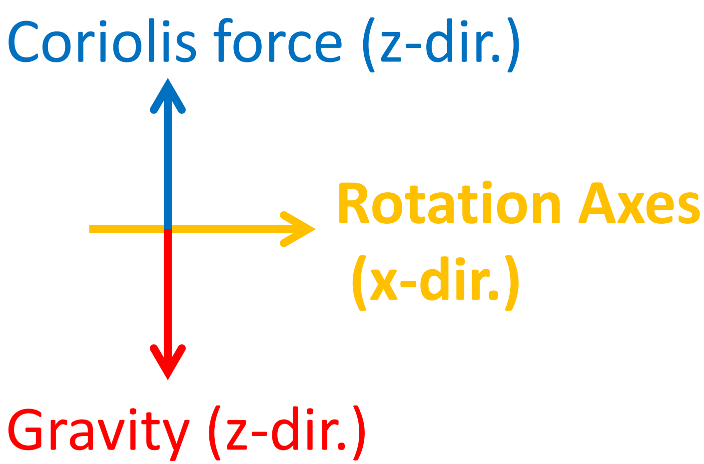


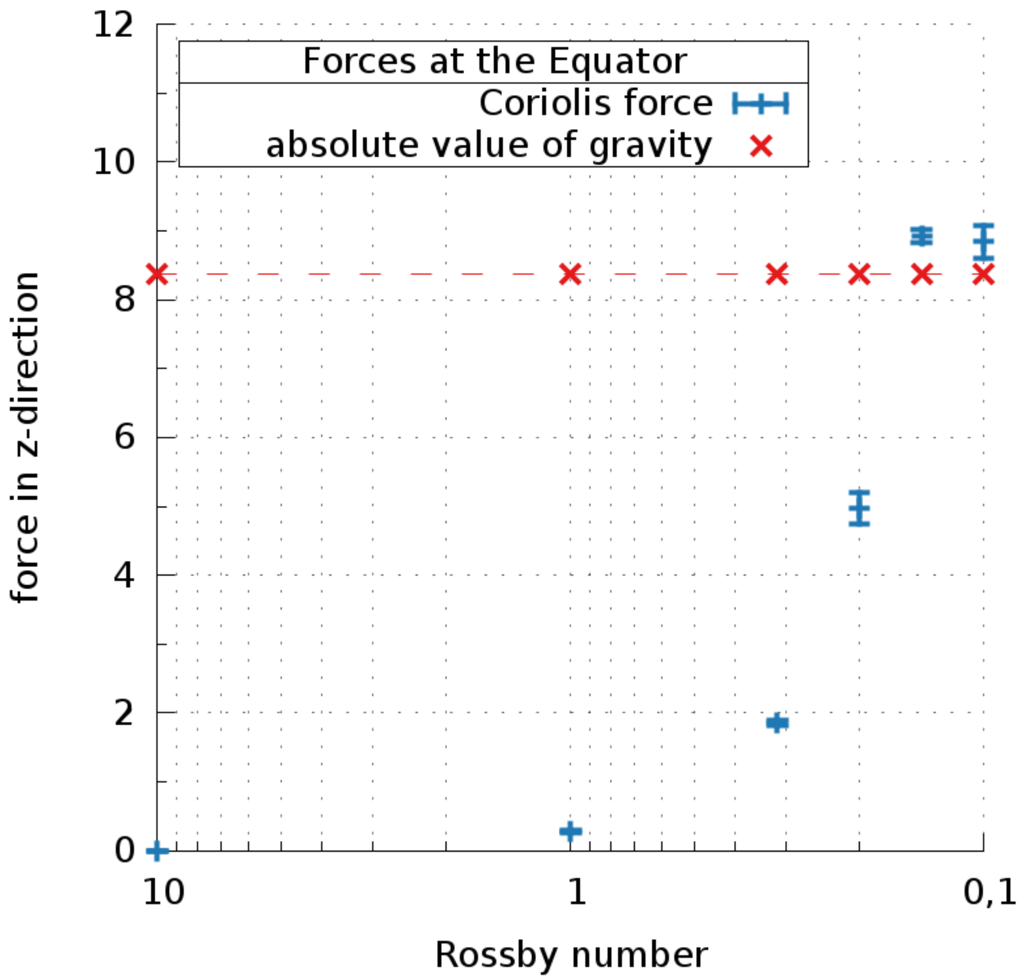


Crystal Settling: Equator

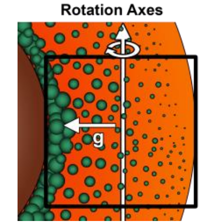


- Coriolis force
  - Prop. & perpendicular to local velocity
- Shear flow:
  - → high y-velocity

