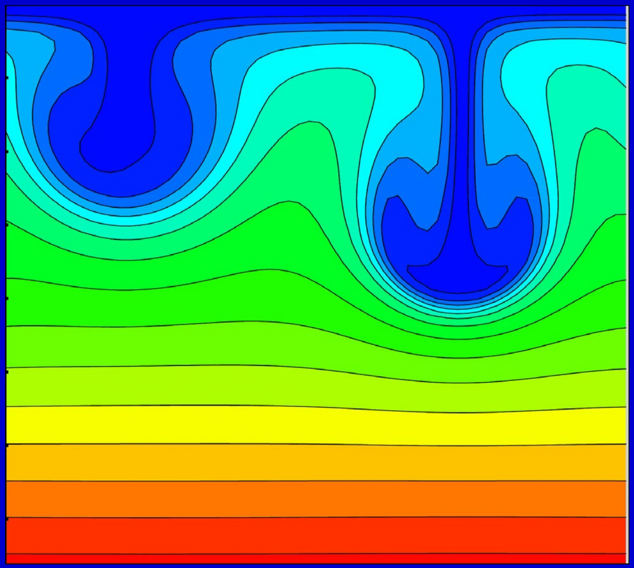
Influences of Compositional Stratification



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Influences of Compositional Stratification

1. Effect on Buoyancy
   a. * Stable Stratification and Thermal Convection
   b. Unstable Stratification driving Convection

2. * Effect on Viscosity

3. Effect on Distribution of Heat-producing (Radiogenic) Elements

* focused on in this talk

Compositional Stratification

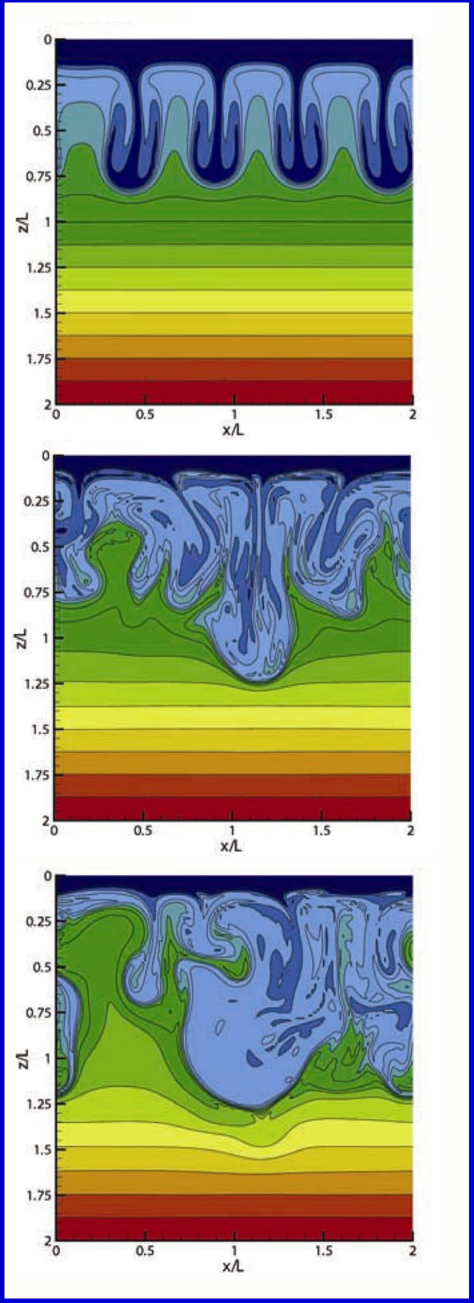
Compositional Stratifications (e.g. core & the mantle) are well recognized as having a major influence on planetary structure.

Density variations due to composition in planetary silicate mantles can be large compared to those temperature difference available to drive thermal convection.

Some cause of compositional stratifications of the mantle are

Crystallization and Overturn of a Magma Ocean

Partial melting such as in the genesis of crust



Convection in Presence of Stable Compositional Stratification

Density variations due to composition in planetary silicate mantles can play a significant role in their thermal evolution.

Temperature-dependent viscosity restricts the temperature difference (available buoyancy) that drives convection and therefore increases the effect of compositional stratification.

The onset of thermally driven convection is delayed or suppressed and the depth scales of convective motions are restricted.

Can predict behaviors with simple scaling laws and parameterizations.

May be important in the thermal evolution of the oceanic upper mantle, formation of subcratonic continental lithosphere, and for planetary evolution, more generally.

2D Numerical Experiments

Thermal Convection w/ Initially Stable Compositional Stratification

$$\frac{\partial c}{\partial t} + \bar{u} \cdot \nabla c = 0$$

$$\frac{\partial T}{\partial t} + \bar{u} \cdot \nabla T = \kappa \nabla^2 T$$

$$-\nabla p + \nabla \cdot \left(\mu (\nabla \bar{u} +' \nabla \bar{u}) \right) + \rho \bar{g} = 0$$

$$\rho = \rho_o (1 + \beta c + \alpha T)$$

$$\nabla \cdot \bar{u} = 0$$

Formulated with 2nd order finite difference approximations on a staggered grid

Multigrid iterative flow solver

Smolarkiewicz method for advection

Initially stable linear density gradient

Cooling from above, No volumetric heating

Temperature dependent rheology using exponential law

Important nondimensional parameters:

