Thermal History Energy Balance Compared with Convection Modeling

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Goal:

- compare and contrast convection and thermal history modeling approaches
- add time-variable terms to convection model step by step
- discuss impact

Compare and Contrast

- thermal history modeling
  - one dimensional
- convection modeling
  - Two or three dimensional

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  - only solves conservation of energy
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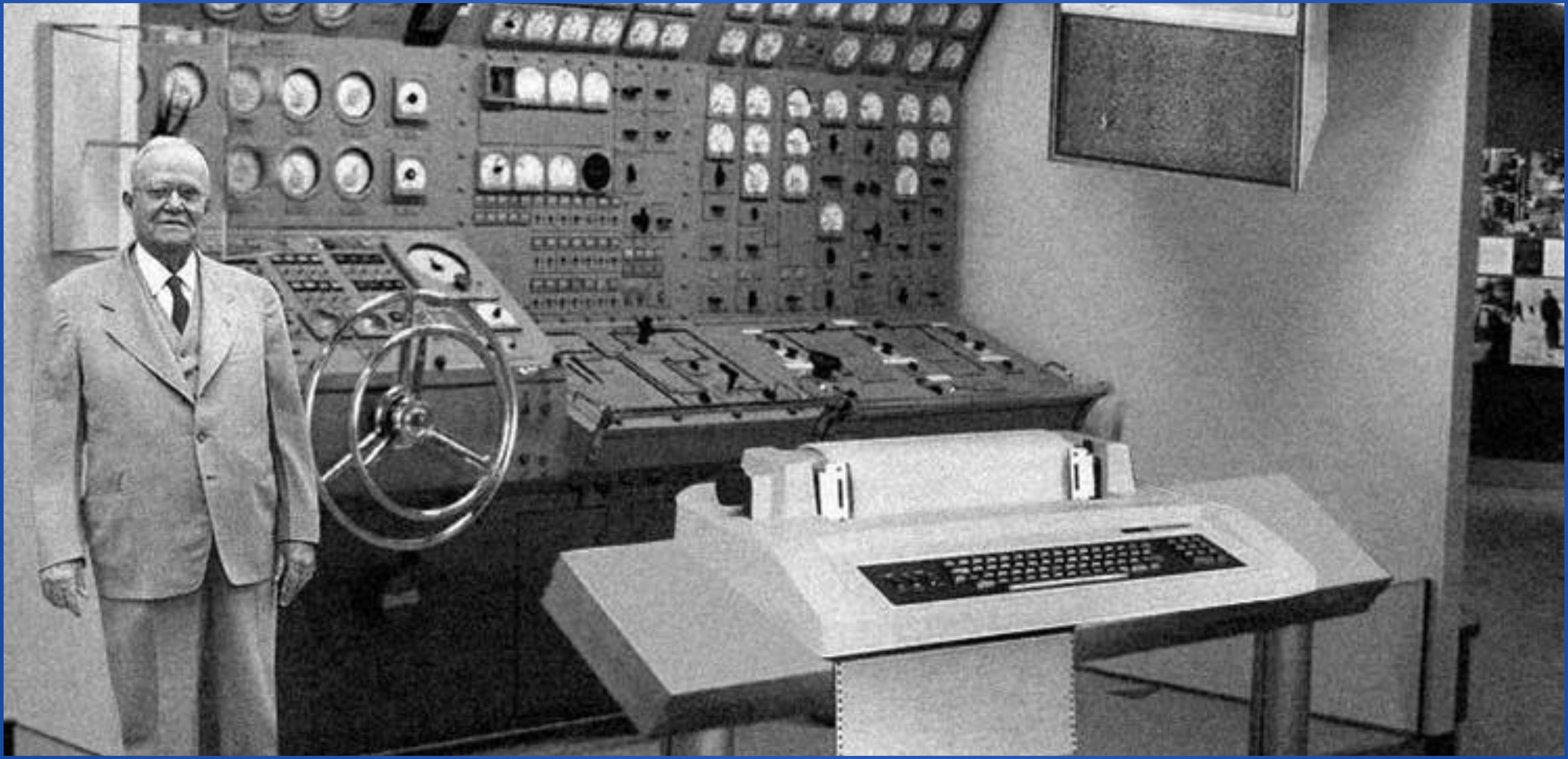
SLOW

Why Thermal History Calculations?



Scientists from RAND Corp. created this model to illustrate how a 'home computer' could look in 2004. However the needed technology will not be economically feasible for the average home. (Actually photo is a hoax!)

Thermal History Calculation

Integrate the energy equation over the whole mantle

$$Mc\frac{\partial T}{\partial t} = MH - Aq$$

M is the mass of Earth
C is the specific heat
H is the concentration of HPE
A is the surface area
q is the surface heat flux

Allow H, to decay with time

$$H = H_0 e^{-\lambda t}$$

Parameterize heat flux out the top of the mantle as

$$q = \frac{k(T - T_s)}{d}\left(\frac{Ra}{Ra_{cr}}\right)^{\beta}$$

Thermal History Calculation

$$\frac{\partial T}{\partial t} = f_1 e^{-\lambda t} - f_2 (T - T_s)^{1+\beta} \exp\left(\frac{-\beta A_0}{T}\right)$$

$$f_1 = H_0 / c$$

$$f_2 = \frac{Ak}{Mcd}\left(\frac{\alpha g d^3}{\kappa \nu_0 Ra_{cr}}\right)^{\beta}$$

The result is an ode with a few adjustable parameters: f_1, f_2, A_0, β, T_s

Nusselt Rayleigh Relationship; The β Saga

- Nu = 0.294 $Ra^{0.333}$ c.f. Turcotte and Schubert
- Christensen xxx
- Nu = xxx Ra xxx Gurnis, 1989
- Moresi and Solomatov, …

So if not thermal history calculations, then how slow is slow?

How Slow Is Slow?

10 steps
write output at final step
includes all phases of solution
default convergence params

Grid	Partition	Total Cores	Wall time (sec)	MG levels	nodes
25x25x25	2x2x2	96	25	3	0.17M
25x25x25	2x2x2	96	90	cg	0.17M
49x49x49	2x2x2	96	67	4	1.35M
49x49x49	2x2x2	96	763	cg	1.35M
49x49x49	3x3x3	324	8	4	1.35M
49x49x49	3x3x3	324	xxx	cg	1.35M
97x97x97	2x2x2	96	368	5	10.7M †
97x97x97	2x2x2	96	456	5	10.7M
97x97x97	2x2x2	96	443	4	10.7M
97x97x97	2x2x2	96	1,081	3	10.7M
97x97x97	2x2x2	96	1,537	2	10.7M
97x97x97	2x2x2	96	4,101	cg	10.7M
97x97x97	3x3x3	324	228	5	10.7M
145x145x145	2x2x2	96	1,921	4	36.1M
145x145x145	3x3x3	324	752	4	36.1M
145x145x145	4x4x2	384	28,336	3	36.1M
145x145x145	4x4x3	576		3	36.1M
193x193x193	3x3x3	324	1,471	6	85.4M
193x193x193	4x4x2	384	1,690	5	85.4M
193x193x193	4x4x3	576	1,097	5	85.4M
241x241x241	4x4x3	576		5	
241x241x241	4x4x4	768		5	

†write to /tmp

Table 1: These scalability tests were run using CitcomS 3.2.0 with default configuration on hess.arc.vt.edu. The mesh for these tests is a sphere with 12 caps. Each cap has n by n by n nodes. The model is run for 11 time steps. The result reported is the total wall clock time.