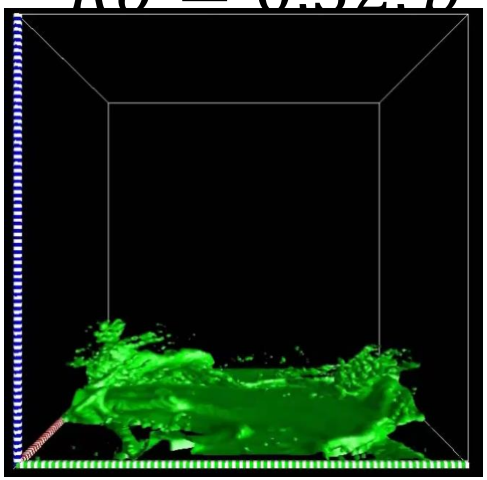




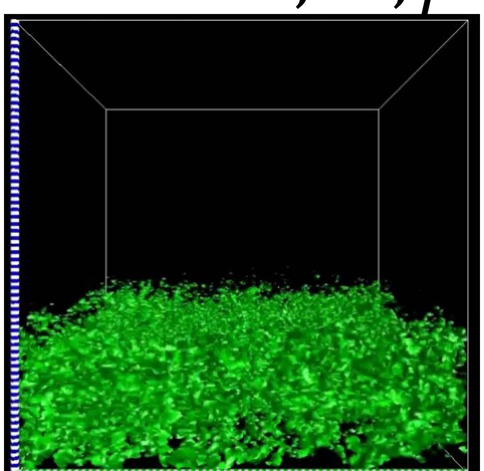
Influence of density: Equator

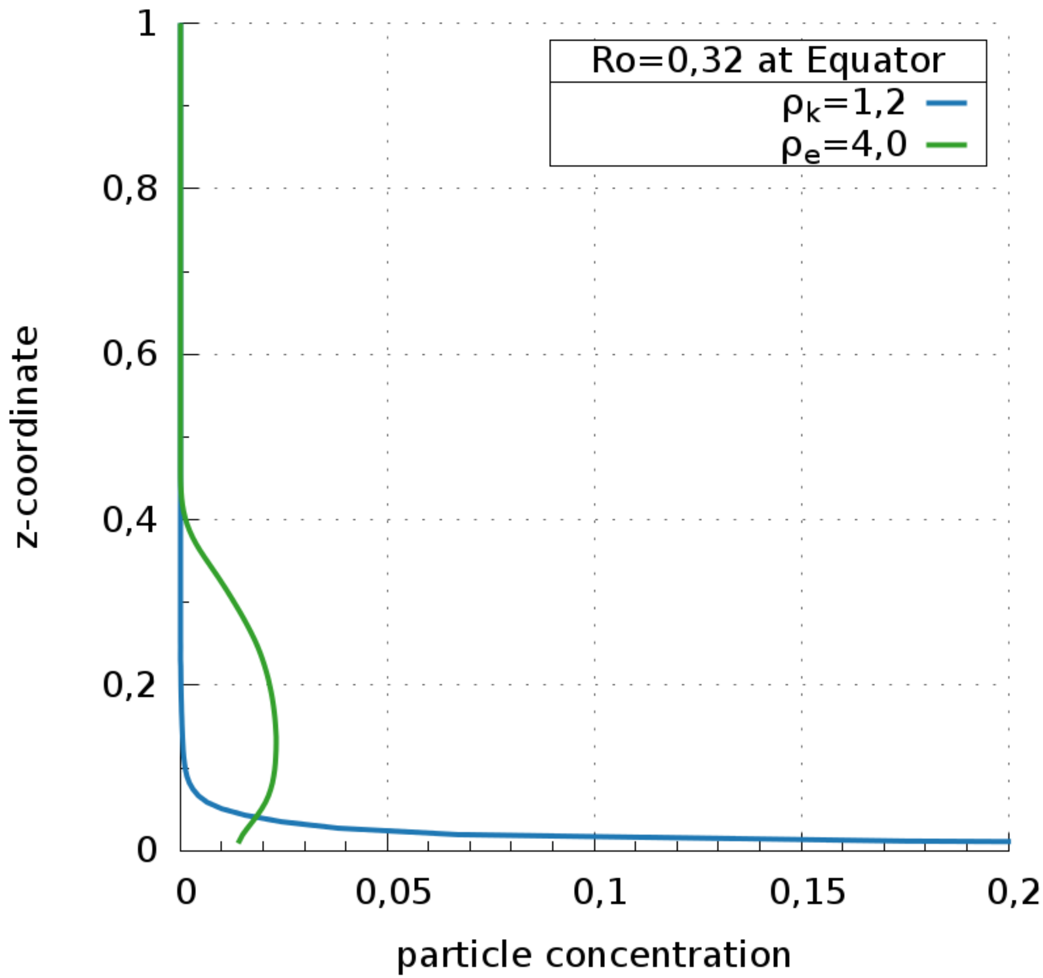


$Ro = 0{,}32, \rho = 1{,}2$



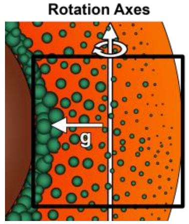
$Ro = 0{,}32, \rho = 4{,}0$



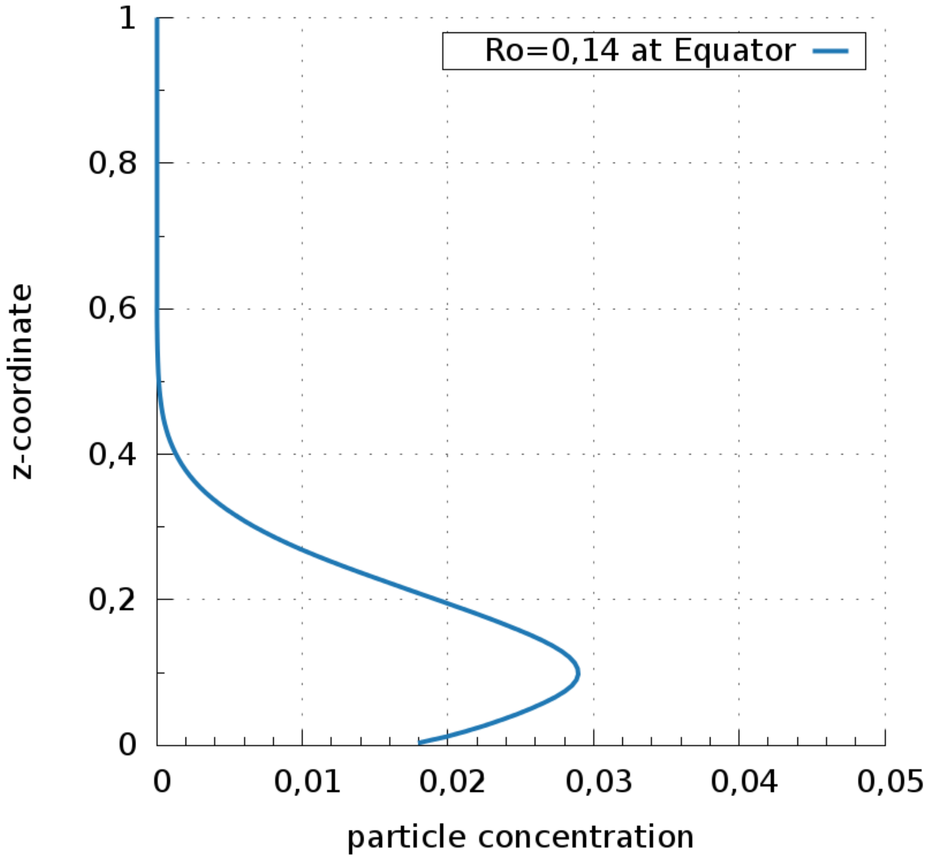


Möller & Hansen, 2013

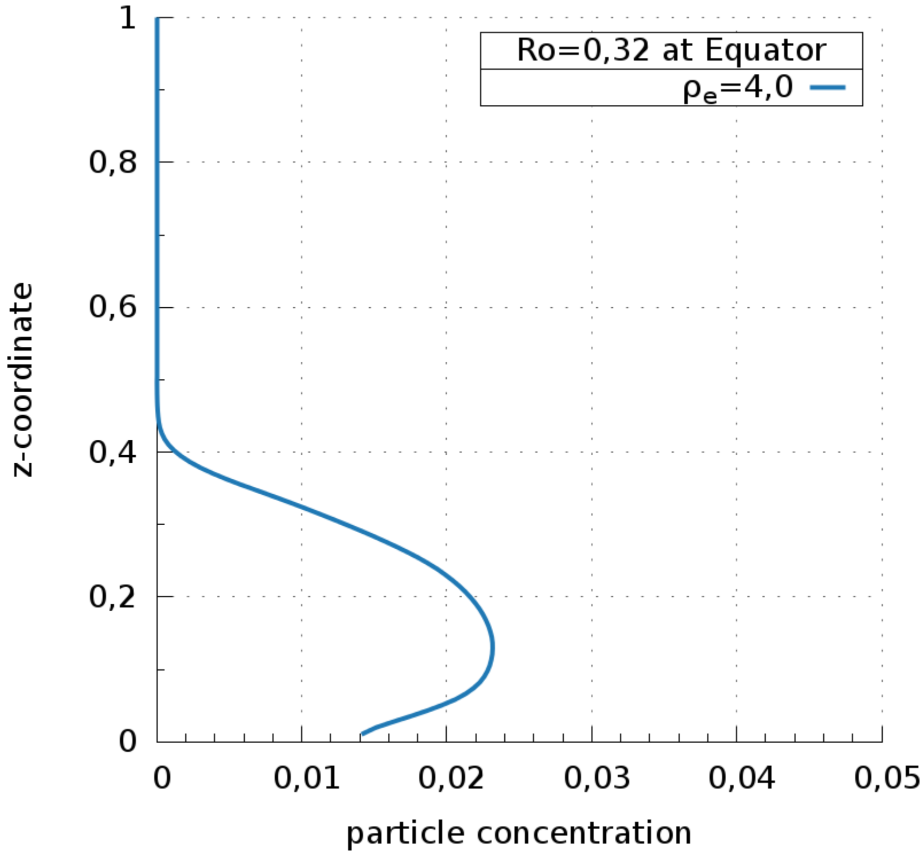
Influence of density: Equator



$Ro = 0{,}14, \rho = 1{,}2$



$Ro = 0{,}32, \rho = 4{,}0$

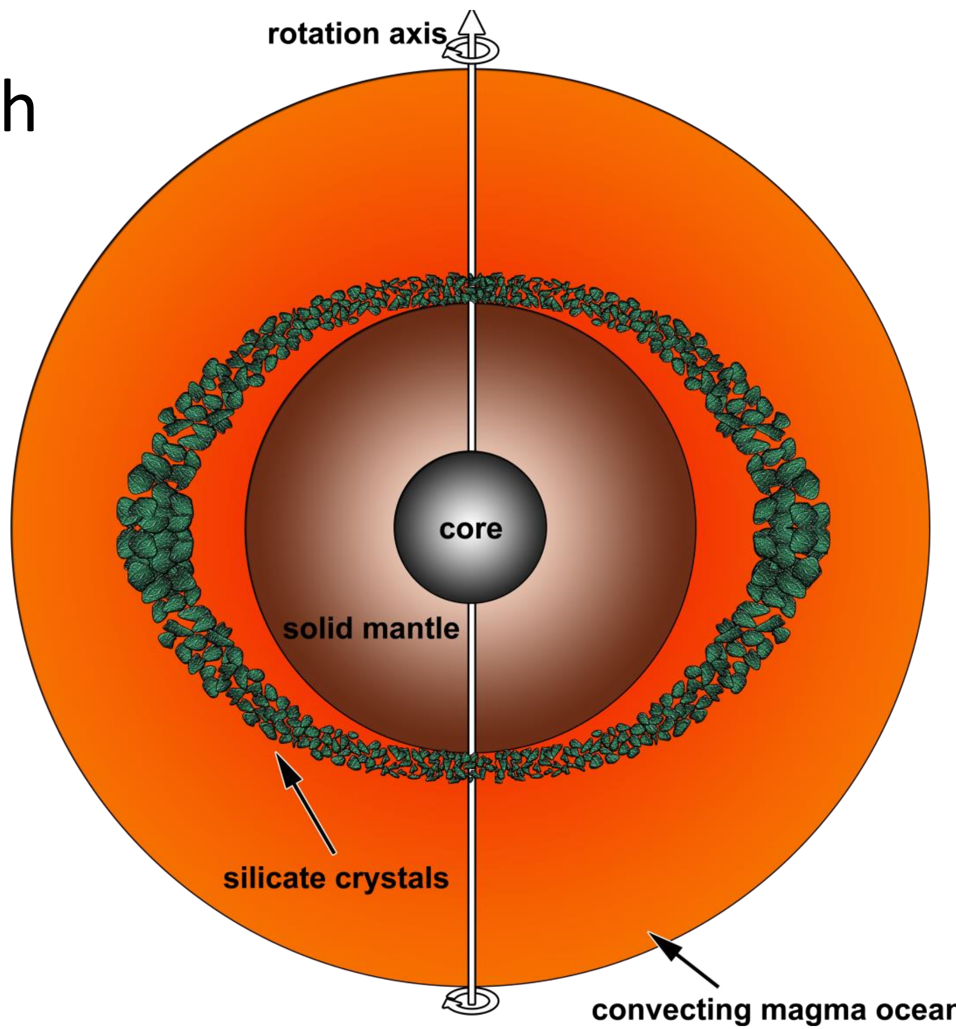


Möller & Hansen, 2013

Summary & Implications

- Crystal settling depends on latitude & rotational strength
- Irregular crystallization:
  - At the Pole:
    - Fast crystal settling
    - Differentiated mantle
  - At the Equator:
    - Settling depends on density
    - Crystallization begins at mid-depth