$$Ra_{box} = \rho_o \alpha \Delta T_i g L^3 / \mu_i \kappa$$

$$\gamma = 1/\rho_o \, d\rho/dz$$

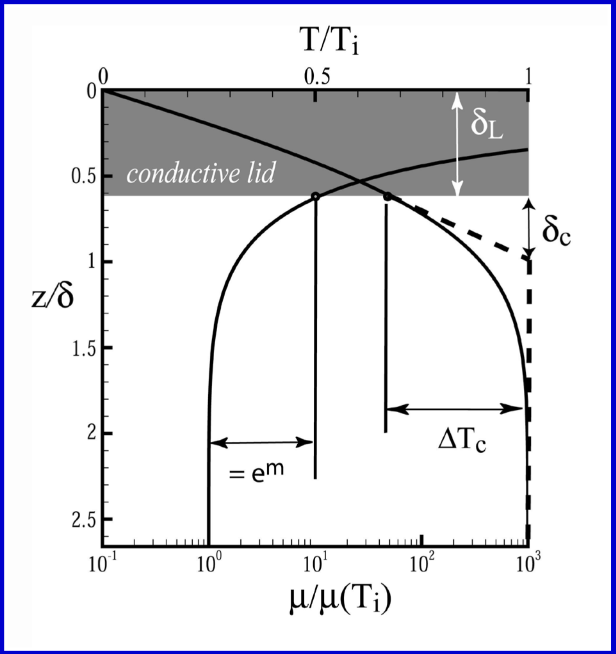
Temperature Dependent Viscosity

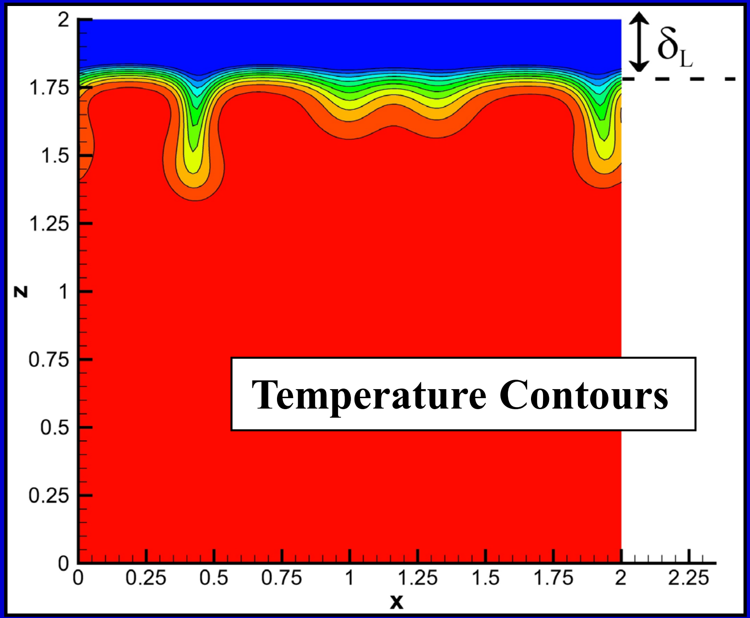
In a fluid with a strongly temperature dependent viscosity:

Convection is confined to a hot, low viscosity region beneath a conducting lid (thickness, δ_L).

Convecting thermal boundary layer (thickness, δ_c) is present between the conducting lid and the isothermal interior.

The temperature difference across the convecting thermal boundary layer denoted as ΔT_c and corresponds to a viscosity increase of ~10.





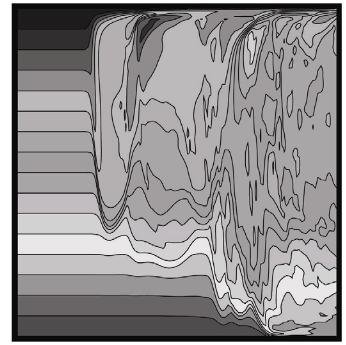
Convective Regimes

1) Convective Overturn Leading to
   Formation of a Mixed Layer
   that Thickens with Time

2) Formation of a Mixed Layer that does
   not Thicken with time

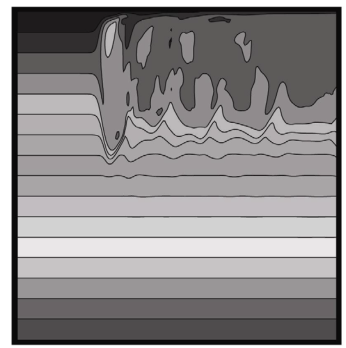
3) Oscillatory Convection where Heat
   is Transferred Faster than
   Conduction alone but with
   Little or No Mixing

4) No Convection.



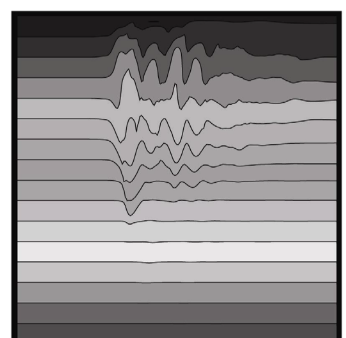
thickening

$\gamma\delta_c / \alpha\Delta T_c < 0.25$



no thickening

$0.25 < \gamma\delta_c / \alpha\Delta T_c < 0.45$



oscillatory

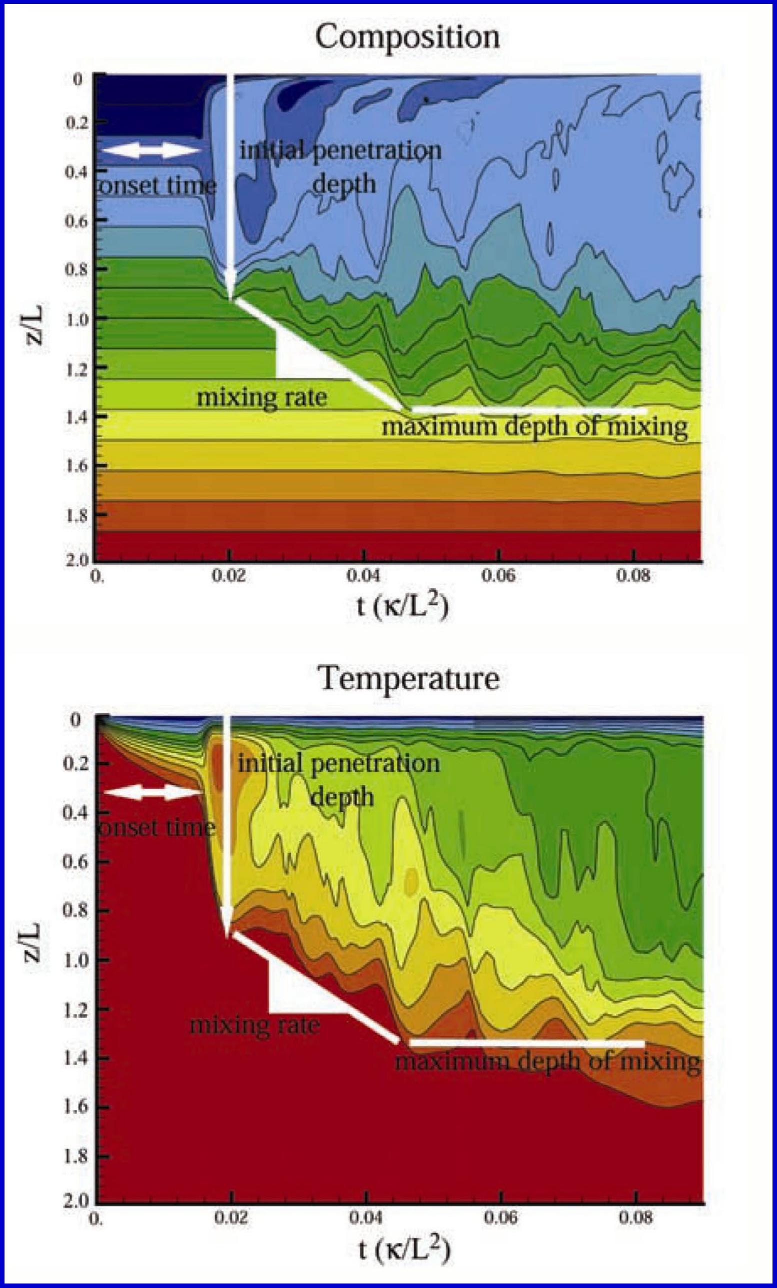
$\gamma\delta_c / \alpha\Delta T_c > 0.45$

