VolkswagenStiftung

CIG/QUEST/IRIS Joint Workshop on
Seismic Imaging of Structure and Source
July 14-17, 2013, Fairbanks, Alaska

Rapid CAD and tetrahedral mesh generation for dynamic rupture problems

SCOREC

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SeisSol - Project overview

LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

Coordination, Host, Physics, Numerics, Algorithm, Pre- and Postprocessing, Application, User support

TUM TECHNISCHE UNIVERSITÄT MÜNCHEN

Technical development, HPC, Optimization, Visualization, Design

IBM

Consulting, Scaling, BlueGene/Q adaption

KAUST King Abdullah University of Science and Technology

Visualization, parallel I/O

SCOREC

Automated CAD generation

Simmetrix Inc. Enabling Simulation-Based Design

Meshing, CAD generation

Caltech

ETH

BSC Barcelona Supercomputing Center Centro Nacional de Supercomputación

lrz

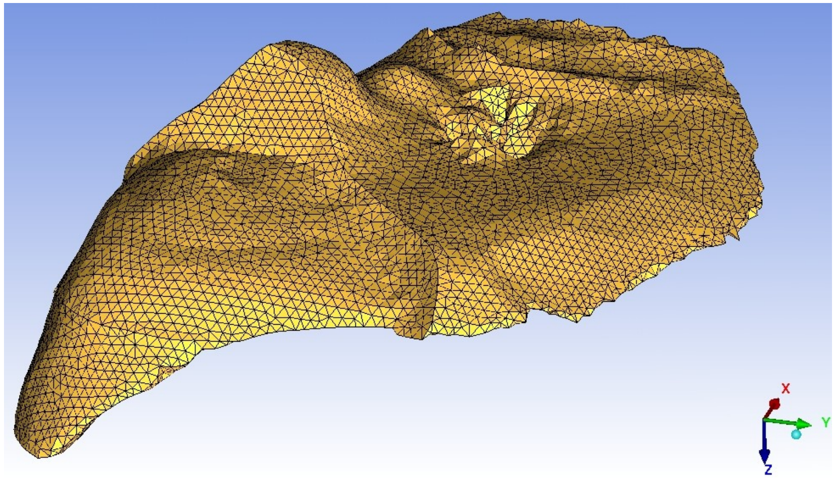
...and others …
Support, Guidance, Experience sharing, Consulting, ...

Goal

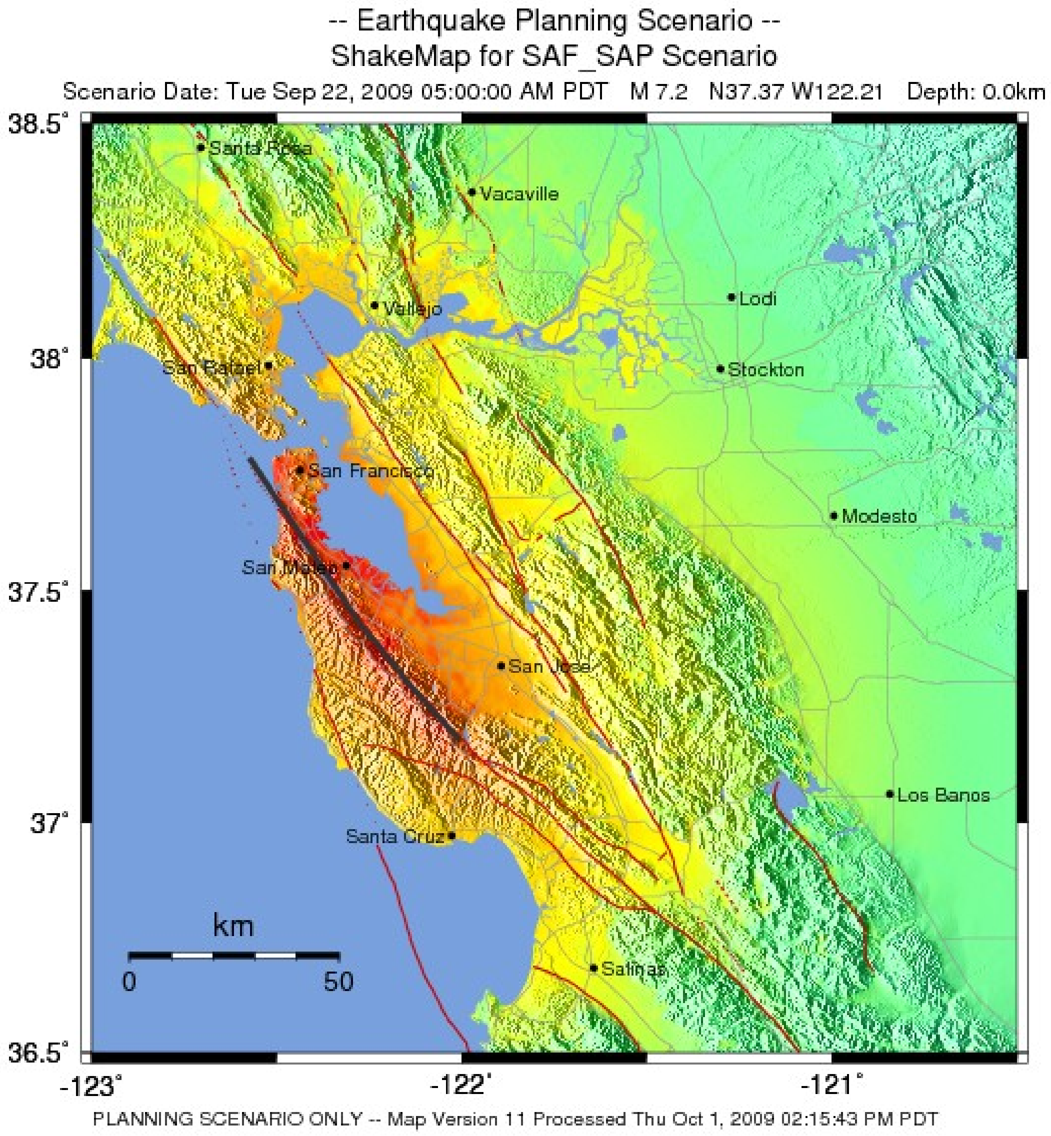
Complete seismic wave propagation package including solutions for

- **dynamic rupture simulations**
- **exploration industry**
- **Seismology**

with complex geometry and heterogeneous medium.



Käser, Martin, Christian Pelties, E. Cristobal Castro, Hugues Djikpesse, and Michael Prange (2010), **Wave Field Modeling in Exploration Seismology Using the Discontinuous Galerkin Finite Element Method on HPC-infrastructure**, *The Leading Edge*



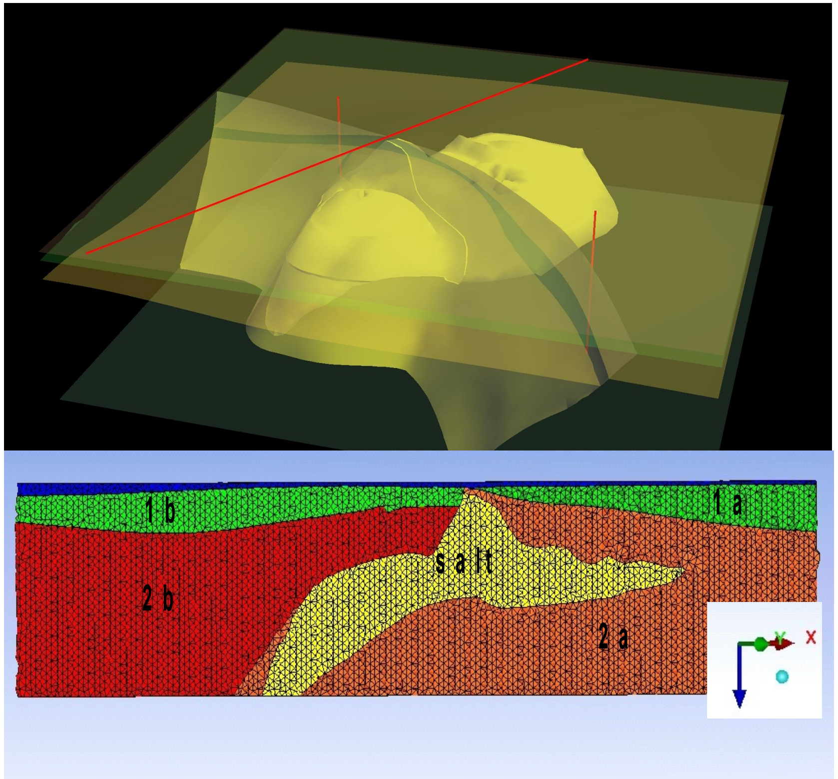
Warmer colors
= higher intensity

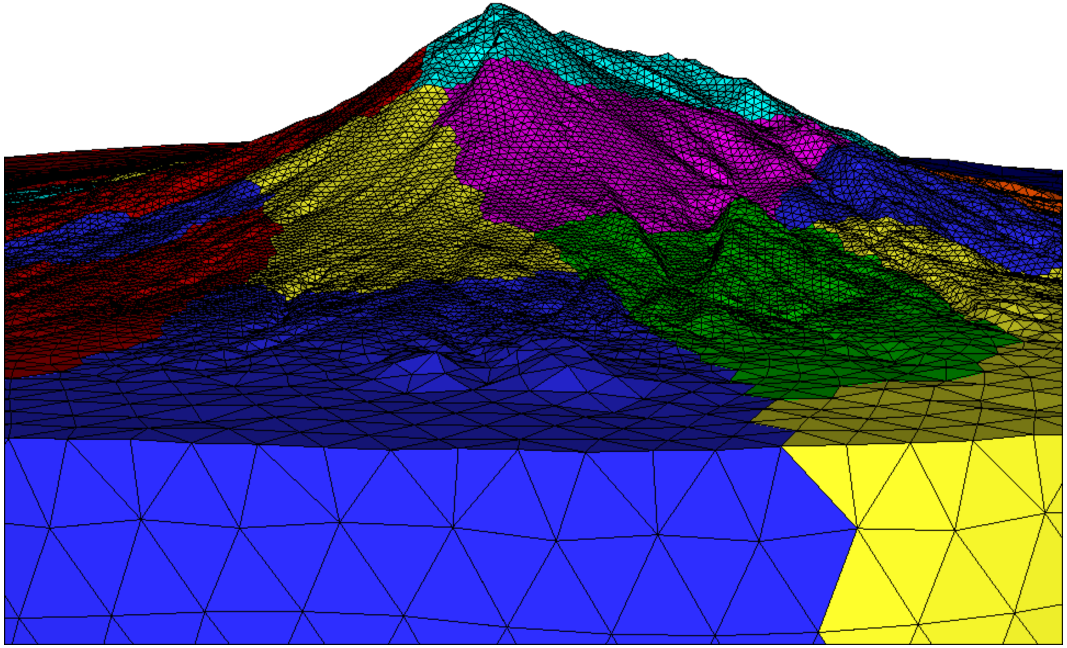
USGS - http://earthquake.usgs.gov/earthquakes/shakemap/ (modified)

Requirements for solver

What do we need for this?

- Accurate numerical methods for reliable results (num. errors, boundary-, initial conditions)
- Proper geometry representation (topography, material interfaces)
- Use of acoustic, elastic, viscoelastic, and anisotropic material to approximate realistic geological subsurface properties
- Scalability on HPC architecture to tackle big problems with high frequency

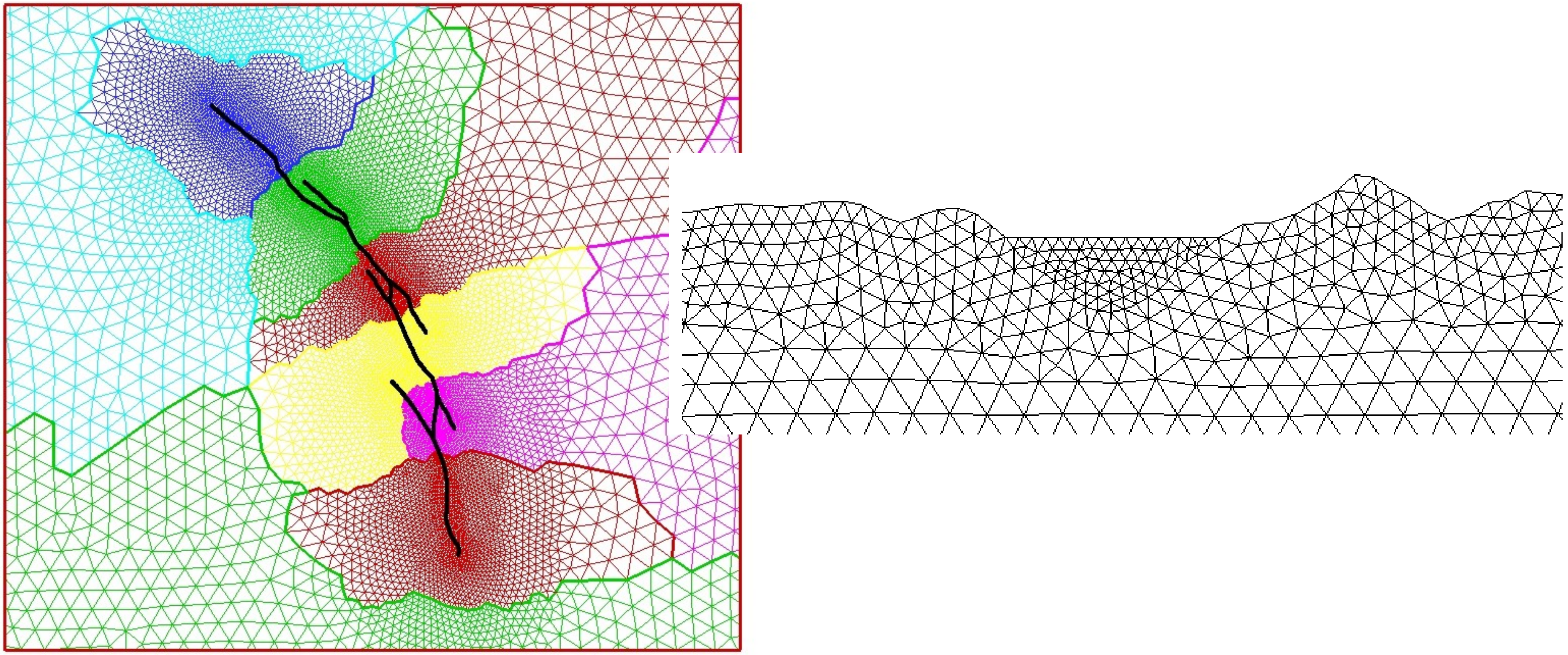




(Merapi model)