How Slow Is Slow?

97 x 97 x 97 = 30 km

Grid	Partition	Total Cores	Wall time (sec)	MG levels	nodes
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write to local scratch vs. write to mounted disk

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145 x 145 x 145 = 20 km

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97x97x97	3x3x3	324	228	5	10.7M
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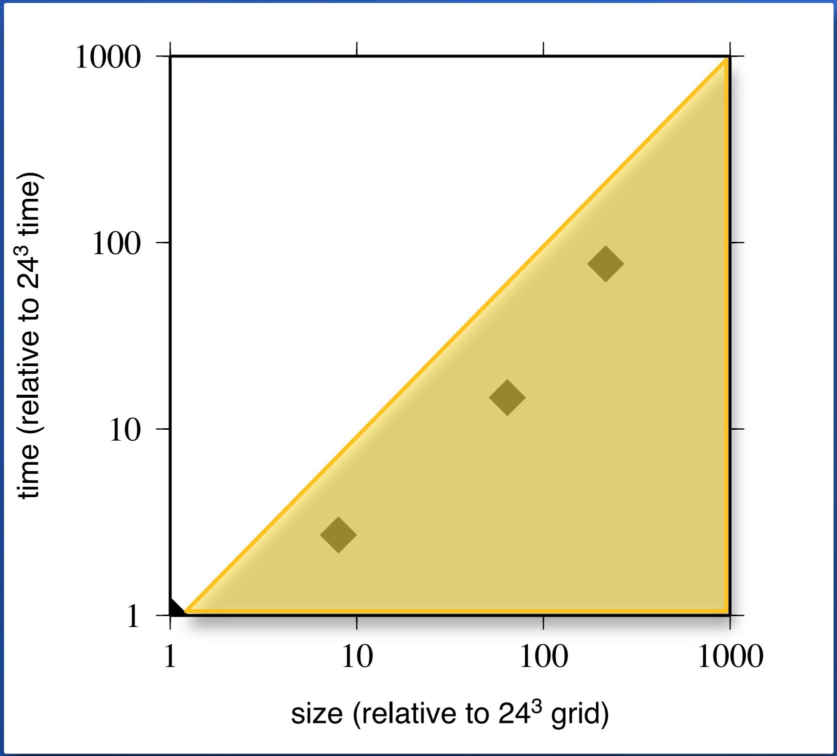
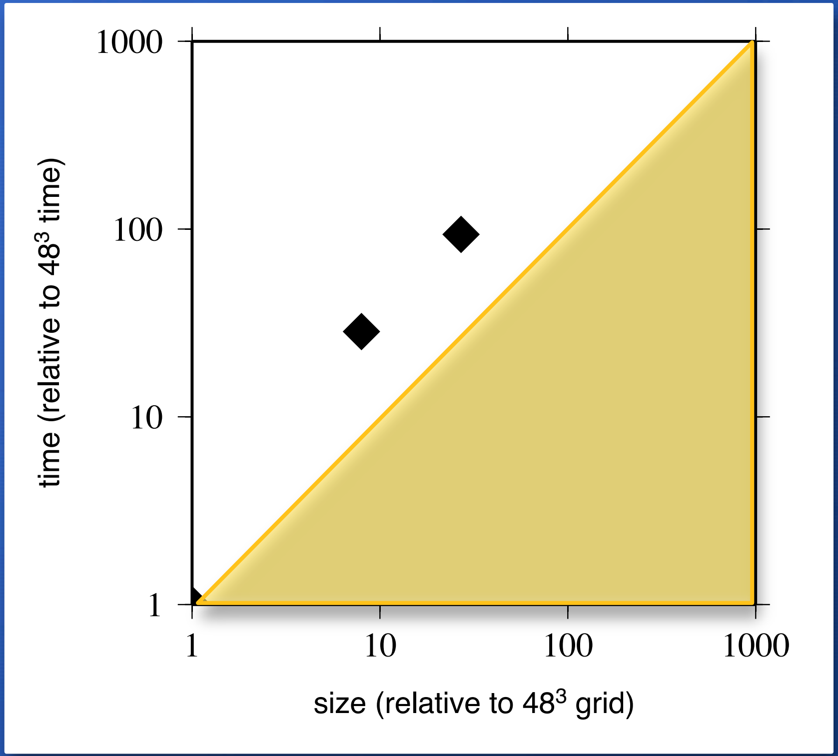
193 x 193 x 192 = 15 km   241 x 241 x 241 = 12 km