      - → Magma Ocean at CMB (Labrosse et al. 2007)

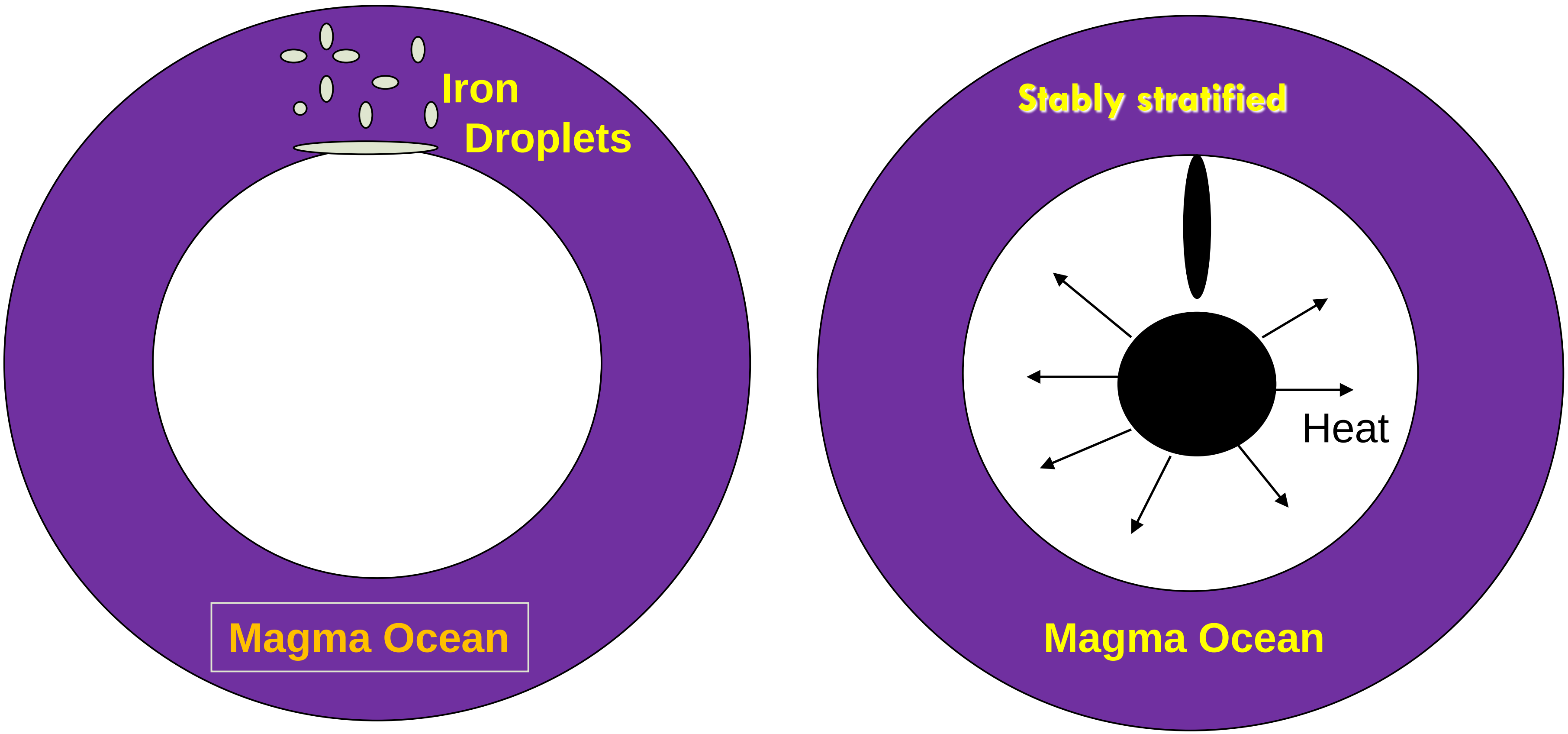
high rotation case:



Outlook

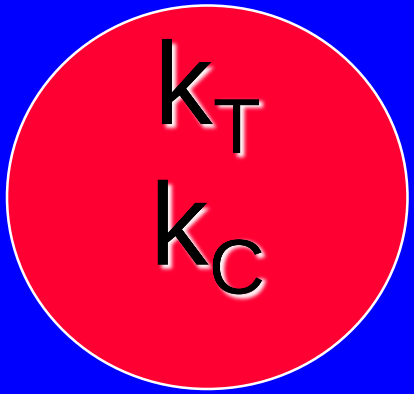
- Geometry: Spherical shell
- Boundary conditions
- Dependence of crystal settling on crystal density
- Two different mineral phases

Heating a stably stratified mantle from below - after core formation



Höink et al (2005)

Basics of Double-diffusive convection



Density: ρ

$$\rho = \rho_0 \left(1 - \alpha T + \beta C \right)$$

T: Temperature
C: Concentration

Fluid parcel with
Thermal diffusivity k_T and
Compositional diffusivity k_C

Lewis number

$$Le = \frac{k_T}{k_c} >> 1$$

Driving – and Restoring Forces are not in phase

Diffusive Regime:
Fast diffusing component (Temperature)
Is **driving** force.
Slowly diffusing component (Salinity)
Is **restoring** force