Advantages of the ADER-DG Method

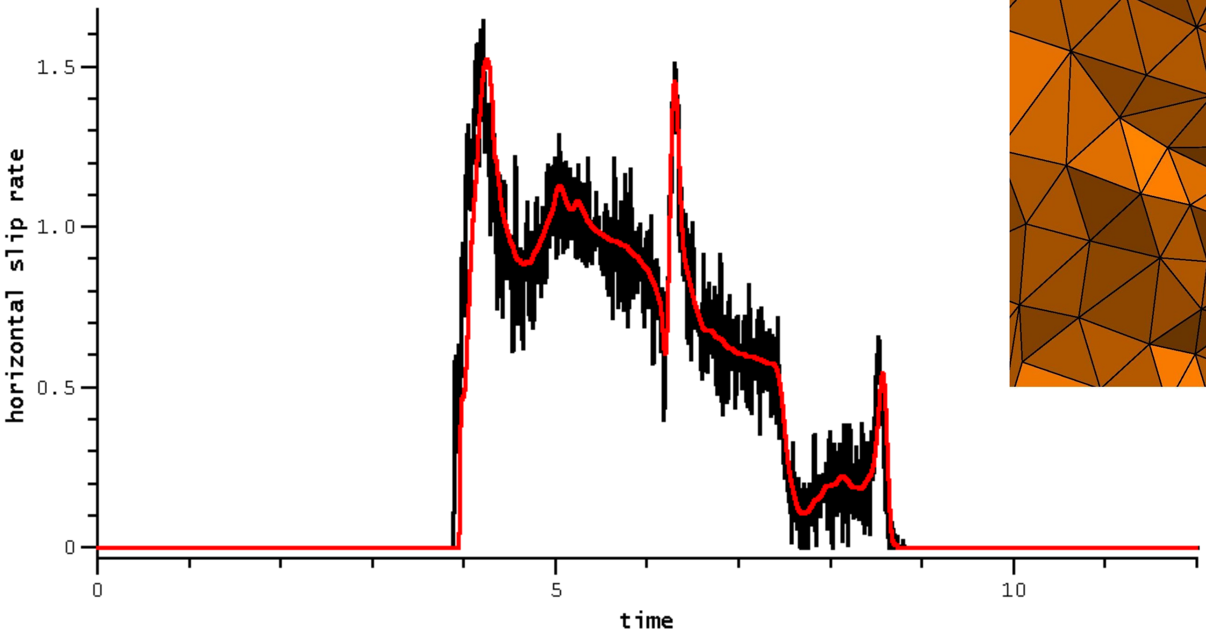
- Enables use of unstructured meshes – low velocity basins, curved or kinked faults, branching, surface rupture, fault interaction
- Mesh coarsening – adjustment of resolution
- High-order accurate simulation of the wave propagation including heterogeneous media and topography

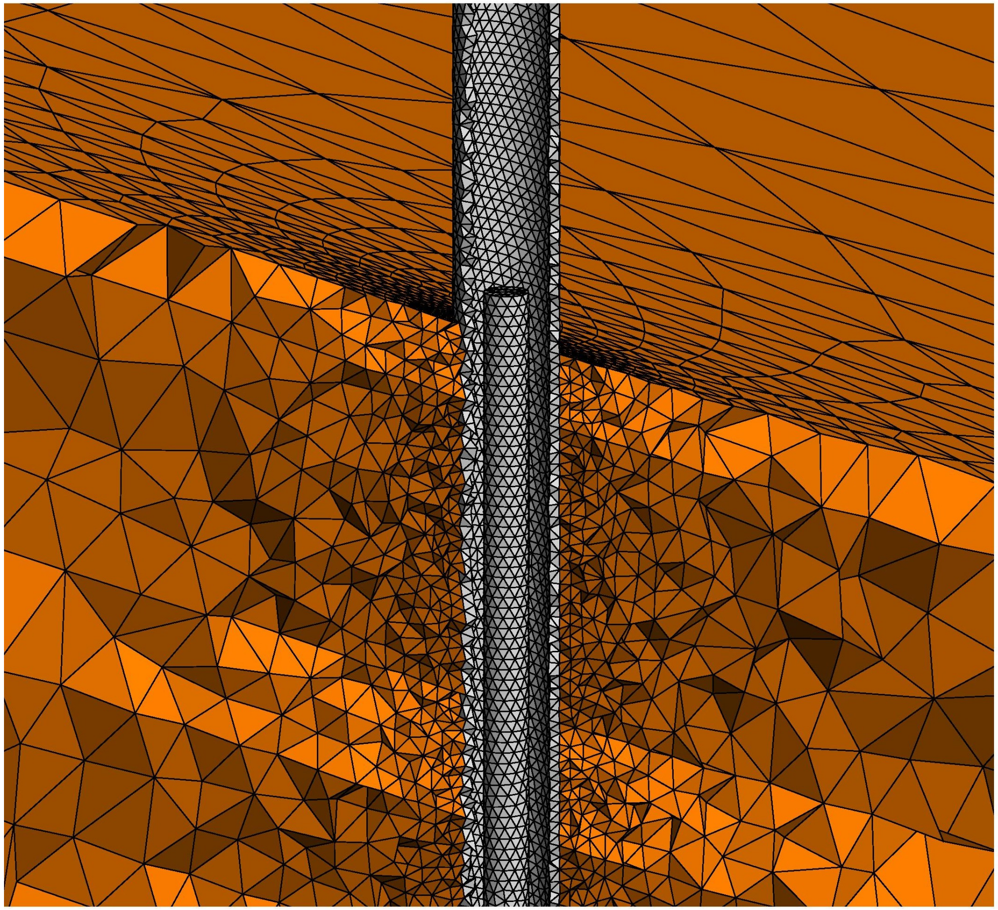


Advantages of the ADER-DG Method

- ADER high-order time integration with local time stepping
- High-accurate results of the rupture process: Oscillation free dynamic rupture

SEM vs ADER-DG





Elastic Wave Equation as a Linear Hyperbolic System:

Vector-matrix notation:

$$\frac{\partial Q_p}{\partial t} + A_{pq}\frac{\partial Q_q}{\partial x} + B_{pq}\frac{\partial Q_q}{\partial y} + C_{pq}\frac{\partial Q_q}{\partial z} = S_p$$

Velocity-stress formulation:

3D: $Q = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, u, v, w)^T$

Mathematical Model

Elastic Wave Equation as a Linear Hyperbolic System:

Vector-matrix notation:

$$\frac{\partial Q_p}{\partial t} + A_{pq}\frac{\partial Q_q}{\partial x} + B_{pq}\frac{\partial Q_q}{\partial y} + C_{pq}\frac{\partial Q_q}{\partial z} = S_p$$

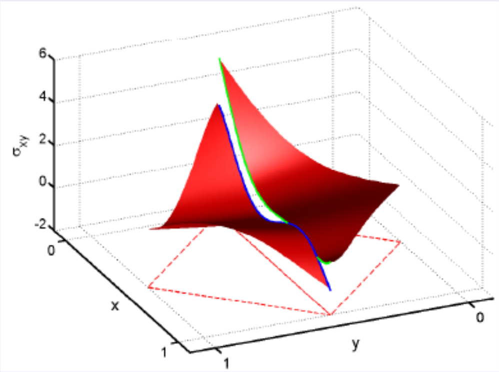
Velocity-stress formulation:

3D: $Q = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}, u, v, w)^T$

Numerical Approximation of the solution

$$\left(Q_h^{(m)}\right)_p (\xi, \eta, \zeta, t) = \hat{Q}_{pl}^{(m)}(t)\Phi_l(\xi, \eta, \zeta)$$

- Φ_l are orthogonal basis functions
- the mass matrix is diagonal



Discontinuous Galerkin Approach – Flux computation

Flux computation

Exact Riemann solver is used to compute the state at the interfaces by upwinding:

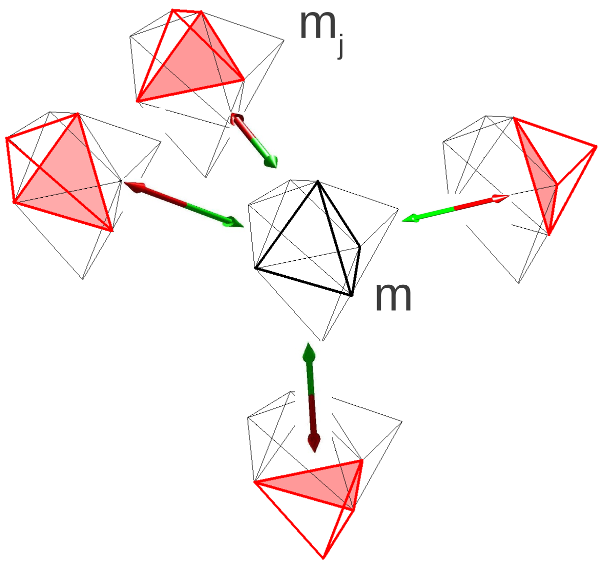
$$F_p^h = \frac{1}{2} T_{pq} \left(A_{qr}^{(m)} + \left| A_{qr}^{(m)} \right| \right) (T_{rs})^{-1} \hat{Q}_{sl}^{(m)} \Phi_l^{(m)}$$
$$+ \frac{1}{2} T_{pq} \left(A_{qr}^{(m)} - \left| A_{qr}^{(m)} \right| \right) (T_{rs})^{-1} \hat{Q}_{sl}^{(m_j)} \Phi_l^{(m_j)}$$

Computation of the line integrals:

- Pre-computed analytically
- Gauss-Legendre integration

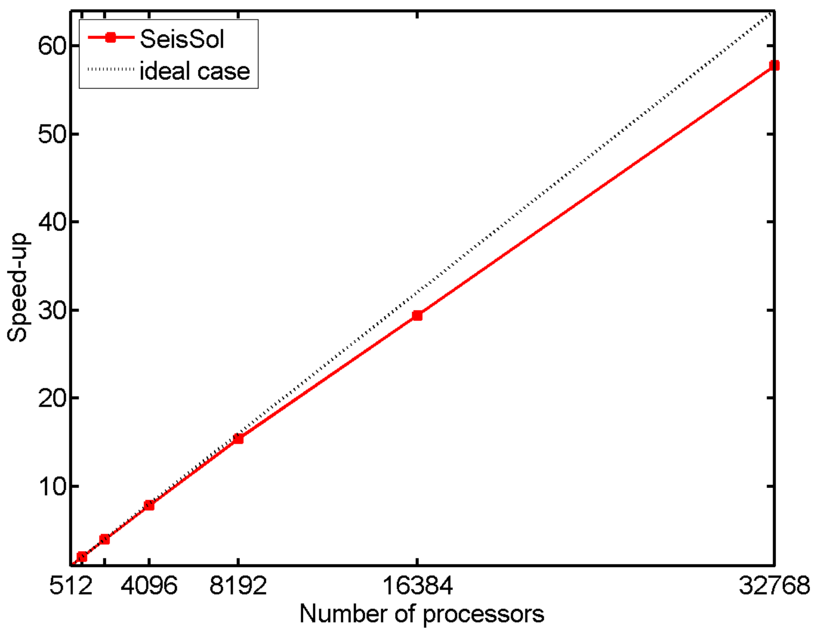
⇨ Opens up new possibilities:
non-conforming meshes, dynamic rupture source type

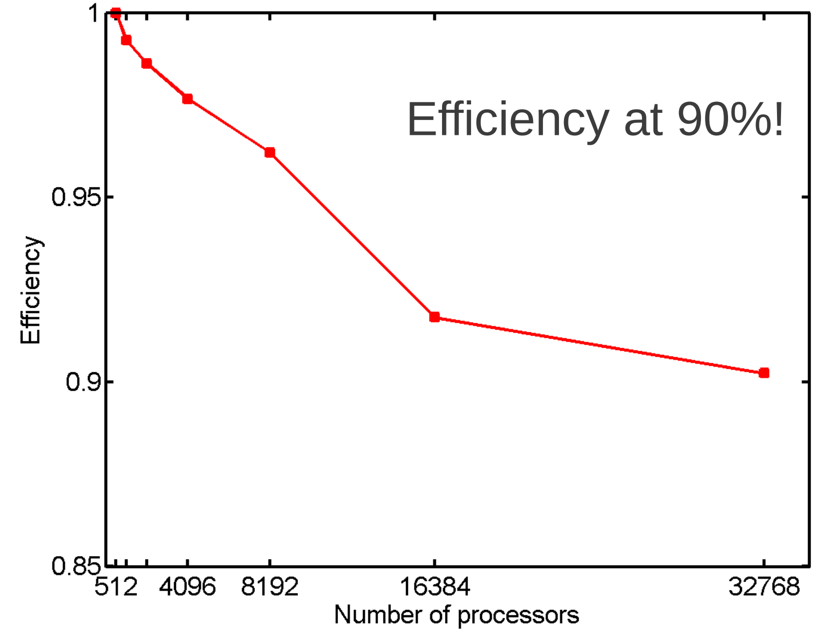
Locality of the computations:
only directly neighboring elements are required to exchange data, which leads to small communication times for parallel calculations



Suitability for large scale HPC infrastructure

Efficiency on the BlueGene/P machine Shaheen at KAUST





- 7,7 Mio. Elements
- Order of accuracy in space and time: O5
- Pure MPI parallelization – code is openMP hybrid now
- Metis partitioning

  http://glaros.dtc.umn.edu/gkhome/metis/metis/overview

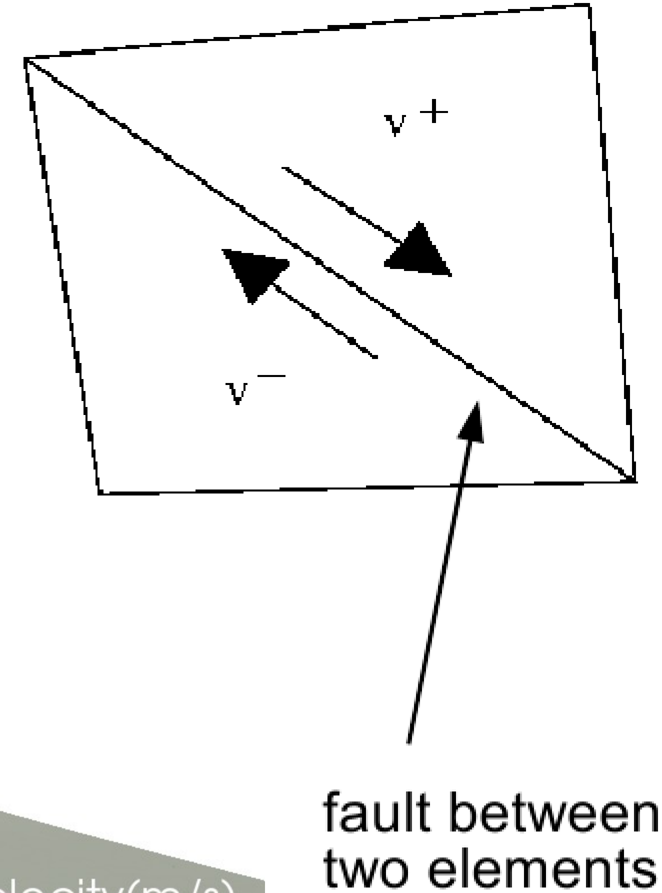
Dynamic Earthquake rupture

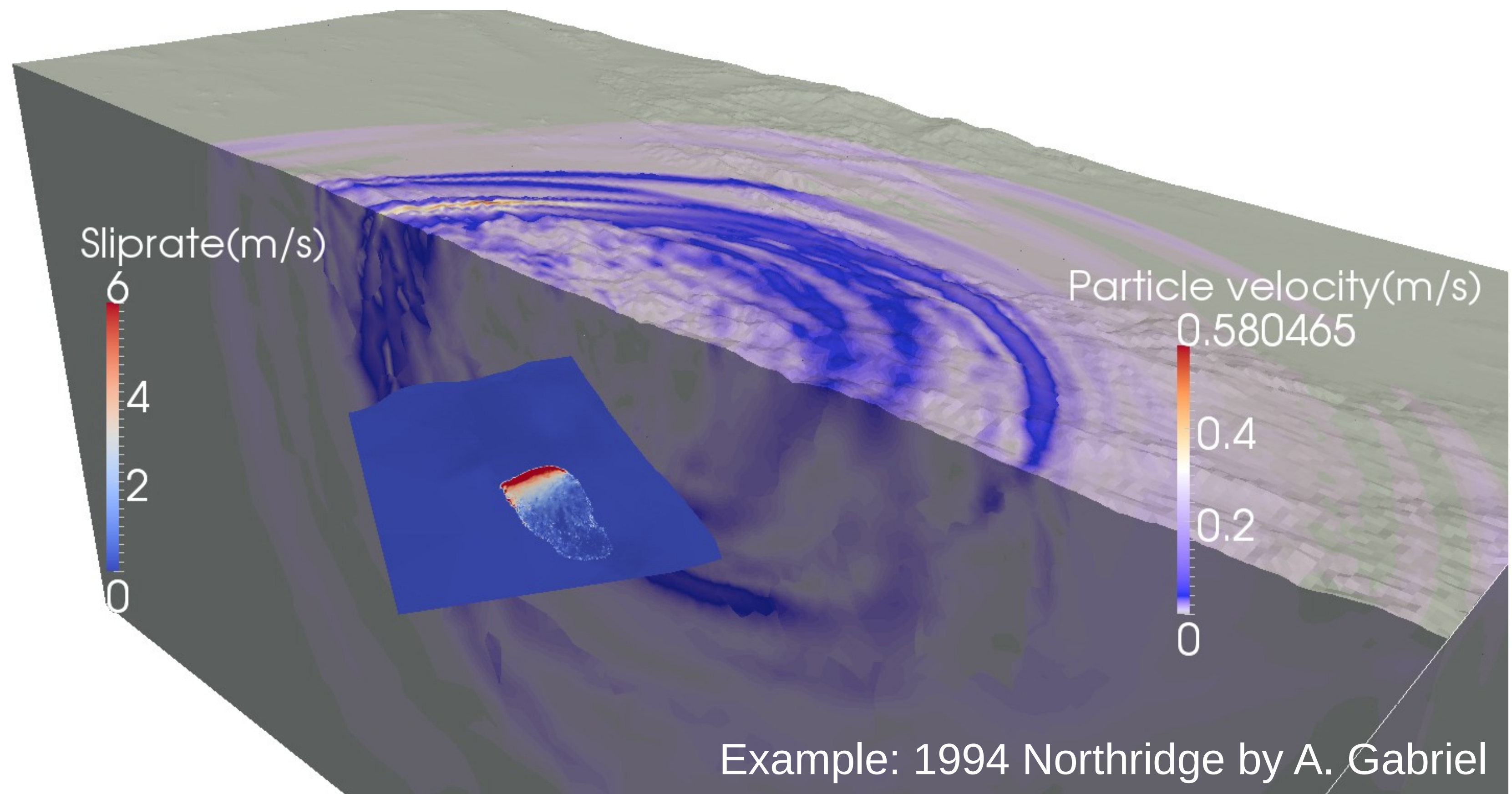
Incorporate source process

- To understand earthquake faulting
- Support physics-based ground motion prediction