Grid Comparisons

fixed 96 cores



fixed 324 cores



Yellow = super speedup

Grid Comparisons

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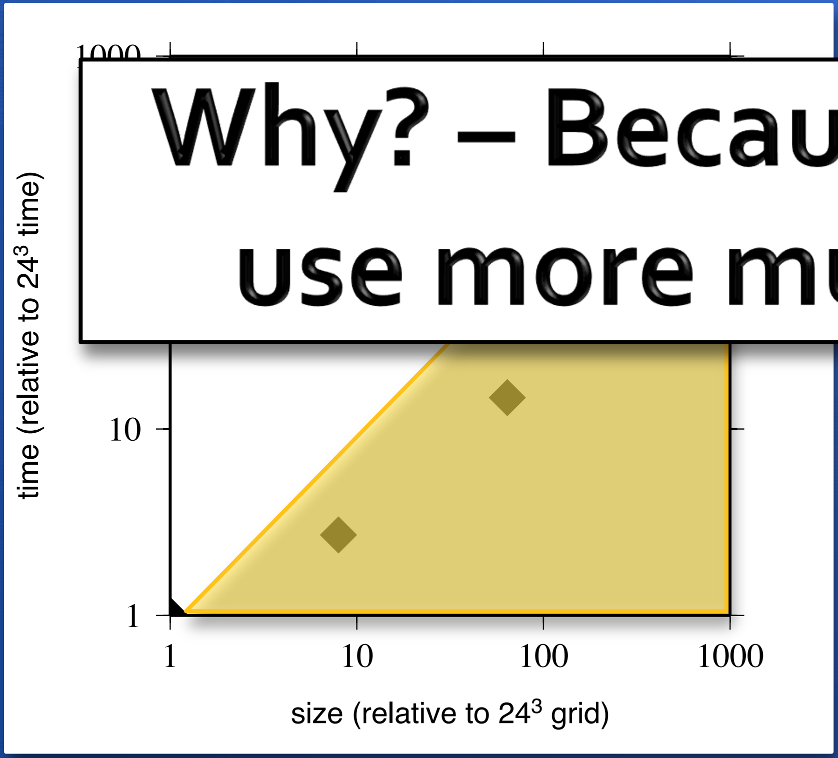
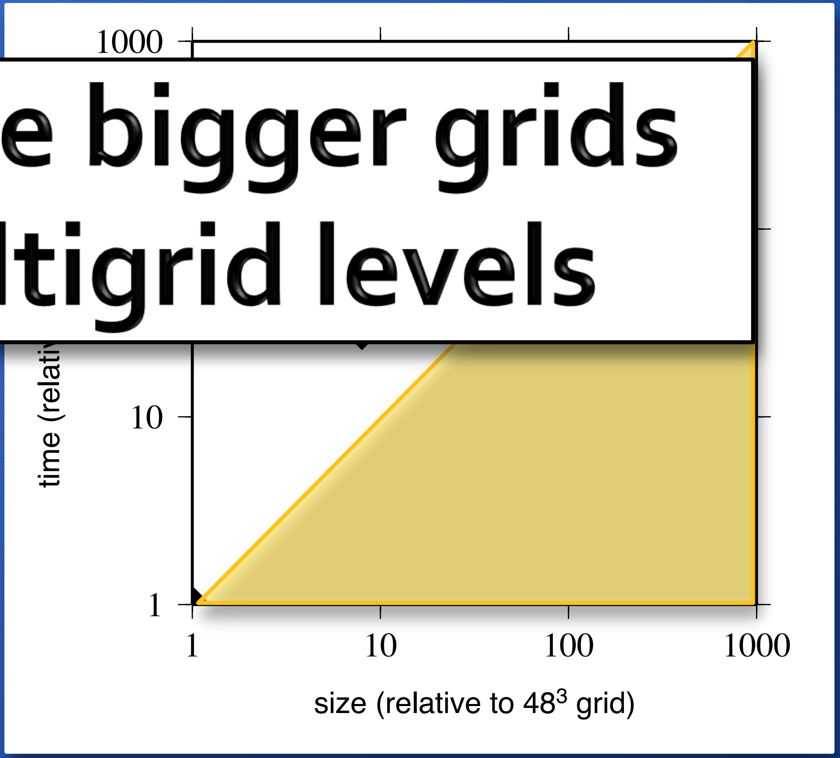


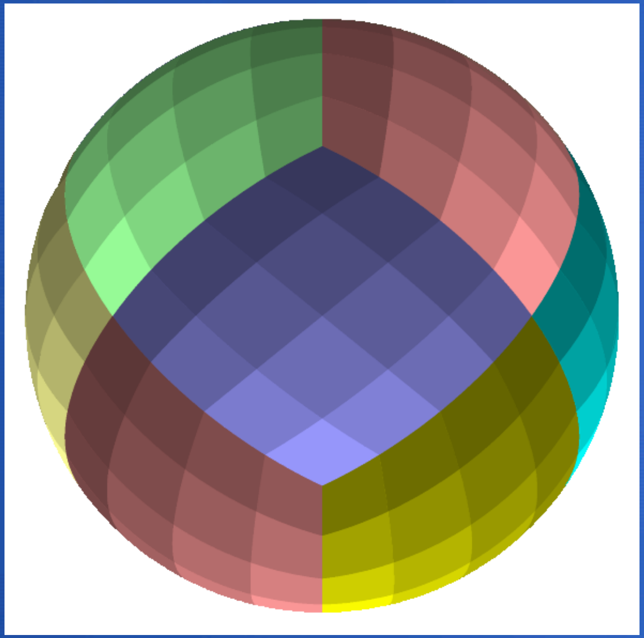
fixed 324 cores

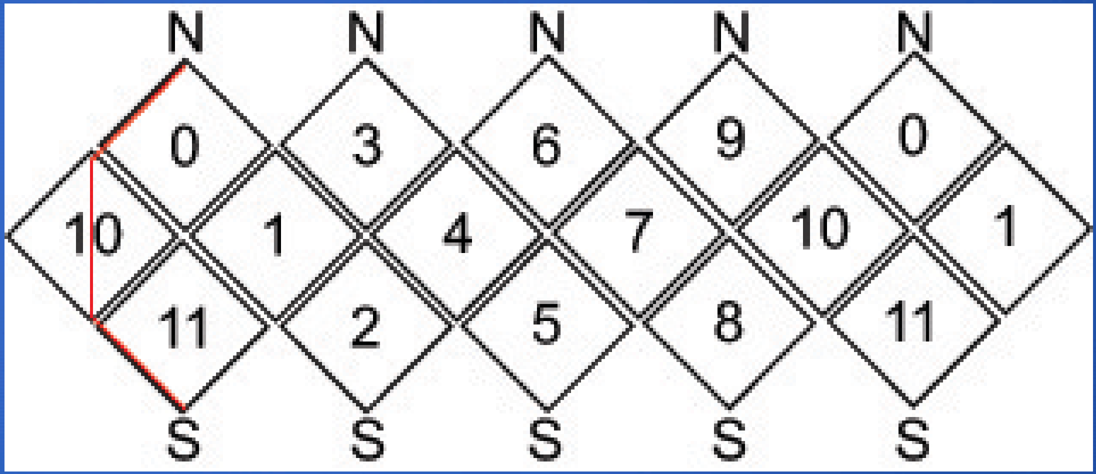


Why? – Because bigger grids use more multigrid levels

Yellow = super speedup

Why? Cube Sphere + 8 cores per node + fixed number of available nodes





- 144 x 144 x 144 cube into 4 x 4 x 2 blocks gives 36 x 36 x 72
- 36 x 36 x 72 block and only be divided twice more to get even multi-grid levels: 18 x 18 x 36 and 9 x 9 x 18