Finger Regime::
Fast diffusing component (Temperature)
Is **restoring** force.
Slowly diffusing component (Salinity)
Is **driving** force

$$Le = \frac{\kappa_T}{\kappa_C} > 1$$

Lewis number,
κ_T thermal diffusivity
κ_C compositional diffusivity

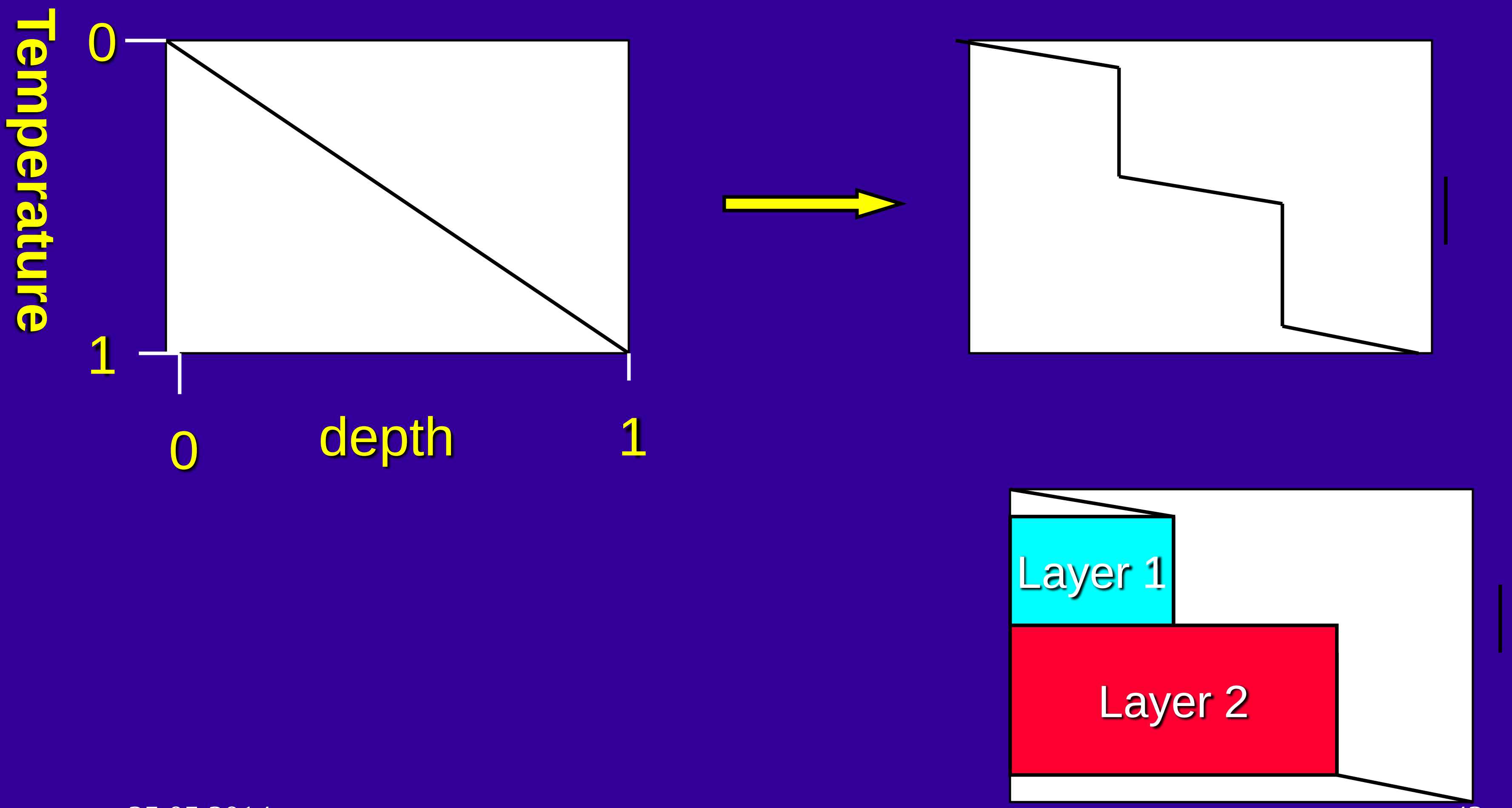
$$Ra_T = \frac{\alpha g \Delta T d^3}{\kappa_T \nu}$$

Thermal Rayleigh number

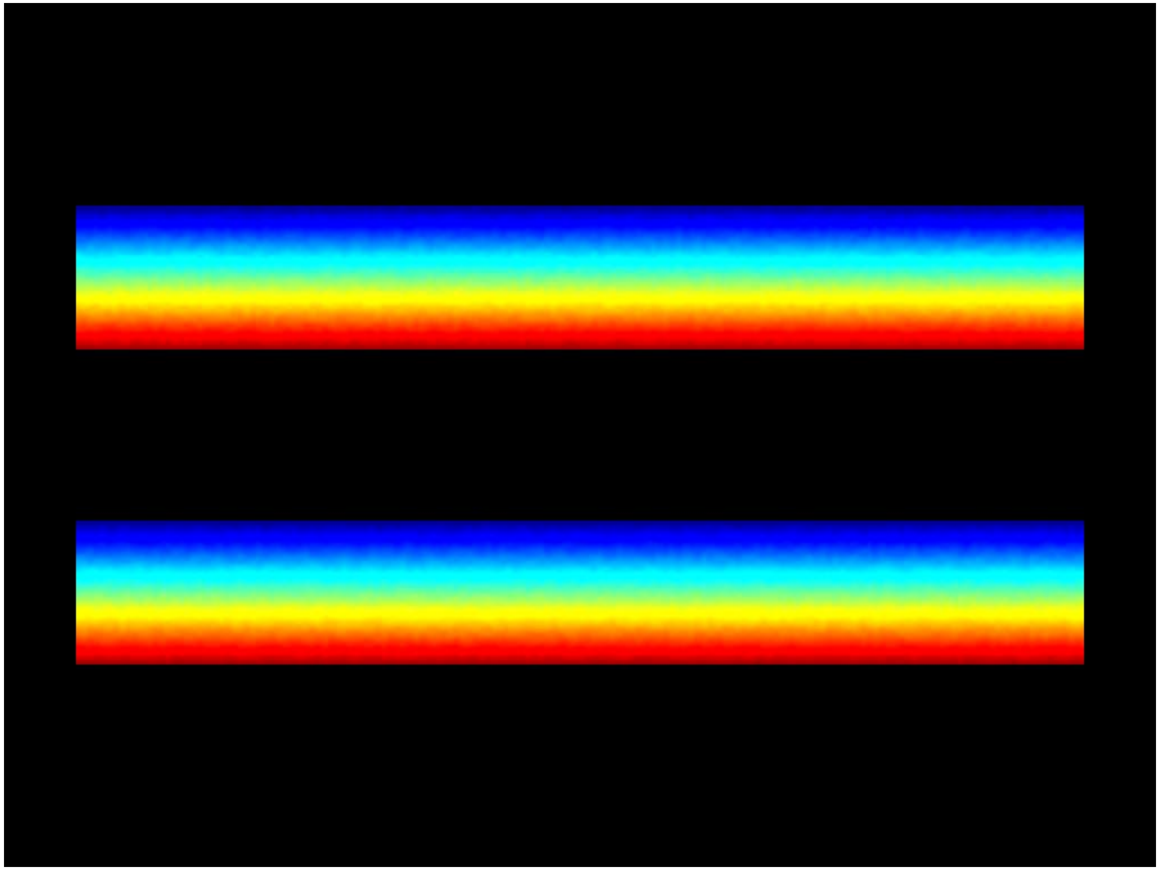
$$Ra_C = \frac{\beta g \Delta C d^3}{\kappa_T \nu}$$