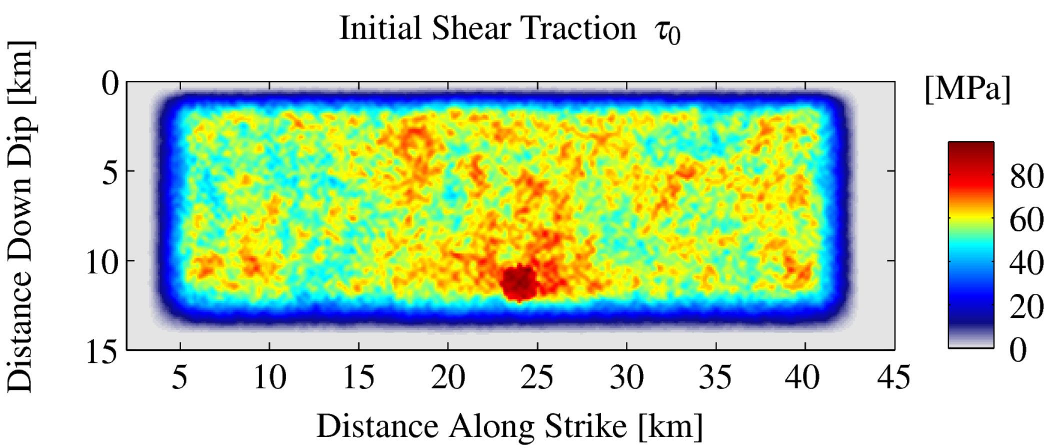
Treat dynamic rupture as an interior time-dependent 'boundary condition' using the flux term!

- Impose new traction following the failure criterion
- Impose fault parallel velocities in opposite directions

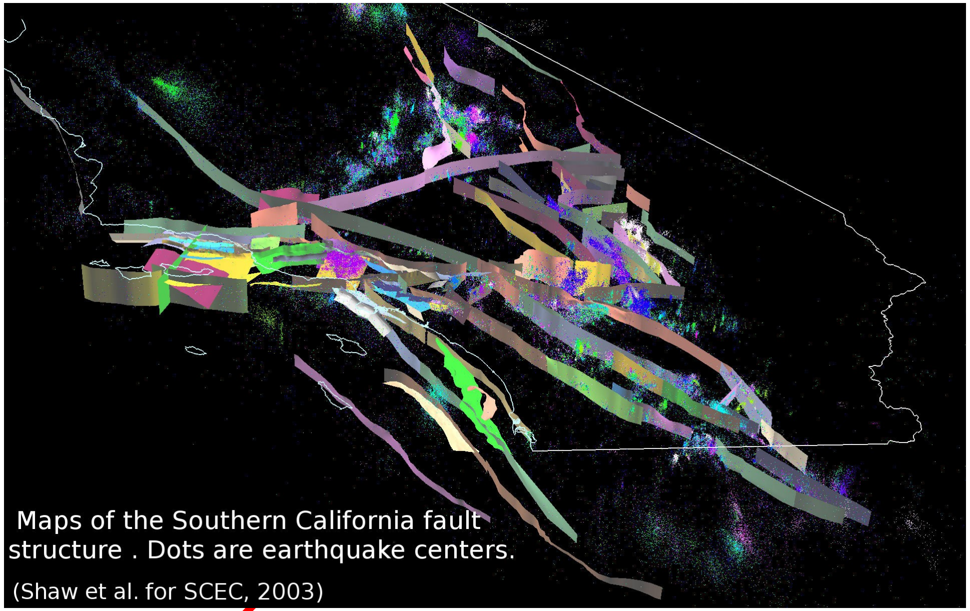


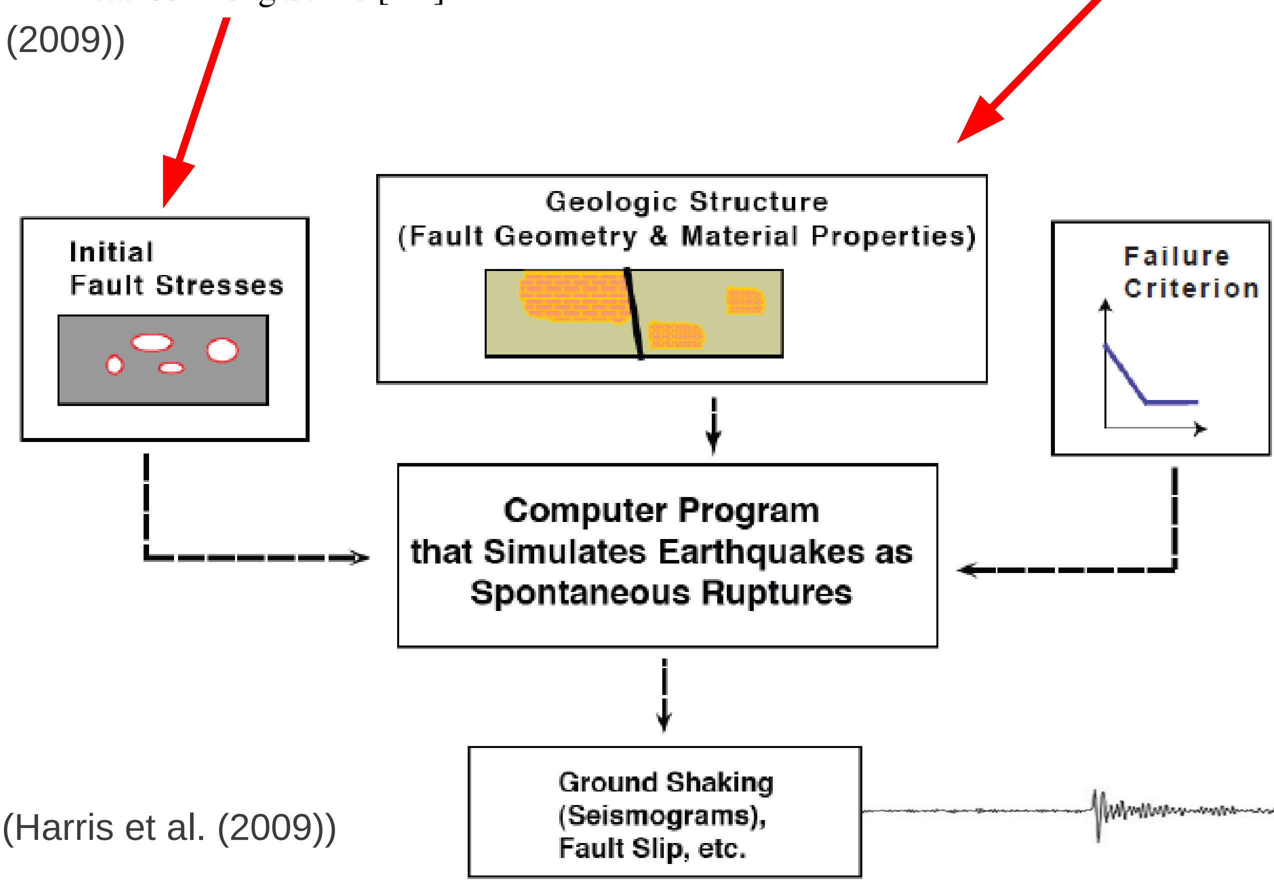


Ingredients



(Brietzke et al. (2009))





friction:
non linear relation between fault stress and slip

(Harris et al. (2009))

Failure criterion:

Coulomb friction model

$$|\sigma_{xy}| \le \mu_f \sigma$$

traction — fault strength

$$\left(|\sigma_{xy}| - \mu_f \sigma\right) \Delta v = 0$$

σ_{xy} traction

μ_f friction coefficient

σ normal stress

Δv slip rate

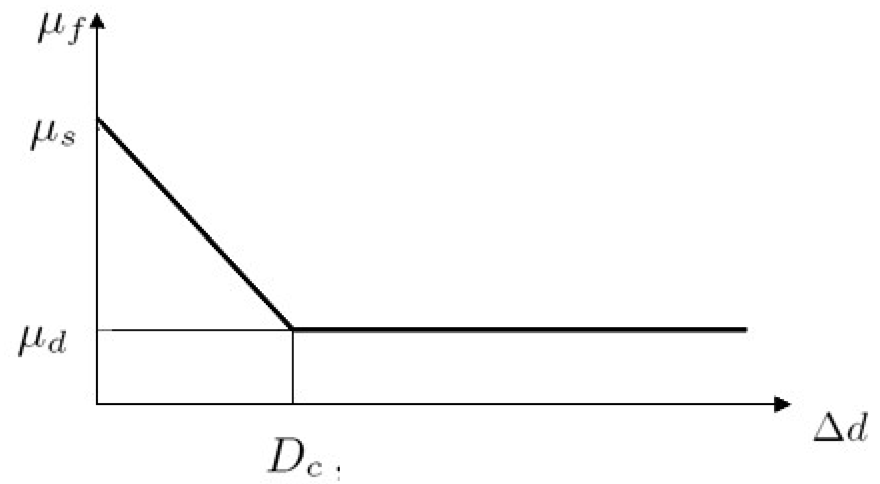
Failure criterion:

Coulomb friction model

$$|\sigma_{xy}| \leq \mu_f \sigma$$

traction fault strength

$$(|\sigma_{xy}| - \mu_f \sigma)\Delta v = 0$$



σ_{xy}	traction
μ_f	friction coefficient
σ	normal stress
Δv	slip rate
Δd	slip
D_c	critical slip distance

Linear Slip Weakening friction law
(laboratory experiments
– rate-and-state also implemented)

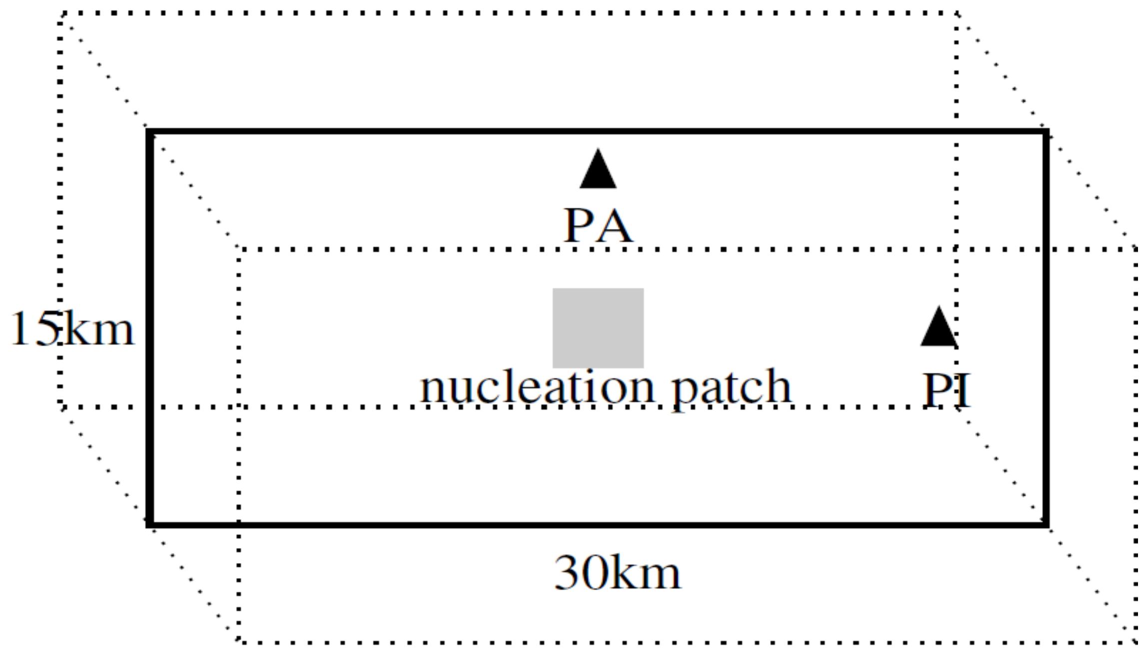
Provides:

- initial rupture
- arrest of sliding
- reactivation of slip

Verification – TPV3 SCEC Test Case

(Harris *et al.*, 2004)

- spontaneous rupture propagation on a straight fault
- homogeneous fullspace
- linear slip weakening friction



Comparison between

ADER-DG method order 4 and 200m triangles at the fault (larger tetrahedrons in bulk)