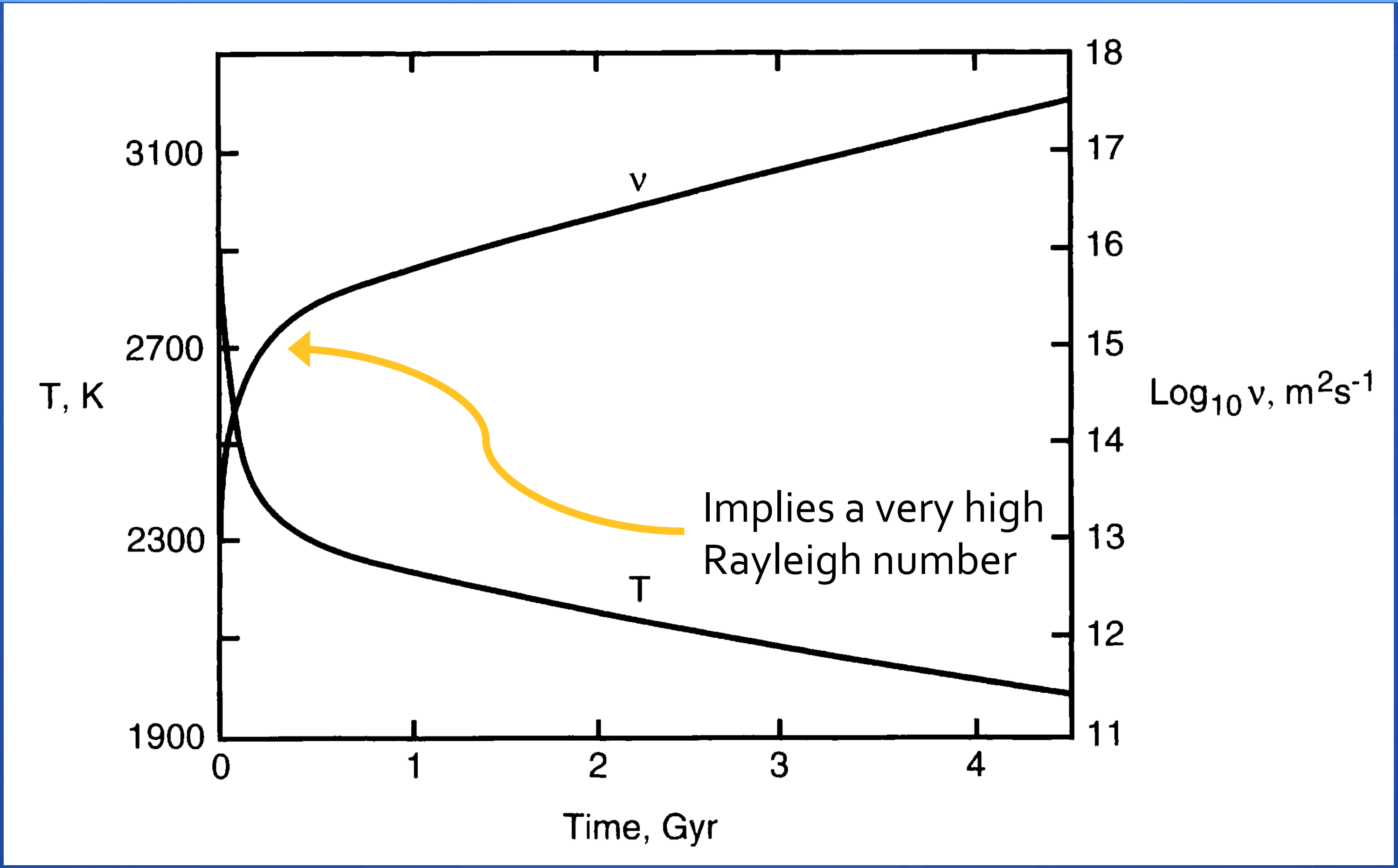
Grid Lessons Learned

- The number of elements per core should be in the 32-cubed to 64-cubed range.
- You are often better off (even in wall clock time) having fewer cores and more multigrid levels as opposed to having more cores and too few MG levels.
- *A more general lesson:* with iterative solvers, the question of best performance is more complex than with traditional direct approaches

Grid

- For this work I settled on 97 x 97 x 97 (30 km grid spacing) both because of time required for solutions and disk space issues
- I tried some 129 x 129 x 129 cases and I don't see significant differences

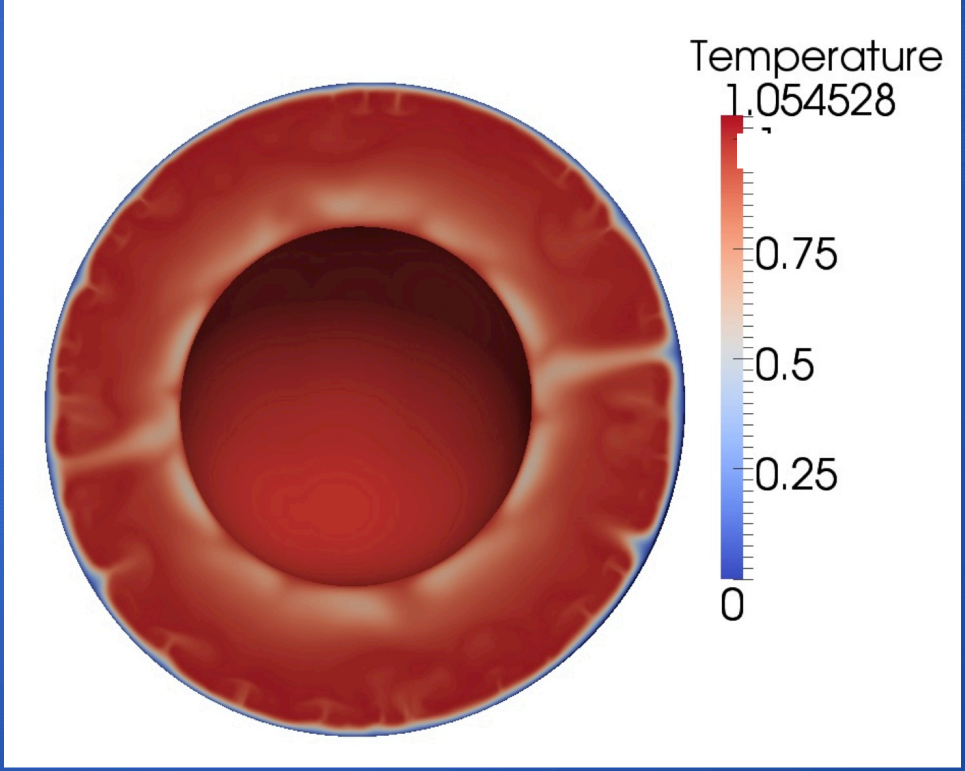
typical thermal history curve



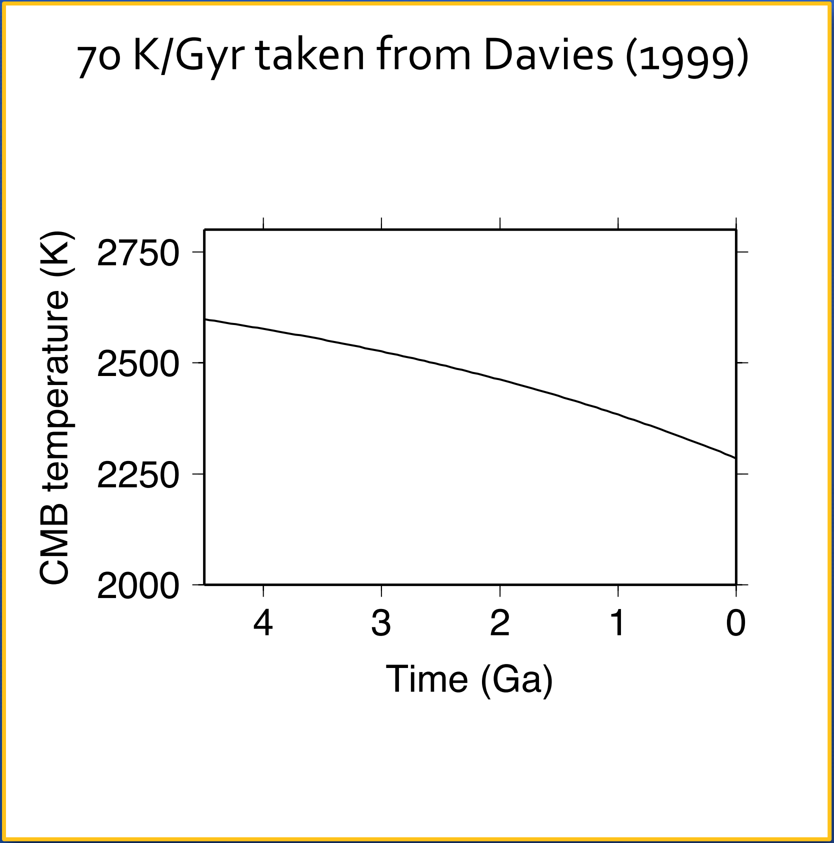
From Schubert, Turcotte and Olson, 2001 Figure 13.1