Horizontally Averaged Contours
Of Composition

Characteristics of Convection in Stably Stratified Fluids

Interested in Characterizing:

1. *Onset Time of Convection*
2. Initial Downwelling Depth
3. *Growth rate of the Mixed Layer*
4. Maximum Depth of Mixing
5. Reduction of Surface Heat Flux



Blue-less compositionally dense material, colder material

Red- more compositionally dense material, hotter material

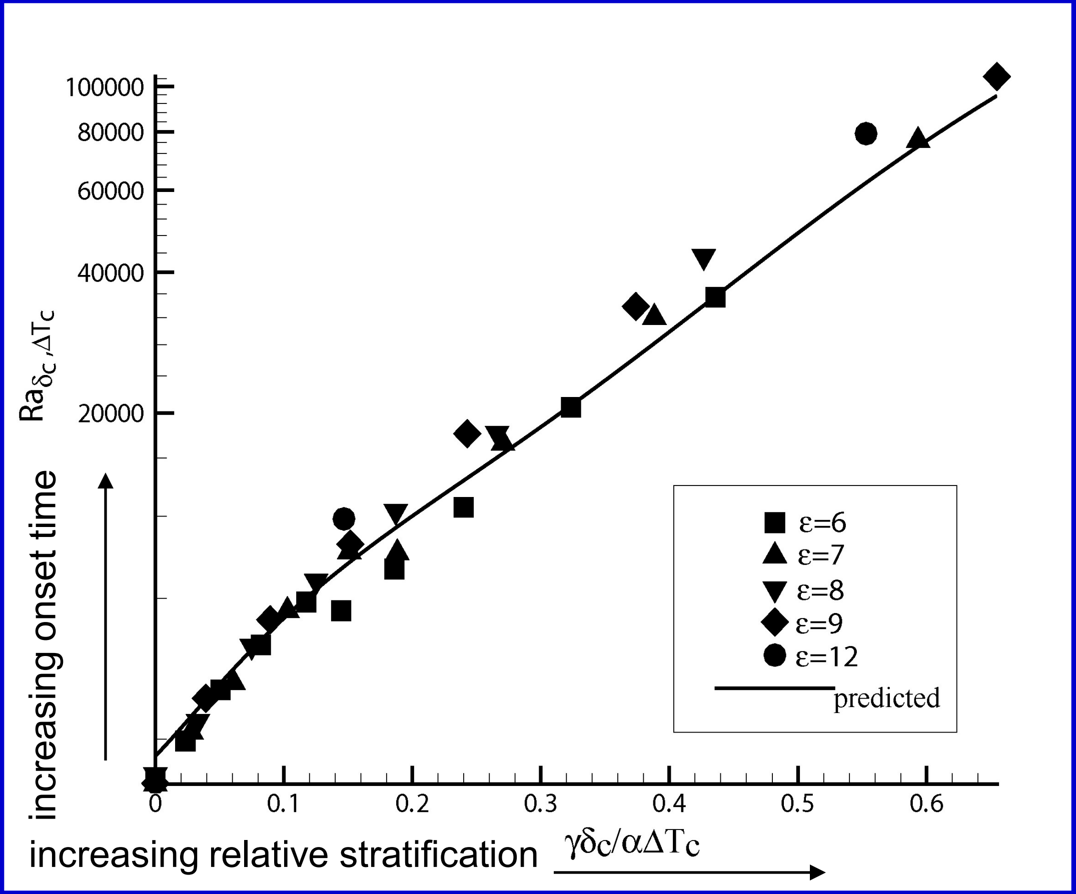
Onset Time of Convection

$$\frac{(\rho_o \alpha \Delta T_c g \delta_c / \mu_i)\, F(\gamma\delta_c / \alpha\Delta T_c)}{\kappa / \delta^2} = Ra_{\delta_c, \Delta T_c}\, F(\gamma\delta_c / \alpha\Delta T_c) = Ra_{crit}$$

Solve for $F(\gamma\delta_c/\alpha\Delta T_c)$ using the RT growthrates from linearized analysis and compare with results from 2D Numerical Experiments