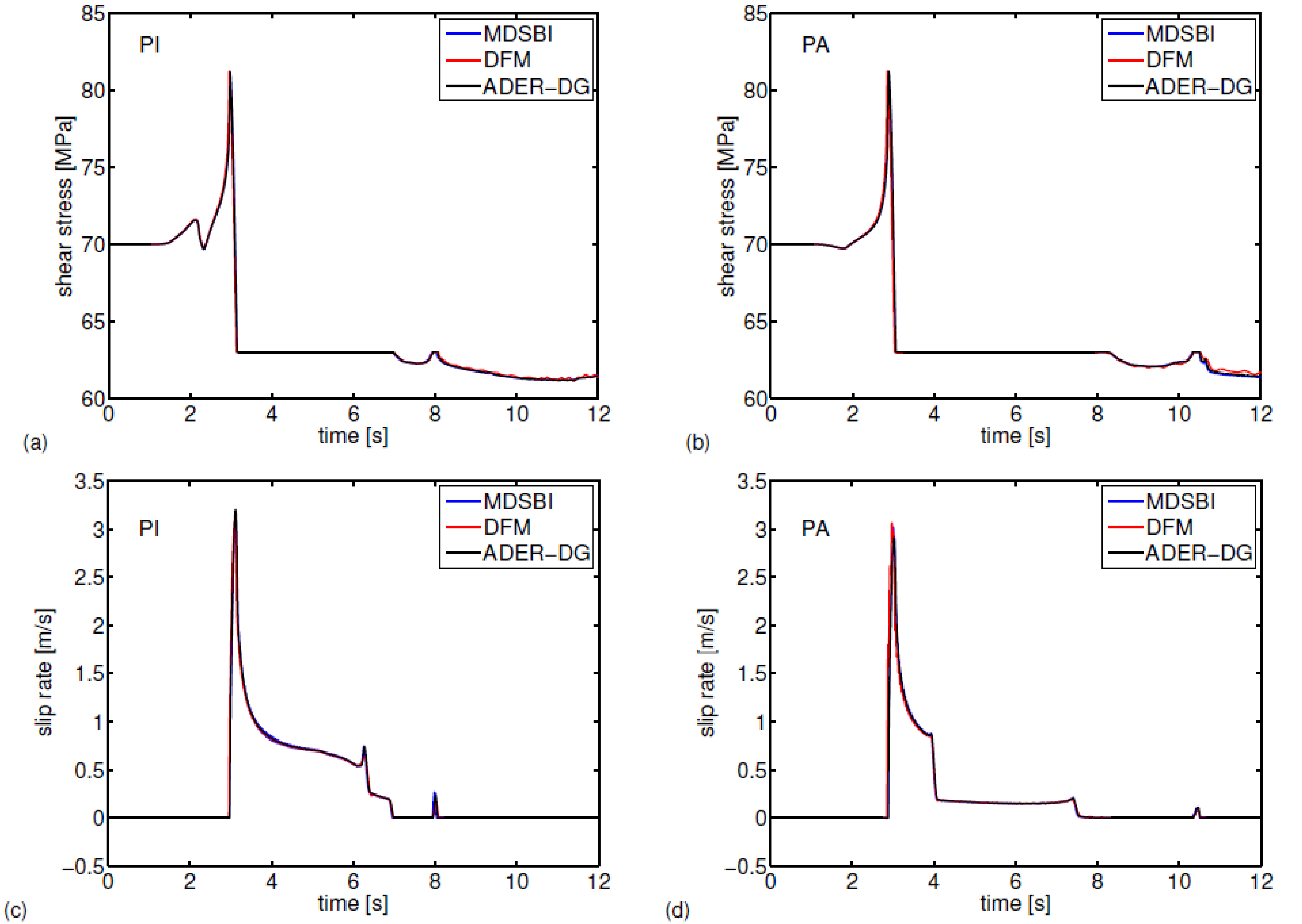
and

DFM - Finite Difference staggered-grid split node order 2 with 50m grid interval

and

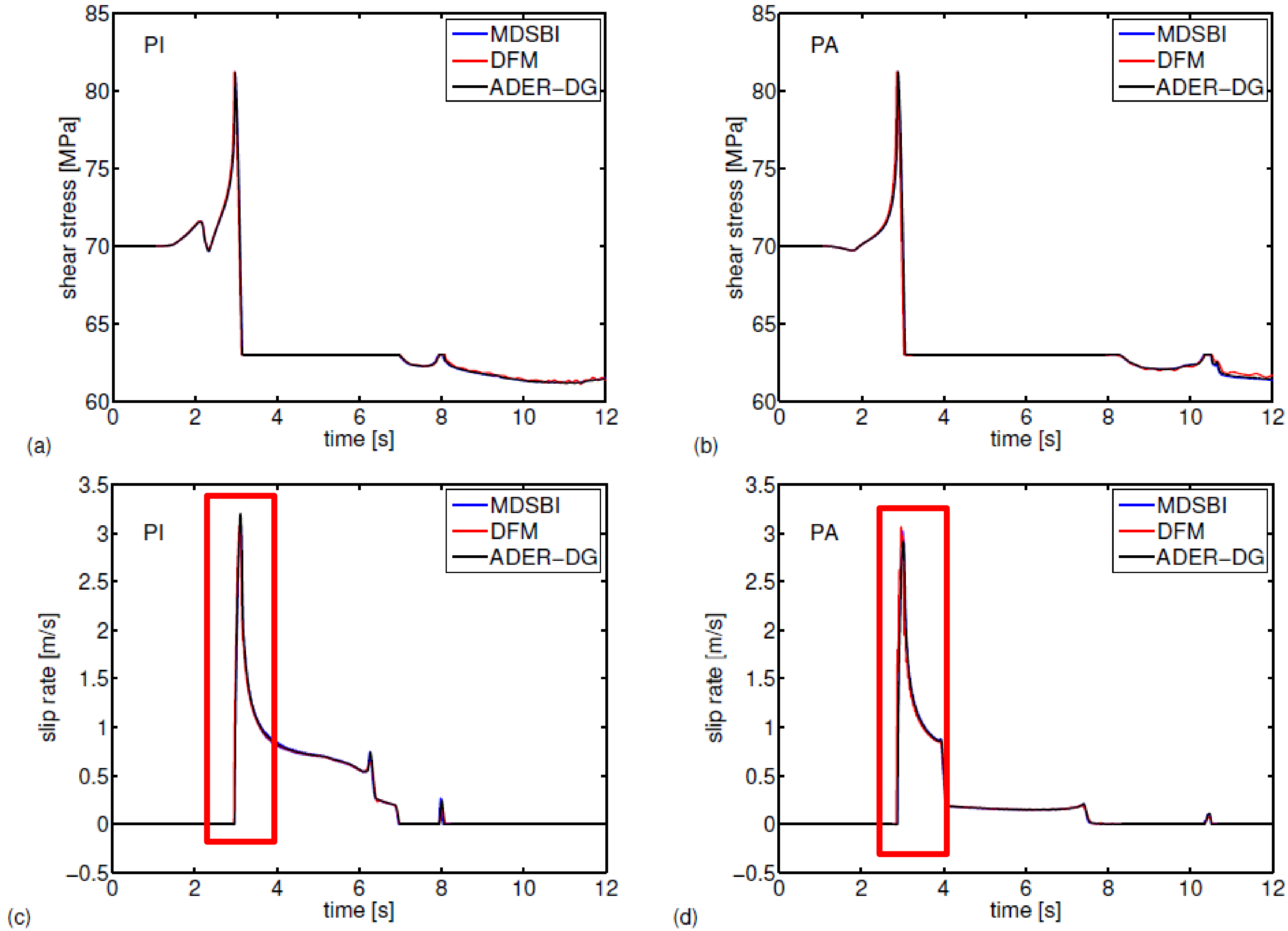
MDSBI - Multidimensional spectral boundary integral with 50m grid interval

Verification – TPV3 SCEC Test Case



Day, S. M., L. A. Dalguer, N. Lapusta, Y. Liu (2005), Comparison of finite difference and boundary integral solutions to three-dimensional spontaneous rupture, J. Geophys. Res., 110, B12307
DFM data provided by Luis Dalguer. MDSBI data computed with the code of E. Dunham (version 3.9.10).

Verification – TPV3 SCEC Test Case



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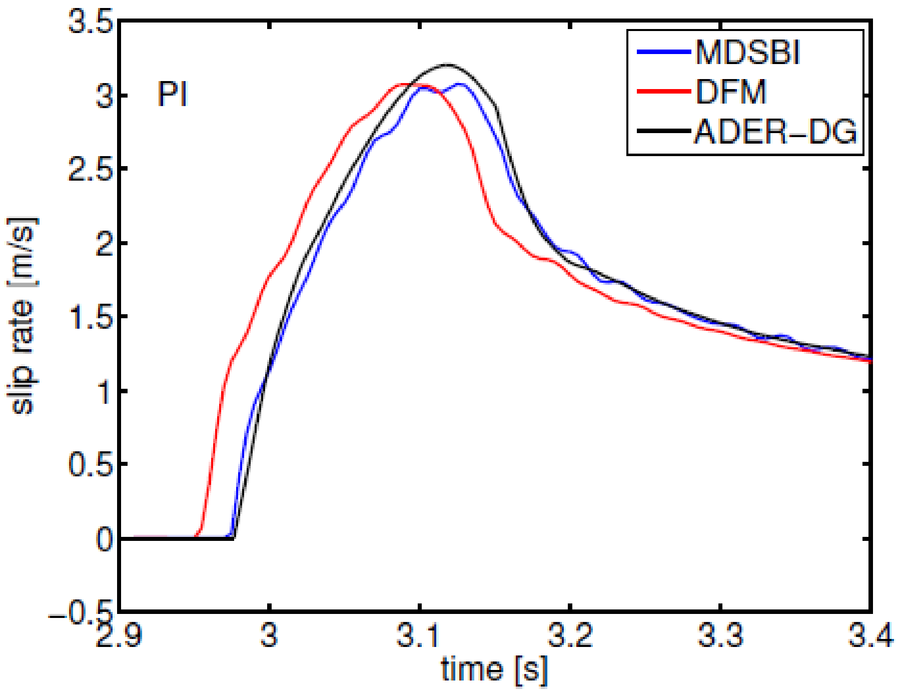
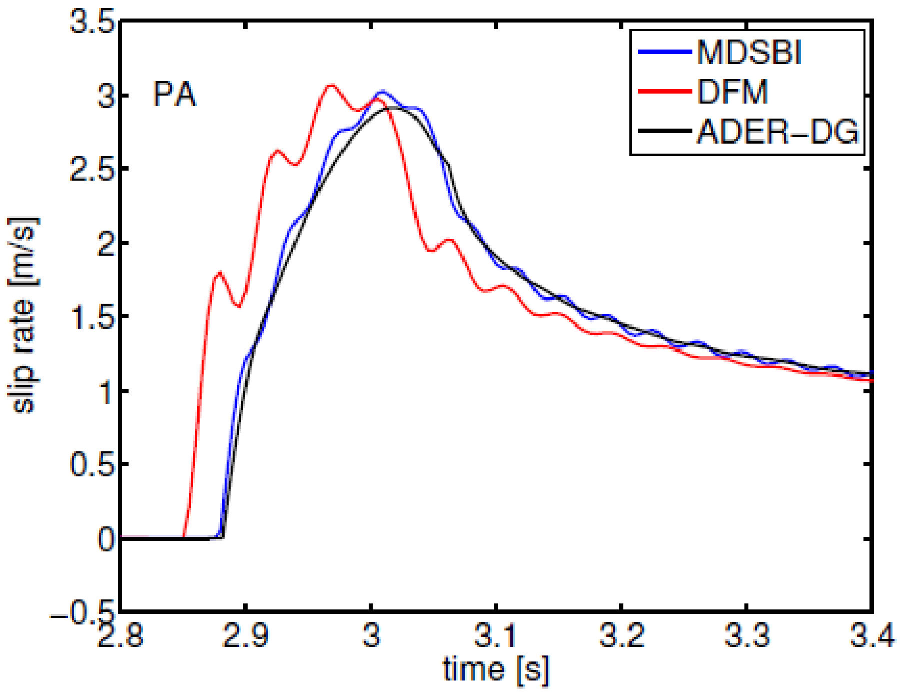
Verification – TPV3 SCEC Test Case



(e)



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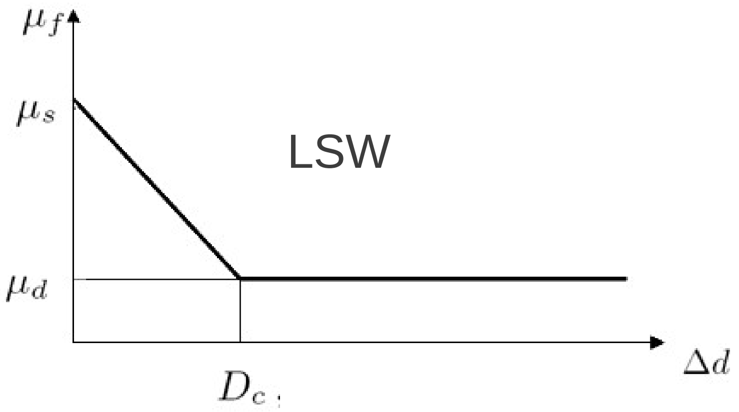


Workflow

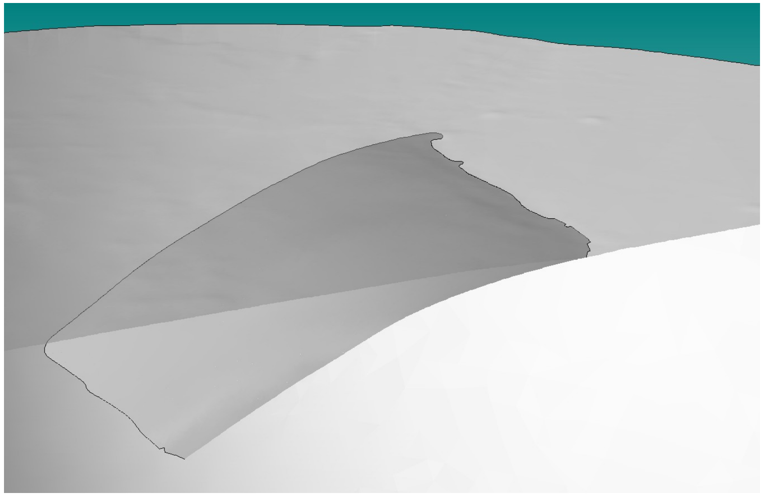
From CAD to seismogram...

"Time to solution!"

- Get geometry and model data
- Assemble CAD model
- Create mesh
- Partitioning
- Set model parameters
- Solve physical equation
- Analysis of output

Pre-processing

Post-processing

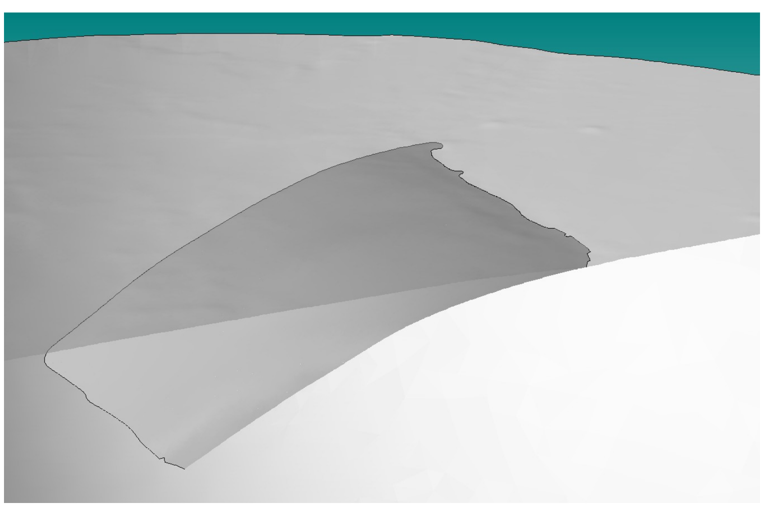


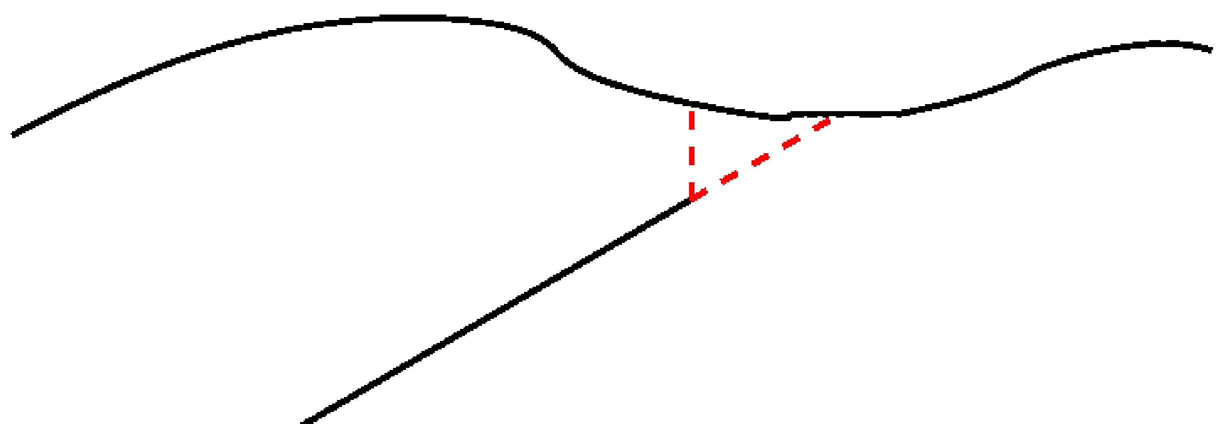
Automated CAD generation

Current bottleneck: CAD generation can easily consume weeks to month

Difficulties:

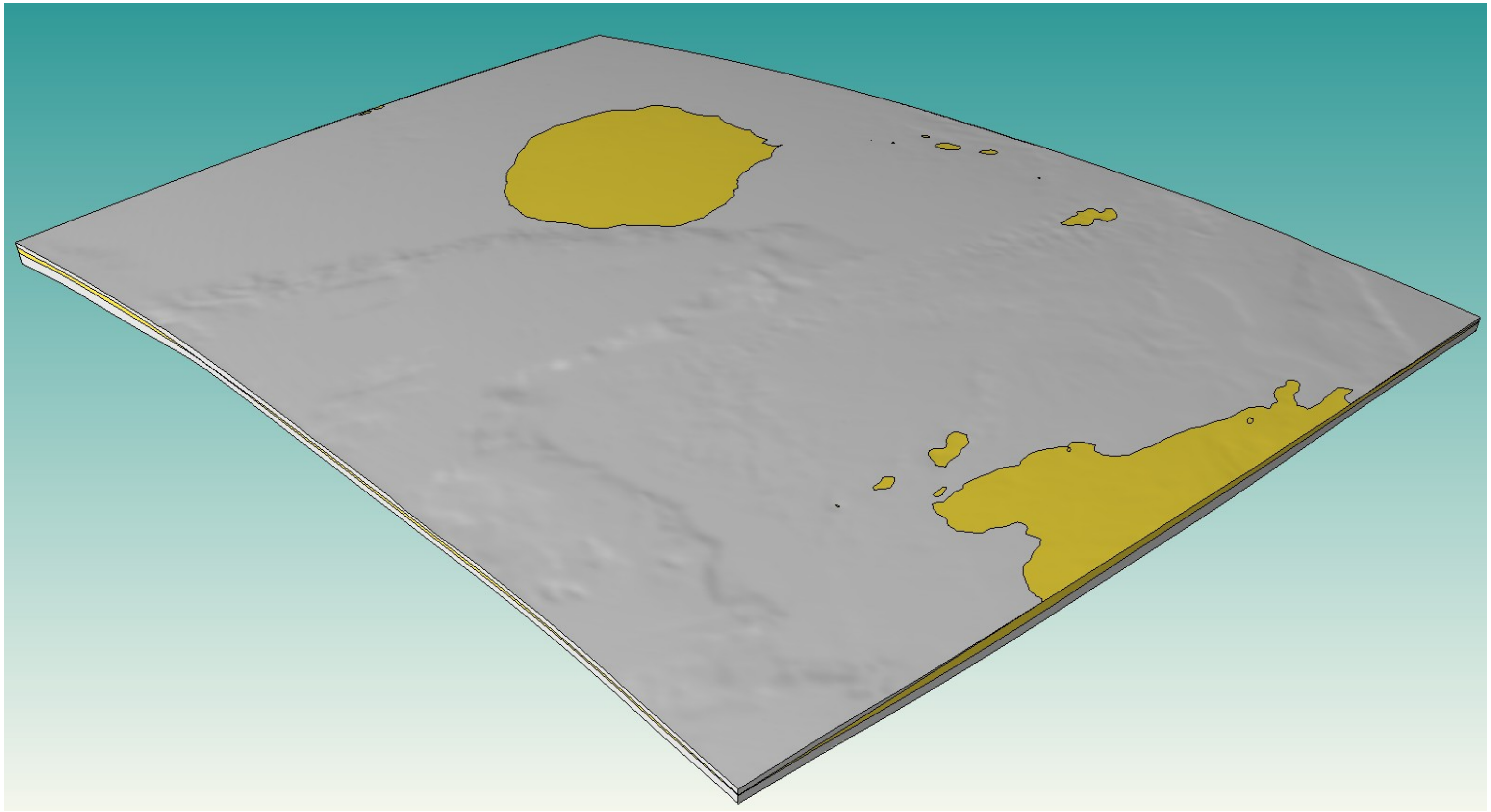
- Surface reconstruction of different types of initial raw data
- Undulating 3D surfaces that merge under shallow angles, intersect
- Remove non-physical features
- Clip too small features depending on the desired mesh size
- Representation by splines as typically used by (commercial) CAD/mesh software unfortunate for geological data
- Watertight model
- Seamless integration into meshing software (avoid format conversion)





Automated CAD generation – preliminary workflow

1. Download topography/bathymetry, e.g. from NOAA's ETOPO data collection
2. Define bounding box: rectangular or spherical
3. Material interfaces: structured grids of points
4. Faults: structured grids of points, gOcad's TS format
5. Check projection
6. (Triangulated) surface generation: Poisson surface reconstruction (MeshLab)
7. Assemble model: apply union, intersection, trimming operations with Simmetrix discrete modeling tools

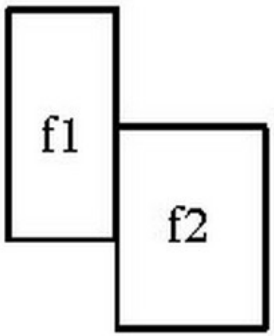


Biscay model, S. Wenk

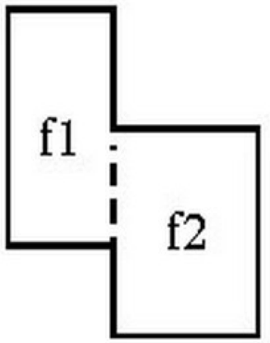
SimModeler

Customized problem definition and mesh generation interface for SeisSol
by **RPI/SCOREC/Simmetrix (C. Smith, M. Shephard)**

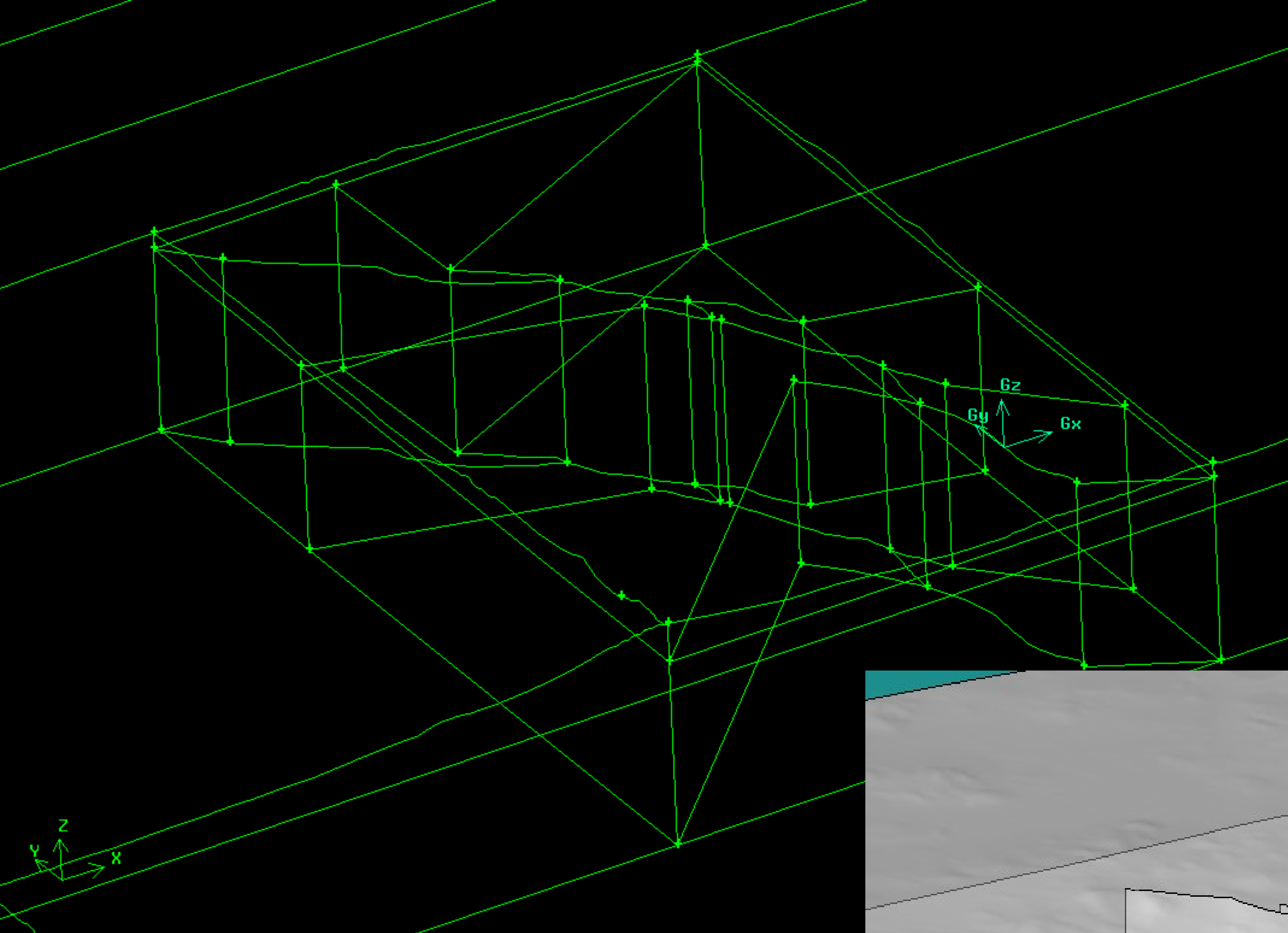
- Accepts e.g. Parasolid, ACIS and STL input
- Trims automatically geometry and creates a watertight model
- Meshes with millions of elements in seconds/minutes
- Mesh coarsening/refining
- Handling complex geometries (no violation)
- user-friendly interface
- Quality metrics
- Exports SeisSol format
- Non-manifold geometry required



Two faces. At the intersection there are two edges overlapping.
= assembly



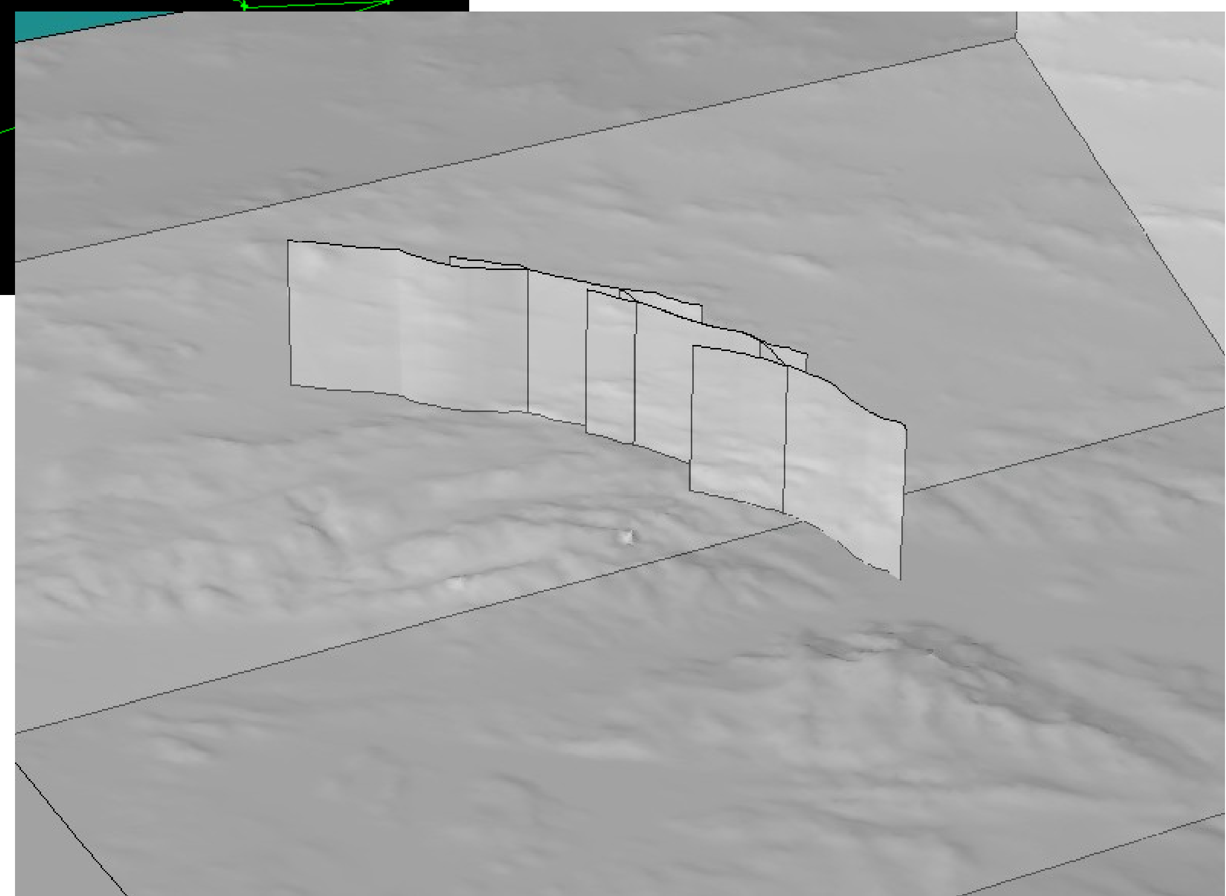
Two faces. At the intersection there is one shared edge.
= non-manifold