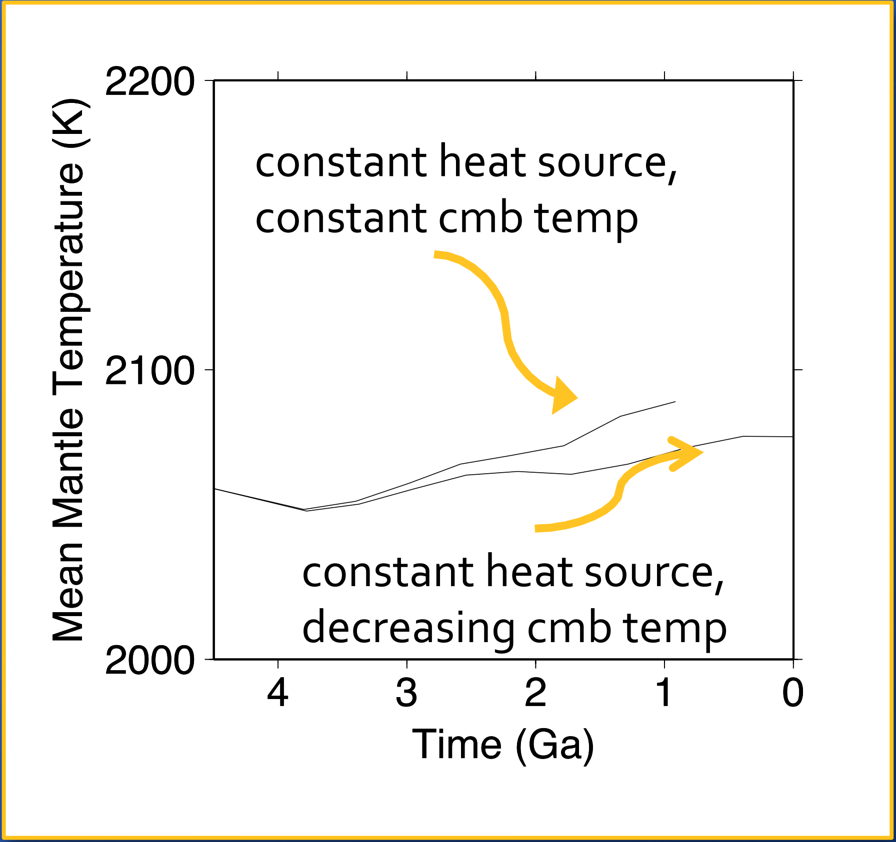
3D spherical convection

- Bousinessq, which means:
  - constant coef thermal expansion, Cp, density
  - no adiabatic heating/gradient
  - no viscous shear heating
- rheology strong function of temperature (Hirth and Kohlstedt, 2003)
- factor of 30 increase in viscosity at 660
- start from *moderately* hot mantle with high frequency perturbation