Important Ratio

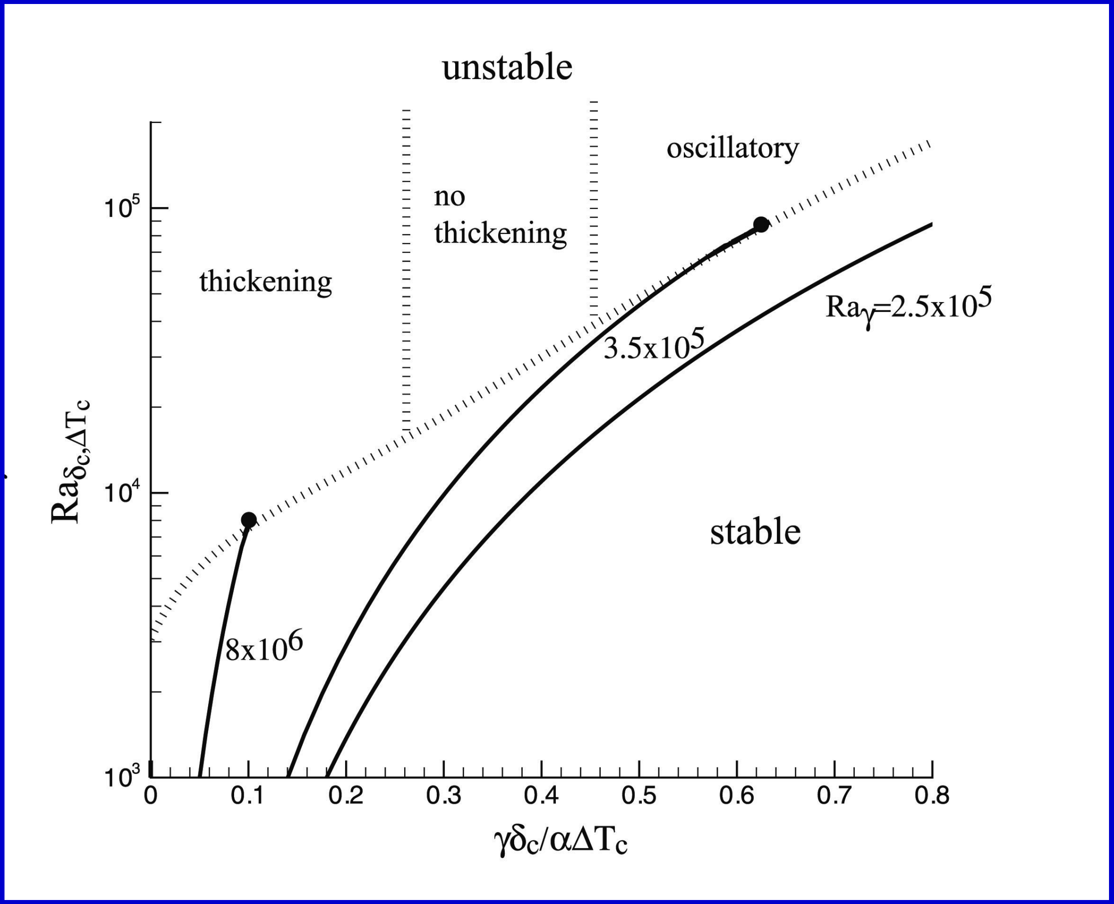
$$\frac{\gamma\delta_c}{\alpha\Delta T_c}$$



Stability Criteria for Convection

$$Ra_\gamma = \frac{(\rho_o \alpha \Delta T_c g)(\alpha \Delta T_c / \gamma)^3}{\mu_i \kappa} \left(\frac{\delta}{\delta_c} \right)^2 = Ra_{\delta c, \Delta Tc} \left(\frac{\gamma \delta_c}{\alpha \Delta T_c} \right)^{-3}$$

For a given value of activation energy (determines ΔT_c, δ_c/δ) and compositional gradient (γ), the value of Ra_γ remains constant as the thermal boundary layer grows.