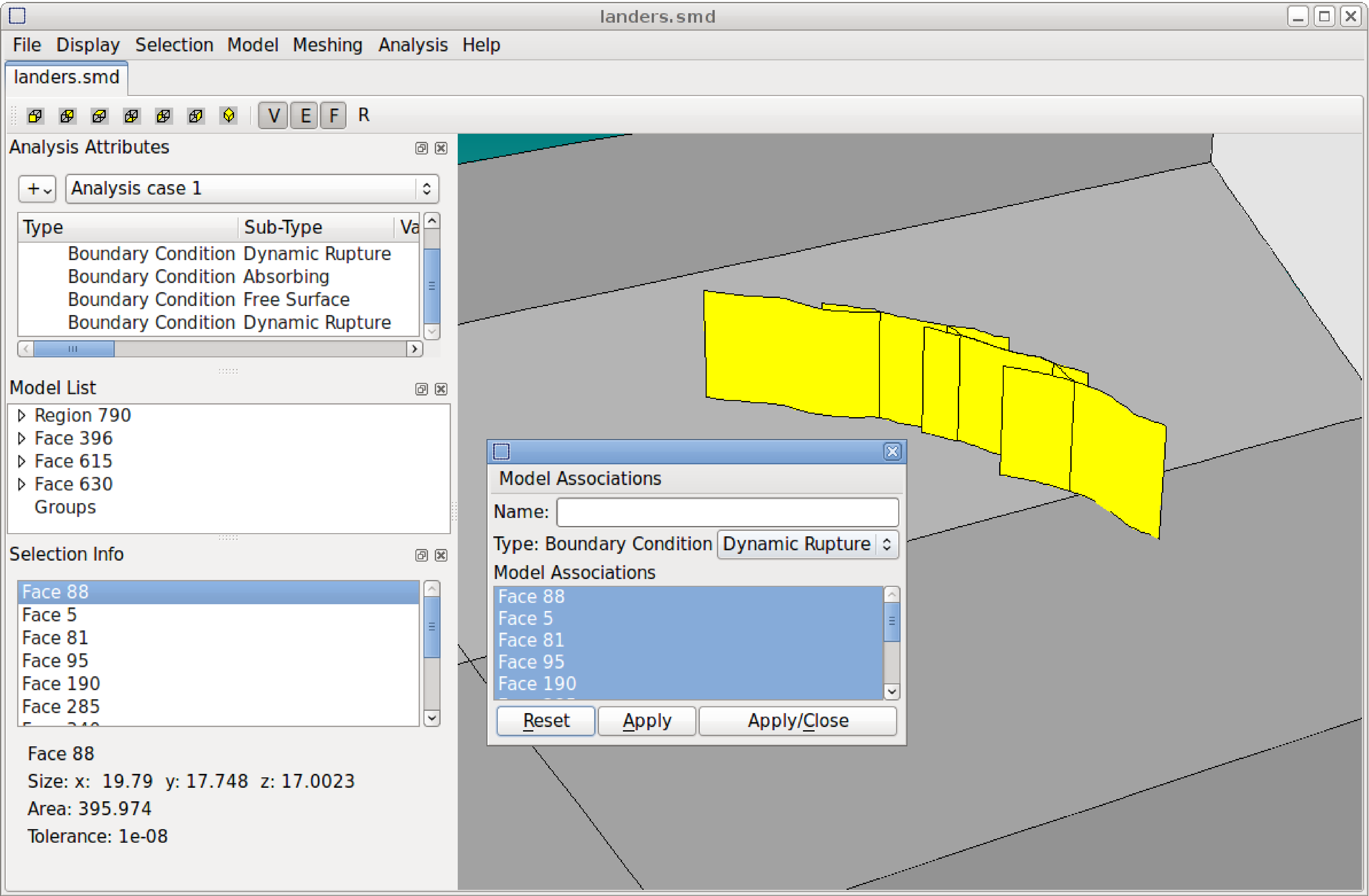
Gambit

vs

SimModeler



SimModeler


landers.smd
File Display Selection Model Meshing Analysis Help
landers.smd
V E F R
Analysis Attributes
Analysis case 1
Type Sub-Type Va
Boundary Condition Dynamic Rupture
Boundary Condition Absorbing
Boundary Condition Free Surface
Boundary Condition Dynamic Rupture
Model List
Region 790
Face 396
Face 615
Face 630
Groups
Selection Info
Face 88
Face 5
Face 81
Face 95
Face 190
Face 285
Face 88
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Area: 395.974
Tolerance: 1e-08
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Name:
Type: Boundary Condition Dynamic Rupture
Model Associations
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Face 5
Face 81
Face 95
Face 190
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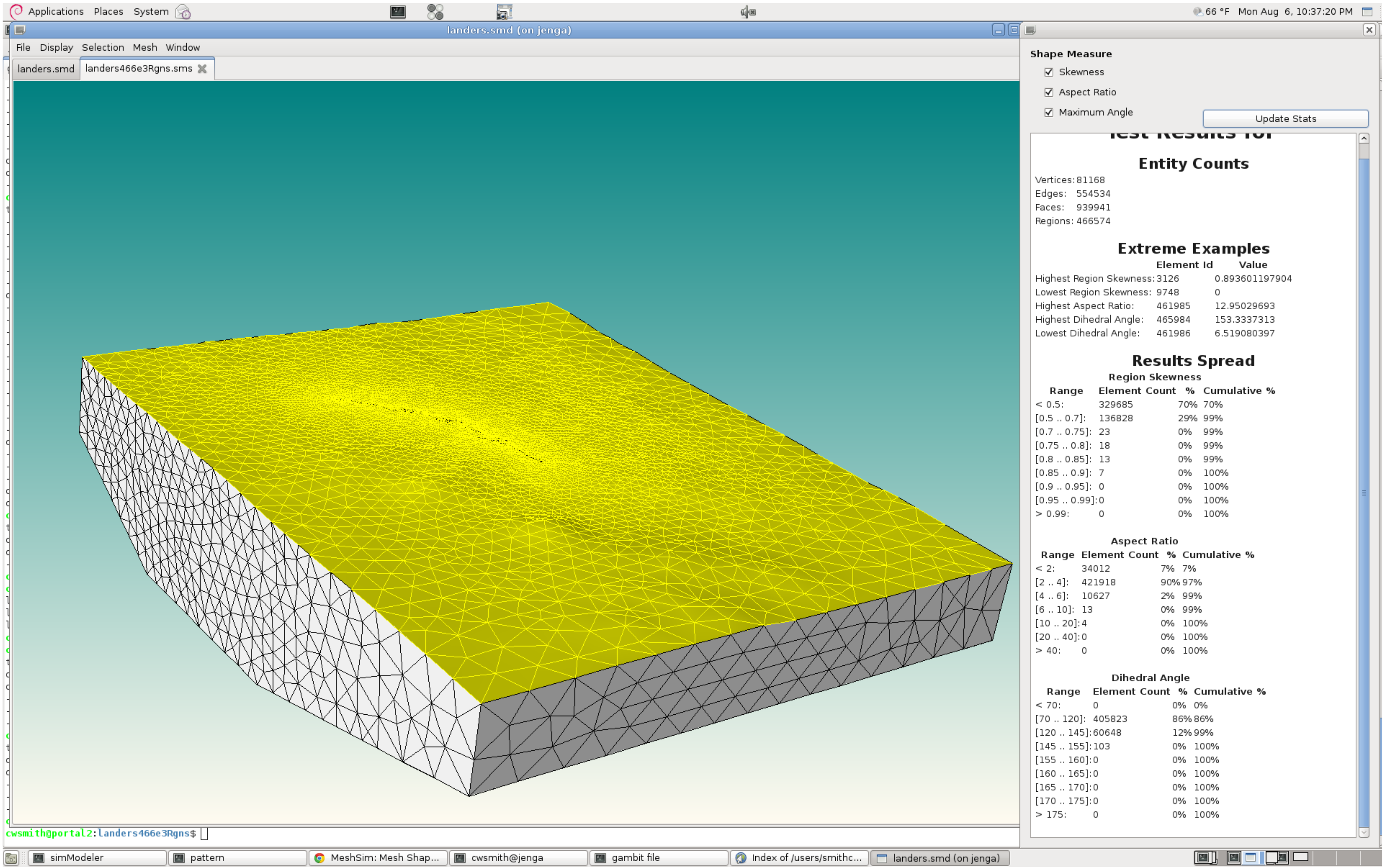
SimModeler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Aspect Ratio
Maximum Angle
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Vertices: 81168
Edges: 554534
Faces: 939941
Regions: 466574
Extreme Examples
Element Id Value
Highest Region Skewness: 3126 0.893601197904
Lowest Region Skewness: 9748 0
Highest Aspect Ratio: 461985 12.95029693
Highest Dihedral Angle: 465984 153.3337313
Lowest Dihedral Angle: 461986 6.519080397
Results Spread
Region Skewness
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Aspect Ratio
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[4 .. 6]: 10627 2% 99%
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> 40: 0 0% 100%
Dihedral Angle
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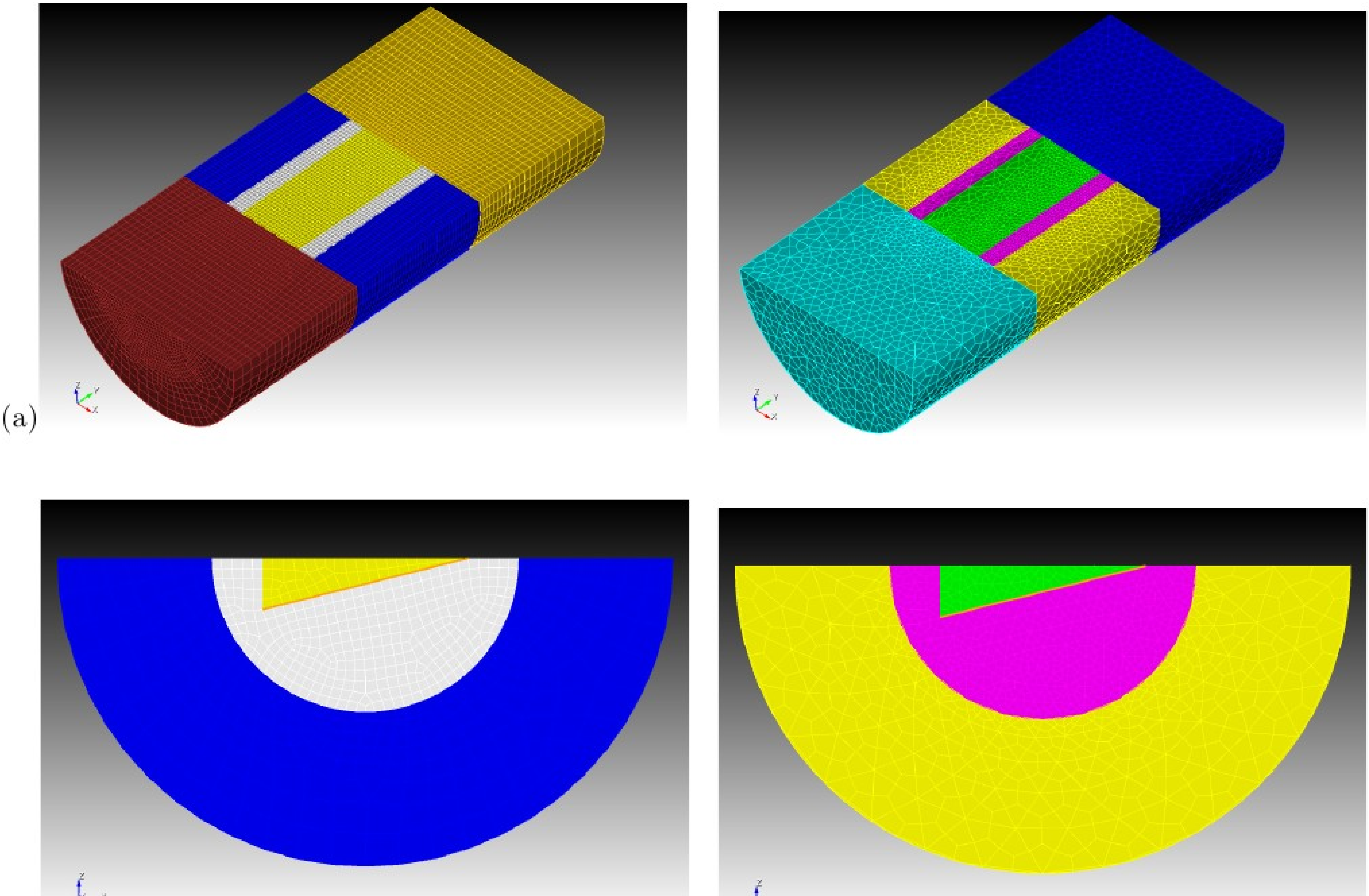
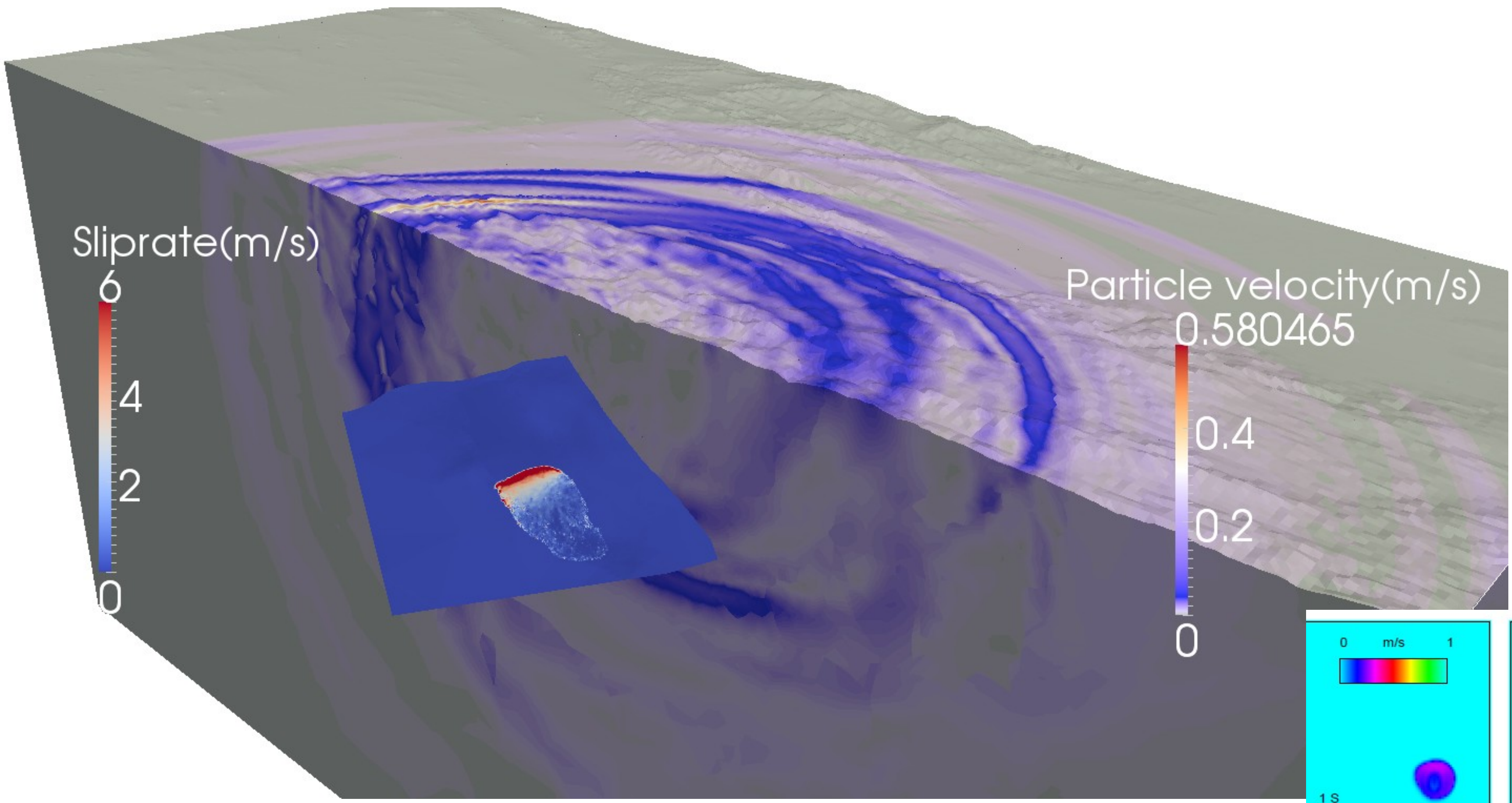
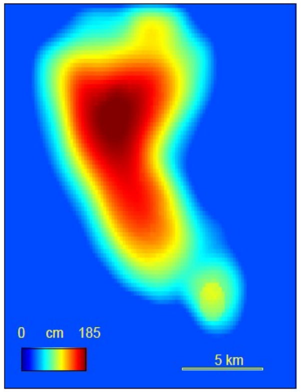
THex approach