Compositional Rayleigh number

From gradual to layered (stair-case) stratification



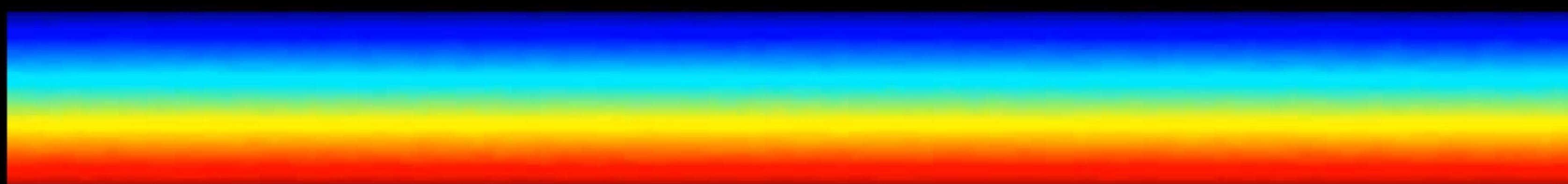
The Onset of Convection in a doubly-diffusive system



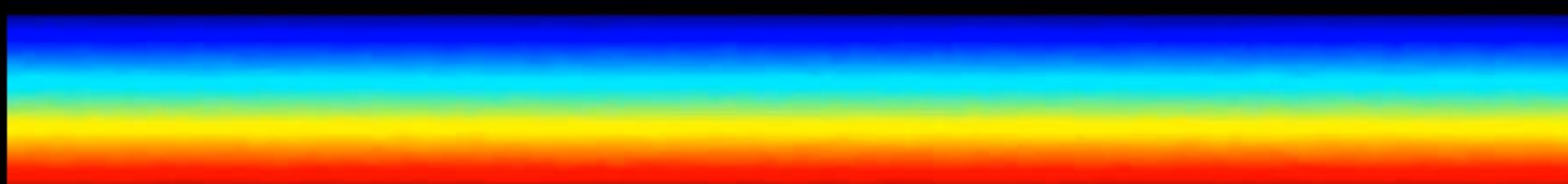
Temperature

Composition

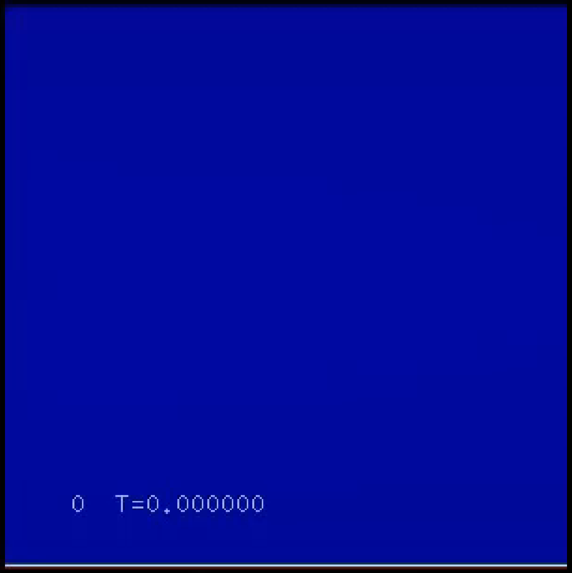
The Onset of Double-diffusive convection