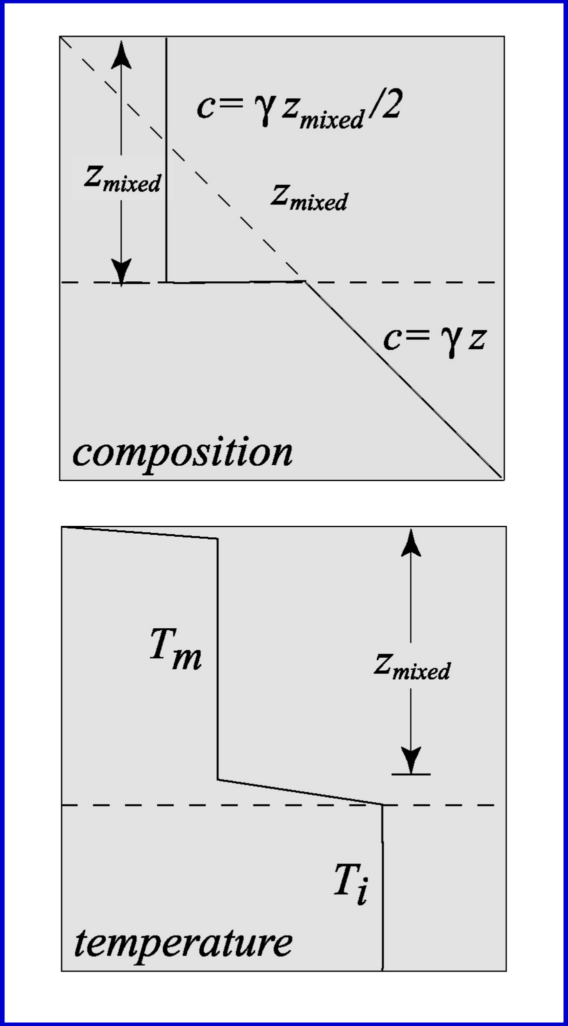


Formulation of Scaling Laws

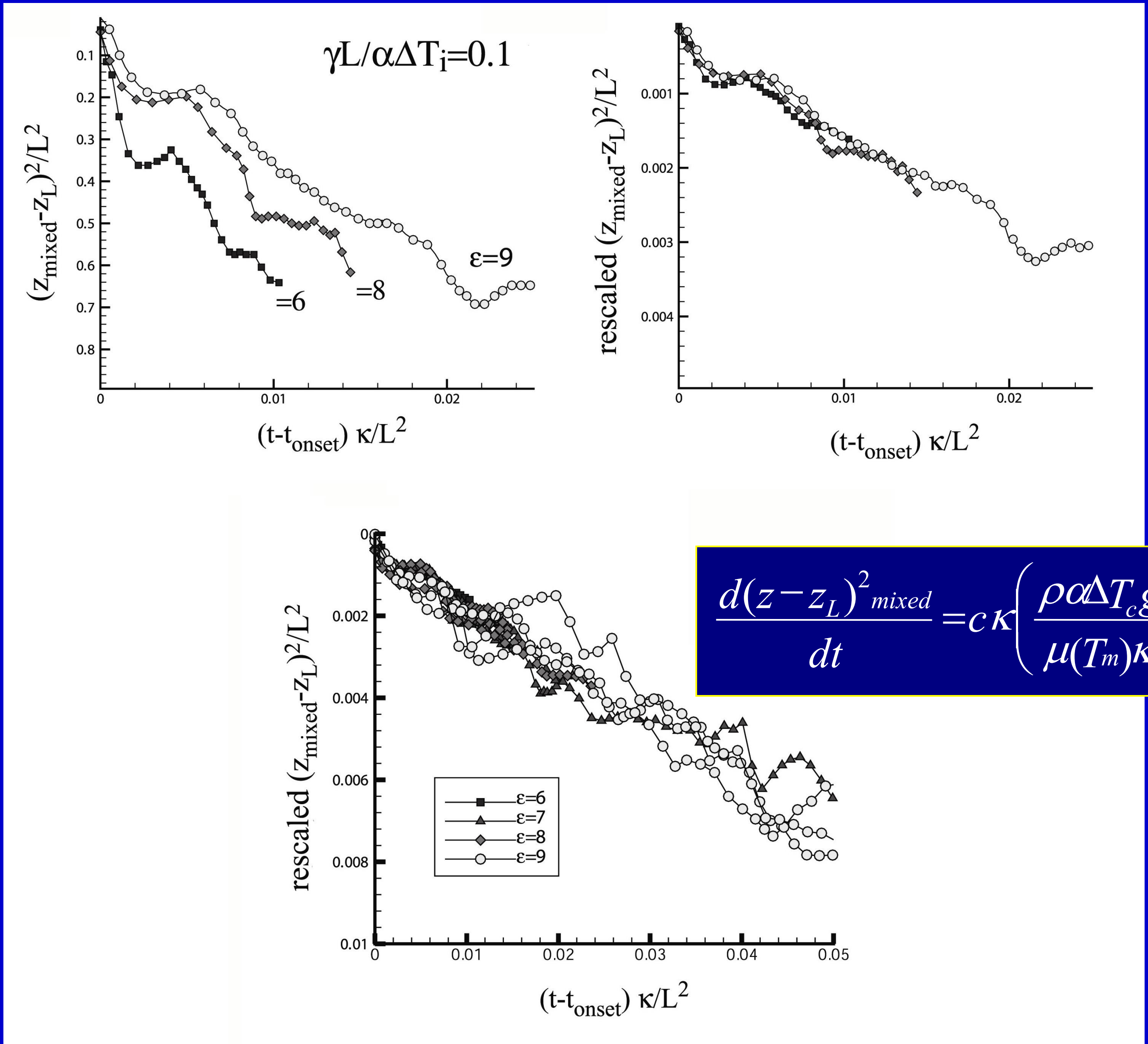
To derive scaling laws:

1. *Composition*: Average of the initial
   over the mixed layer depth.
   Temperature: Uniform value T_m
   outside thin thermal boundary layers at
   the top and bottom.
2. Beneath the mixed layer, temperature
   and composition retain their initial values.
3. Density continuity (buoyant rather than
   viscous entrainment) at the interface between
   the mixed and unmixed layers
4. Conservation of heat
5. Parameterized convective heat flux
   (e.g. Davaille and Jaupart, 1993) at the base
   of the conductive lid