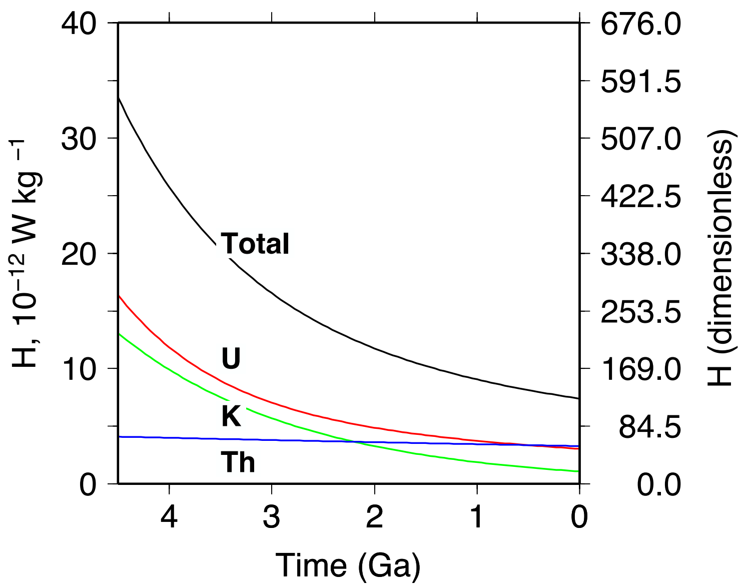
add decreasing CMB temperature

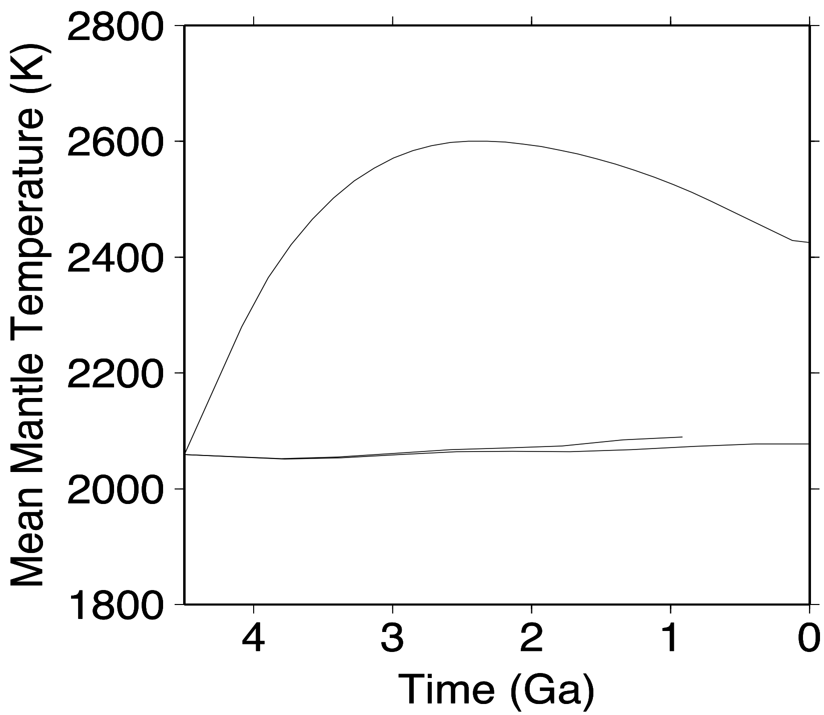




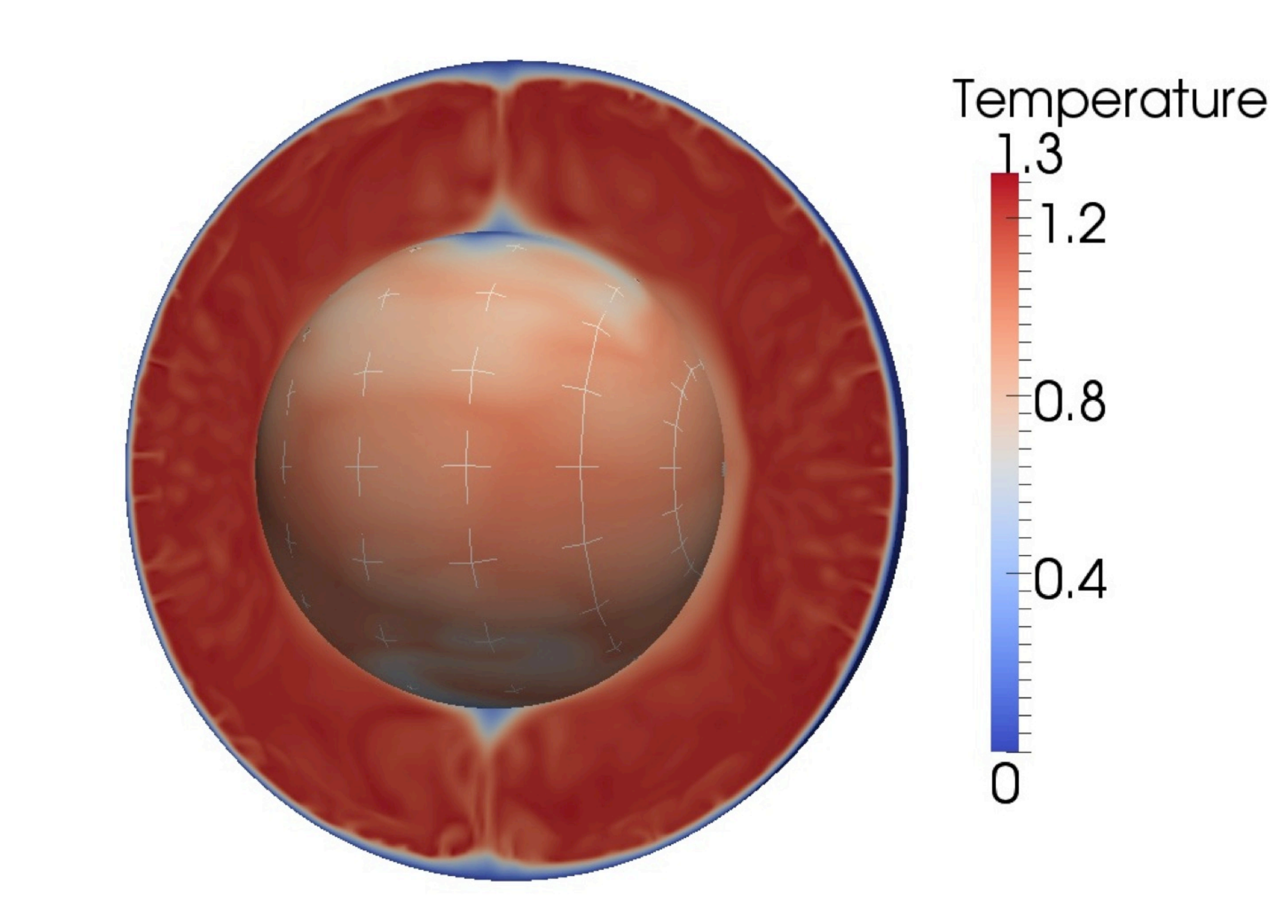
add decaying HPE

taken from Turcotte and Schubert





mantle hotter than core!!



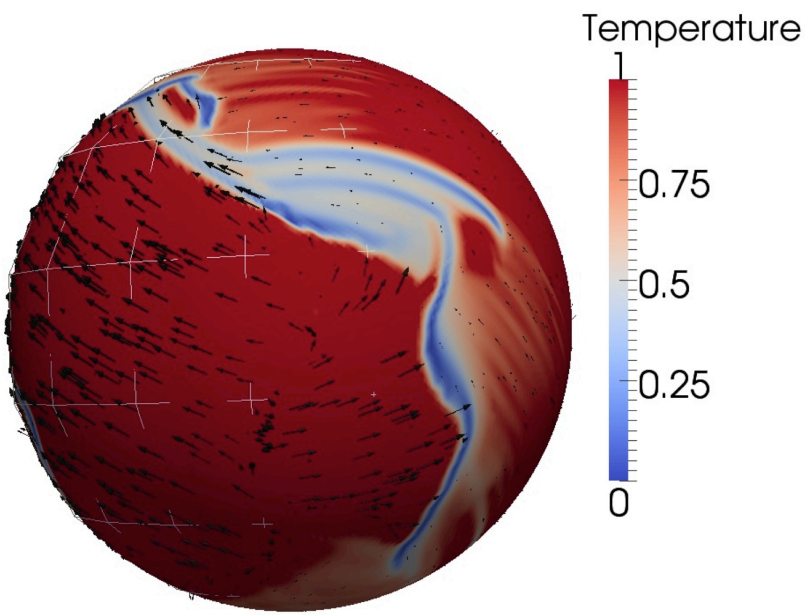
A brief (albeit biased*) history of U estimates in BSE:

- Urey (56) 16 ppb
- Wasserburg et al (63) 33 ppb
- Ganapathy & Anders (74) 18 ppb
- Ringwood (75) 20 ppb
- Jagoutz et al (79) 26 pp
- Schubert et al (80) 31 ppb
- Davies (80) 12-23 ppb
- Wanke (81) 21 ppb
- Turcotte & Schubert (82;03) 31 ppb
- Hart & Zindler (86) 20.8 ppb
- McDonough & Sun (95) 20 ppb ± 20%
- Allegre et al (95) 21 ppb
- Palme & O'Neill (03) 22 ppb ± 15%
- Lyubetskaya & Korenaga (05) 17 ppb ± 17%
- O'Neill & Palme (08) 10 ppb
- Javoy et al (10) 12 ppb