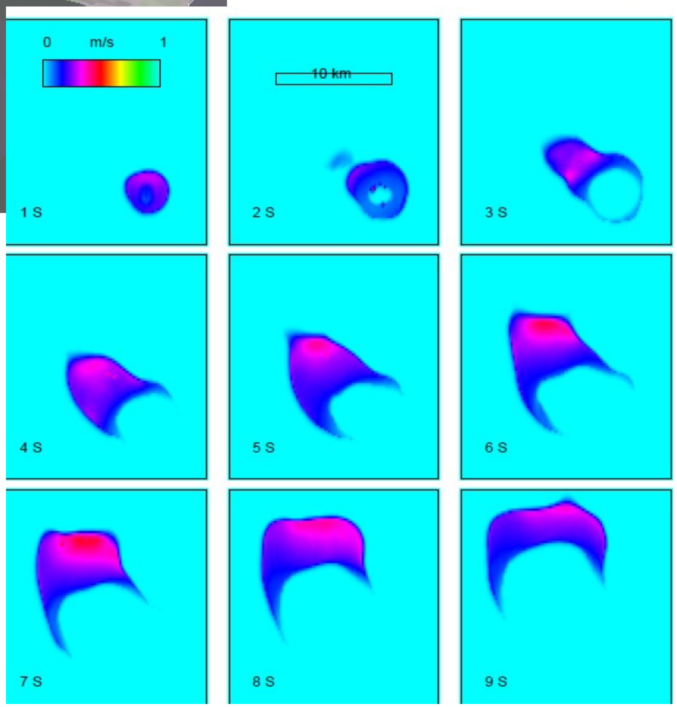


Work by Surendra Nadh Somala and Jean-Paul Ampuero

Example – The Mw 6.7 1994 Northridge earthquake







Compare final slip and slip rate from homogeneous dynamic rupture simulation on planar dipping fault with rate-and-state friction (Olsen et al., 1998)

Work by A. Gabriel

Conclusion & Outlook

- ADER-DG solver ready, functional and benchmarked
- Bring all features into production version (under construction)
- Combine dynamic rupture with local time stepping

- Current bottleneck CAD generation (under construction)
- Use CAD for quality control

- Open Source (soon), already available through http://verce.eu/

http://seissol.geophysik.uni-muenchen.de/

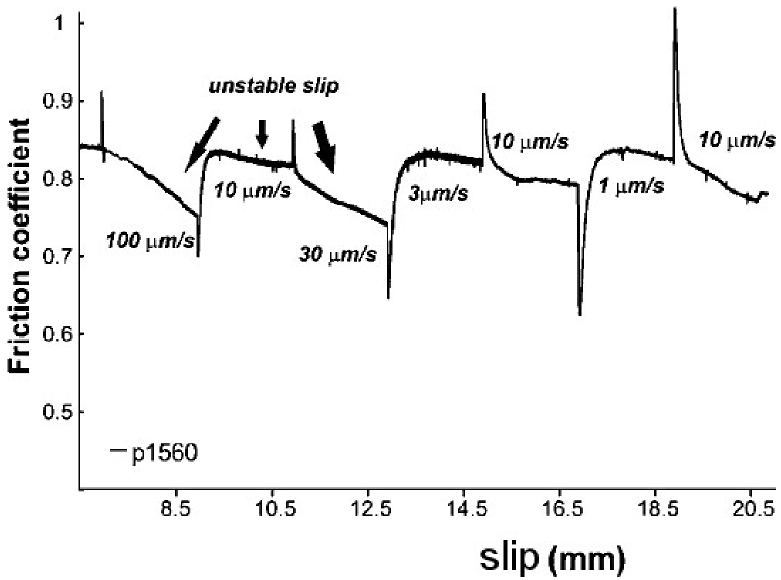
Failure criterion

Rate-and-state dependent friction

Implementation of rate-and-state friction

- Updating scheme includes Newton-Raphson search for slip rate and two iterations for state variable (Kaneko et al., 2008)

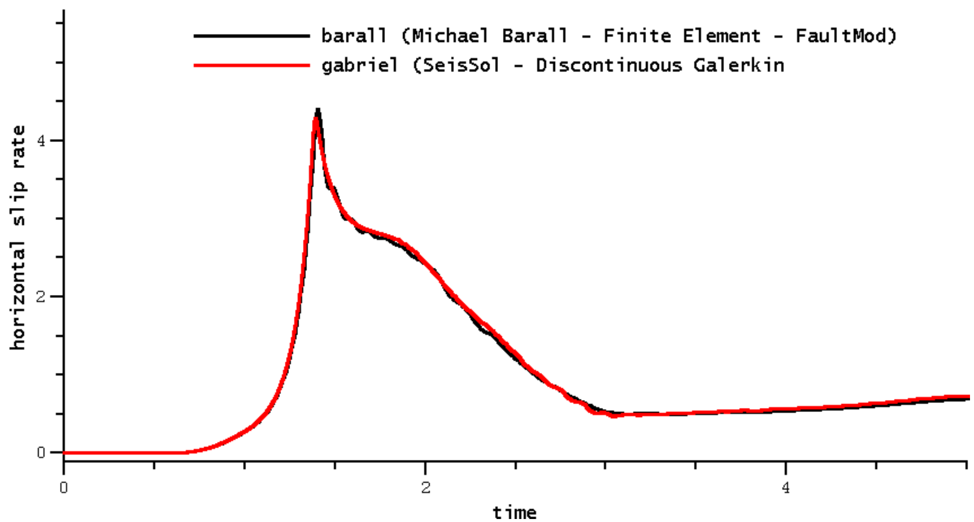
Velocity-stepping experiment of Niemeijer et al. (2010)



$$\mu_f = \mu_0 + a \ln \frac{v}{v_0} + b \ln \frac{v_0 \theta}{D_c}$$

$$\dot{\theta} = 1 - \frac{v\theta}{D_c}$$

θ state variable
a direct effect
b evolution effect
v_0 steady-state reference velocity
μ_0 steady-state reference friction

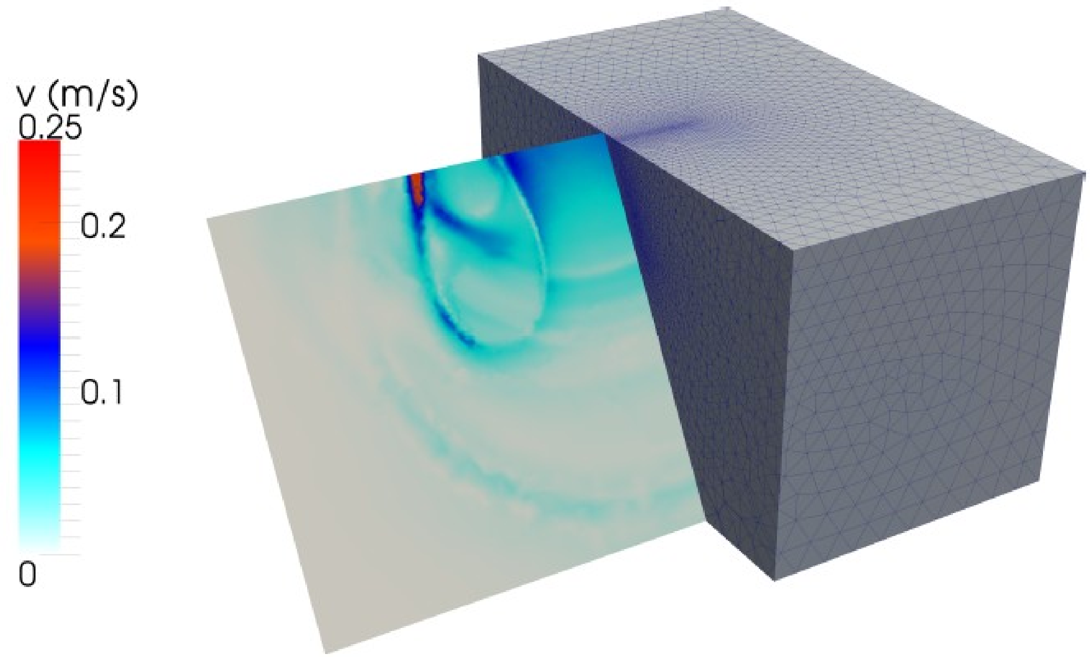


SCEC TPV102 benchmark

Dipping fault geometry

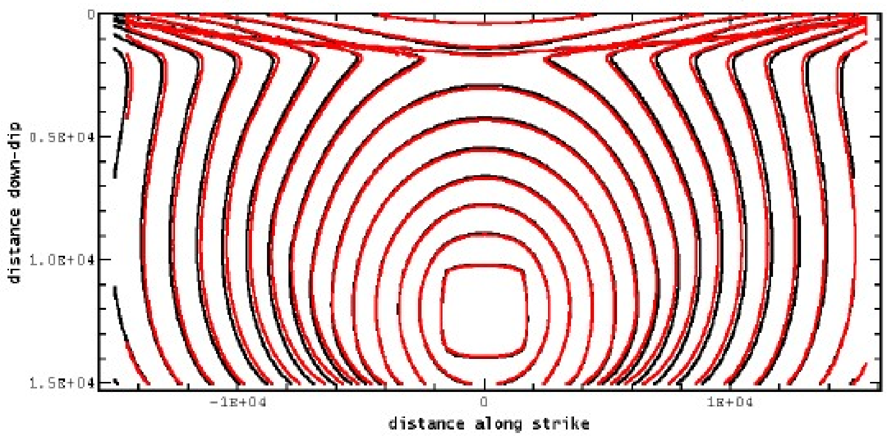
(SCEC Test Cases **TPV10** and TPV11)

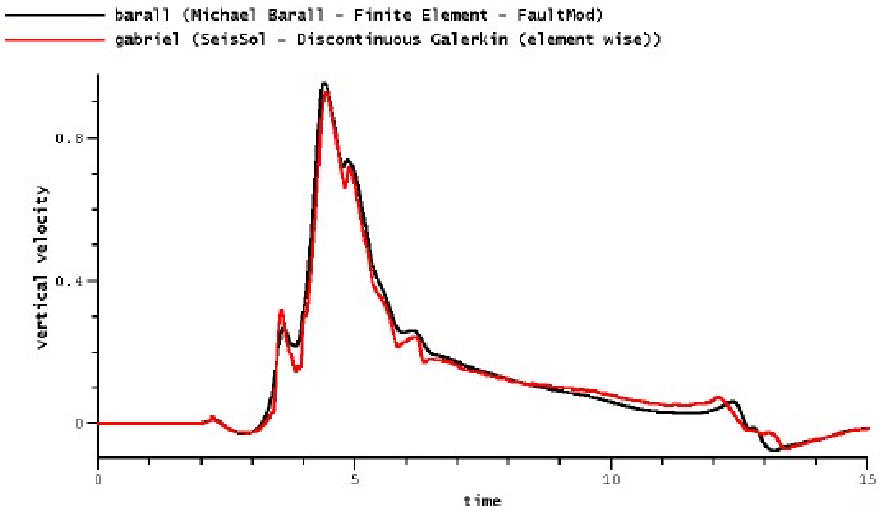
- 60 degree dipping normal fault geometry
- Initial stress linearly depth dependent
- **Subshear** / supershear rupture conditions

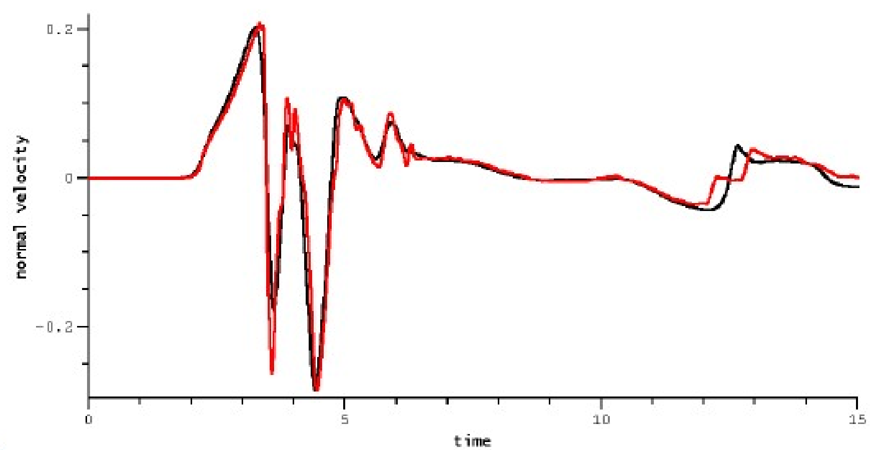


Mesh geometry, computational domain and particle velocity on the fault plane after ~9.6 s

Rupture time – contour plot (each 0.5 s)







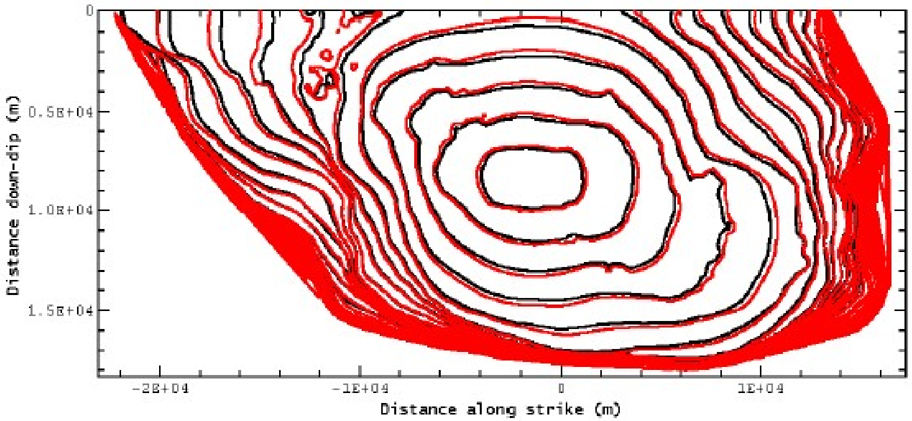
Off- fault station
(body 1.0 km, strike 0.0 km, depth 0.0 km)

Heterogeneous background stress

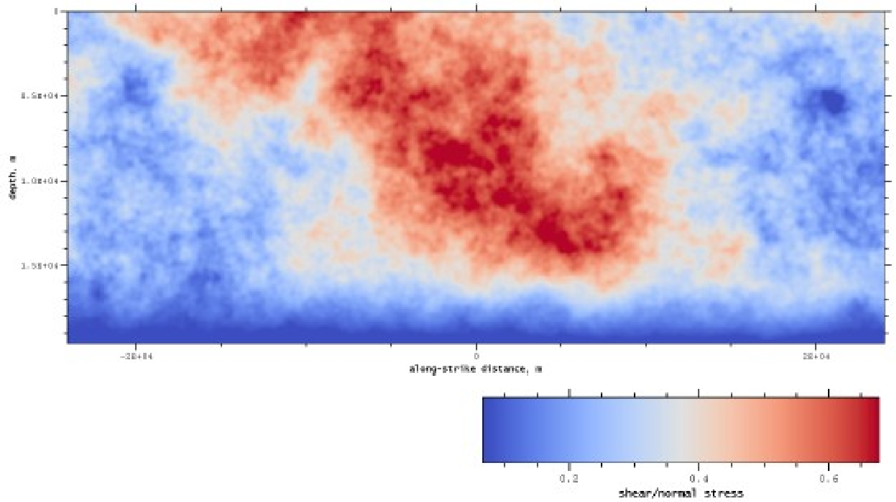
(SCEC Test Cases TPV16 and **TPV17**)

- Vertical strike-slip fault
- Randomly-generated heterogeneous initial stress conditions
- Trilinear interpolation to map background values on irregular distributed integration points

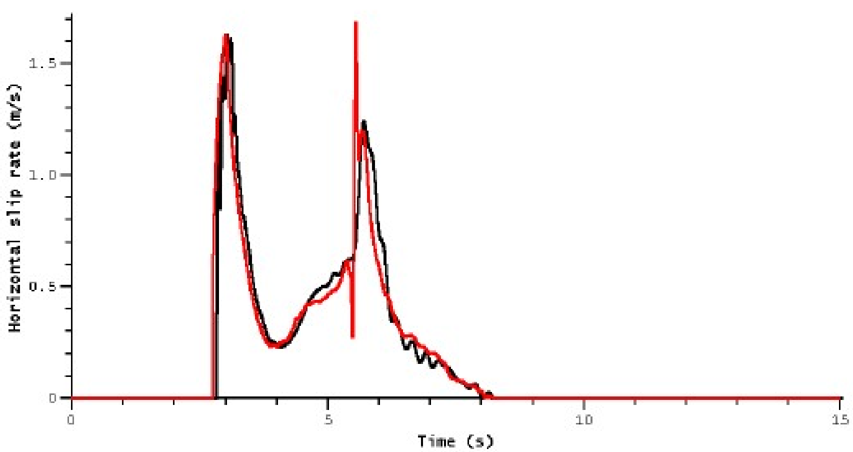
Rupture time – contour plot (each 0.5 s)