Schematic of the Idealization of a Well-mixed Layer



Thickening of the Mixed Layer

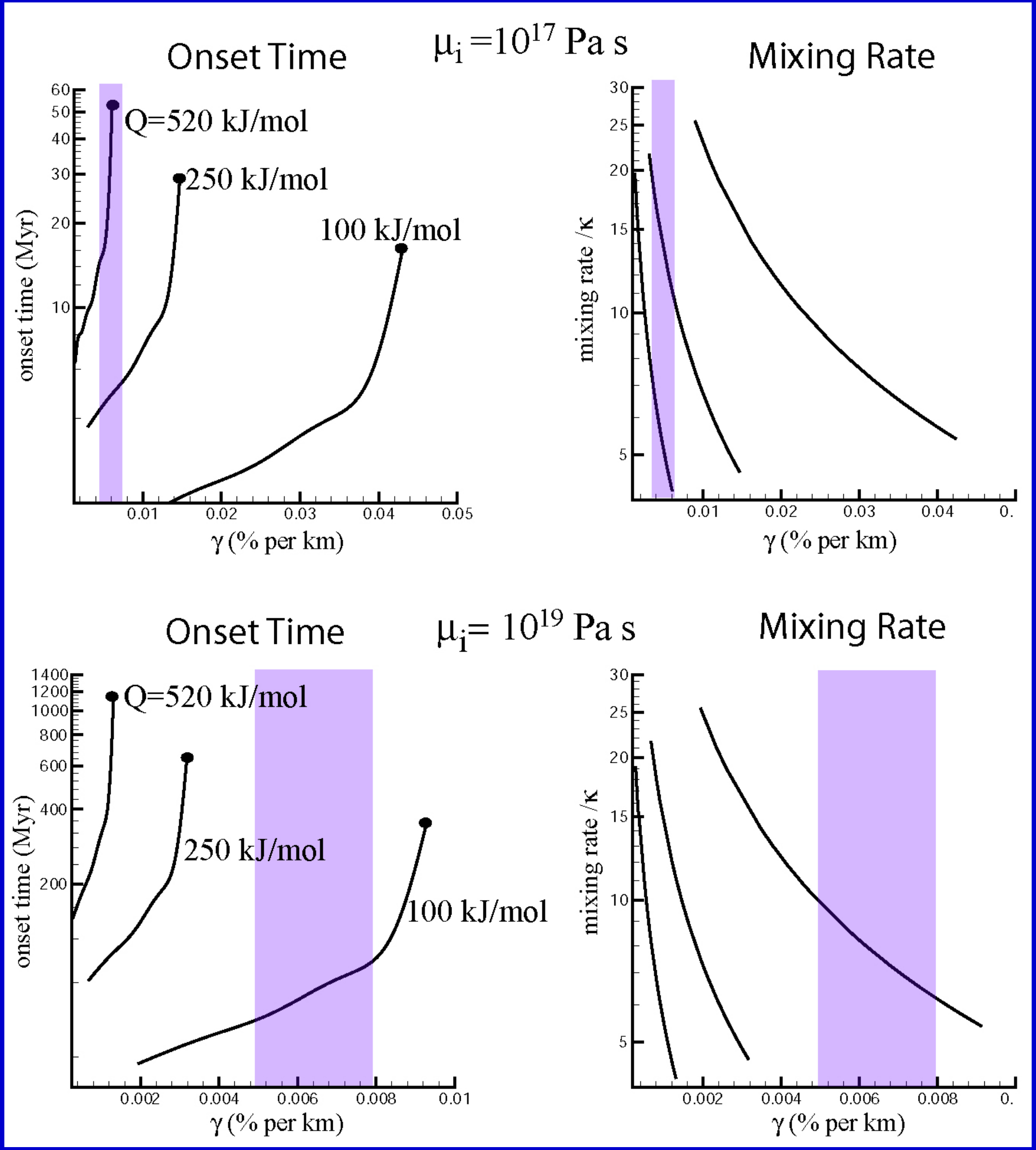


$$\frac{d(z-z_L)^2{}_{mixed}}{dt} = c\kappa\left(\frac{\rho\alpha\Delta T_c g}{\mu(T_m)\kappa}\right)^{1/3}\left(\frac{\alpha\Delta T_c}{\gamma}\right)\left(1-\frac{\gamma z_{mixed}}{2\alpha T_i}\right)^{4/3}$$

The mixed layer thickens by the penetration of plumes into the still stratified layer.

For planetary scales, thickening rate ~5-25 κ

Application to Subcratonic Lithospheric Evolution



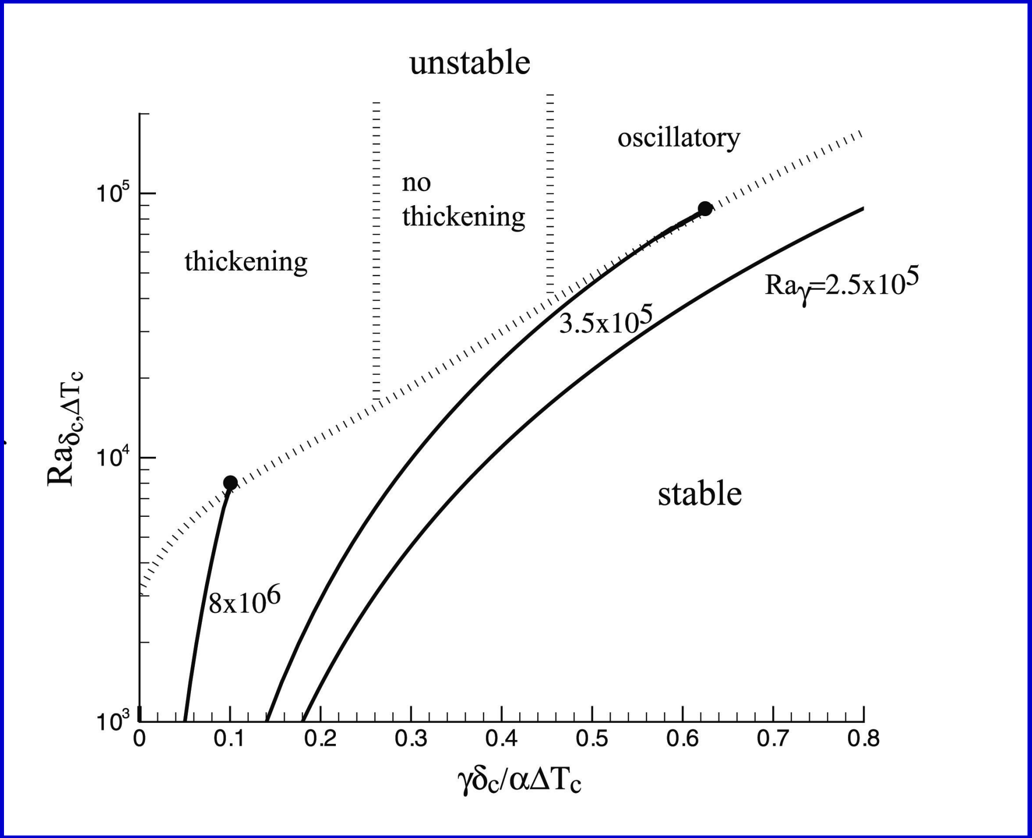
Application to Subcratonic Lithospheric Evolution

Can the Lithosphere Experience an Oscillatory Mode of Convection?

* $Ra_\gamma = \sim 2.5 \times 10^5 - 6.5 \times 10^6$ ($\mu_i = 10^{17}$ Pa s)