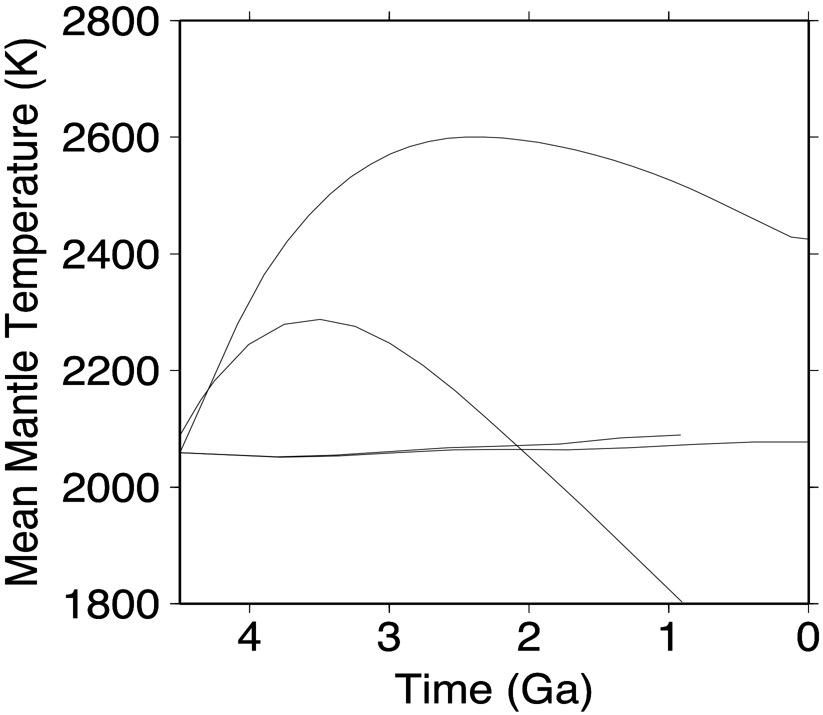
* From Bill McDonough

add imposed plates

present day plate motions

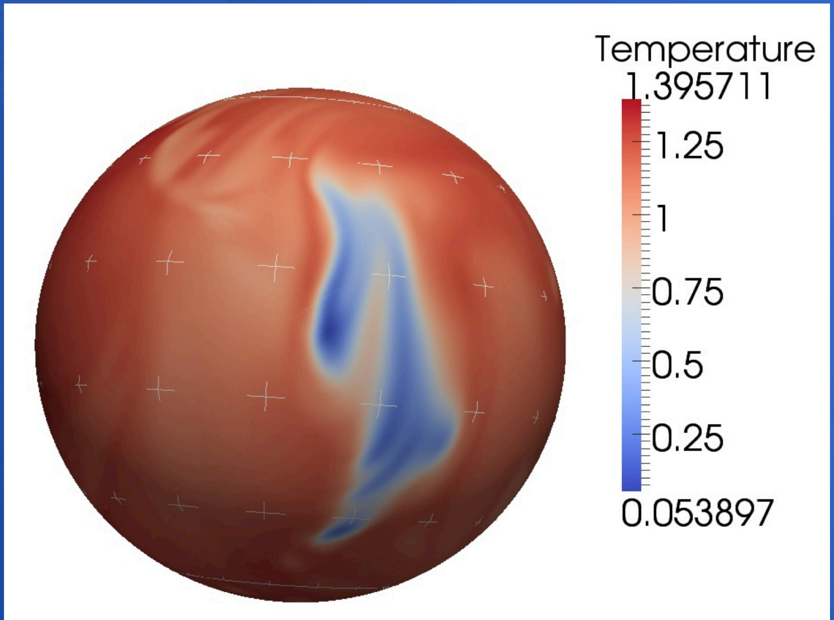


temperatures at 300 km depth

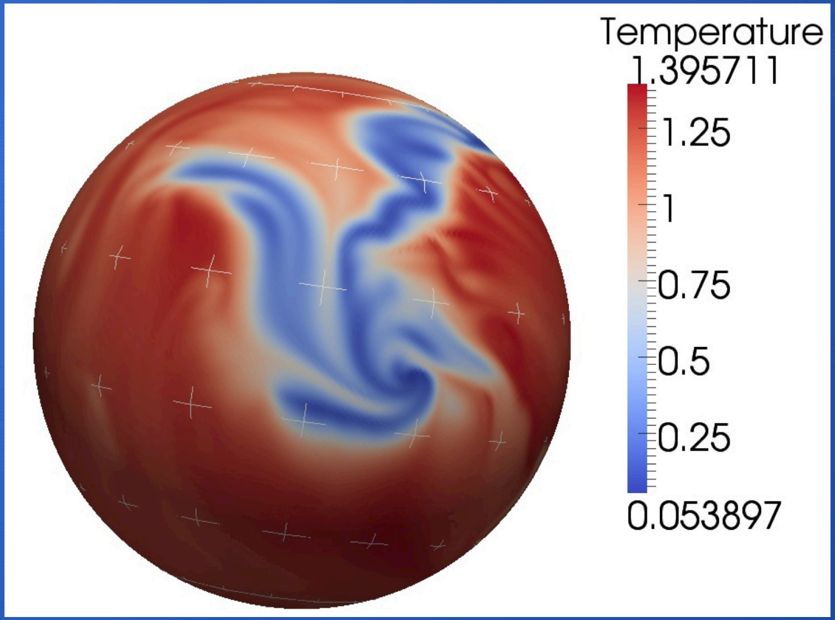


Temperature at 650 km at 2.7 Ga

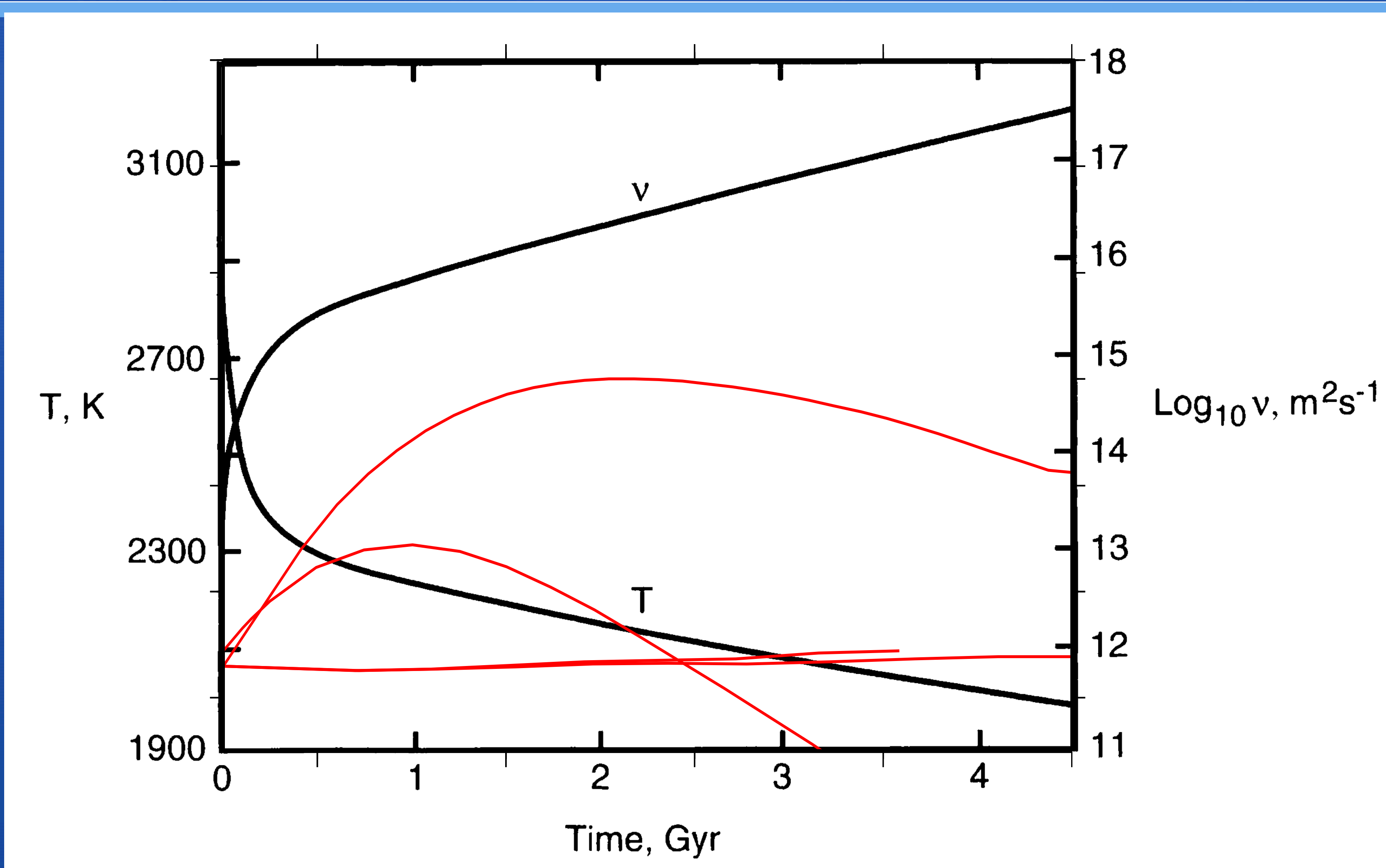
Farallon Slab Hemisphere



Western Pacific Hemisphere



compare



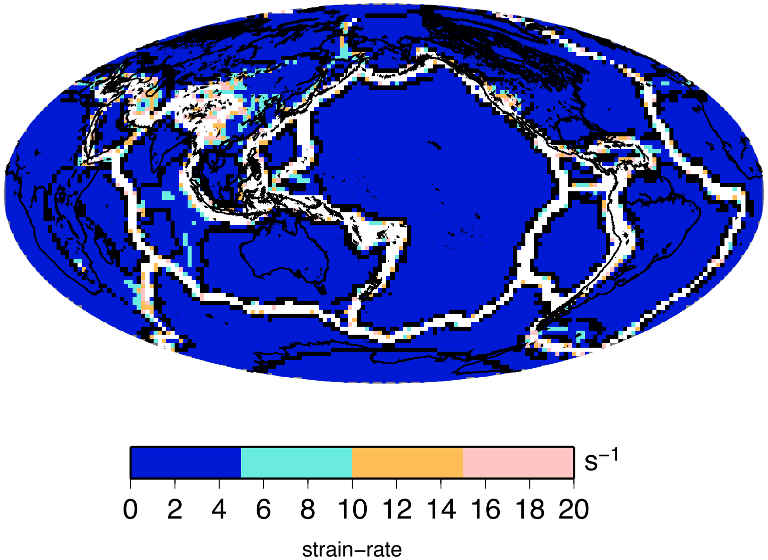
3100
2700
2300
1900
T, K
ν
T
0
1
2
3
4
Time, Gyr
18
17
16
15
14
13
12
11
Log10 ν, m2s-1

observations

- cmb temperature plays little role -- 90% of surface heat flow is from radiogenic heating and/or secular cooling, you have to do a lot to the 10% to get a noticeable effect