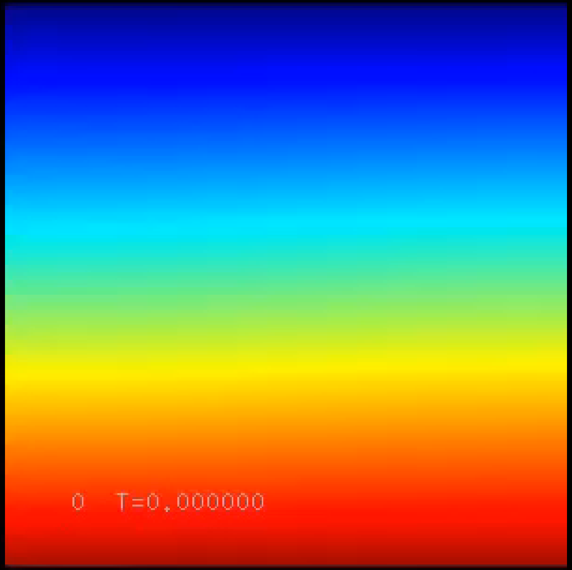
Temperatue



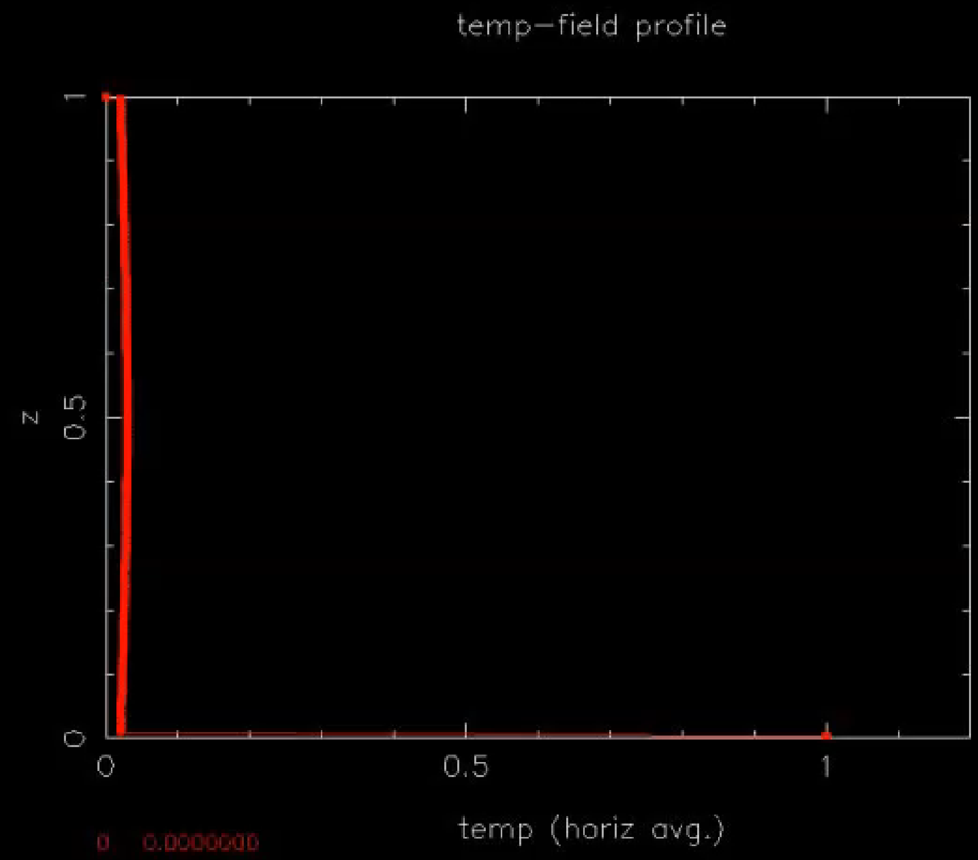
Composition



temperature



composition



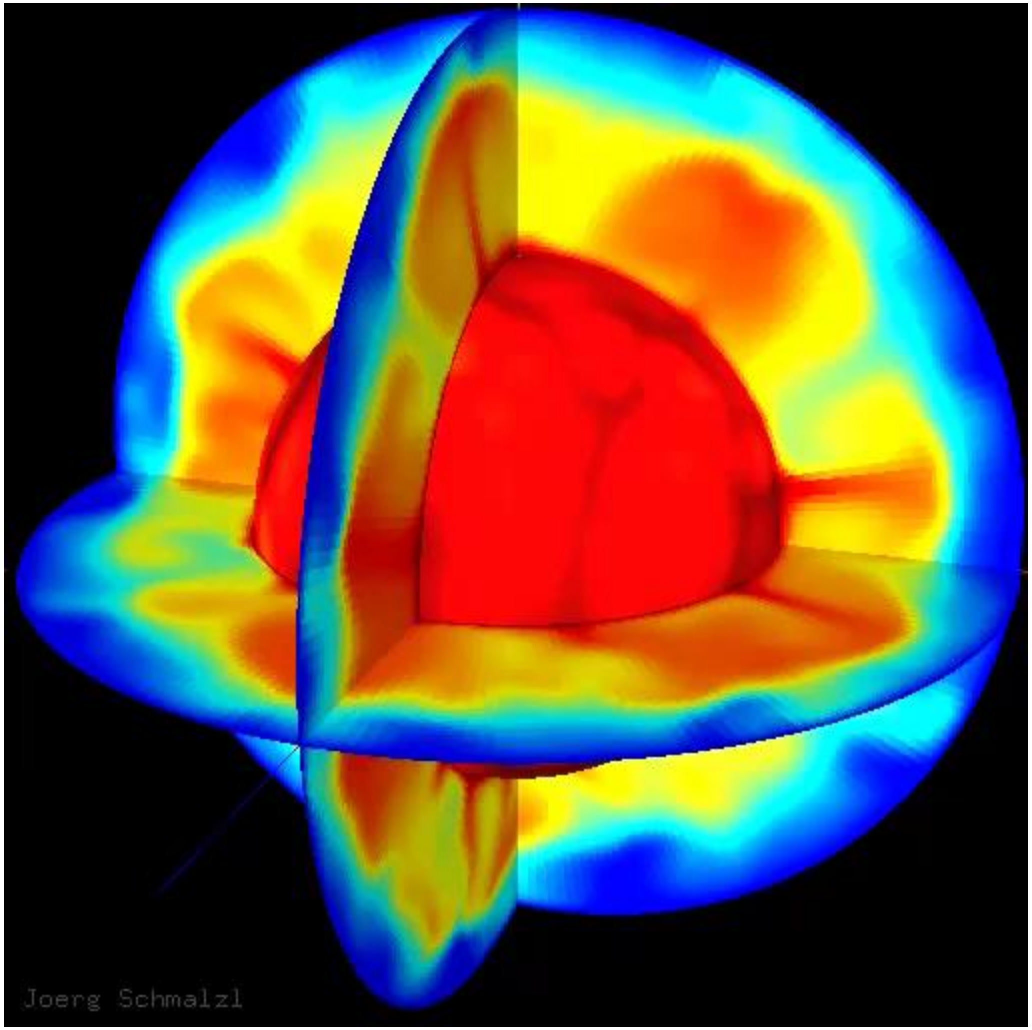
$Ra_T = 10^7$

$Le = 70$

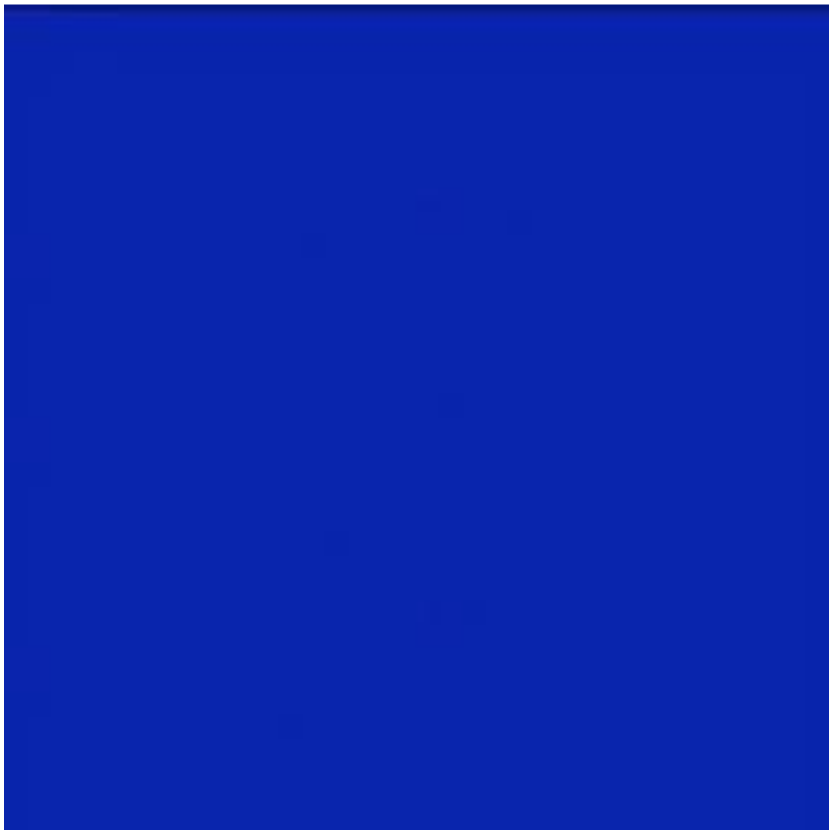
$Ra_s = 2 \cdot 10^7$

$vt = 1000$

Doublr-diffusive Convection in a Sphere



A Simple Mantle Model



Temperature

$Ra_T = 10^6$,