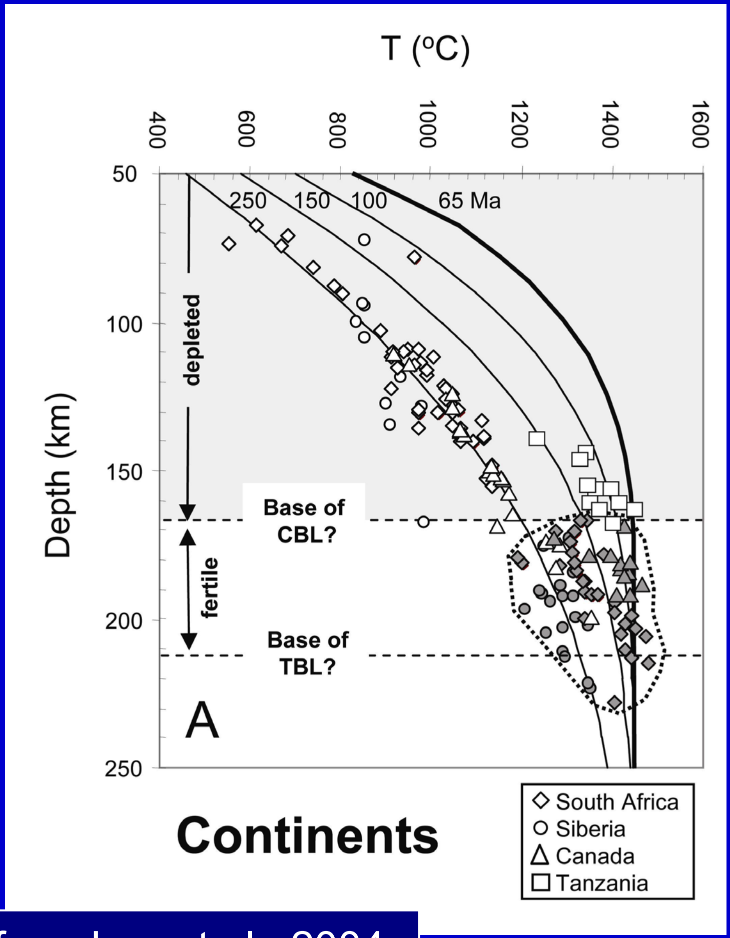
* $Ra_\gamma = \sim 2.5 \times 10^4 - 6.5 \times 10^5$ ($\mu_i = 10^{18}$ Pa s)

For γ = 5-8 x 10^{-3} % per km
Q = 300-520 kJ/mol

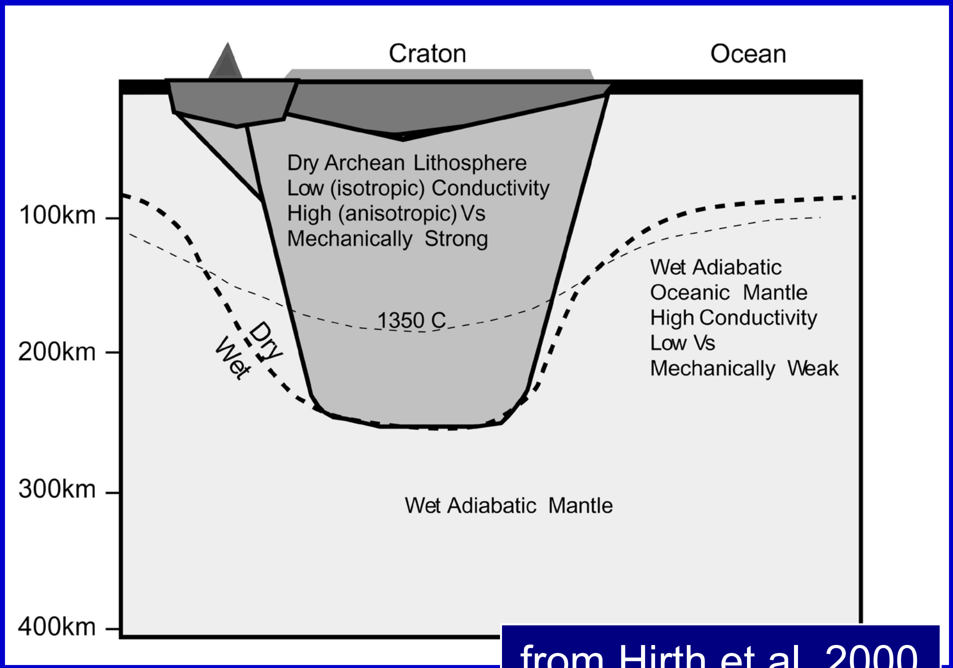


Yes – trade off between Q and μ_i, but possible

Compositional Viscous Lid



from Lee et al., 2004



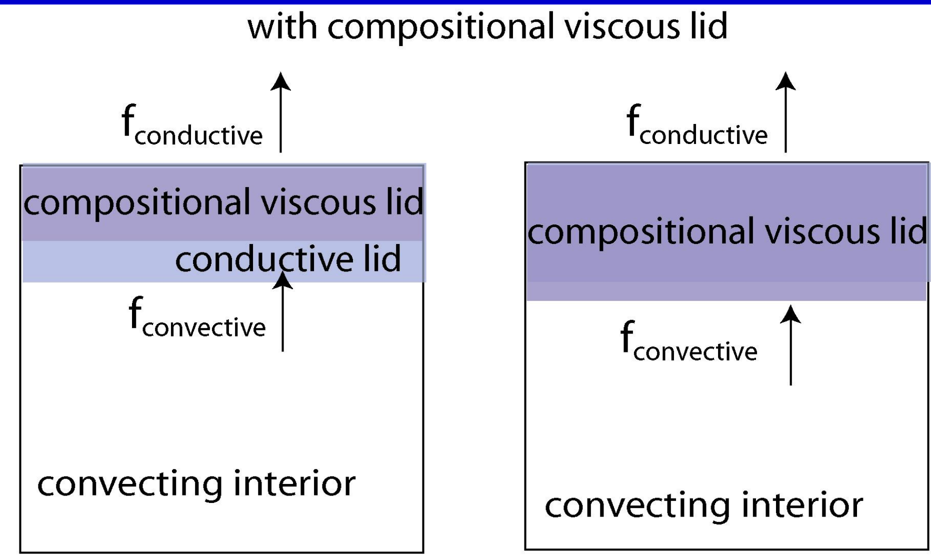
from Hirth et al, 2000

(from electrical conductivity profiles)

Influence of Compositional Viscous Lid

$$f_{convect} = c_1 k \left(\frac{\rho \alpha g}{\mu_{min} \kappa} \right)^{1/3} \Delta T_c^{\,4/3}$$