- must have a mobile lid -- stagnant lid convection just gets/stays too hot (what was Earth's lithosphere like before plate tectonics?)
- where are the plumes? - don't yet have a high enough starting temperature-> low enough viscosity for early earth-> not enough early mantle cooling

add mobile lithosphere

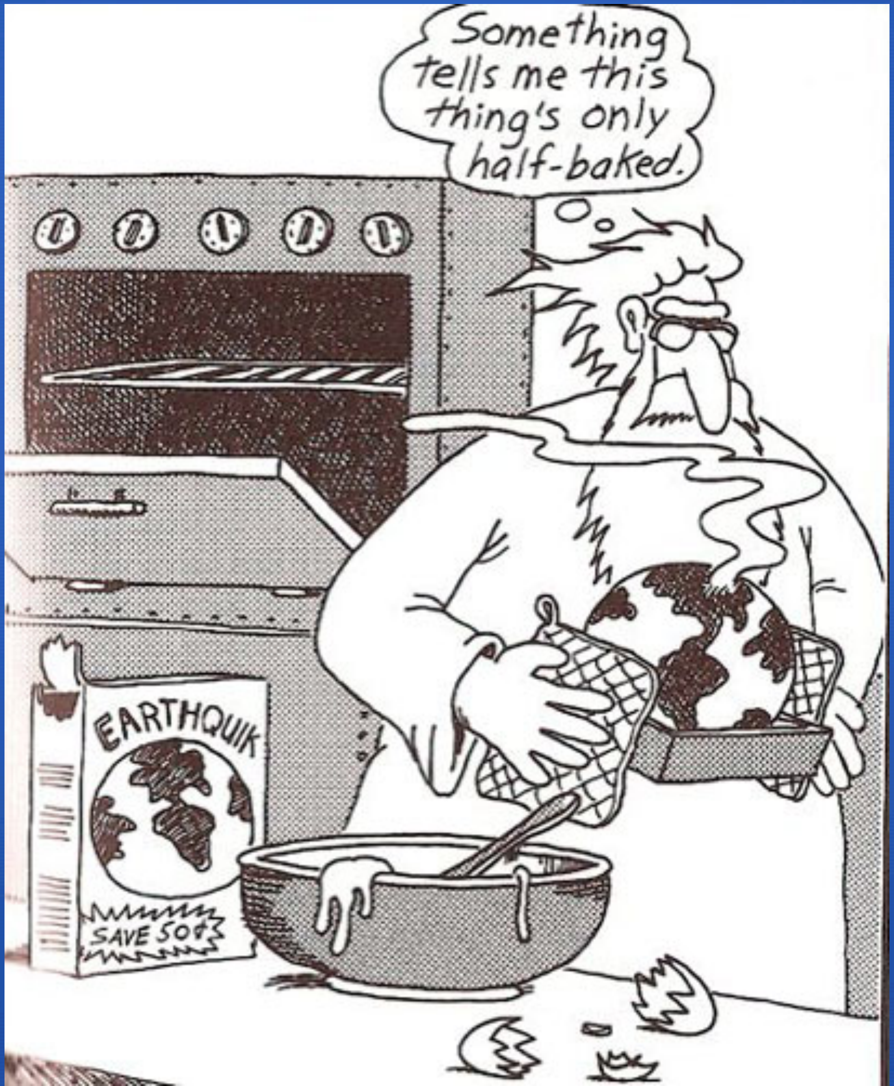
Global Strain-Rate Map
(Kreemer et al., 2003)



high strain -> weak zone

- Still a work in progress…

In God's Kitchen




IN GOD'S KITCHEN
WHAT ARE YOU COOKING?
...AUSTRALIA... WHY?
Hidden / 9GAG