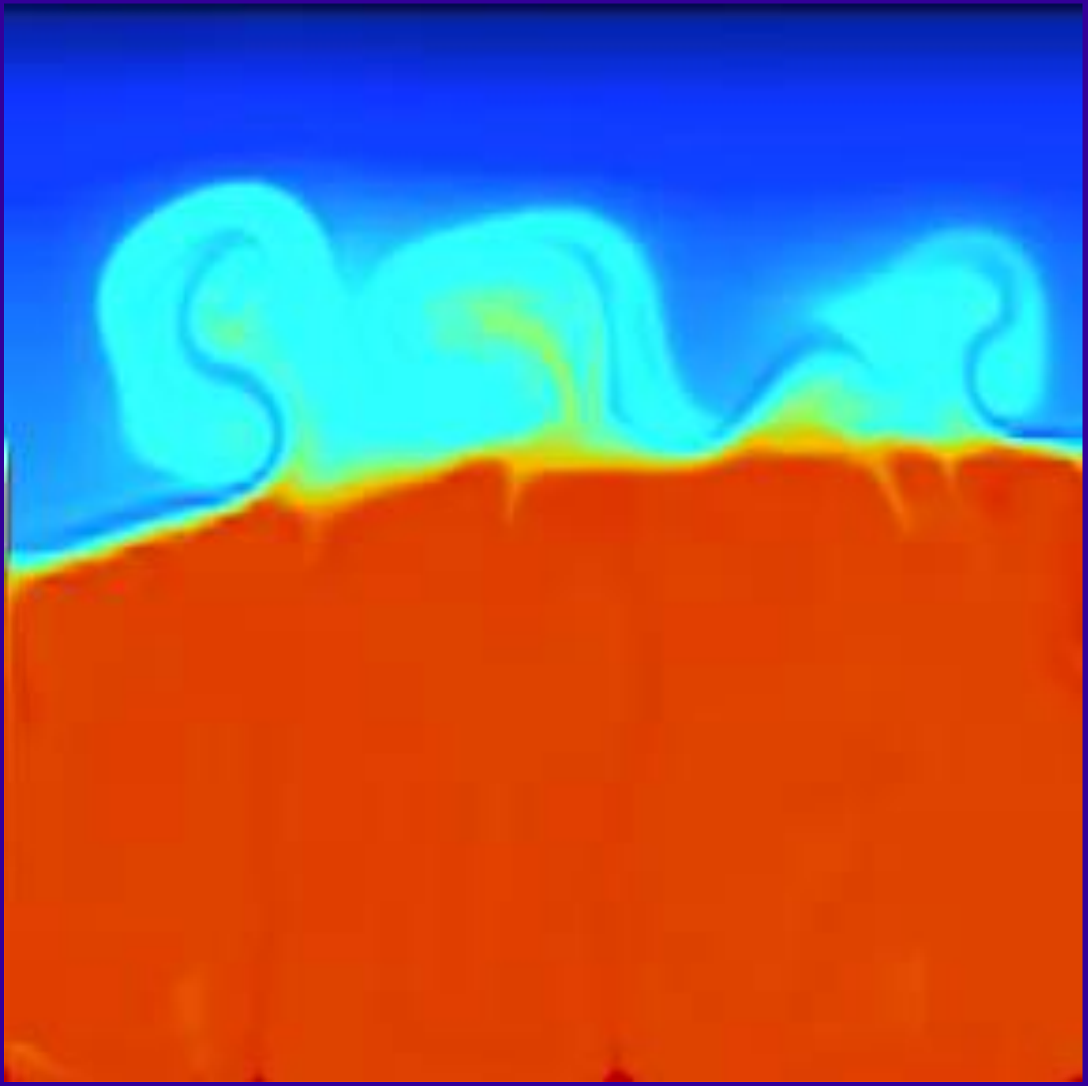
$Ra_C = 1.5x10^6$

$Le = 100$

$Vt = 10^4$

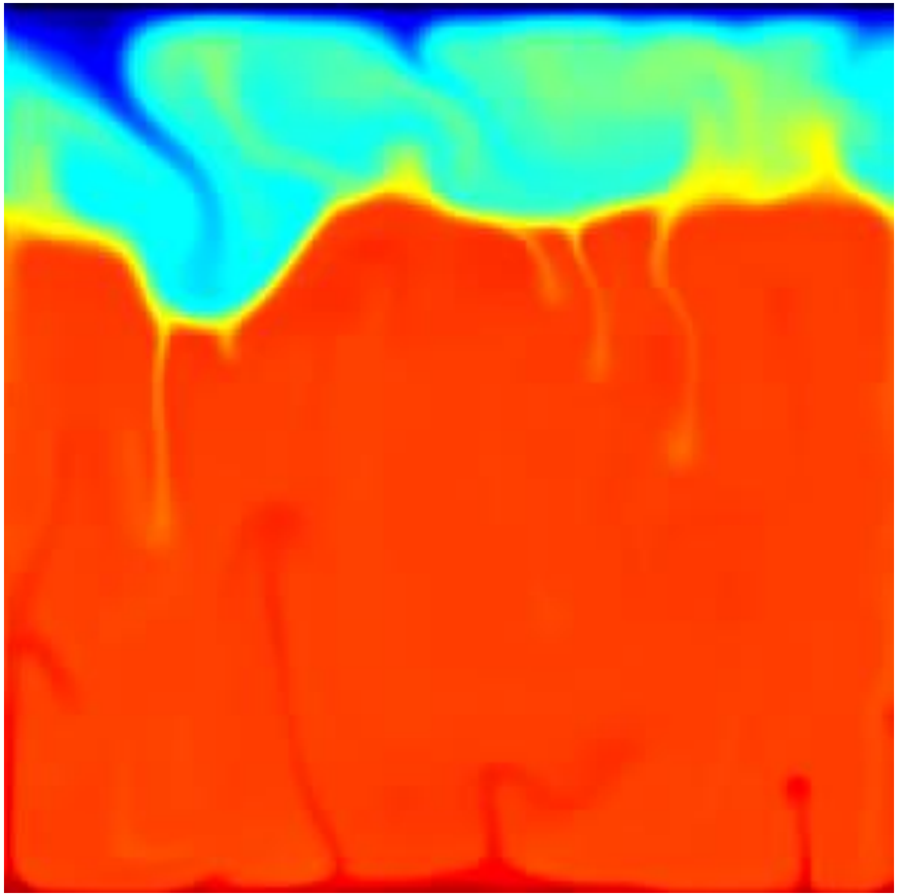
Temperature and composition

Fixed - Dirichlet conditions



Composition

A Simple mantle Model 3

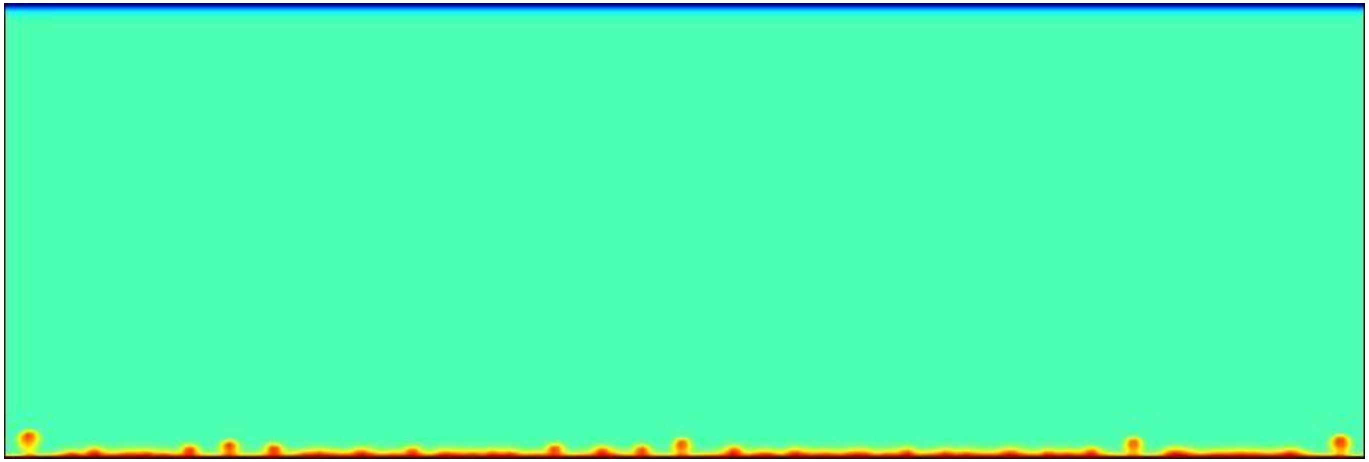


Temperature

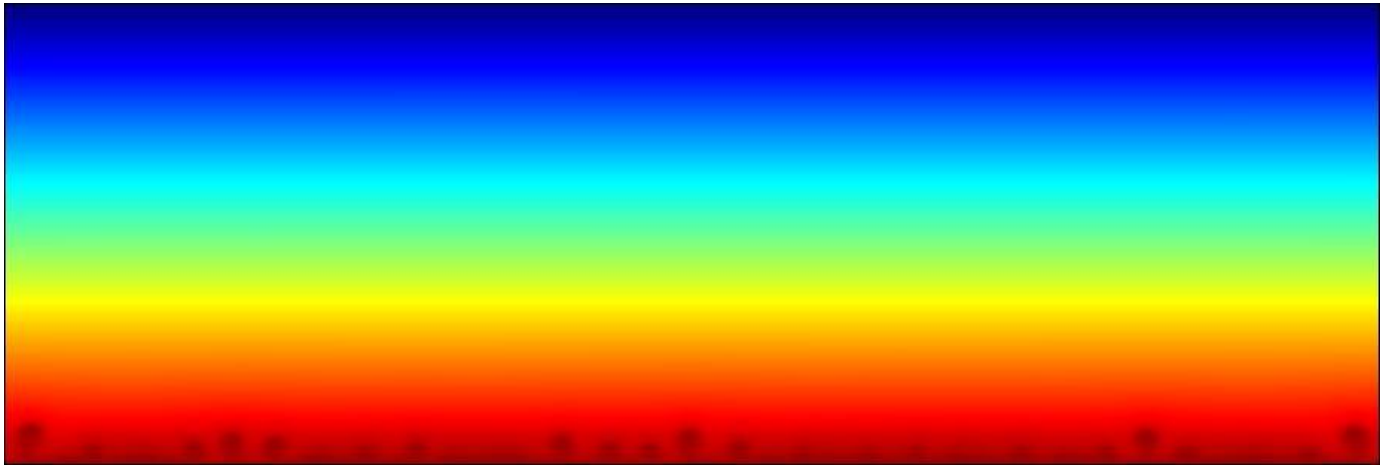
Late phase Transition from layered to non-layerrf flow:

Stably stratified, heated from below and cooled from above, Bingham rheology

T

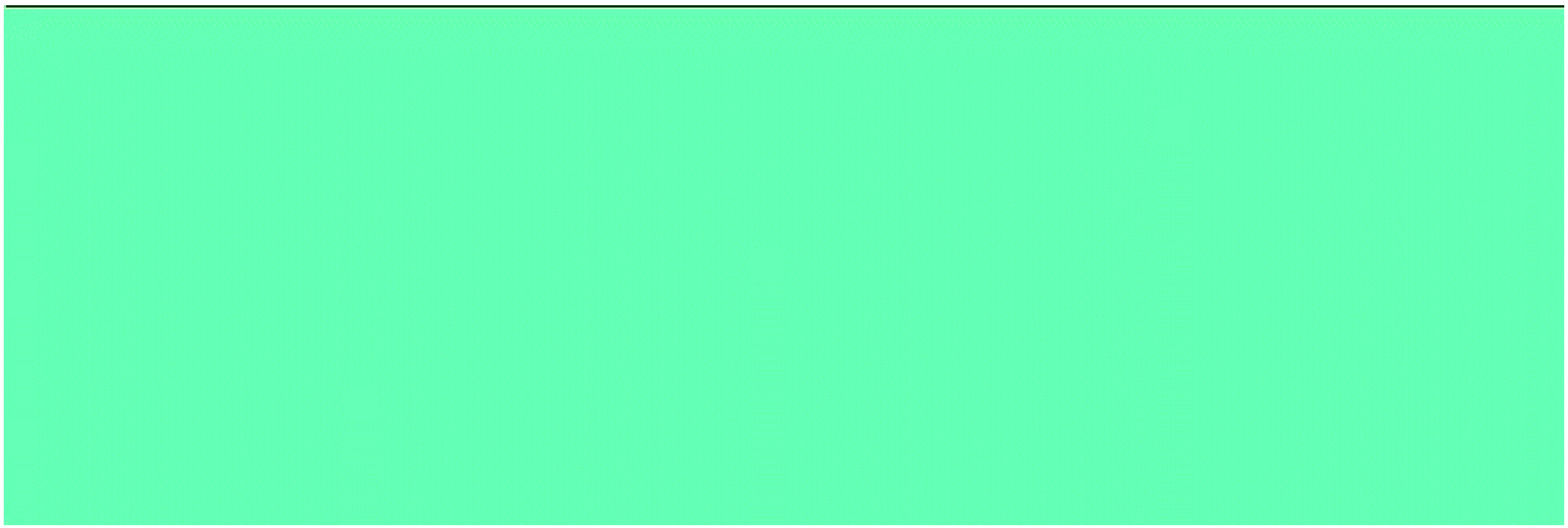


C



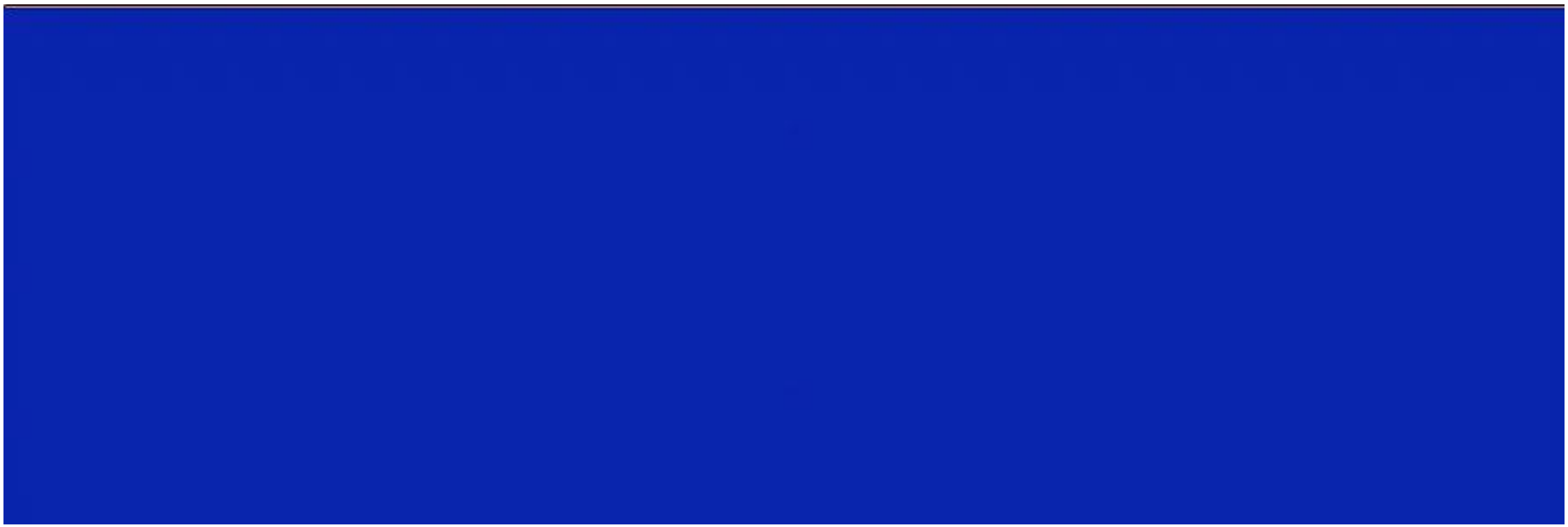
$Ra_T = 10^5, Ra_C = 1.5x10^5$, VT= 10^6

Onset of Plate Tectonics



Temperature

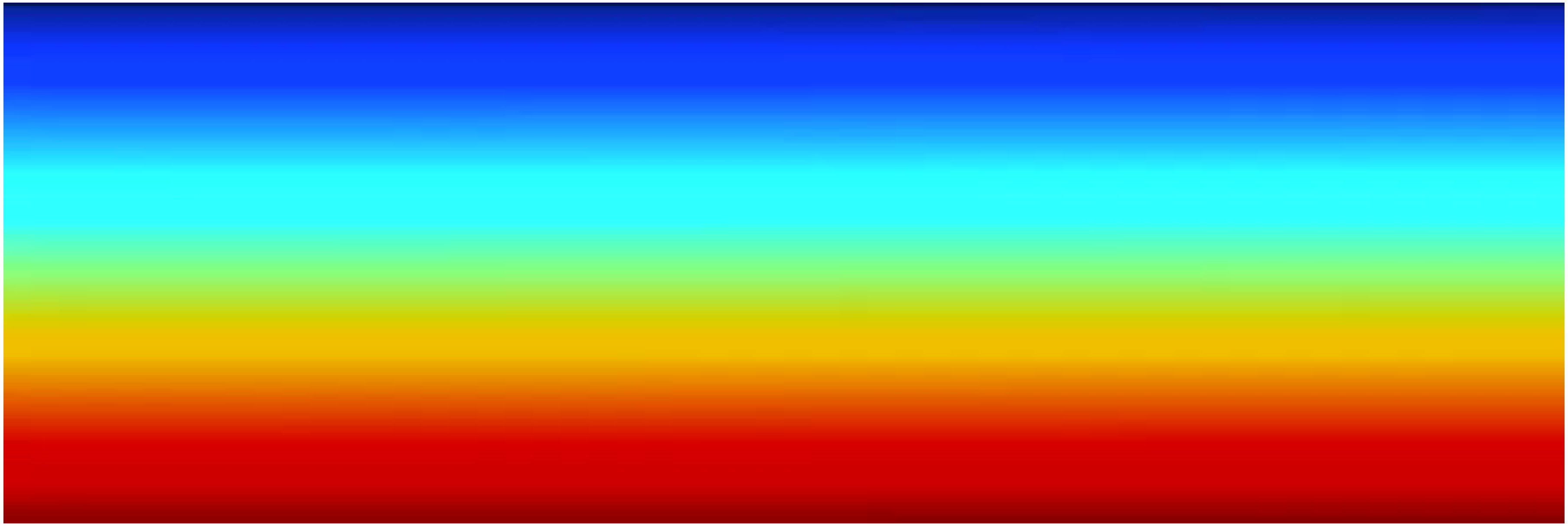
A more extreme case



Temperature

$Ra_T = 10^6$, $Ra_C=1.5\text{x}10^6$, $Vt = 10^6$, $V_p = 50$

A more extreme case 2



Composition

Summary

A stably stratified compositional gradient, overlying a heat reservoir may resemble the situation of the Early Earth, after Core-formation.

Under such conditions, in a wide parameter range, a layered flow pattern develops. The range includes temperature- and pressure dependence of the viscosity, internal heat generation and temporally decaying heat sources.

The layer formation is observed in 2D and 3D Cartesian geometry as well as in fully spherical domains.
The results indicate that the formation of layers and thus of discontinuities are a typical feature resulting from planetary formation