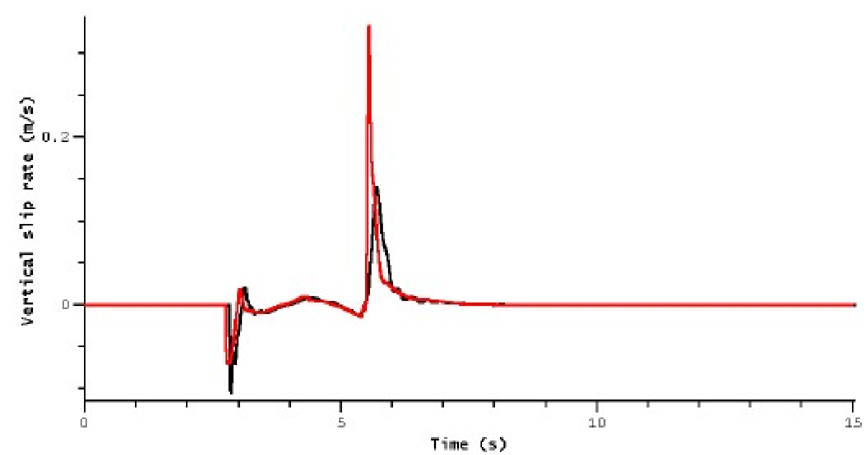


barall (Michael Barall - Finite Element - FaultMod)
pelties (Christian Pelties - Discontinuous Galerkin (GP wise))



Initial shear stress on the fault plane



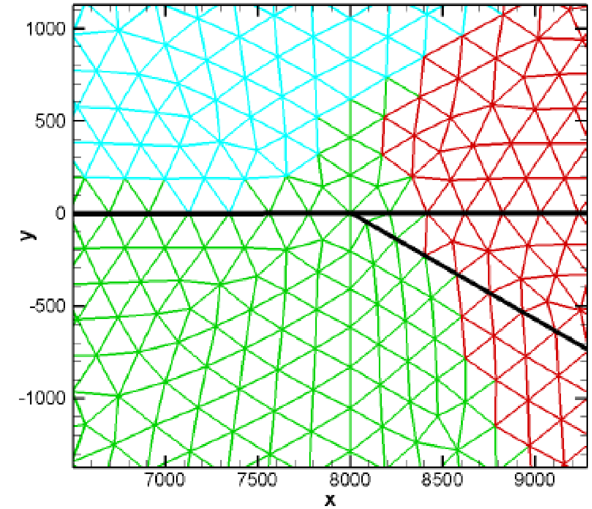


On- fault station
(strike -9.0 km, dip 9.0 km)

Fault branching geometry

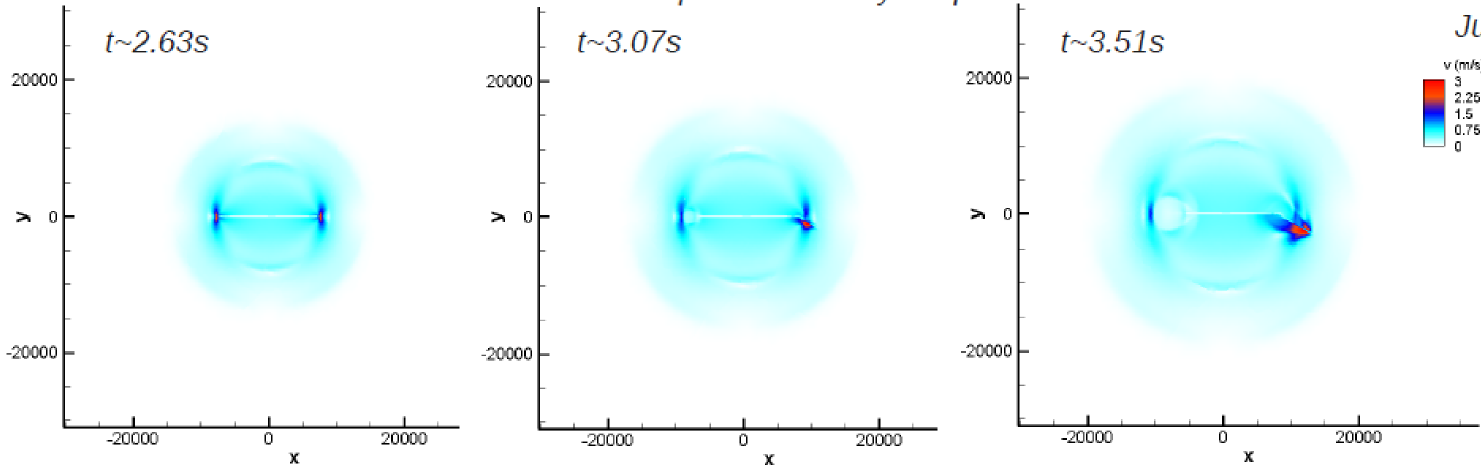
(2D SCEC Test Cases TPV14 and **TPV15**)

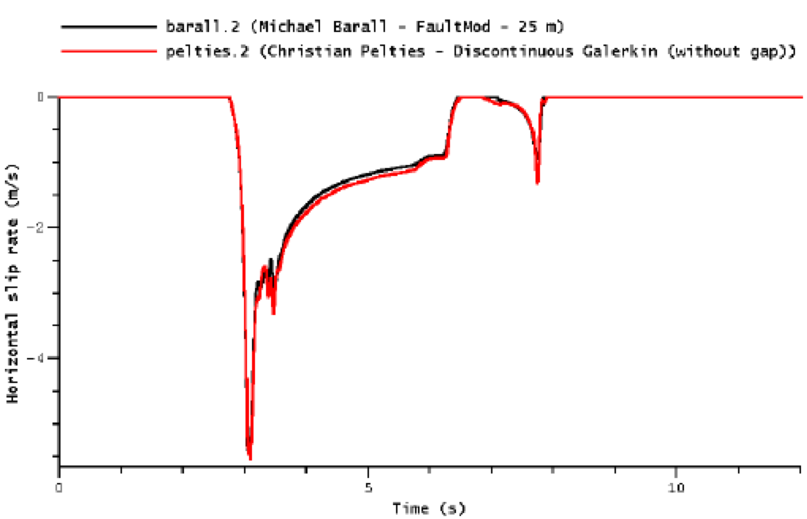
- Left-lateral, vertical, strike-slip fault with a rightward branch forming a 30 degree angle
- Slightly stress-heterogeneous
- High resolution required

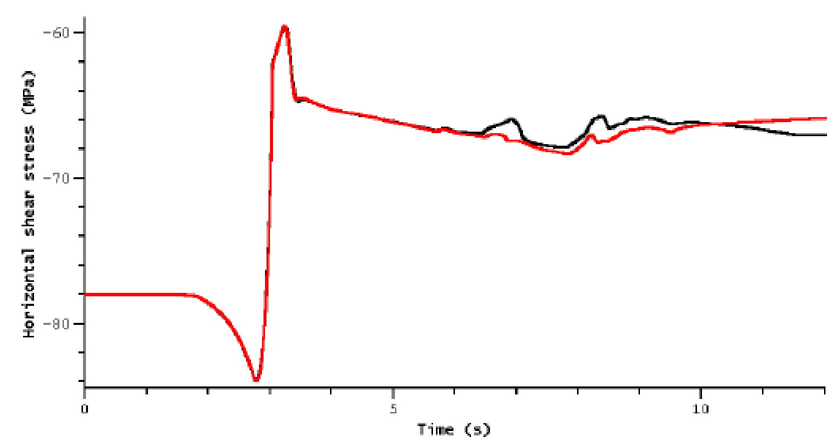


Junction point domain partitioning

Absolute particle velocity snapshots



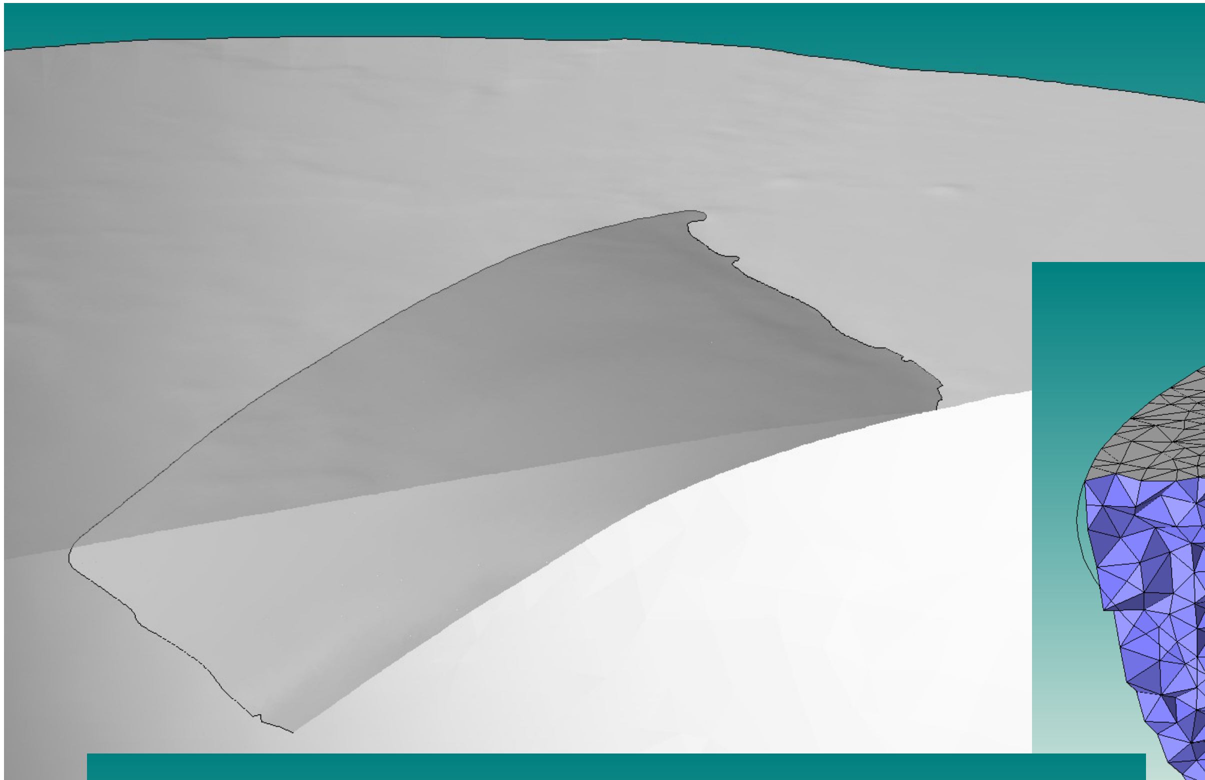


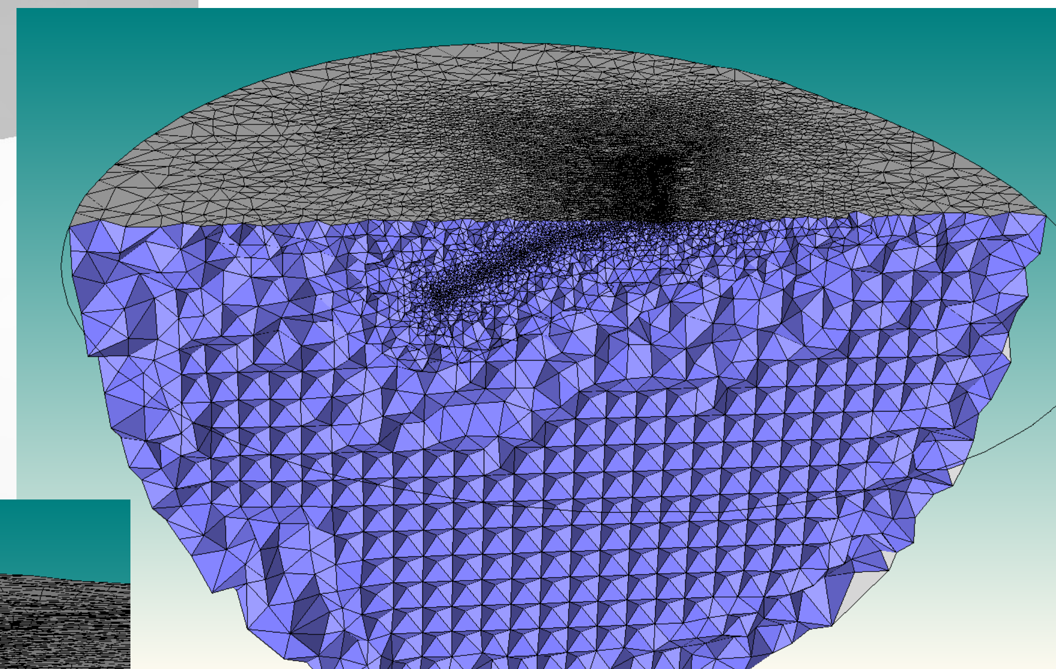


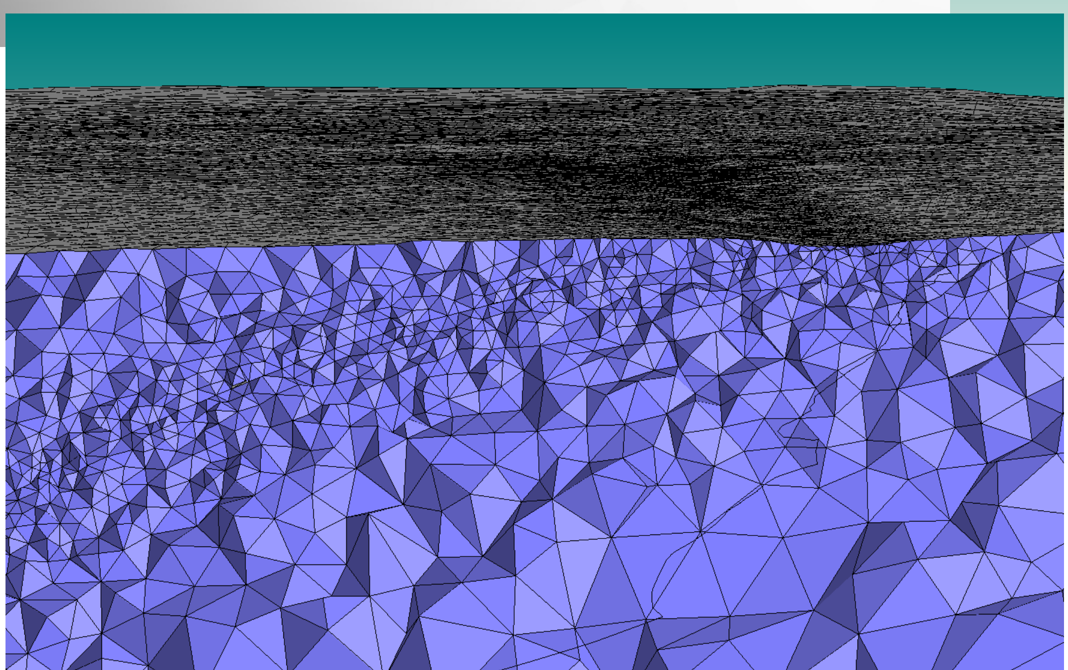
On- fault station
(branch, strike 2.0 km, dip 7.5 km)

By A. Nerger

Tohoku







- CAD generation difficult
- Extremely shallow angle at trench
- Skewed elements