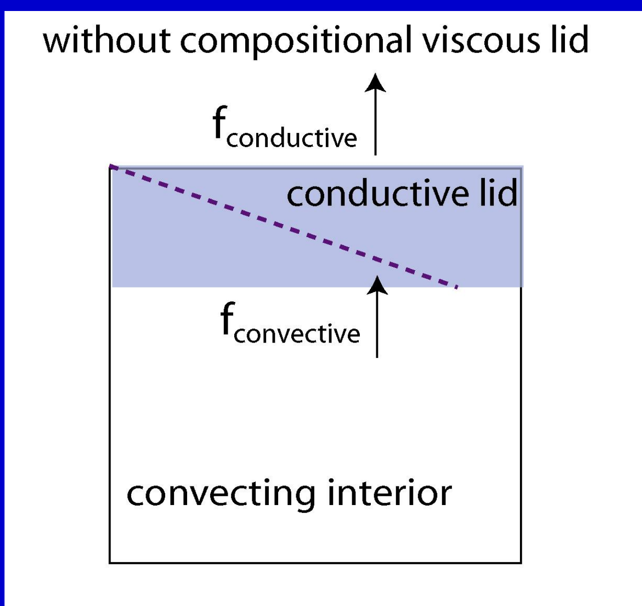
$$f_{convect} = f_{conduct}$$



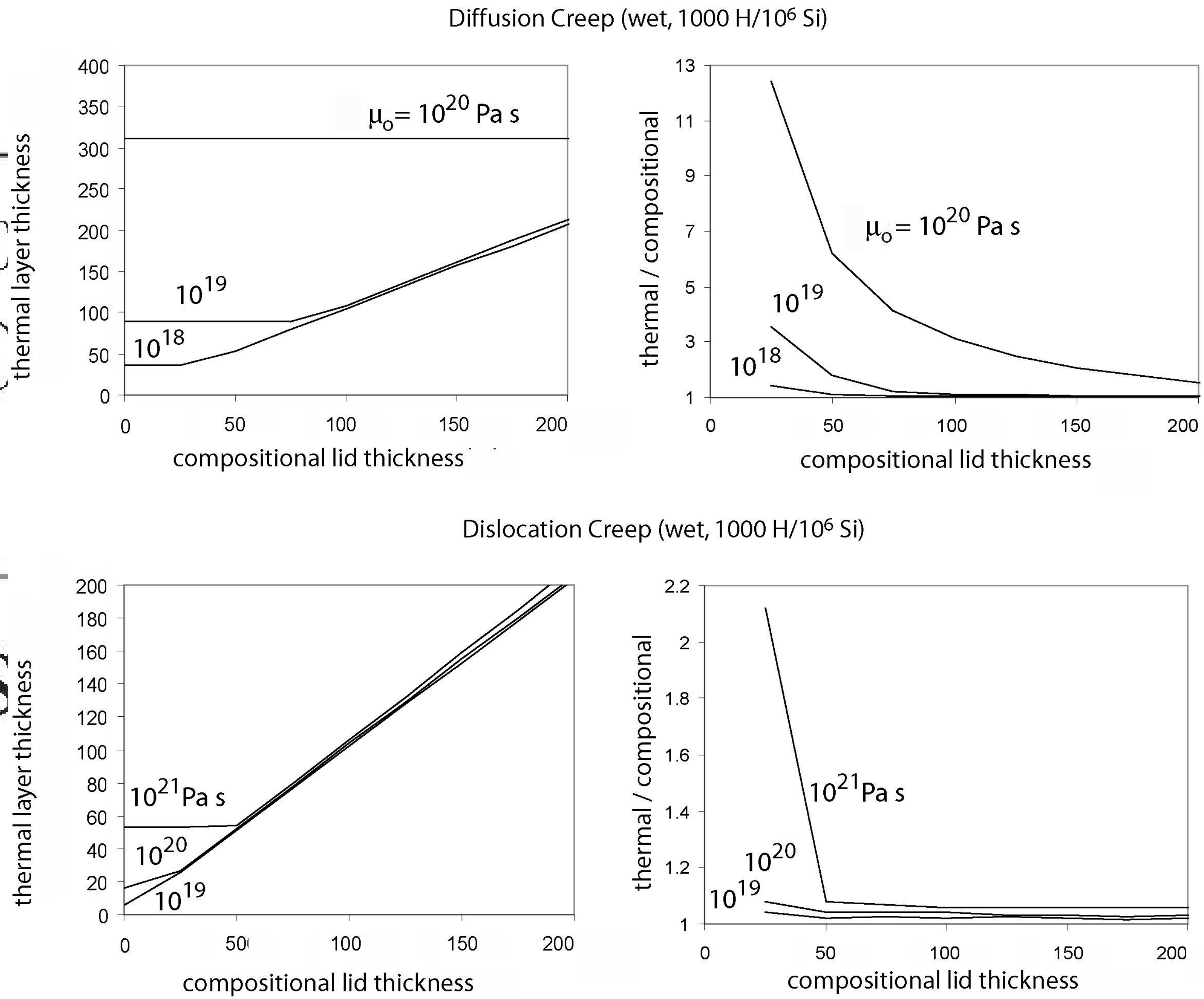
CASE 1:
Compositional viscous lid THINNER than conductive lid determined by just temp-depend viscosity

CASE 2:
Compositional viscous lid THICKER than conductive lid determined by just temp-depend viscosity

ΔT_C driving convection reduced to match the reduced $f_{conductive}$



Influence of A Viscous Compositional Lid



Conclusions

Compositional Stratification may significantly alter convection by affecting the buoyancy forces, the viscosity structure and the distribution of radioactive elements.

Buoyancy:

With a stable compositional stratification, the onset of thermally driven convection is delayed or suppressed and the depth scales of convective motions are restricted.

Convective behavior can be predicted and parameterized, where one of the key ratios is $\alpha\Delta T_c/\gamma\delta_c$.

Predicted stratifications for subcratonic lithosphere is significant enough to strongly influence the convective behavior. Oscillatory style convection is possible.

Conclusions

Compositional Stratification may significantly alter convection by affecting the buoyancy forces, the viscosity structure and the distribution of radioactive elements.

Viscosity:

Compositional stratification may also be accompanied by viscosity stratification (dehydration?)

A compositionally viscous lid can either be too thin to effect convection (still within the conductive/stagnant lid of temp-dep viscosity) or thick enough to define a new stagnant lid.

A near constant ratio of thermal thickness to composition thickness for specific values of Q, V, and μ_{ref}.

This ratio ranges from 1.09-1.2, indicating (at maximum) a 175 km compositional viscous lid would produce a thermal thickness of 210 km.